18th International Conference on
RF Superconductivity
Lanzhou China
July 17-21, 2017

A unified theory of surface resistance and the residual resistance of SRF cavities at low temperatures

Takayuki Kubo and Alex Gurevich

KEK / SOKENDAI
Tsukuba, Japan

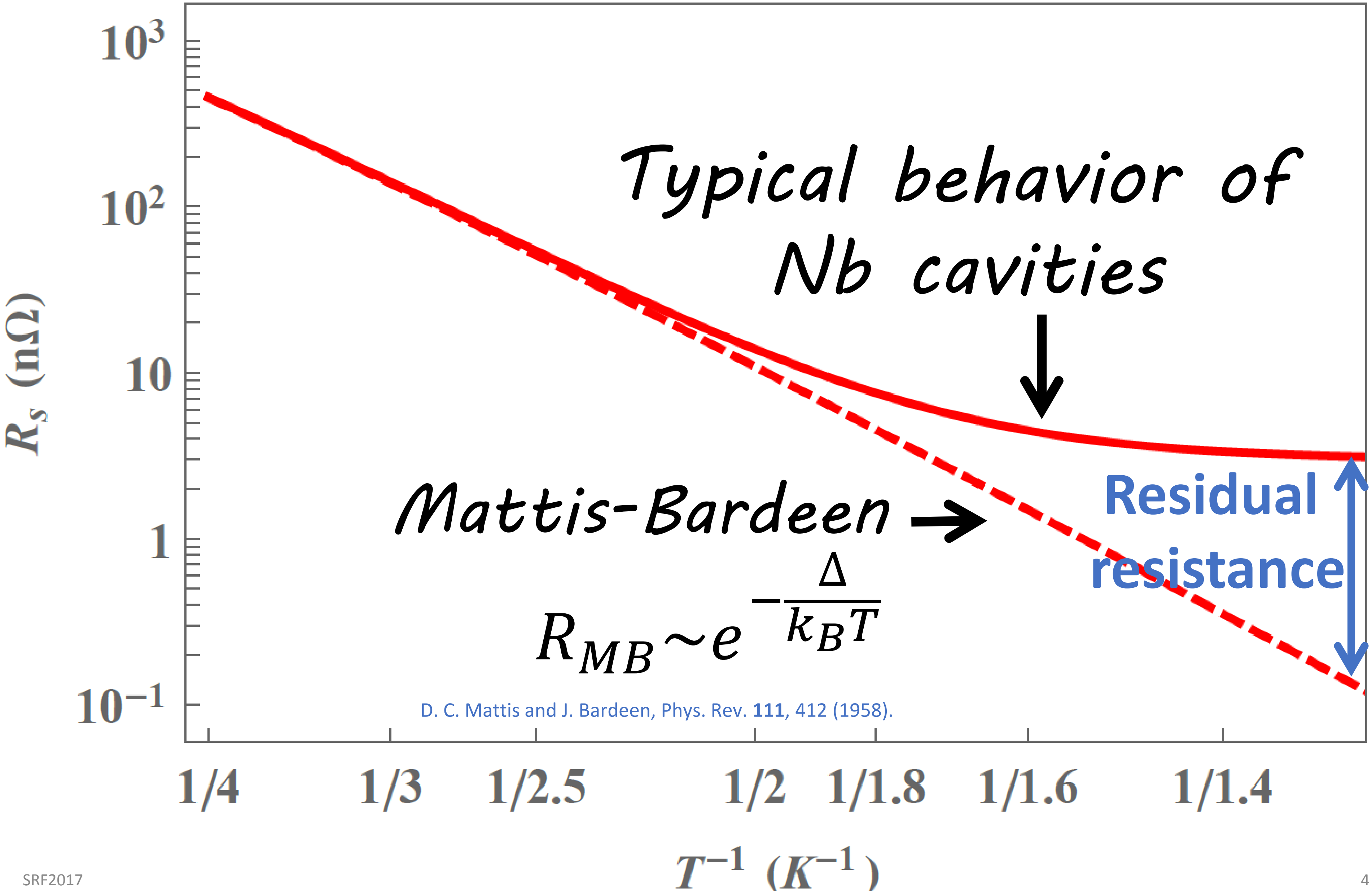
National University
The Graduate University
for Advanced Studies [SOKENDAI]

Old Dominion University,
Department of Physics and
Center for Accelerator Science,
Norfolk, VA 23529, USA

科研費 KAKENHI #17H04839

Supported by NSF under grant PHY-1416051

Surface resistance of Nb cavity



$$R_{MB} \sim e^{-\frac{\Delta}{k_B T}}$$

D. C. Mattis and J. Bardeen, Phys. Rev. **111**, 412 (1958).

To explain this shift, we have summed different contributions so far

R_{MB} : Mattis-Bardeen surface resistance

R_{others} : others

Damaged layer

Metallic sub-oxide

Subgap states

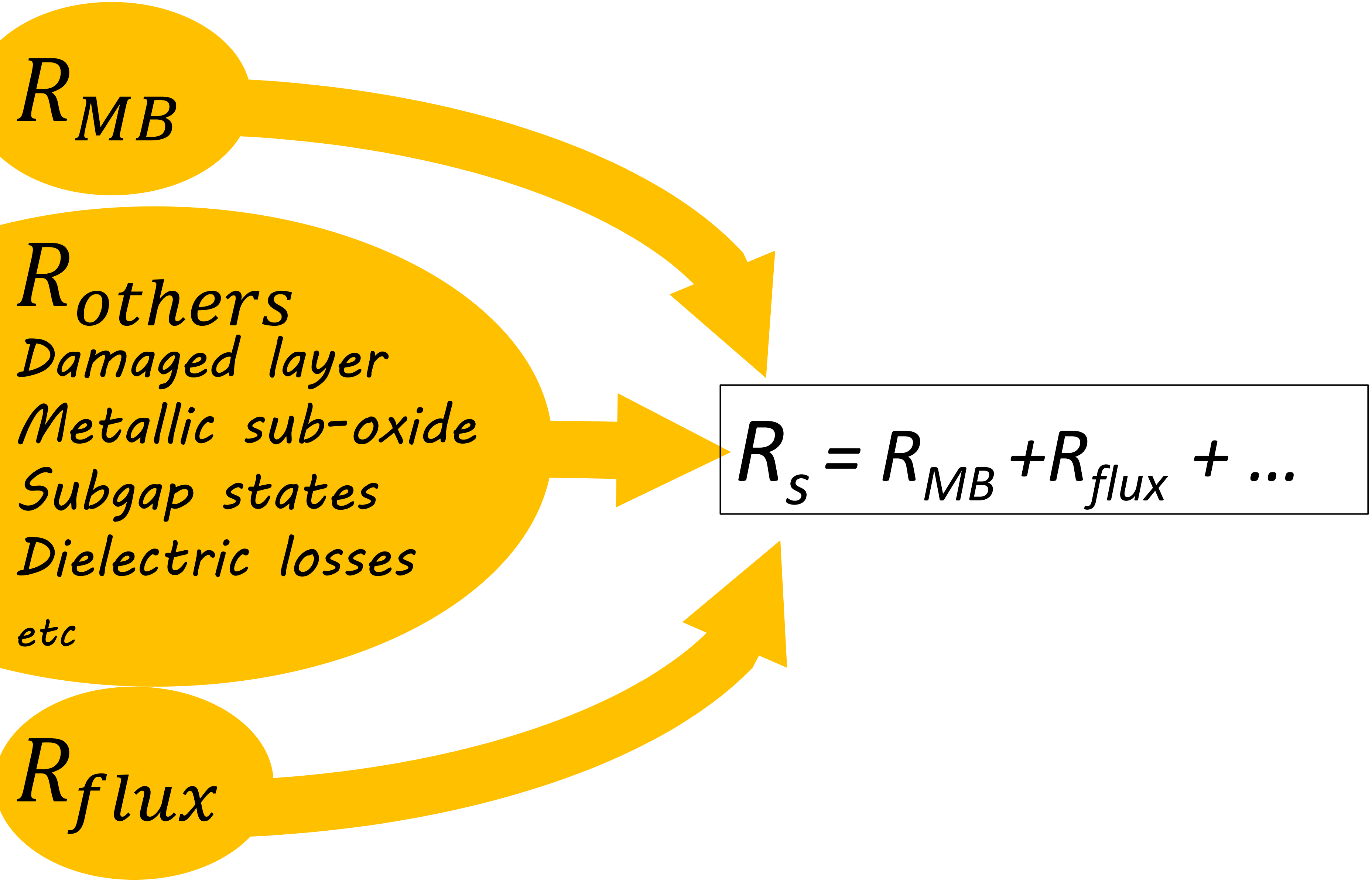
Dielectric losses

etc

R_{flux} : trapped flux contribution

A. Gurevich and G. Ciovati, Phys. Rev. B **87**, 054502 (2013).

To explain this shift, we have summed different contributions so far



R_{MB}

R_{others}

Damaged layer

Metallic sub-oxide

Subgap states

Dielectric losses

etc

R_{flux}

$$R_s = R_{MB} + R_{flux} + \ldots$$

Today, R_{flux} can be substantially reduced by cooling down a cavity with a large temperature gradient.

- A. Romanenko, et al., Appl. Phys. Lett. **105**, 234103 (2014).
- S. Posen et al., J. Appl. Phys. **119**, 213903 (2016)
- S. Huang, T. Kubo, and R. Geng, Phys. Rev. Accel. Beams **19**, 082001 (2016)

R_{MB}

R_{others}

Damaged layer

Metallic sub-oxide

Subgap states

Dielectric losses

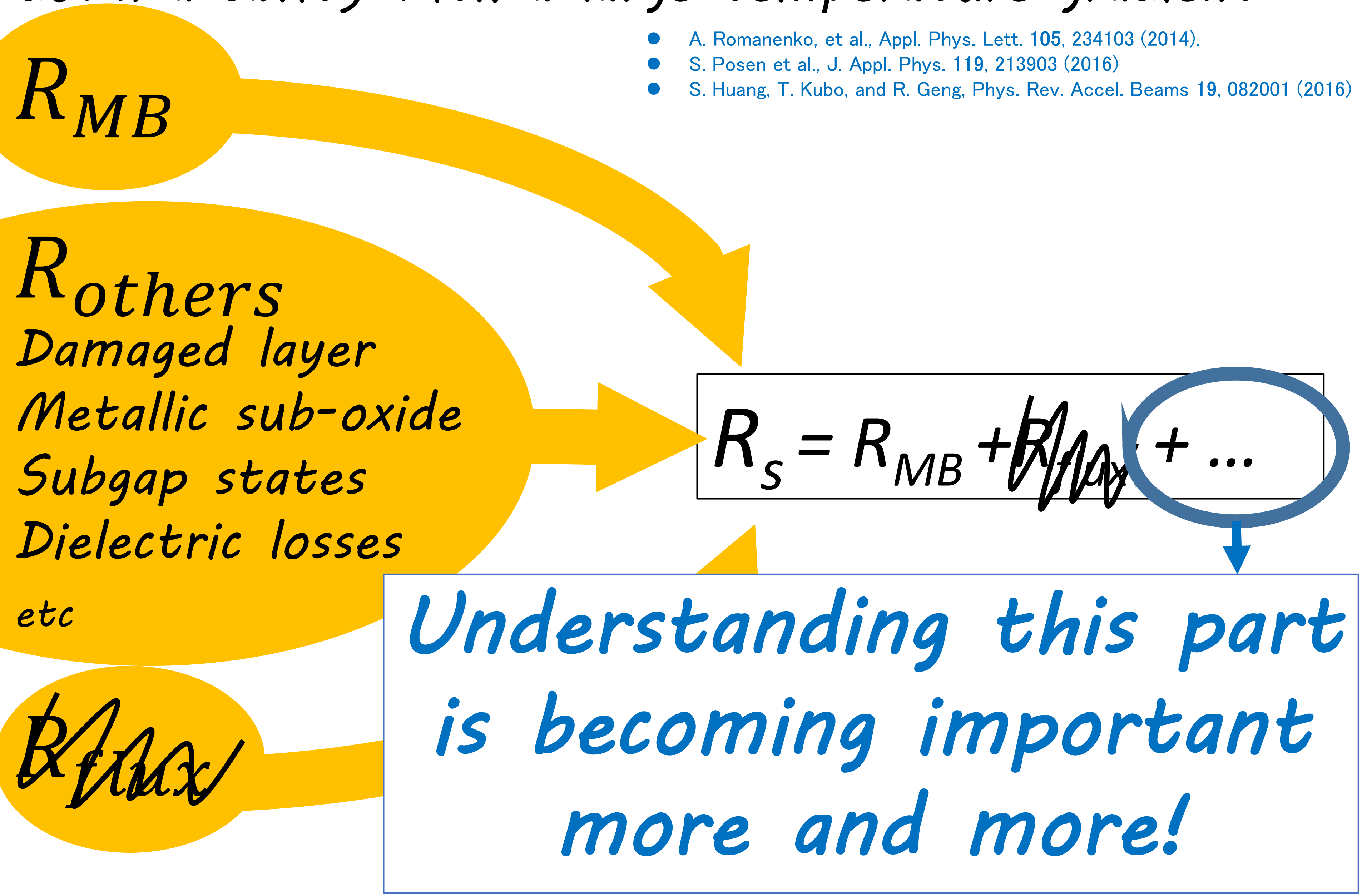
etc

~~R_{flux}~~

$$R_s = R_{MB} + \cancel{R_{flux}} + \ldots$$

Today, R_{flux} can be substantially reduced by cooling down a cavity with a large temperature gradient.

- A. Romanenko, et al., Appl. Phys. Lett. **105**, 234103 (2014).
- S. Posen et al., J. Appl. Phys. **119**, 213903 (2016)
- S. Huang, T. Kubo, and R. Geng, Phys. Rev. Accel. Beams **19**, 082001 (2016)



R_{MB}

R_{others}

Damaged layer

Metallic sub-oxide

Subgap states

Dielectric losses

etc

$\cancel{R_{flux}}$

$$R_s = R_{MB} + \cancel{R_{flux}} + \ldots$$

Understanding this part is becoming important more and more!

In the present study, we incorporate these effects based on the BCS theory

R_{MB}

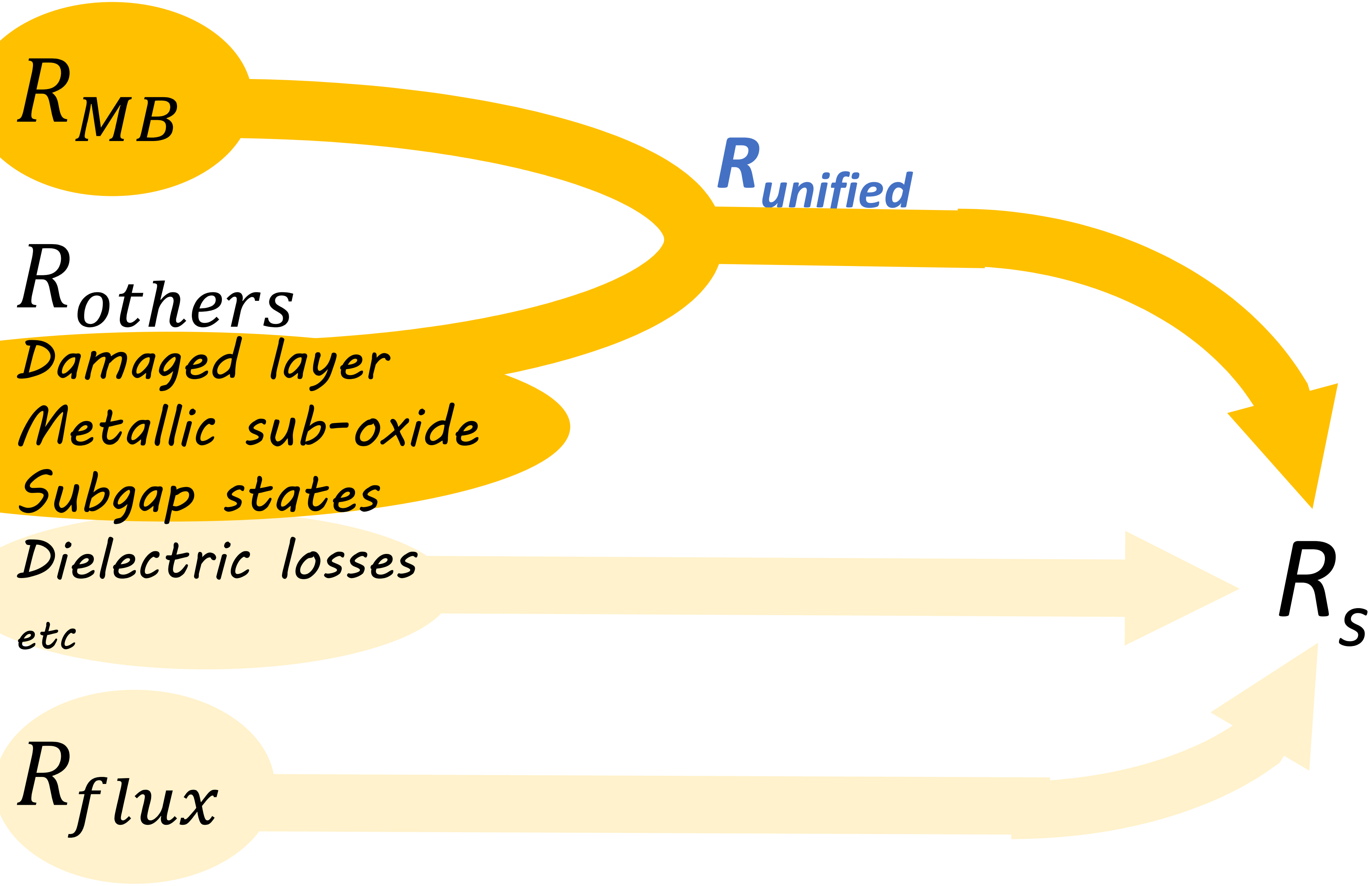
R_{others}

Damaged layer
Metallic sub-oxide
Subgap states
Dielectric losses

etc

R_{flux}

and obtain a theory that unifies R_{MB} and the effects of real materials.



R_{MB}

R_{others}

Damaged layer

Metallic sub-oxide

Subgap states

Dielectric losses

etc

R_{flux}

$R_{unified}$

R_s

and obtain a theory that unifies R and

It should be noted that

We <u>do not</u> propose
"a new BCS" or "new MB"

We <u>do not</u> propose
"a new model"

and obtain a theory that unifies R_{MB} and the effects of real materials.

R

R

D

M

S

D

et

We incorporate

the realistic surface

and bulk properties

based on the BCS theory.

R_{flux}

At the end of this talk we will have a unified theory,

which will provide us with clues to understanding what makes the differences of the low field R_s among

- EP only
- EP + 120℃ baking
- N or Ti doping
- Nitrogen infusion

Etc

and tell us how to engineer the cavity surface to minimize R_s

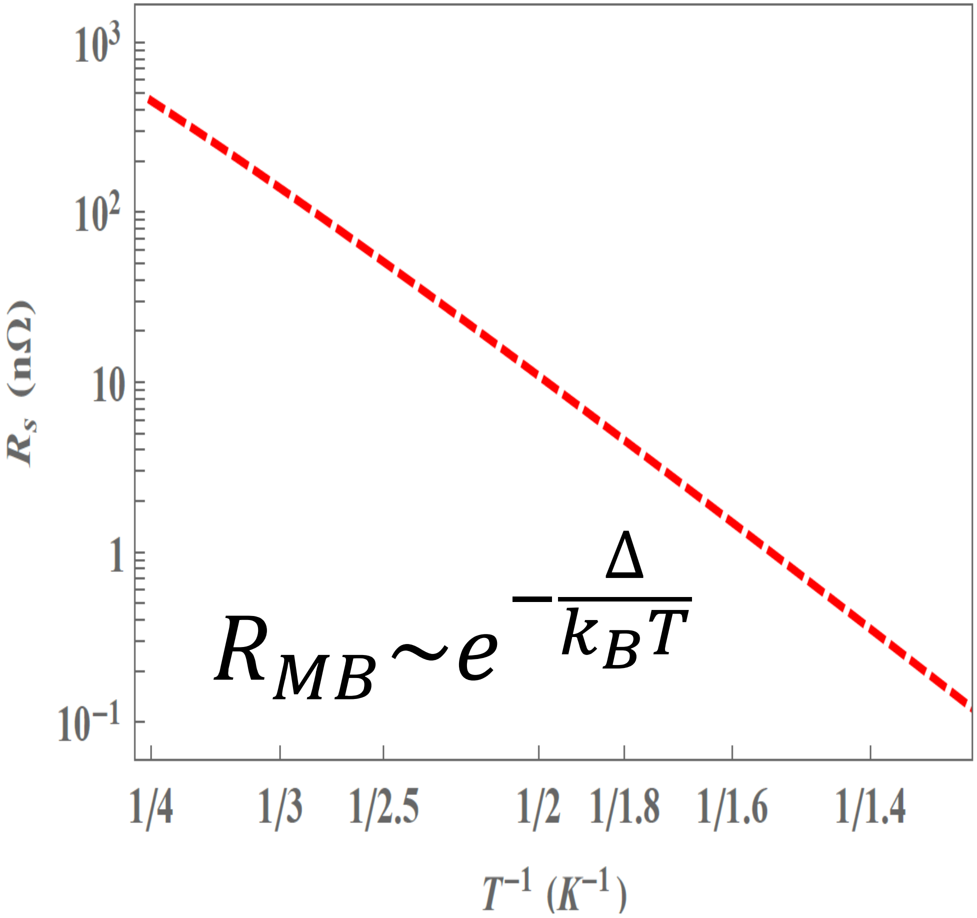
Overview

Strategy

If we start from

BCS model of SC with an idealized surface

Mattis-Bardeen



$$R_{MB} \sim e^{-\frac{\Delta}{k_B T}}$$

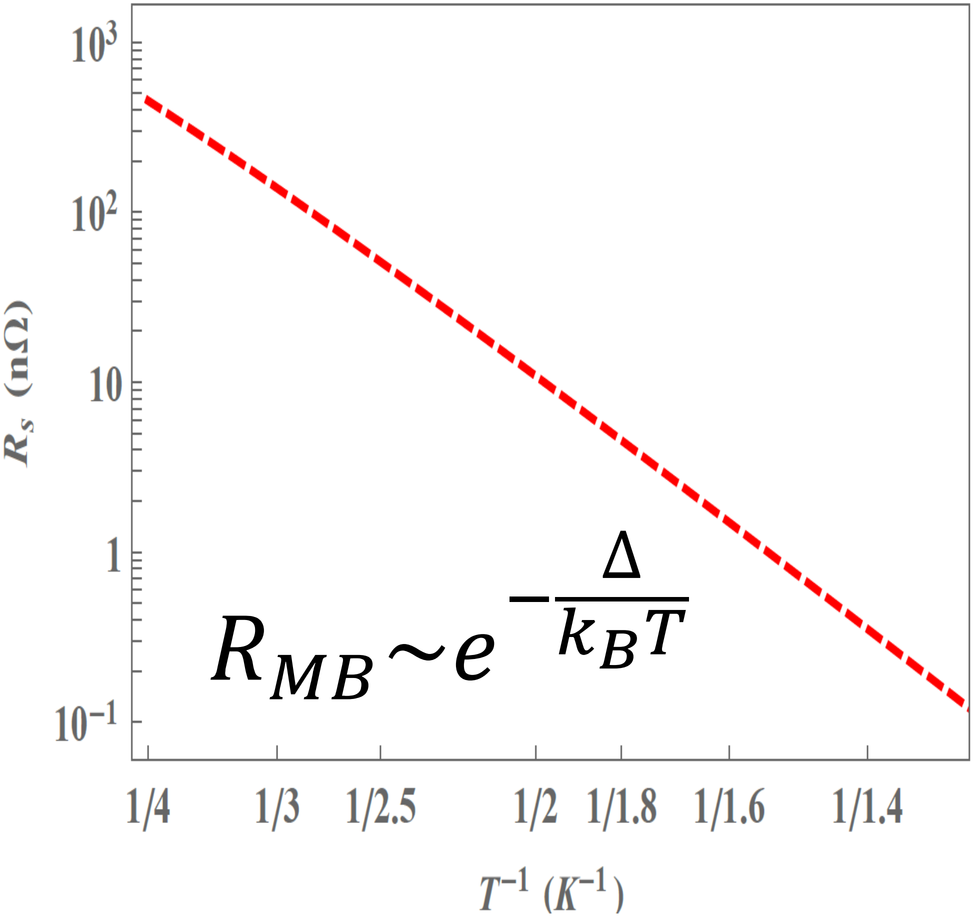
Strategy

~~If we start from~~

~~BCS model of SC with an idealized surface~~

~~Mattis-Bardeen~~

Fails



$$R_{MB} \sim e^{-\frac{\Delta}{k_B T}}$$

Strategy

If ~~we start from~~

~~BCS model of SC with an idealized surface~~

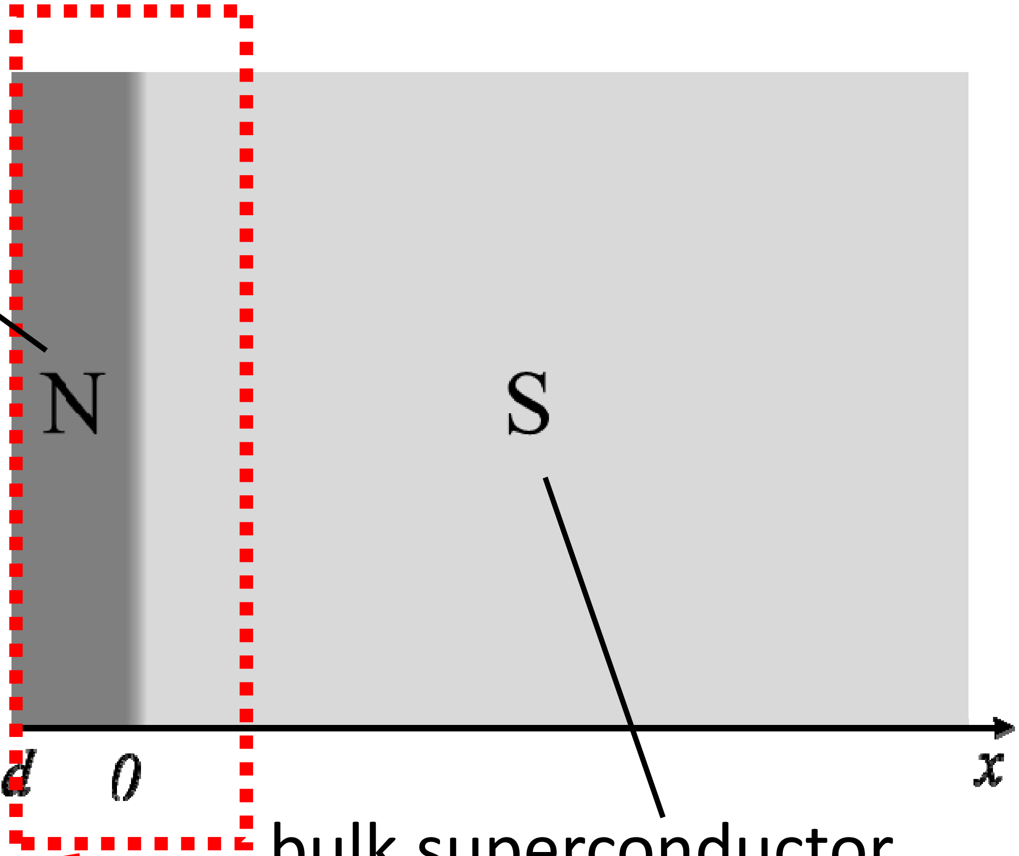
~~Mattis-Bardeen~~

Fails

We start from

BCS model of “a more realistic” SC

"More realistic" SC



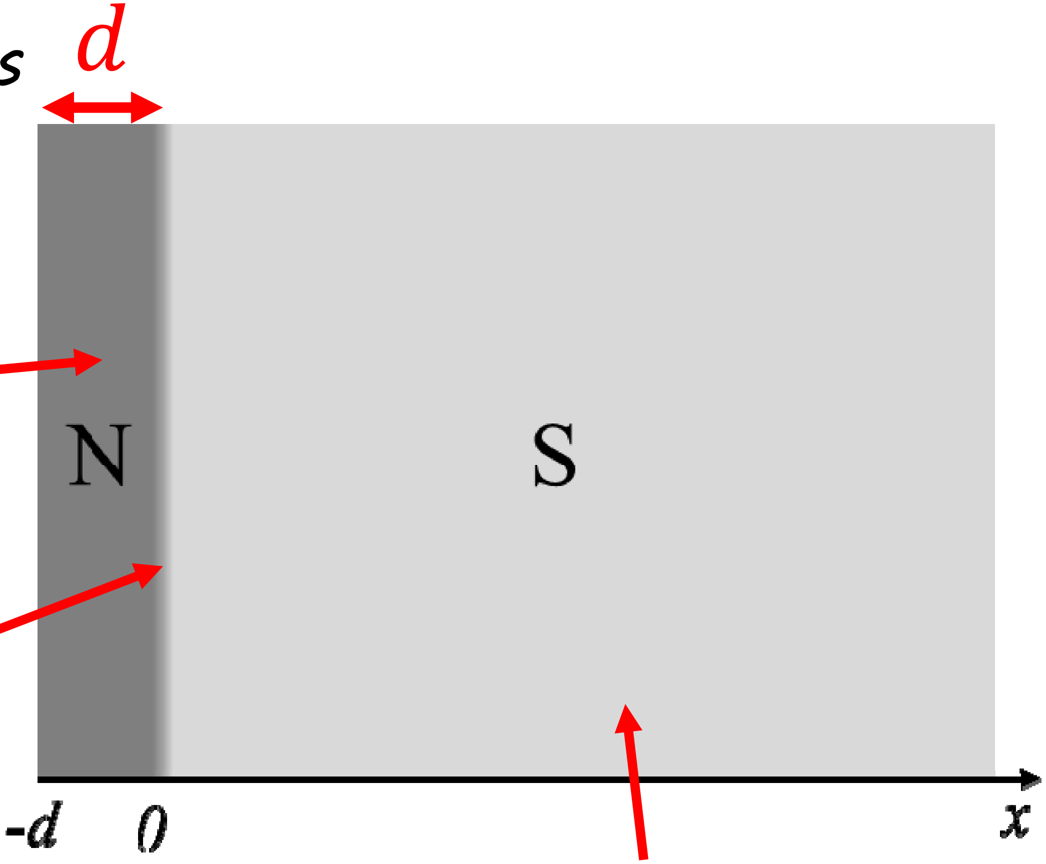
normal conductor
with thickness d<<ξ
(e.g., damaged Nb or
metallic sub-oxide with
a few nm thickness)

bulk superconductor
(e.g., Nb)

Proximity
effect

“More realistic” SC

Main physical parameters of this system



Normal layer thickness d

Normal conductance for N layer σ_n^N

Interface resistance R_B

Normal conductance for bulk region σ_n^S

Theoretical tool

We use the quasiclassical theory in the diffusive limit.

- Usadel equation $\xi_j^2\theta'' = -\frac{\Delta}{\Delta_\infty}\cos\theta + \frac{\hbar\omega_n}{\Delta_\infty}\sin\theta$ $\qquad \xi_j \equiv \sqrt{\hbar D_j/2\Delta_\infty}\ (j=\mathrm{N},\,\mathrm{S})$
- Self-consistency condition $\Delta(x) = 2\pi k_B T g(x) \sum_{\omega_n}^{\Omega} \sin\theta(x)$
- Boundary conditions $\theta'|_{\rm surface} = 0, \qquad \gamma_B \xi_{\rm N} \theta'_- = \sin(\theta_0 - \theta_-)$

$$\theta(\infty) = \theta_\infty, \qquad \gamma \xi_{\rm N} \theta'_- = \xi_{\rm S} \theta'_0,$$

K. D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
M. Yu. Kuprianov and V. F. Lukichev, Sov. Phys. JETP **67**, 1163 (1988).

where $\gamma \equiv \sigma_n^{\rm N}\xi_{\rm S}/\sigma_n^{\rm S}\xi_{\rm N} \qquad \gamma_B = \sigma_n^{\rm N} R_B/\xi_{\rm N}$

Normal and anomalous
Quasiclassical Matsubara Green functions $\quad G = \cos\theta \qquad F = \sin\theta$

T. Matsubara, Prog. Theor. Phys. **14**, 351 (1955).

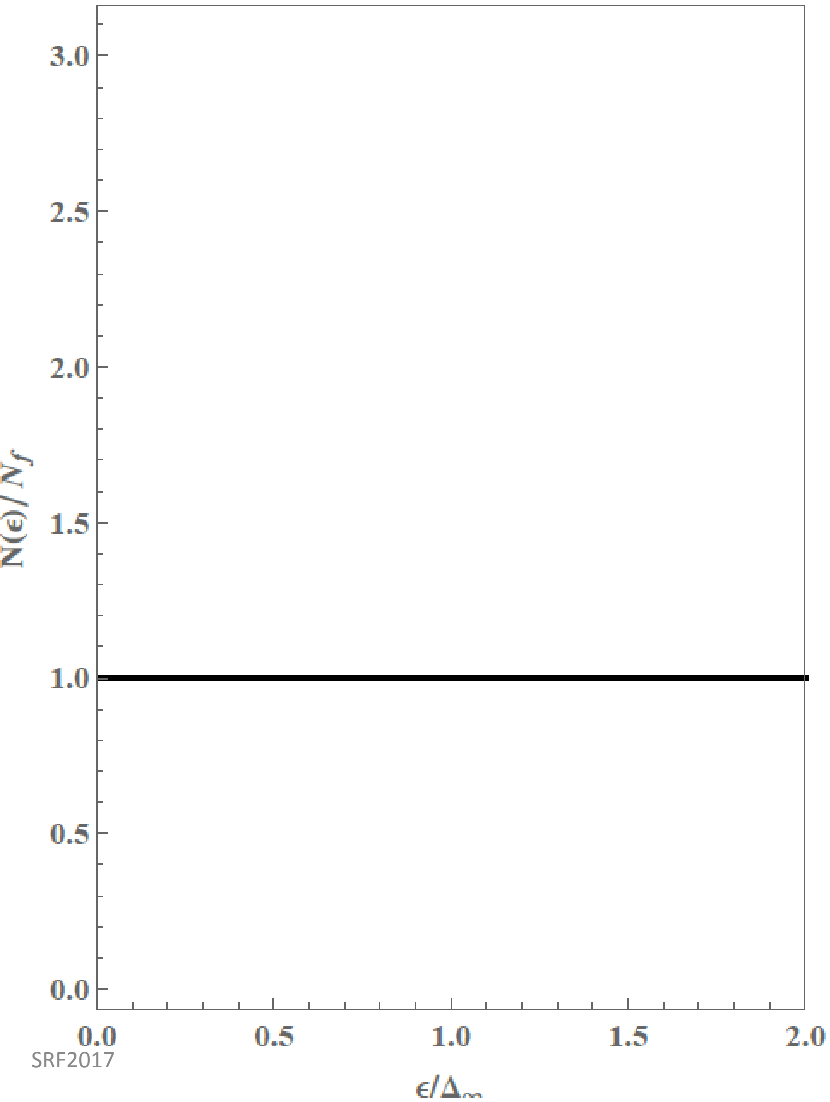
Retarded normal and anomalous
Quasiclassical Green functions $\quad G^R = \cosh\theta \qquad F^R = \sinh\theta$

Physical quantities: Density of states and surface resistance

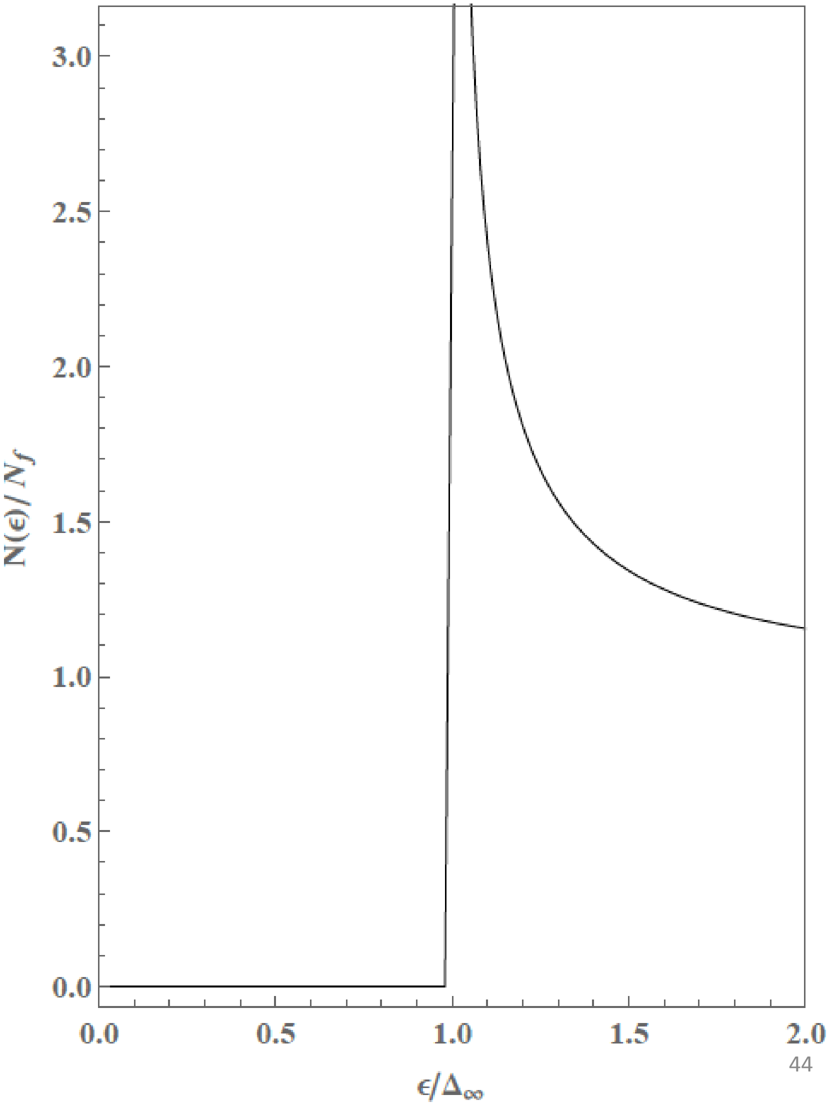
Results

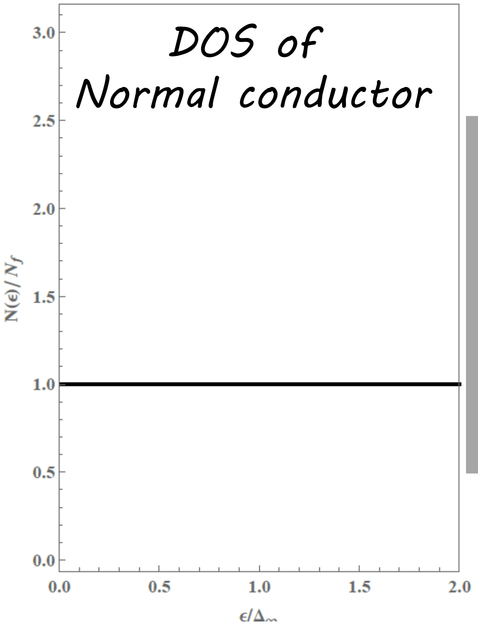
Density of states: $n=\mathrm{Re}[G^R]$

DOS of Normal conductor



DOS of BCS superconductor

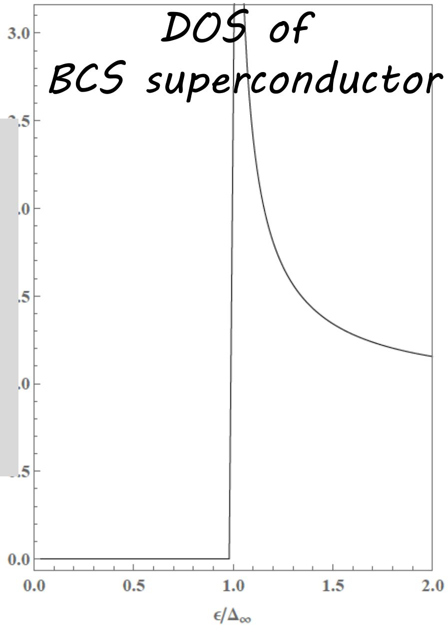




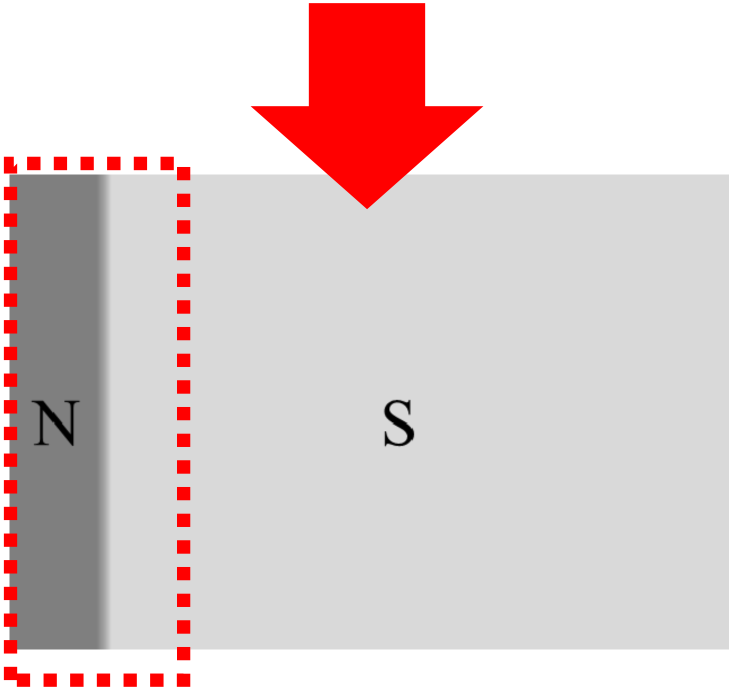
DOS of
Normal conductor

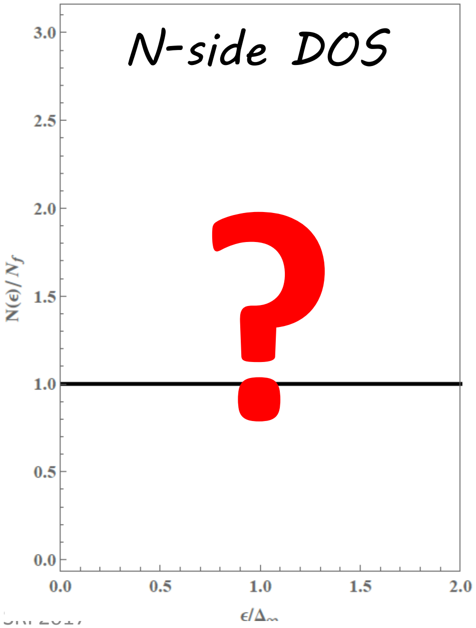
Normal
conductor

BCS
superconductor

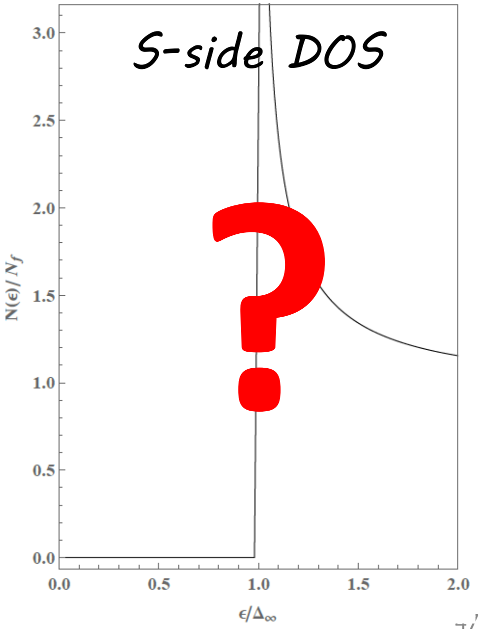


DOS of
BCS superconductor





N-side DOS

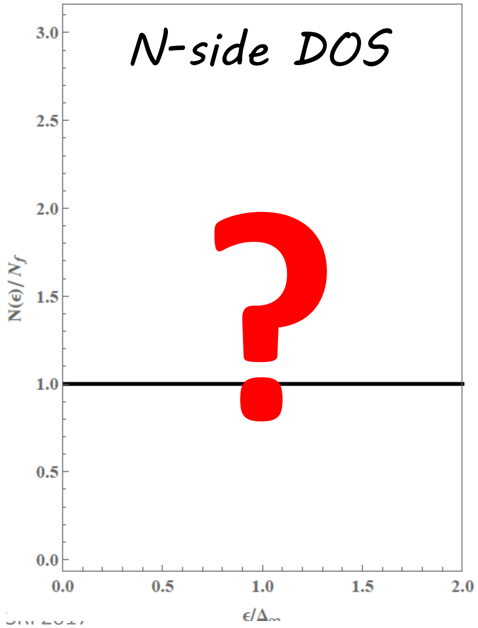


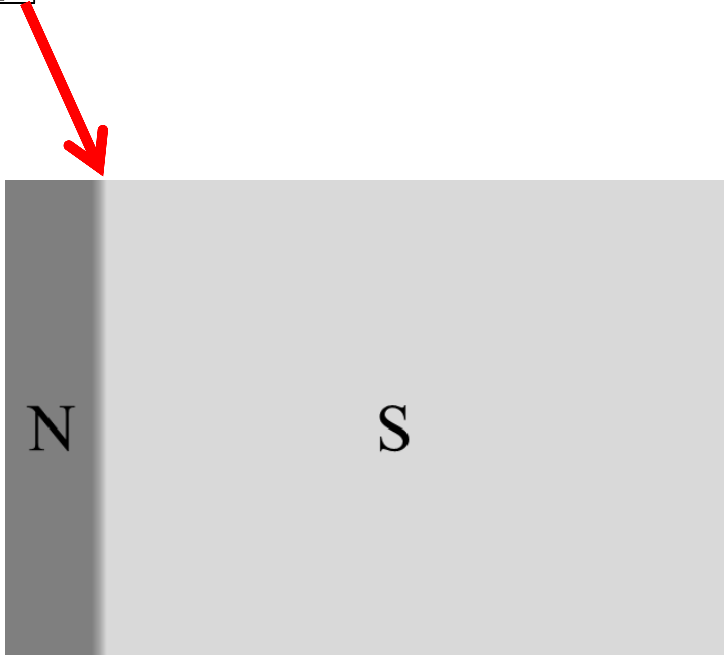
S-side DOS

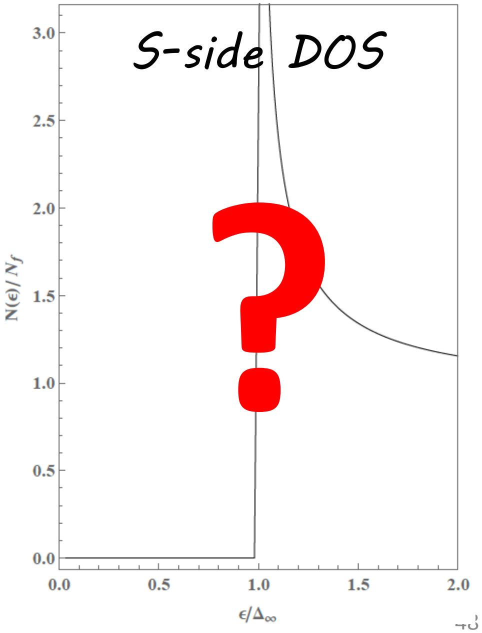
The proximity effect
changes DOS

The proximity effect and resultant DOS depend on the boundary parameters

$$\boxed{\alpha \equiv \gamma \frac{d}{\xi_N}}$$ where $\gamma = \frac{\sigma_n^N}{\sigma_n^S}\frac{\xi_S}{\xi_N}$ is proportional to the conductance ratio between N and S.





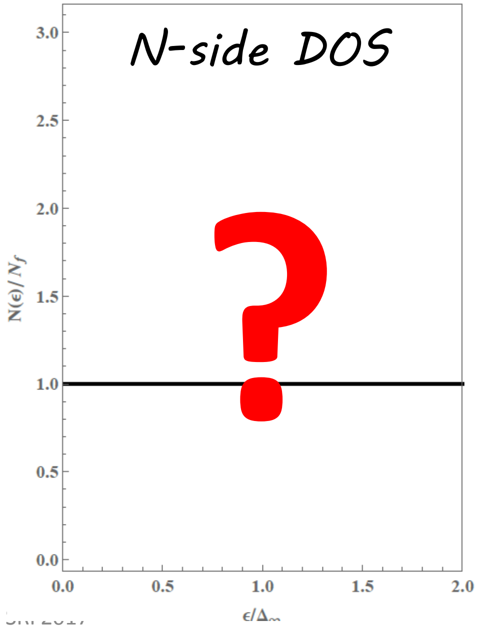


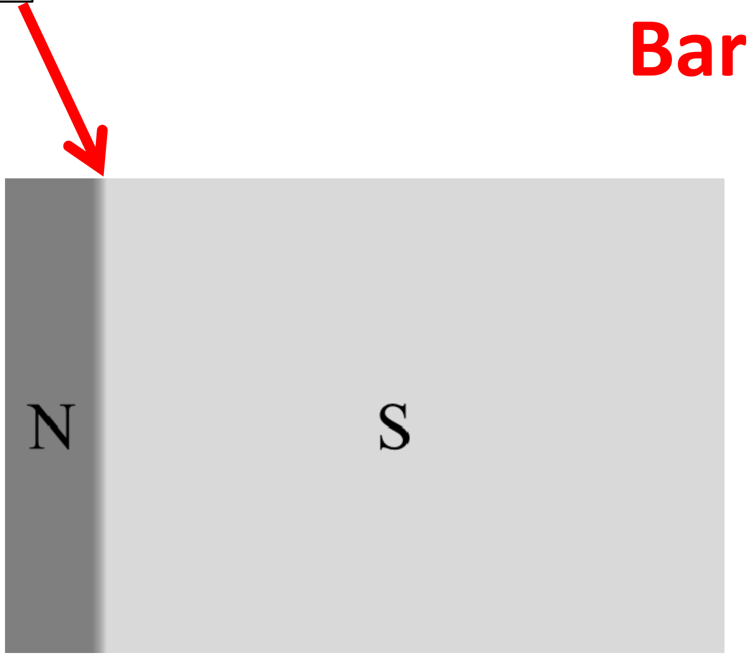
The proximity effect and resultant DOS depend on the boundary parameters

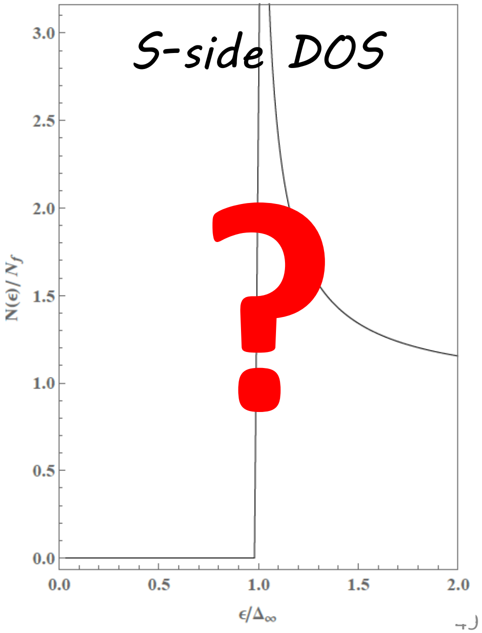
$$\boxed{\beta \equiv \gamma_B \frac{d}{\xi_N}}$$

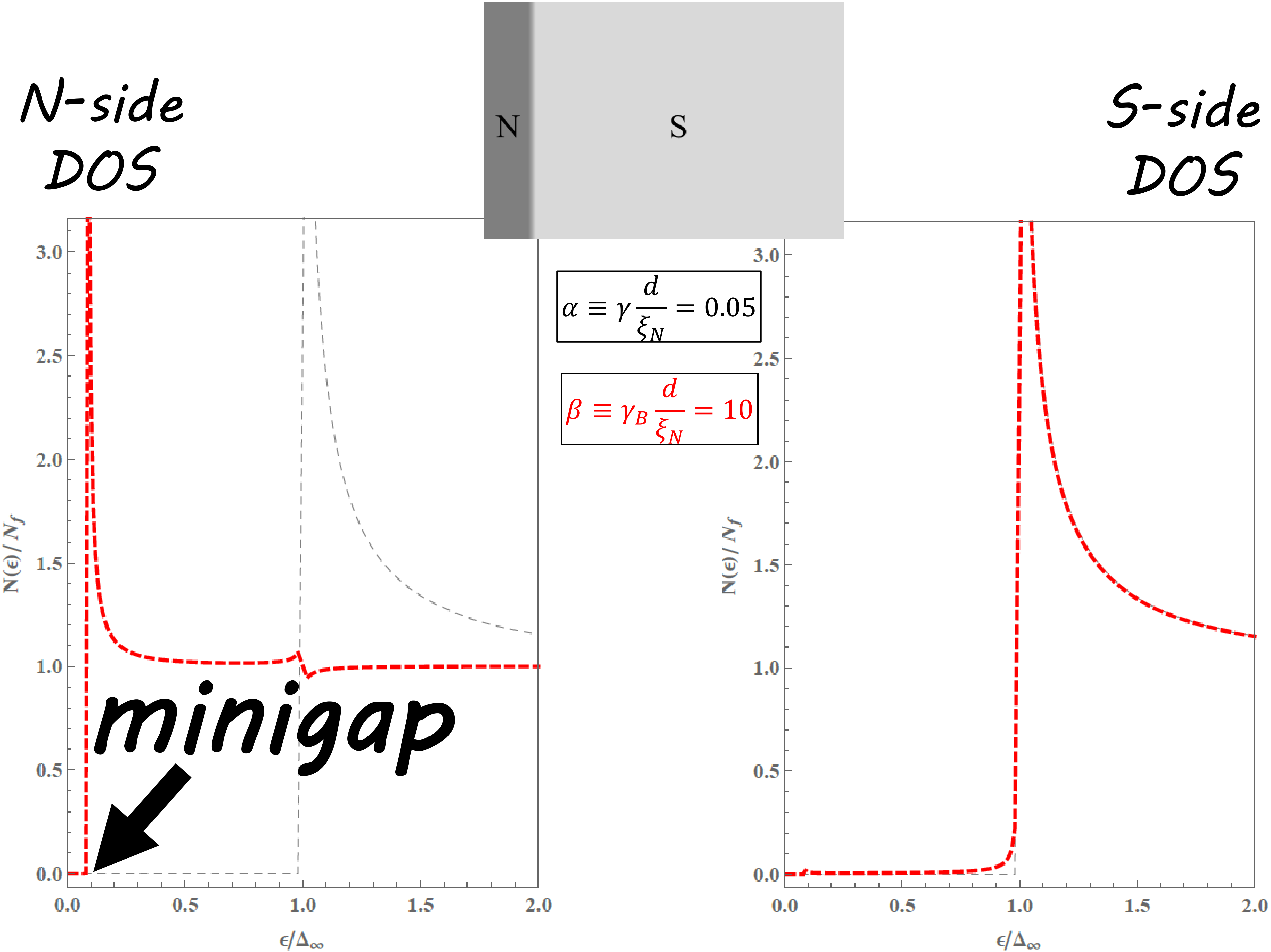
where $\gamma_B = \frac{\sigma_n^N}{\xi_N} R_B$ is proportional to the interface resistance between N and S.

Barrier parameter





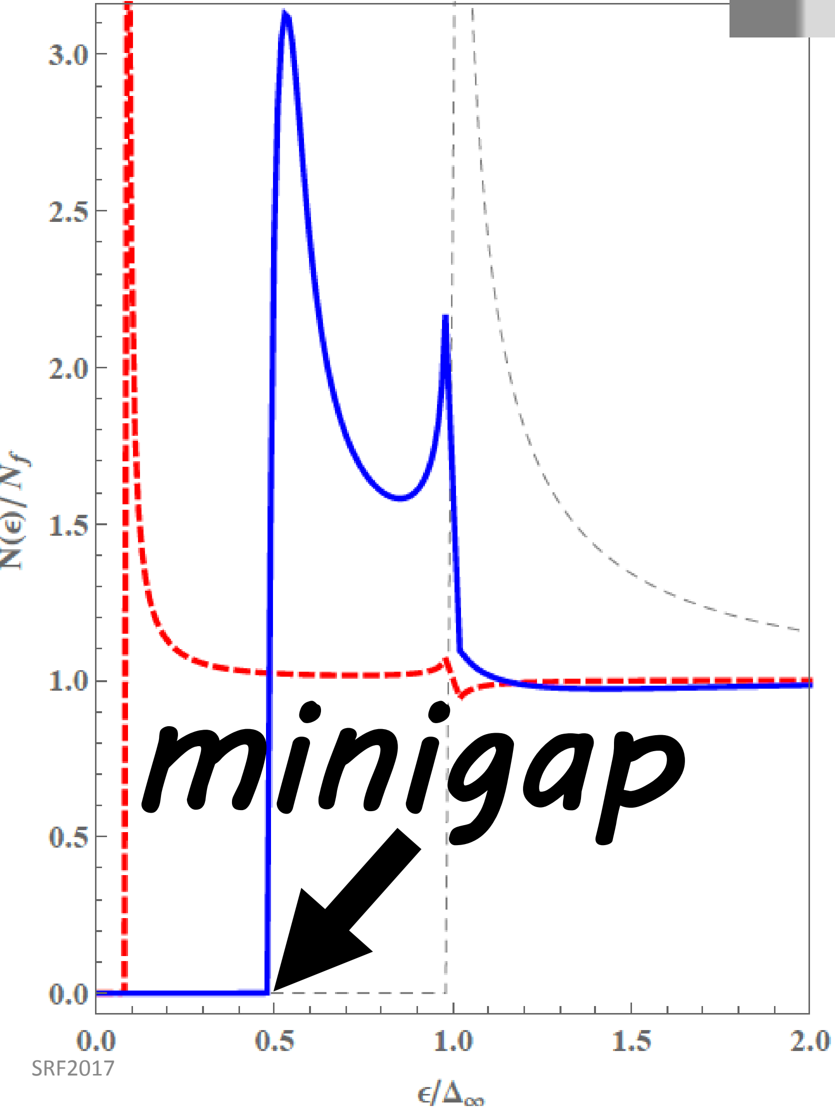


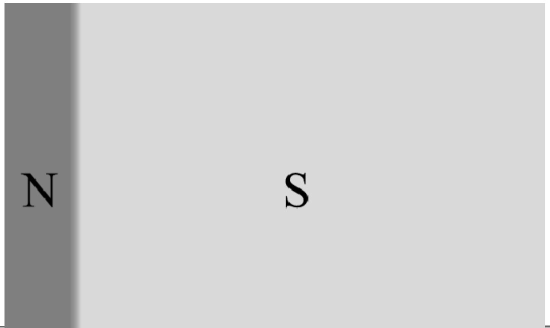


$$\alpha \equiv \gamma \frac{d}{\xi_N} = 0.05$$

$$\beta \equiv \gamma_B \frac{d}{\xi_N} = 10$$

N-side DOS

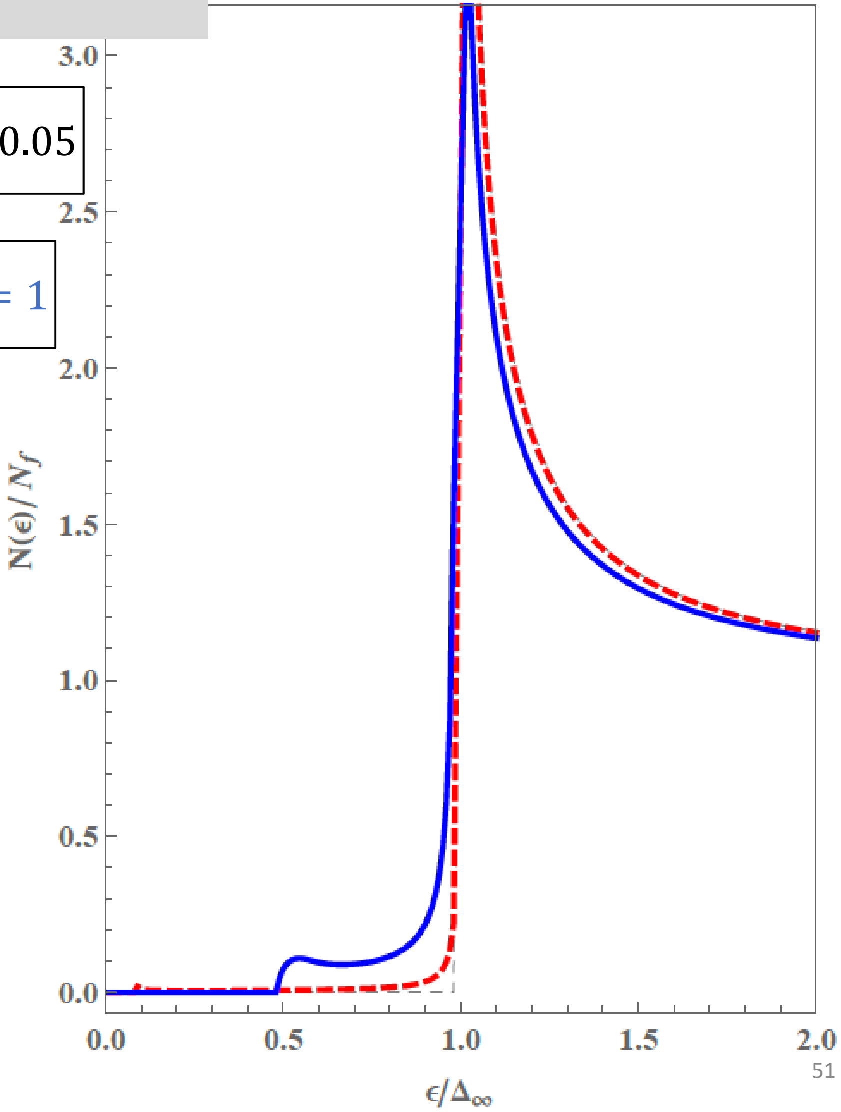




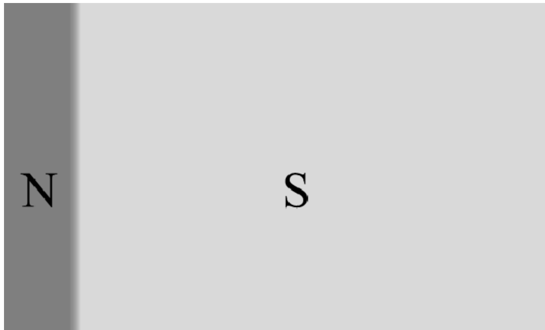
$$\alpha \equiv \gamma \frac{d}{\xi_N} = 0.05$$

$$\beta \equiv \gamma_B \frac{d}{\xi_N} = 1$$

S-side DOS



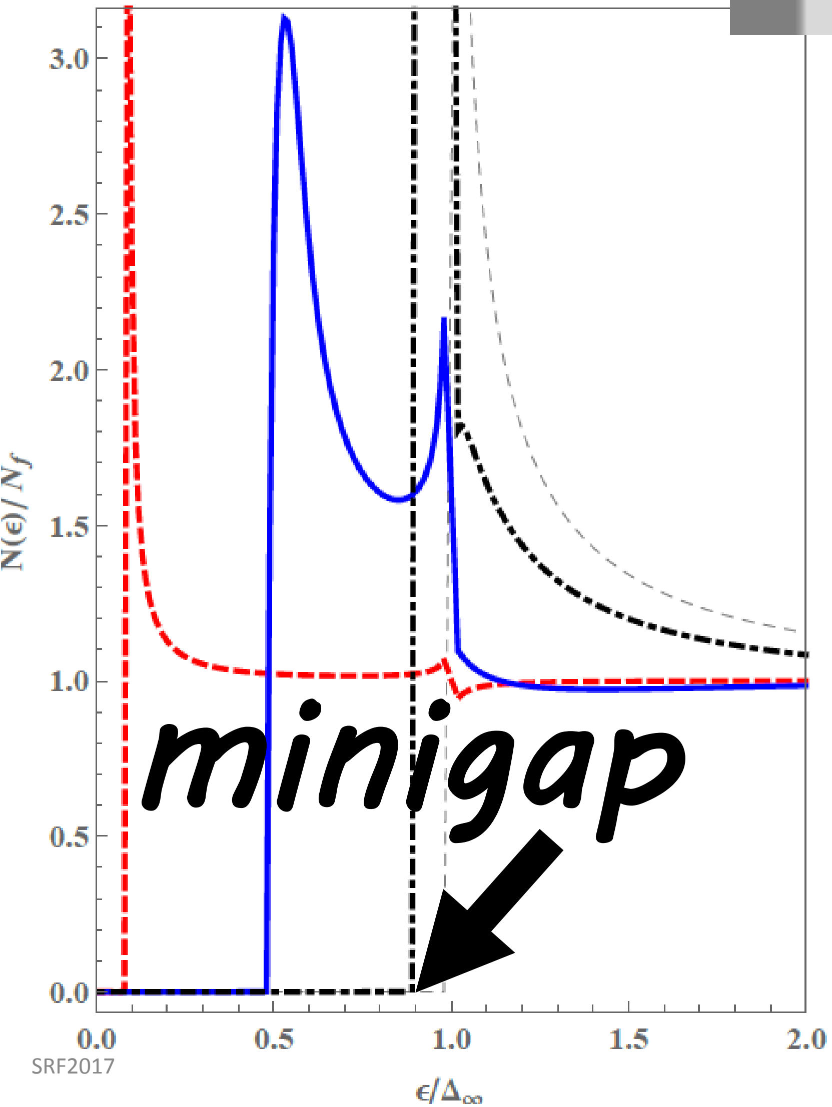
N-side DOS

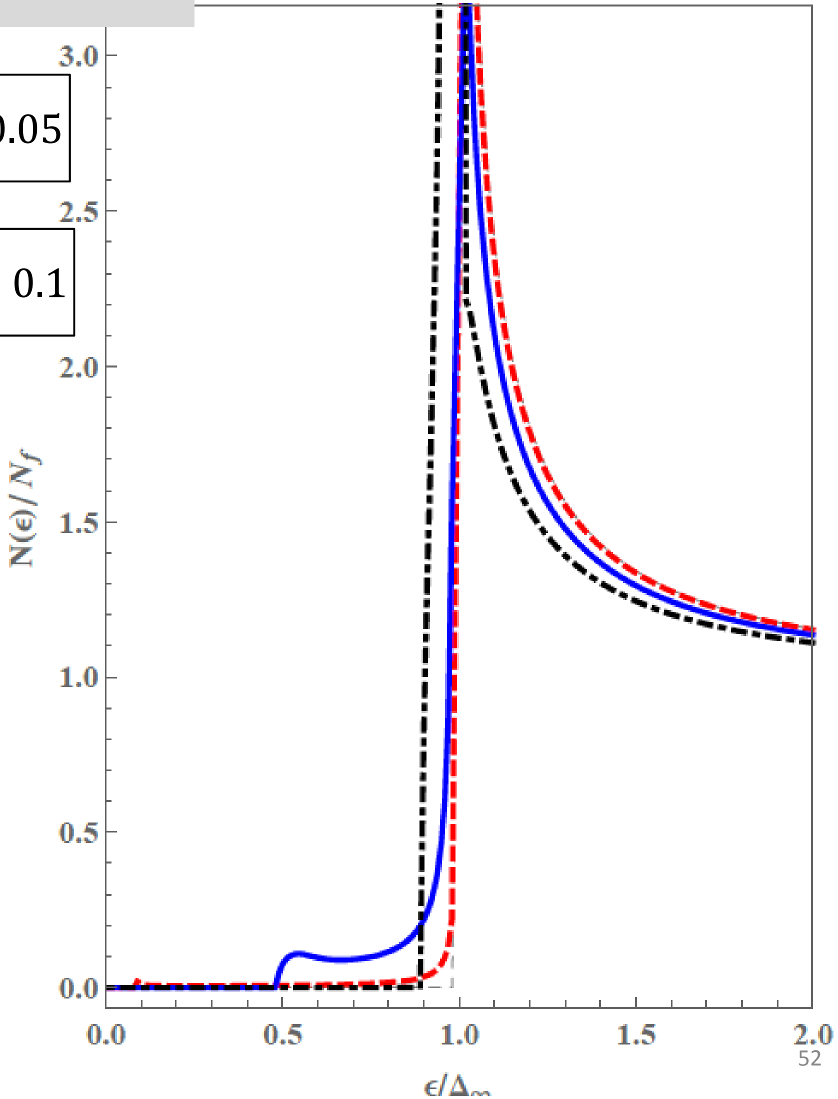


S-side DOS

$$\alpha \equiv \gamma \frac{d}{\xi_N} = 0.05$$

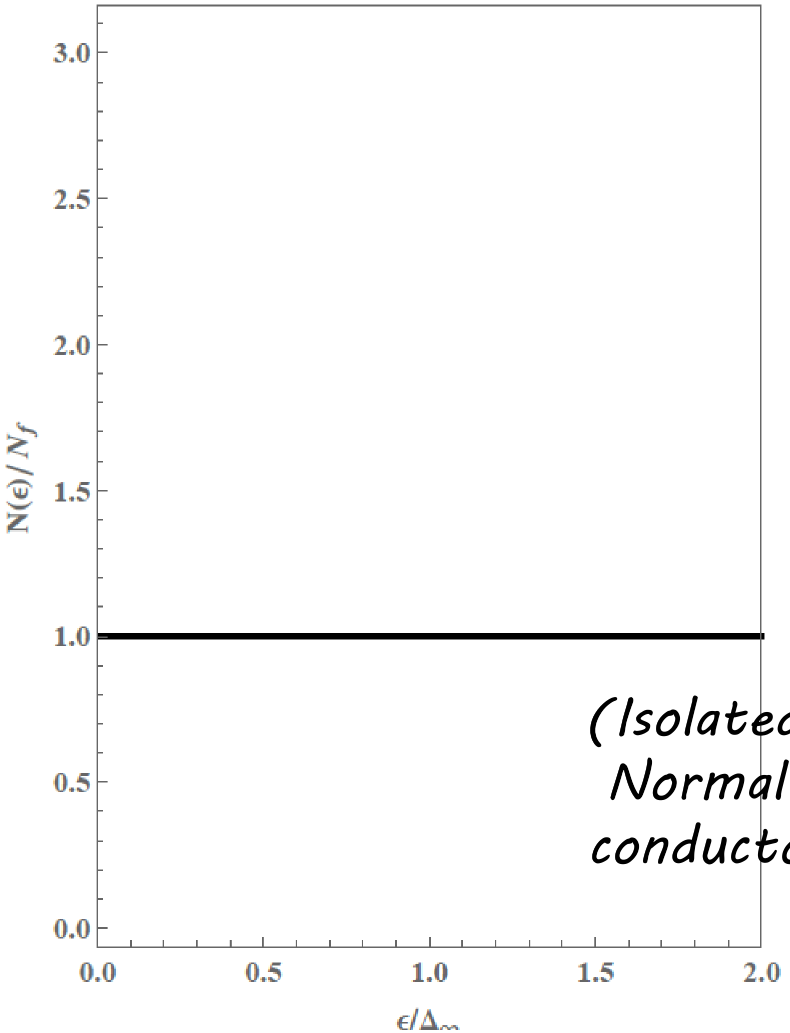
$$\beta \equiv \gamma_B \frac{d}{\xi_N} = 0.1$$



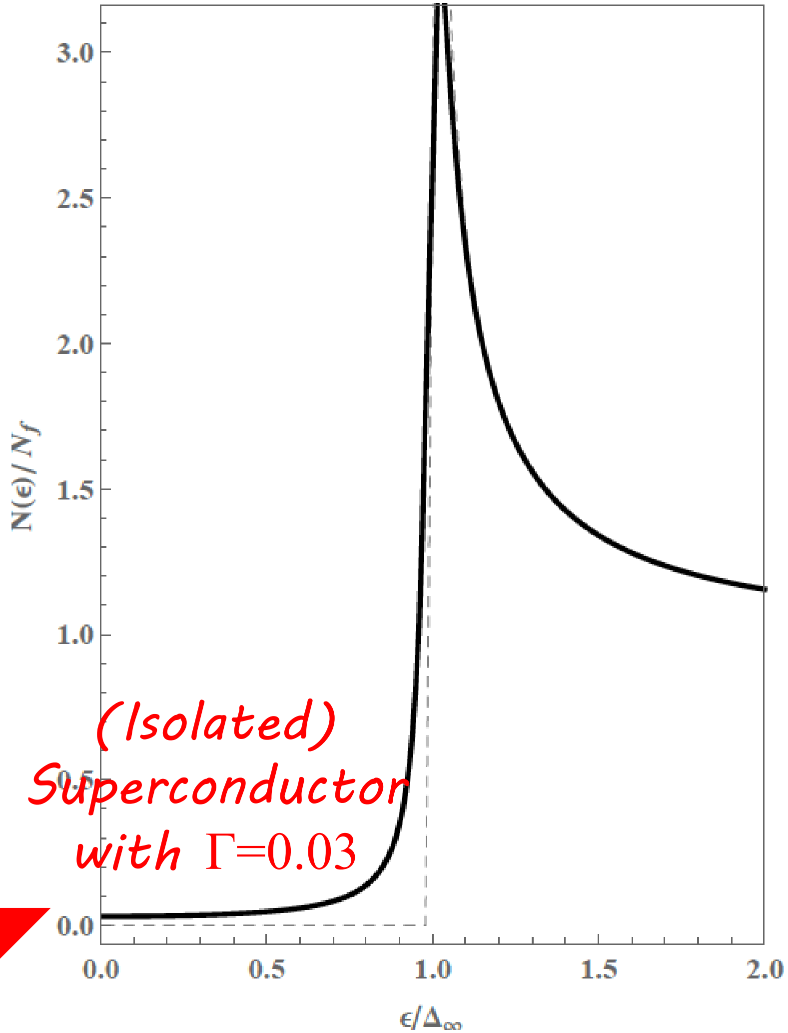


So far, in order to explicitly show the proximity effect and resultant minigap, we have neglected the bulk DOS broadening (due to an inlelastic scattering, magnetic impurities, random inhomogeneities of the BCS pairing constant etc): Γ=0.

Now we incorporate a finite Γ.

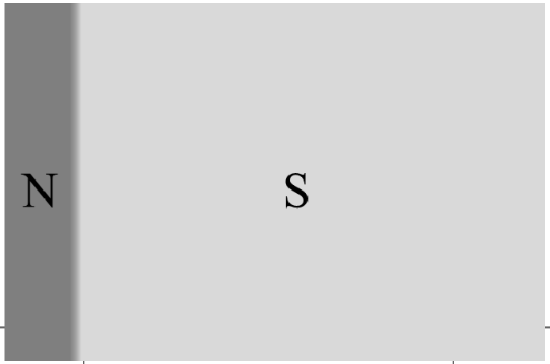


(Isolated)
Normal
conductor

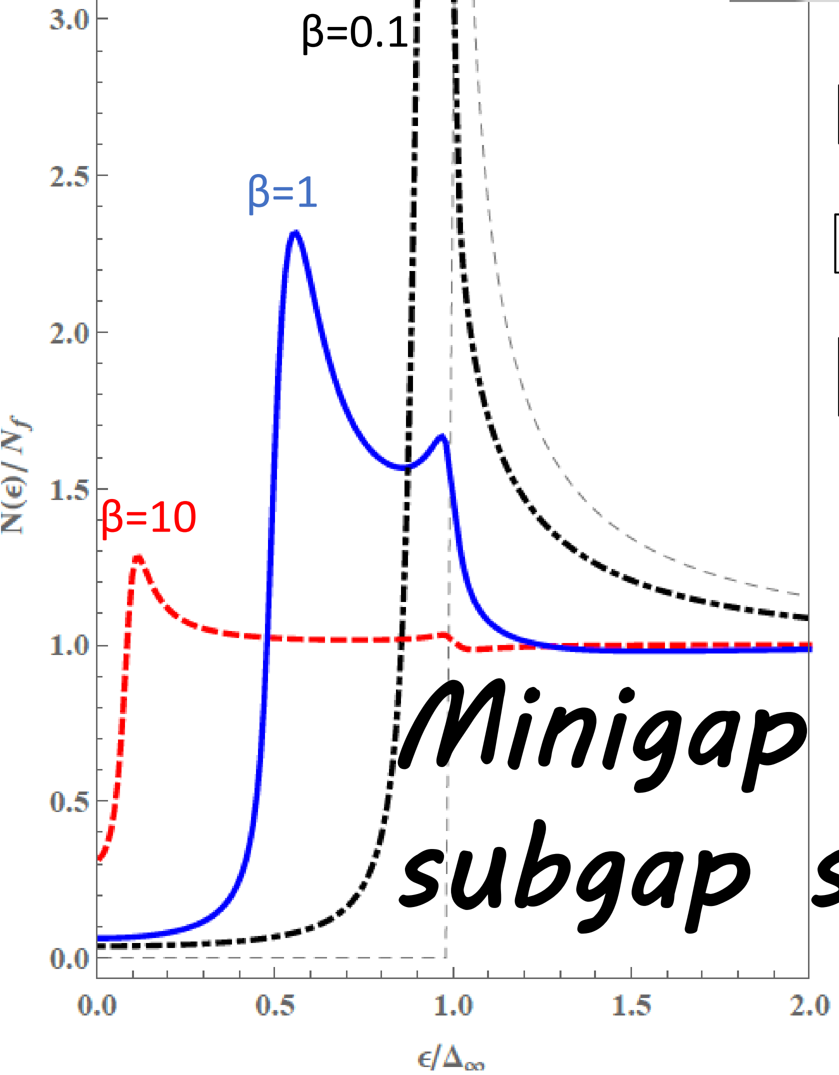


(Isolated)
Superconductor
with Γ=0.03

N-side DOS



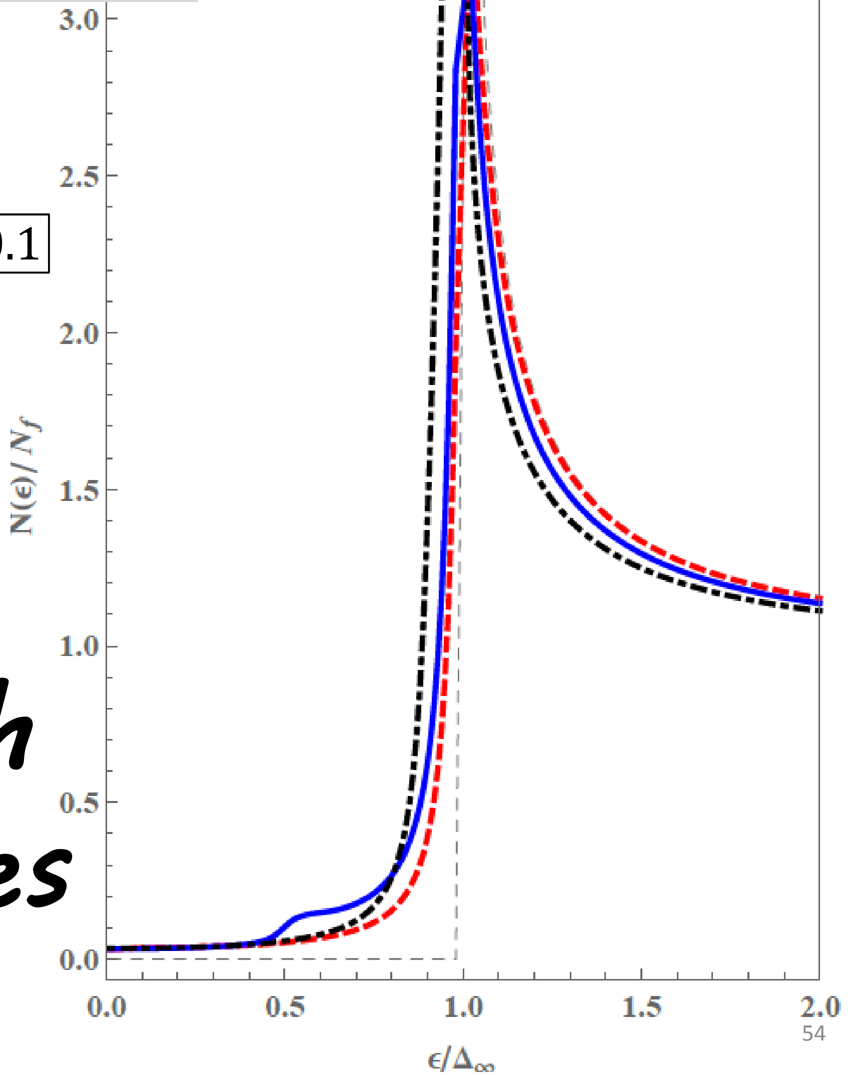
S-side DOS



$\alpha = 0.05$

$\beta = 10, 1, 0.1$

Γ=0.03

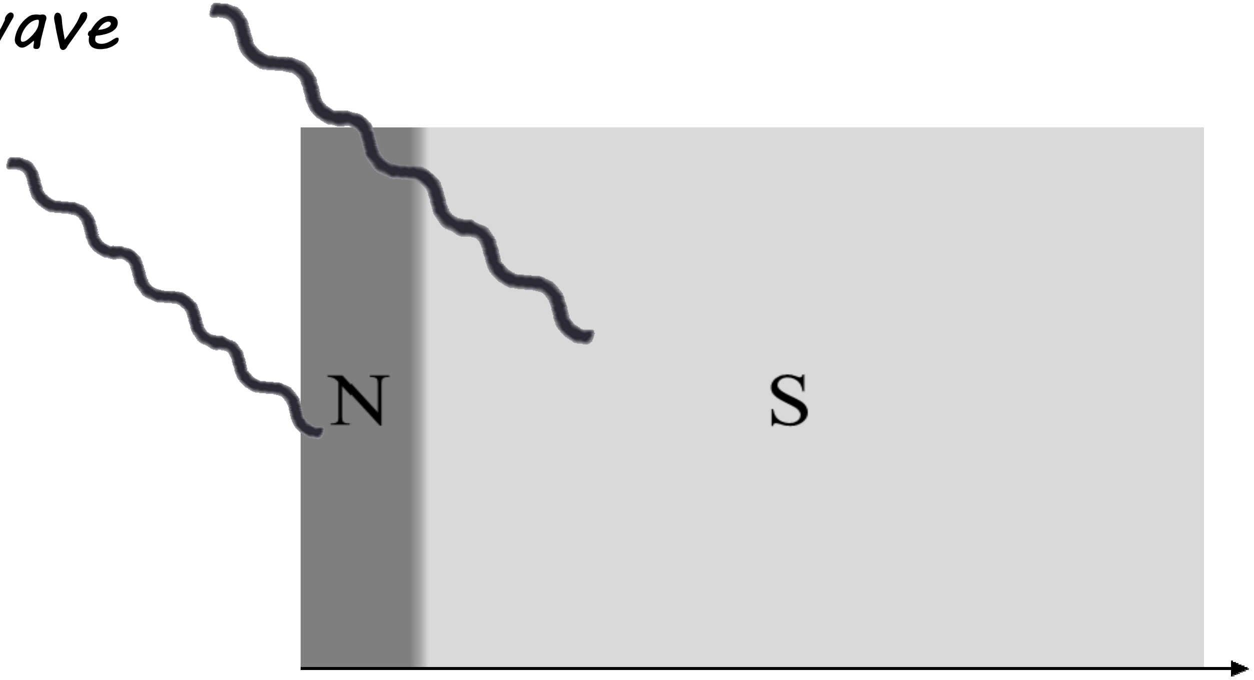


Minigap with subgap states

Surface resistance

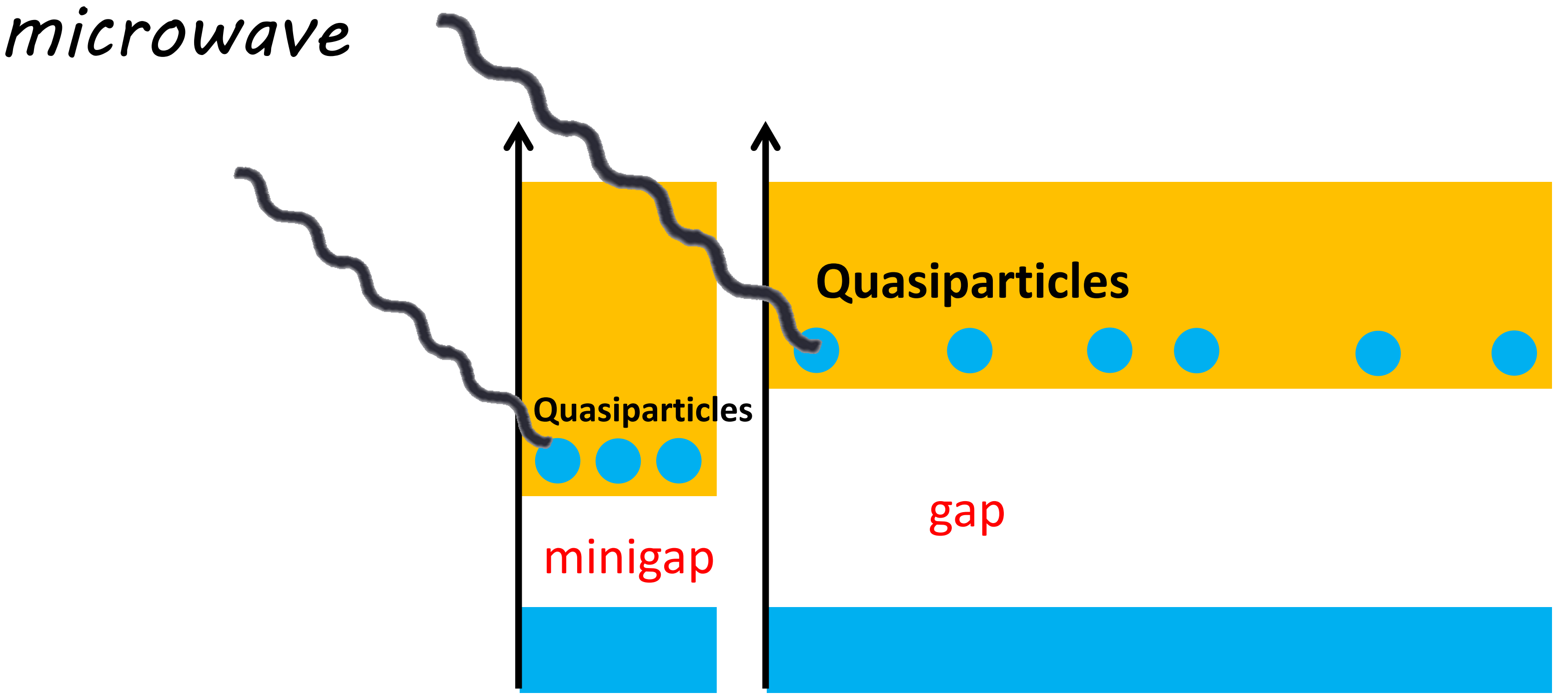
Now we can calculate the surface resistance of the proximity coupled NS system taking the minigap induced at the N layer into account.

microwave



Surface resistance

Now we can calculate the surface resistance of the proximity coupled NS system taking the minigap induced at the N layer into account.



Surface resistance

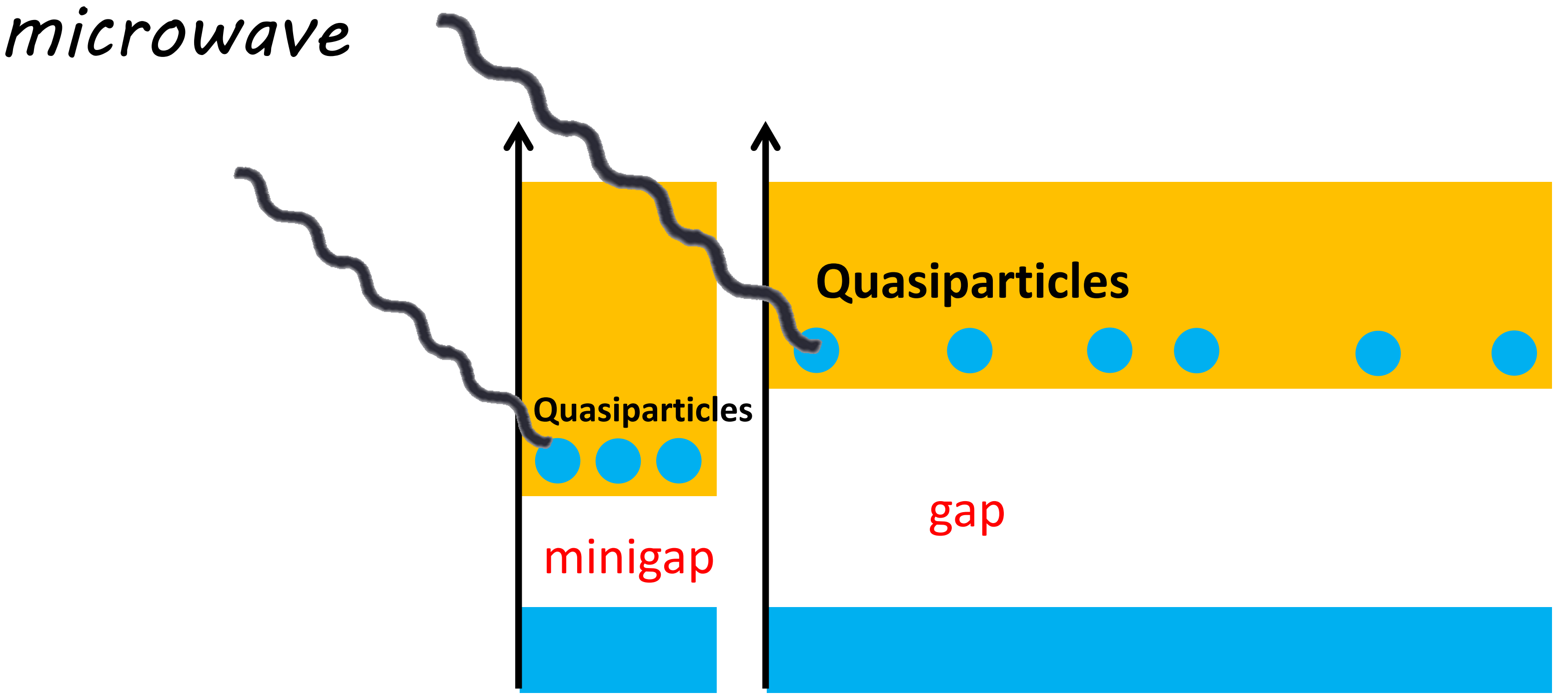
General formula

$$R_s = \frac{1}{2}\mu_0^2\omega^2\lambda^3\sigma_n^{\mathrm{S}}(I_{\mathrm{N}} + I_{\mathrm{S}})$$

$$I_{\mathrm{N}} = \frac{4}{u\lambda}\frac{\sigma_n^{\mathrm{N}}}{\sigma_n^{\mathrm{S}}}\int_0^\infty d\epsilon[f(\epsilon) - f(\epsilon+u)] \times \int_{-d}^{0} dx[n(\epsilon,x)n(\epsilon+u,x) + m(\epsilon,x)m(\epsilon+u,x)]$$

$$I_{\mathrm{S}} = \frac{4}{u\lambda}\int_0^\infty d\epsilon[f(\epsilon) - f(\epsilon+u)] \times \int_0^\infty dx[n(\epsilon,x)n(\epsilon+u,x) + m(\epsilon,x)m(\epsilon+u,x)]e^{-\frac{2x}{\lambda}}$$

A. Gurevich, Phys. Rev. Lett.
A. Gurevich, Supercond. Sci. Technol. **30**, 034004 (2017).



Surface resistance

Approximate formula
(for $\alpha \ll 1$ and $\Gamma \to 0$)

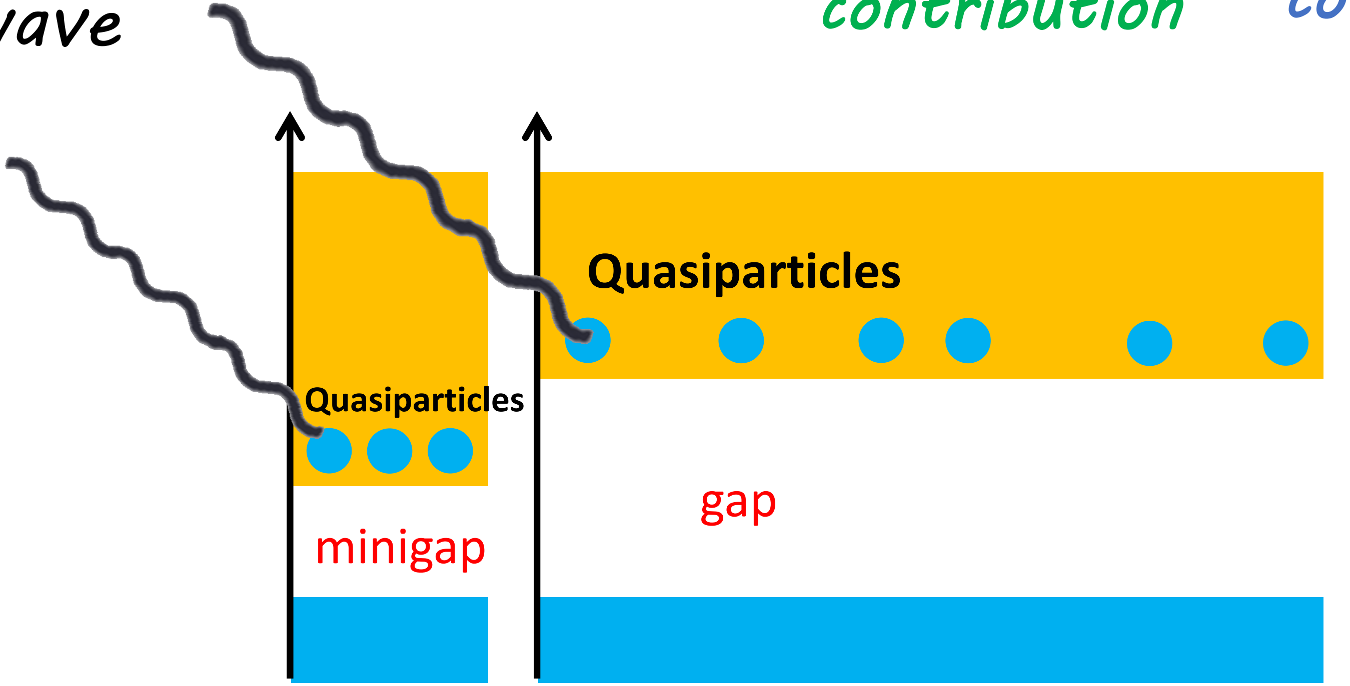
$$g(\beta) = \frac{1}{2\epsilon_0} \frac{1+\epsilon_0^2(1+\beta\sqrt{1-\epsilon_0^2})^2}{1+\beta^2-2\beta^2\epsilon_0^2+2\beta\sqrt{1-\epsilon_0^2}-\frac{\beta\epsilon_0^2}{\sqrt{1-\epsilon_0^2}}}$$

$$R_s = \frac{1}{2}\mu_0^2\omega^2\lambda^3\sigma_n^{\mathrm{S}}\frac{2\Delta_\infty}{k_B T}\ln\frac{4k_B T}{\hbar\omega e^{\gamma_E}}\left(\boxed{\frac{2d}{\lambda}\frac{\sigma_n^{\mathrm{N}}}{\sigma_n^{\mathrm{S}}}g(\beta)e^{-\frac{\epsilon_0}{k_B T}}}+\boxed{e^{-\frac{\Delta_\infty}{k_B T}}}\right)$$

N layer contribution

Bulk S contribution

microwave



Surface resistance

Approximate formula
(for α<<1 and Γ→0)

$$g(\beta) = \frac{1}{2\epsilon_0} \frac{1+\epsilon_0^2(1+\beta\sqrt{1-\epsilon_0^2})^2}{1+\beta^2-2\beta^2\epsilon_0^2+2\beta\sqrt{1-\epsilon_0^2}-\frac{\beta\epsilon_0^2}{\sqrt{1-\epsilon_0^2}}}$$

$$R_s = \frac{1}{2}\mu_0^2\omega^2\lambda^3\sigma_n^{\mathrm{S}}\frac{2\Delta_\infty}{k_B T}\ln\frac{4k_B T}{\hbar\omega e^{\gamma_E}}\left(\boxed{\frac{2d}{\lambda}\frac{\sigma_n^{\mathrm{N}}}{\sigma_n^{\mathrm{S}}}g(\beta)e^{-\frac{\epsilon_0}{k_B T}}}+\boxed{e^{-\frac{\Delta_\infty}{k_B T}}}\right)$$

N layer contribution

Bulk S contribution

1. The first term is tiny due to its small thickness and small normal-conductivity.
2. As T decreases, the second term decreases rapidly rather than the first term due to its large gap.
3. At a low temperature, the first term becomes dominant, which looks like the residual resistance.

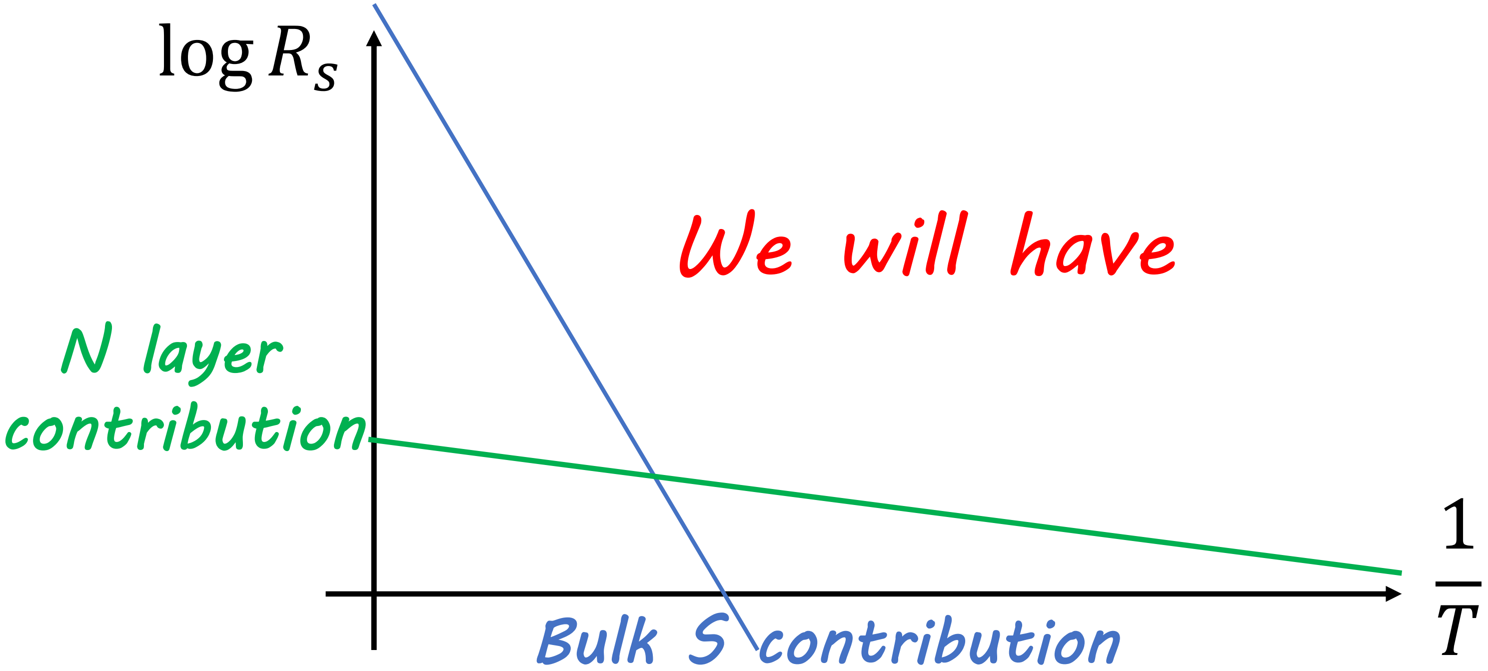
Surface resistance

Approximate formula

(for α<<1 and Γ→0)

$$g(\beta) = \frac{1}{2\epsilon_0}\frac{1+\epsilon_0^2(1+\beta\sqrt{1-\epsilon_0^2})^2}{1+\beta^2-2\beta^2\epsilon_0^2+2\beta\sqrt{1-\epsilon_0^2}-\frac{\beta\epsilon_0^2}{\sqrt{1-\epsilon_0^2}}}$$

$$R_s = \frac{1}{2}\mu_0^2\omega^2\lambda^3\sigma_n^{\mathrm{S}}\frac{2\Delta_\infty}{k_BT}\ln\frac{4k_BT}{\hbar\omega e^{\gamma_E}}\left(\underbrace{\boxed{\frac{2d}{\lambda}\frac{\sigma_n^{\mathrm{N}}}{\sigma_n^{\mathrm{S}}}g(\beta)e^{-\frac{\epsilon_0}{k_BT}}}}_{\text{N layer contribution}}+\underbrace{\boxed{e^{-\frac{\Delta_\infty}{k_BT}}}}_{\text{Bulk S contribution}}\right)$$



We will have

Surface resistance

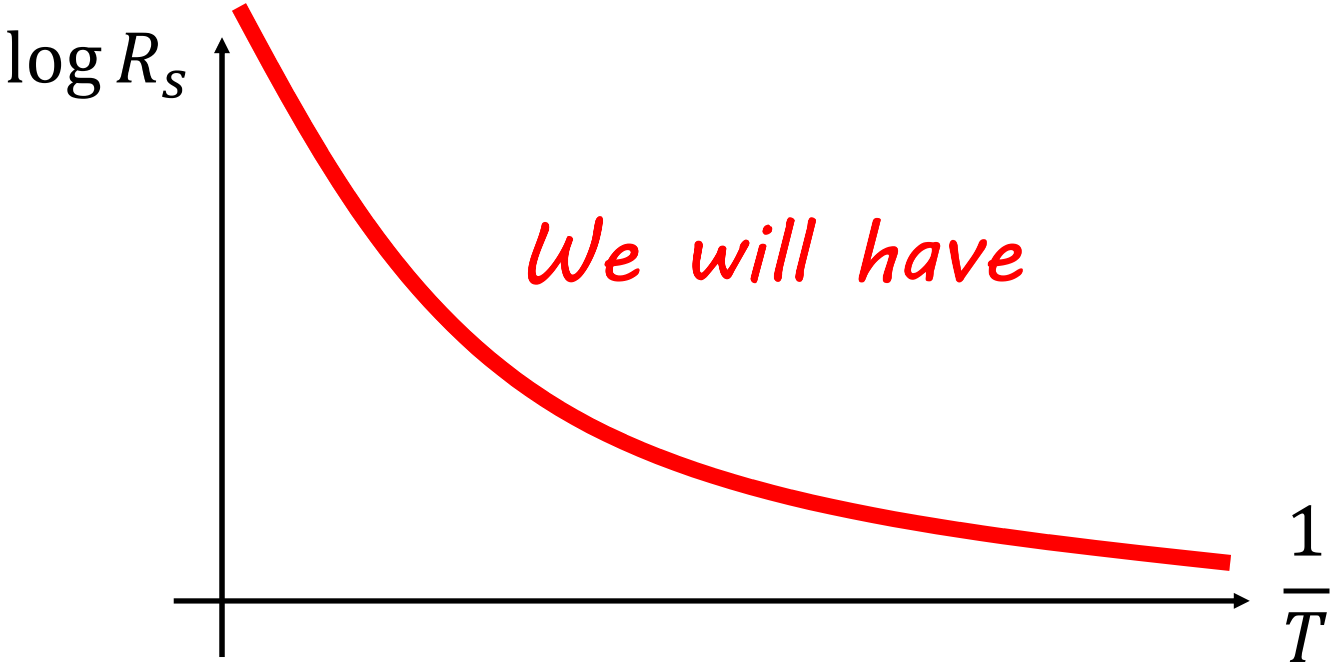
Approximate formula
(for α<<1 and Γ→0)

$$g(\beta) = \frac{1}{2\epsilon_0} \frac{1+\epsilon_0^2(1+\beta\sqrt{1-\epsilon_0^2})^2}{1+\beta^2-2\beta^2\epsilon_0^2+2\beta\sqrt{1-\epsilon_0^2}-\frac{\beta\epsilon_0^2}{\sqrt{1-\epsilon_0^2}}}$$

$$R_s = \frac{1}{2}\mu_0^2\omega^2\lambda^3\sigma_n^{\rm S}\frac{2\Delta_\infty}{k_B T}\ln\frac{4k_B T}{\hbar\omega e^{\gamma_E}}\left(\boxed{\frac{2d}{\lambda}\frac{\sigma_n^{\rm N}}{\sigma_n^{\rm S}}g(\beta)e^{-\frac{\epsilon_0}{k_B T}}}+\boxed{e^{-\frac{\Delta_\infty}{k_B T}}}\right)$$

N layer contribution

Bulk S contribution

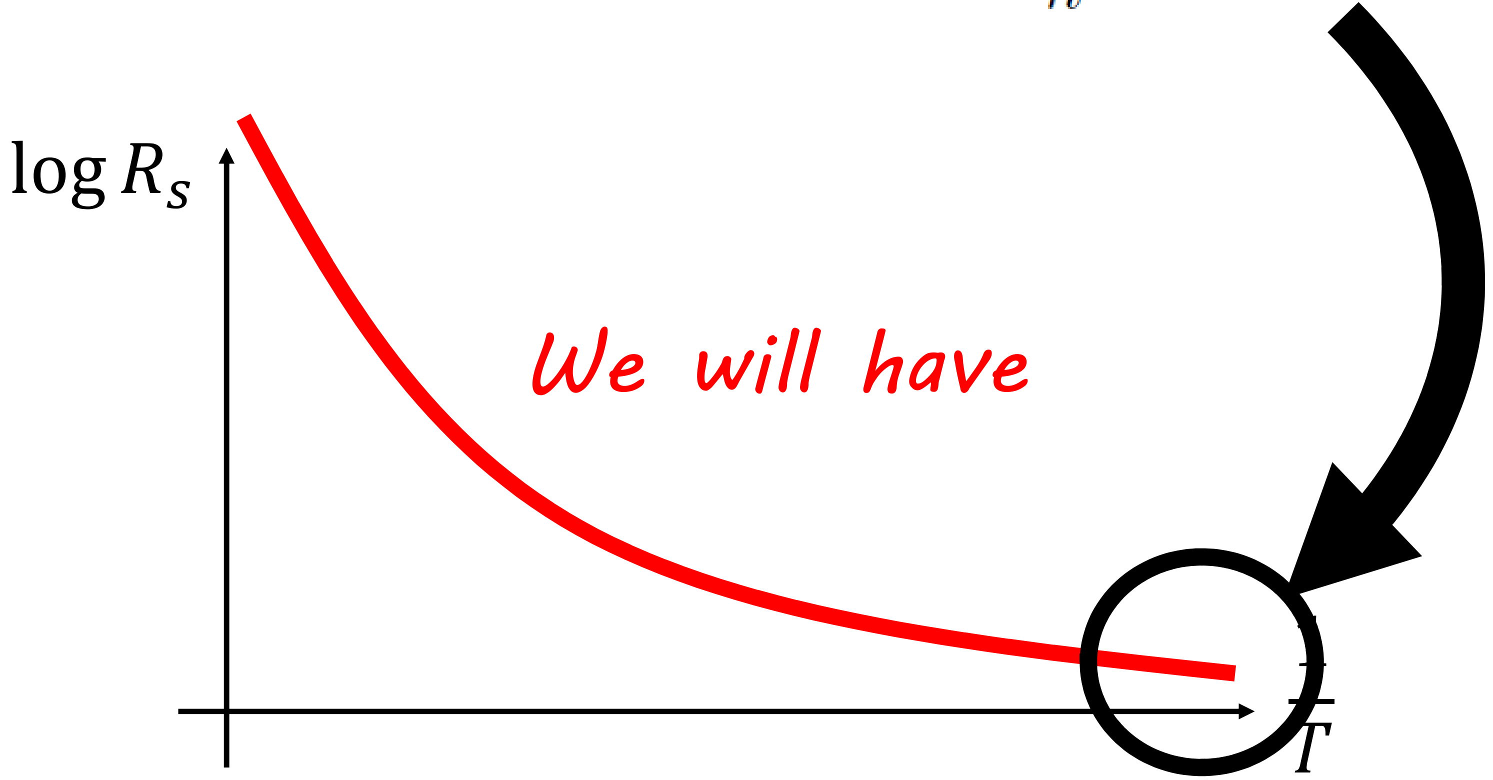


We will have

Surface resistance

Approximate formula
(for α<<1 and Γ→0)

$$R_i = \frac{1}{2}\mu_0^2\omega^2\lambda^3\sigma_n^{\mathrm{S}}\frac{4\Delta_\infty}{k_BT}\ln\frac{4k_BT}{\hbar\omega e^{\gamma_E}}\frac{d}{\lambda}\frac{\sigma_n^{\mathrm{N}}}{\sigma_n^{\mathrm{S}}}g(\beta,\epsilon_0)e^{-\frac{\epsilon_0}{k_BT}}$$



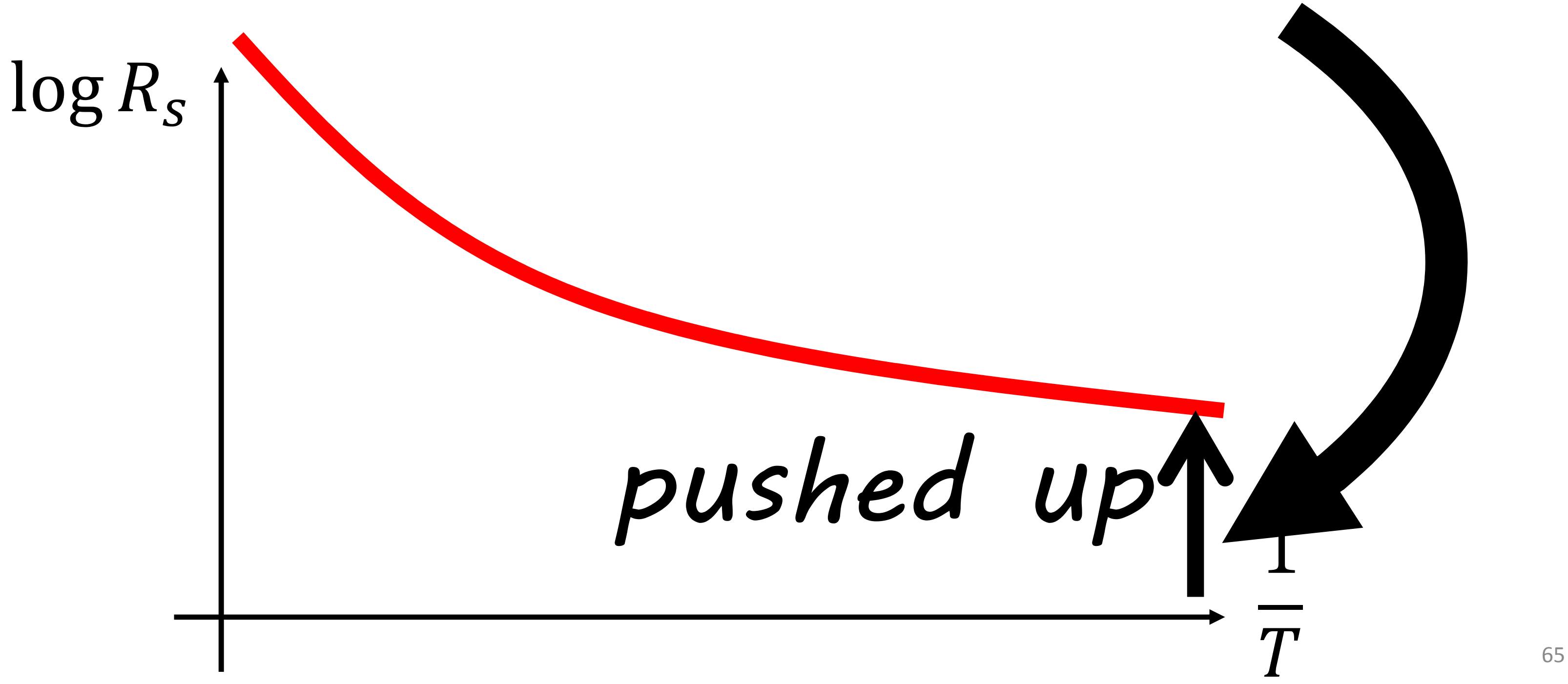
We will have

Surface resistance

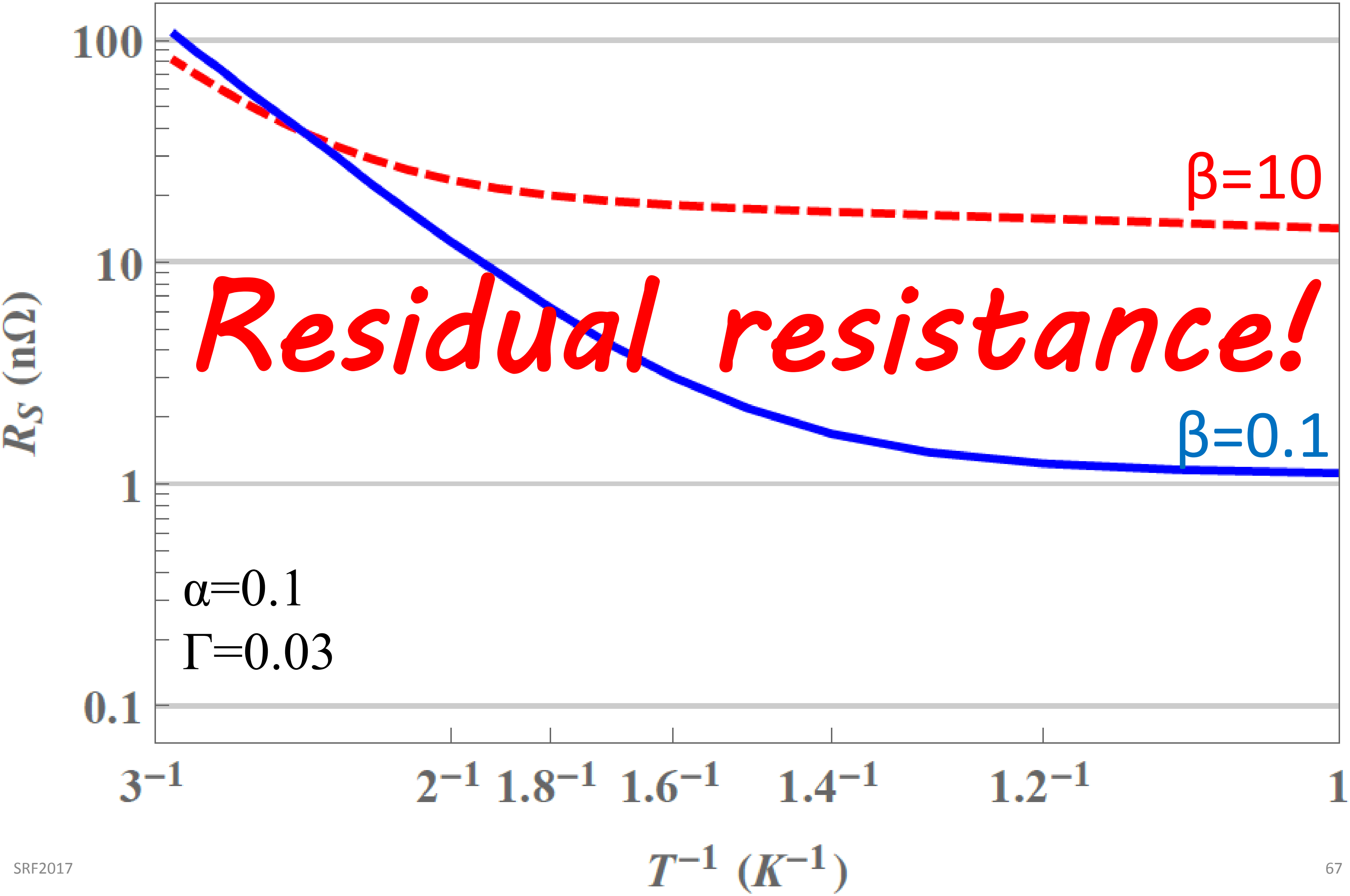
As the DOS broadening parameter increases, the subgap state contribution pushes up the residual resistance.

Approximate formula (for finite Γ)

$$R_i = \frac{1}{2}\mu_0^2\omega^2\lambda^3\sigma_n^{\rm S}\frac{4k_BT}{\hbar\omega}\frac{\Gamma^2}{\Delta_\infty^2}\ln\frac{2}{1+e^{-\hbar\omega/k_BT}}\left(\frac{d}{\lambda}\frac{\sigma_n^{\rm N}}{\sigma_n^{\rm S}}\frac{\left(1+\beta\sqrt{1+(\frac{\Gamma}{\Delta_\infty})^2}\right)^2}{Z_\Gamma}+\frac{1}{2[1+(\frac{\Gamma}{\Delta_\infty})^2]}\right)$$

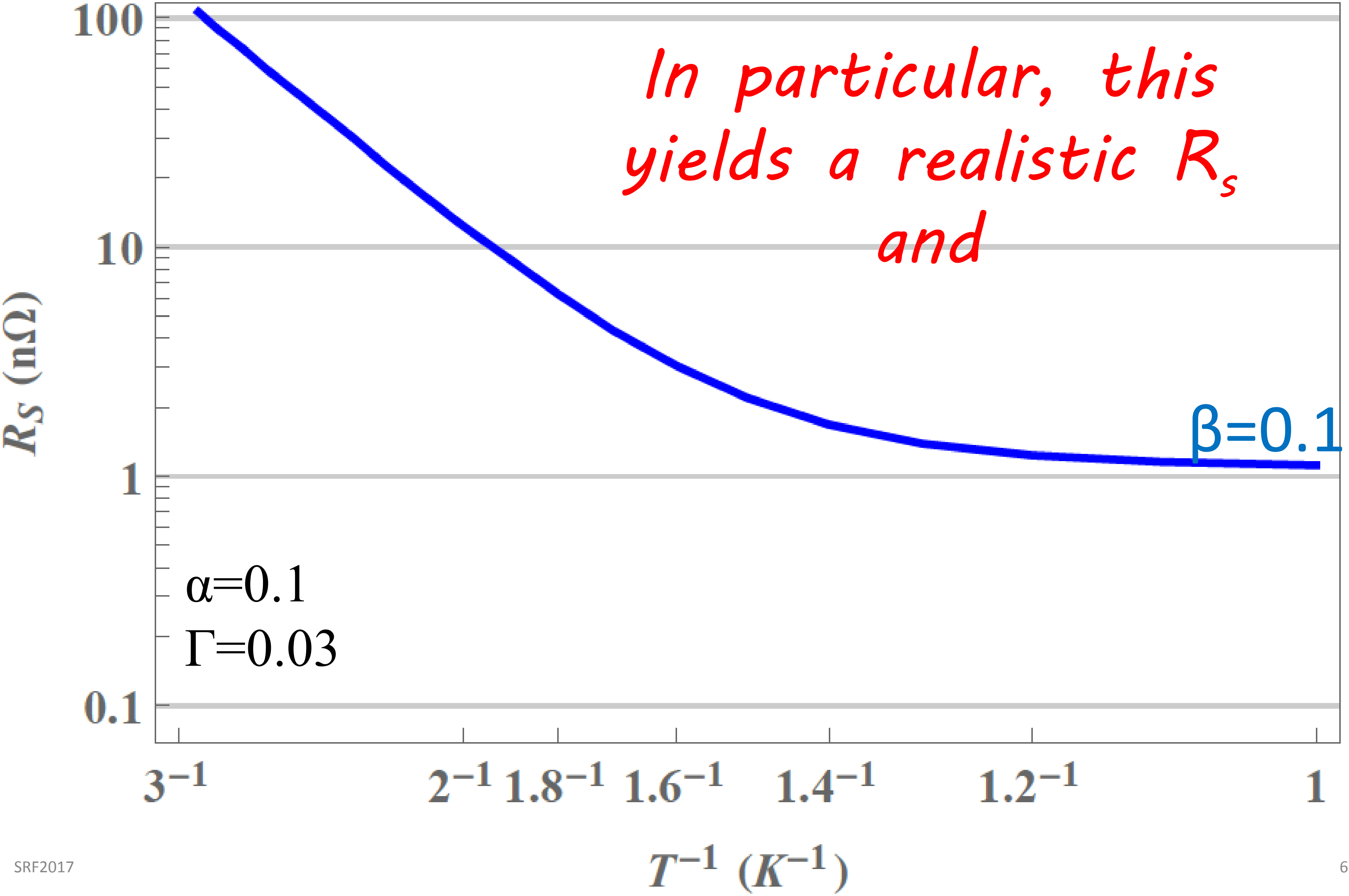


Surface resistance: example



Residual resistance!

Surface resistance: example

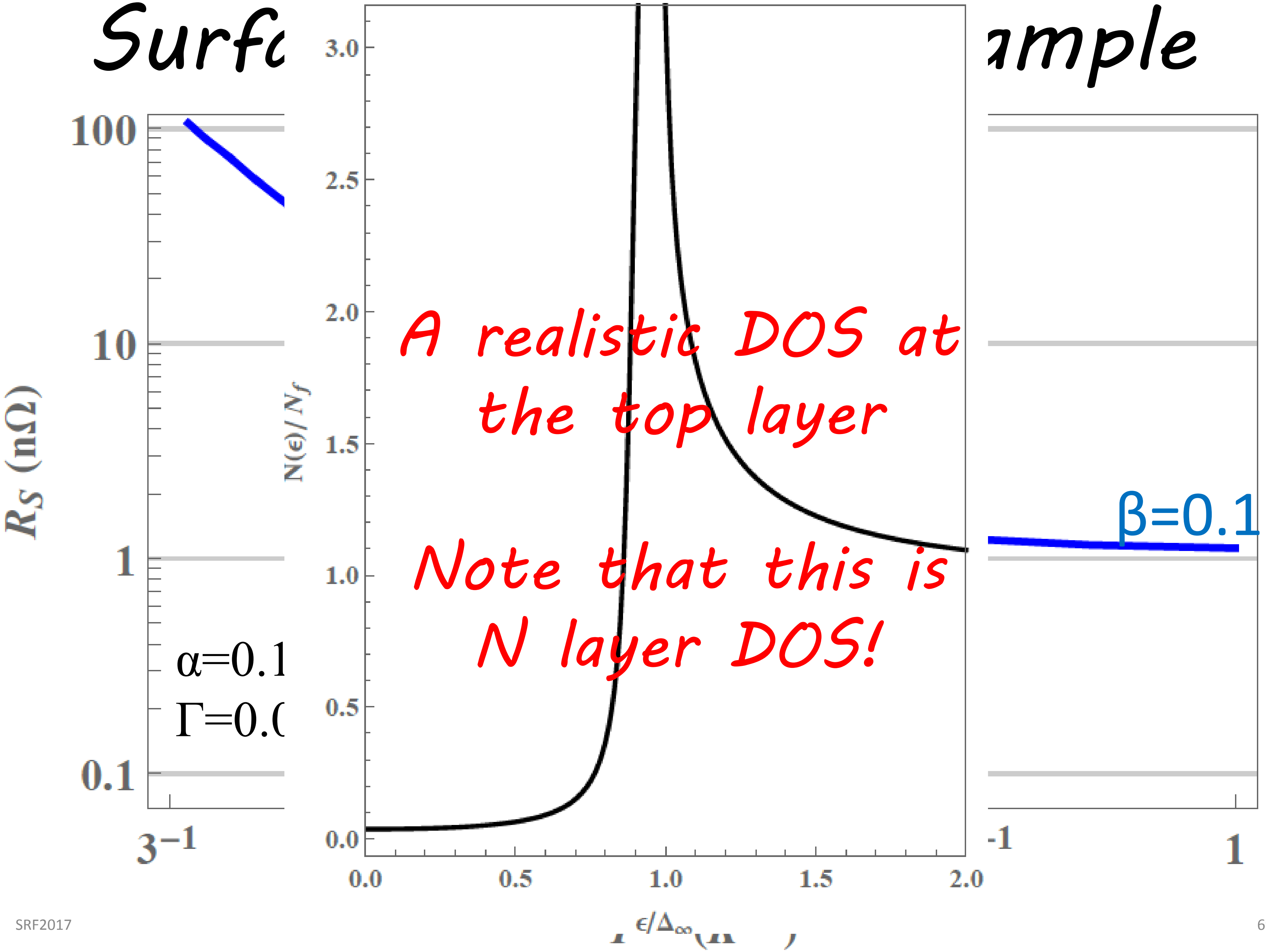


In particular, this yields a realistic R_s and

β=0.1

α=0.1
Γ=0.03

Surf ample



A realistic DOS at the top layer

Note that this is N layer DOS!

Probably you are thinking

- OK, such a thin normal layer would exist and the DOS is broadened.

- Development of the theory with proximity coupled NS system with a finite density of subgap states is natural.

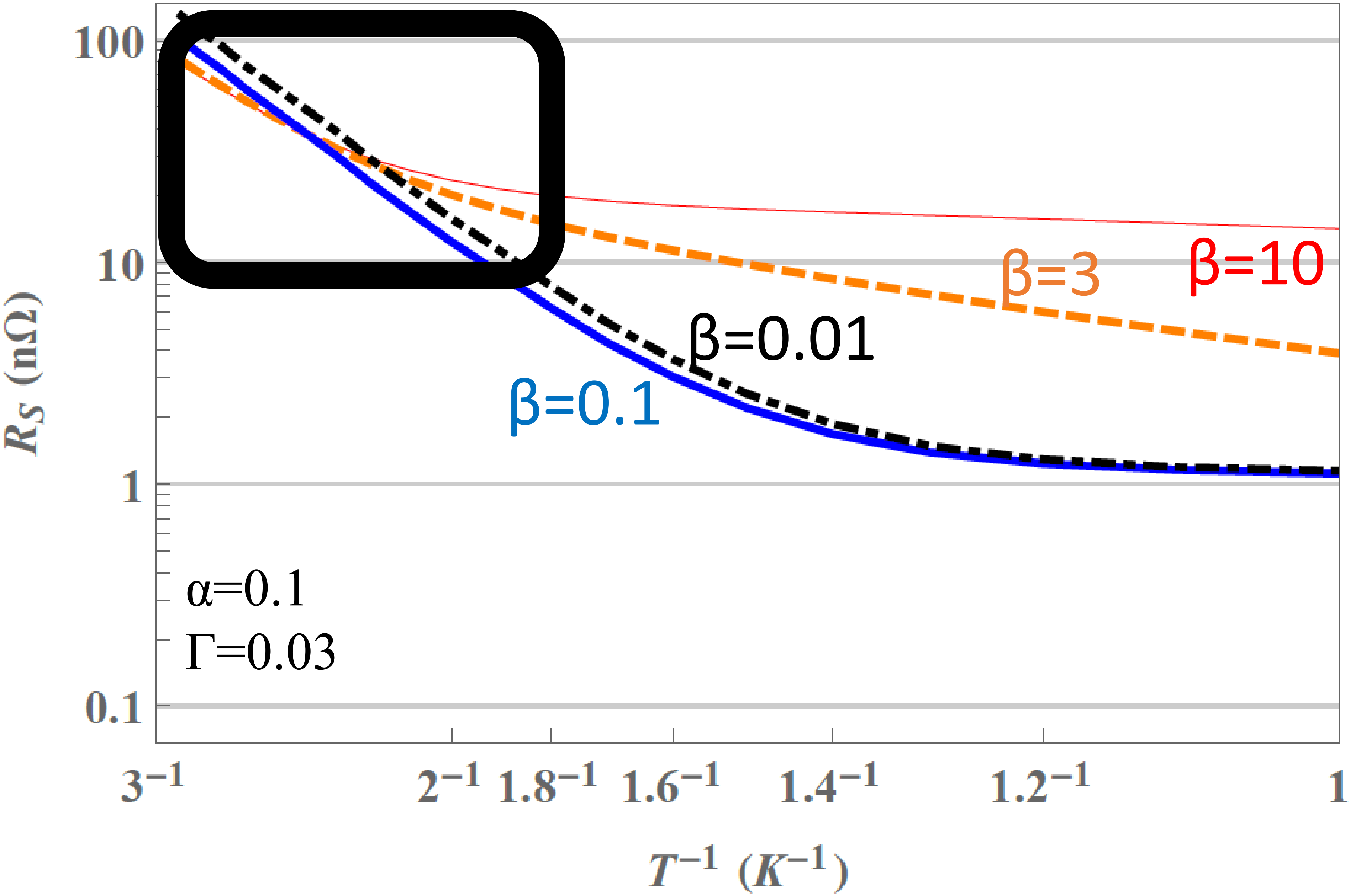
- The results seem to be nice.

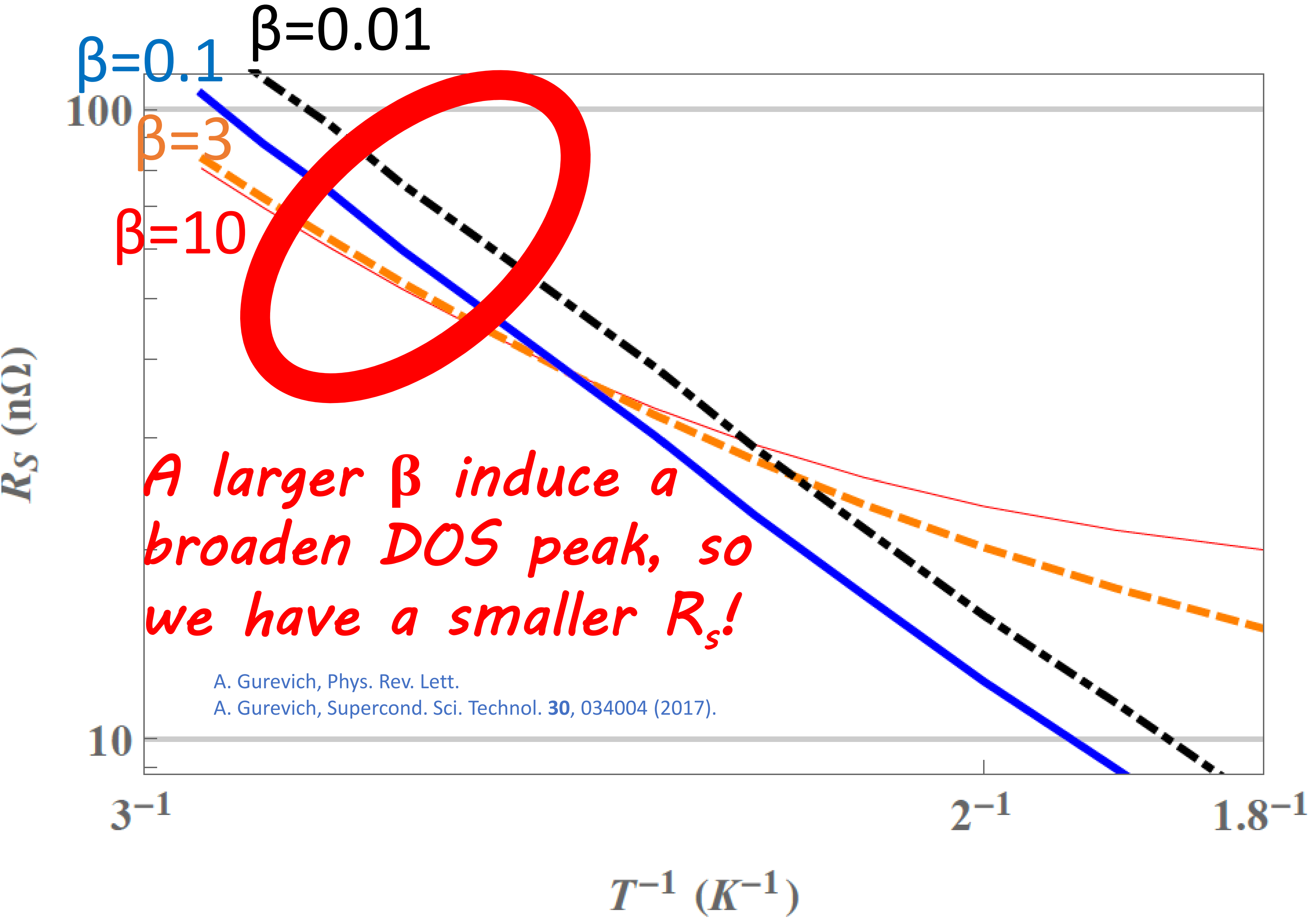
Then tell me

- how to confirm the theory by experiments?
- how to use the theory in order to improve Q_0?

Tune the surface layer towards higher Q at 2K

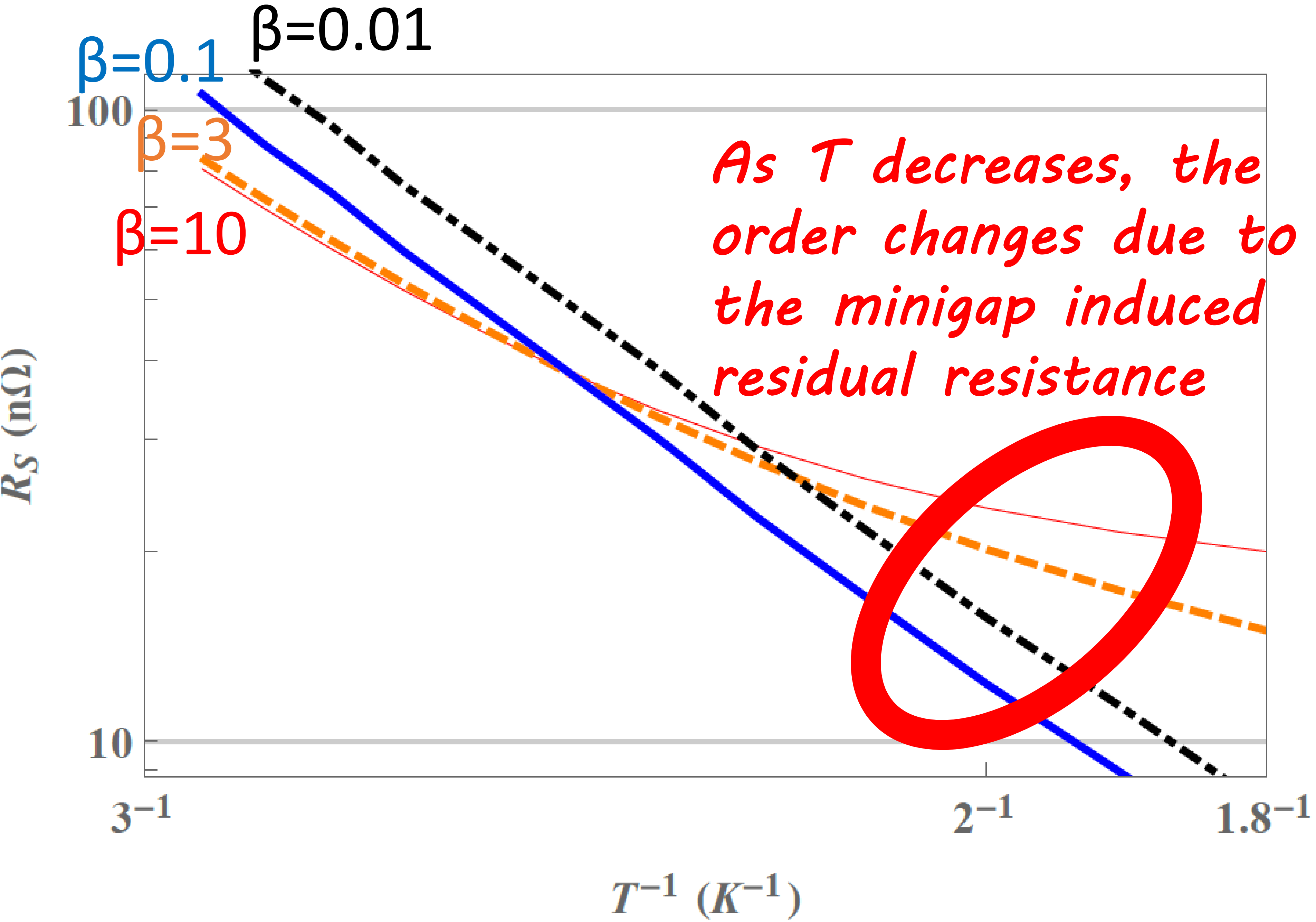
At $T \simeq 2K$


100
10
1
0.1
R_S (nΩ)
3^{-1}
2^{-1}
1.8^{-1}
1.6^{-1}
1.4^{-1}
1.2^{-1}
1
T^{-1} (K^{-1})
β=3
β=10
β=0.01
β=0.1
α=0.1
Γ=0.03

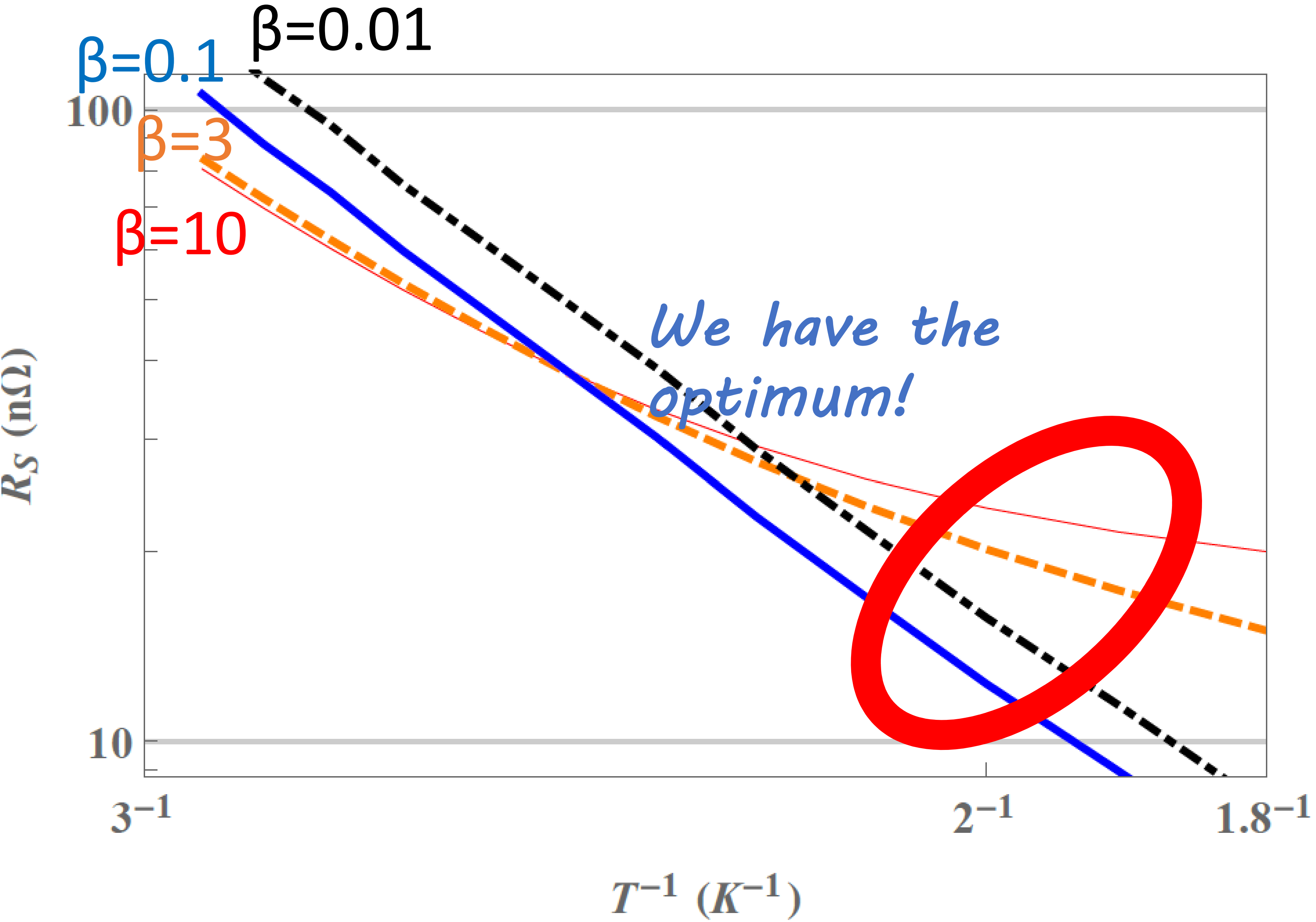


A larger β induce a broaden DOS peak, so we have a smaller R_s!

A. Gurevich, Phys. Rev. Lett.
A. Gurevich, Supercond. Sci. Technol. **30**, 034004 (2017).

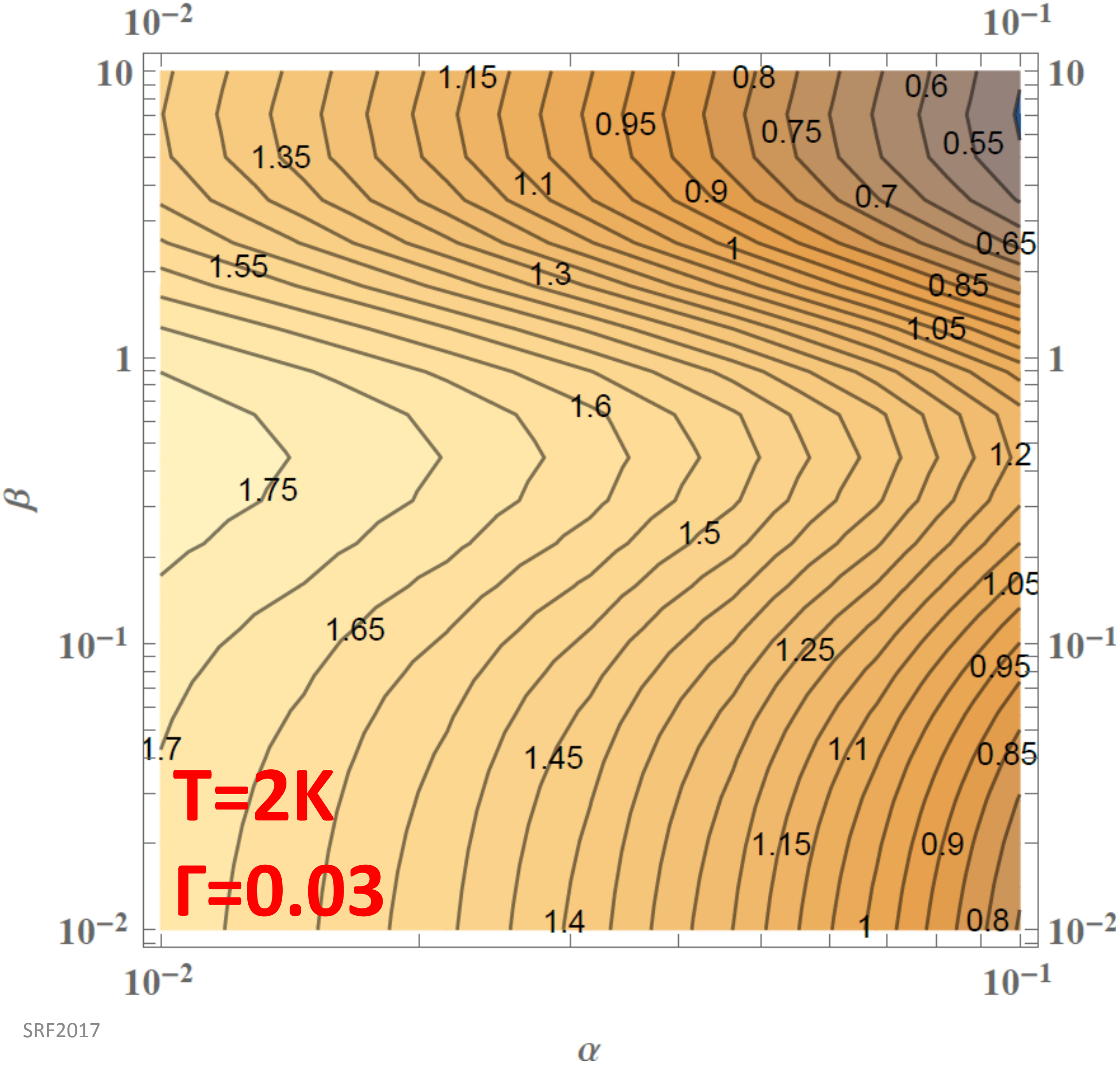


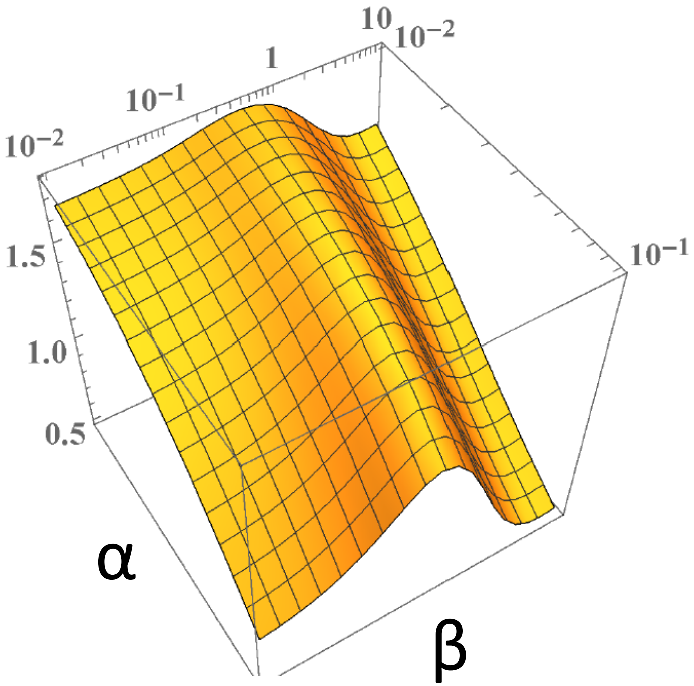
As T decreases, the order changes due to the minigap induced residual resistance



We have the optimum!

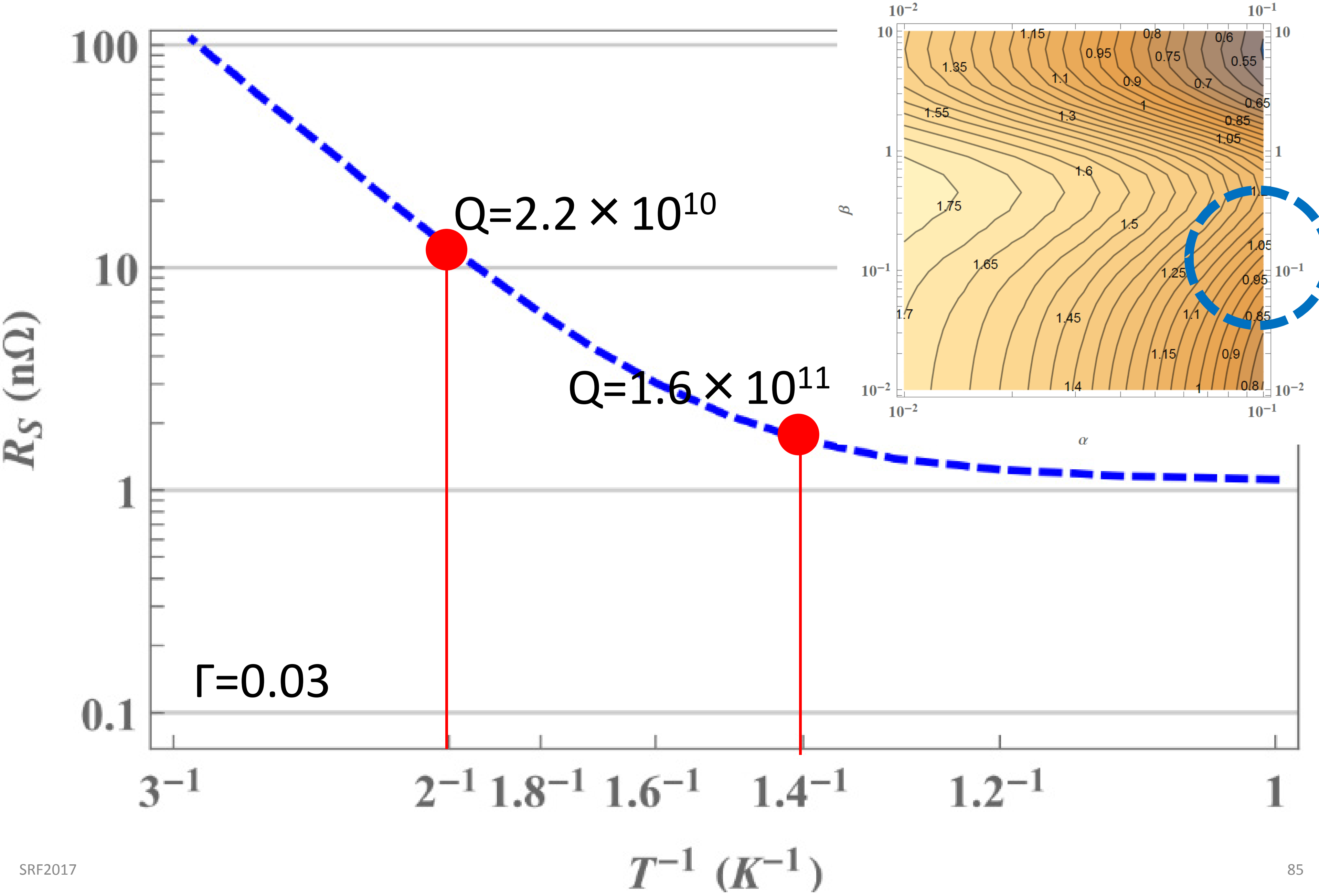
We can tune R_s by varying α and β.





$$\frac{Q_0}{Q_0^{MB}} = \frac{\mathrm{R}_{\mathrm{MB}}}{\mathrm{R}_s}$$

$$= \frac{I_{MB}}{I_N+I_S}$$

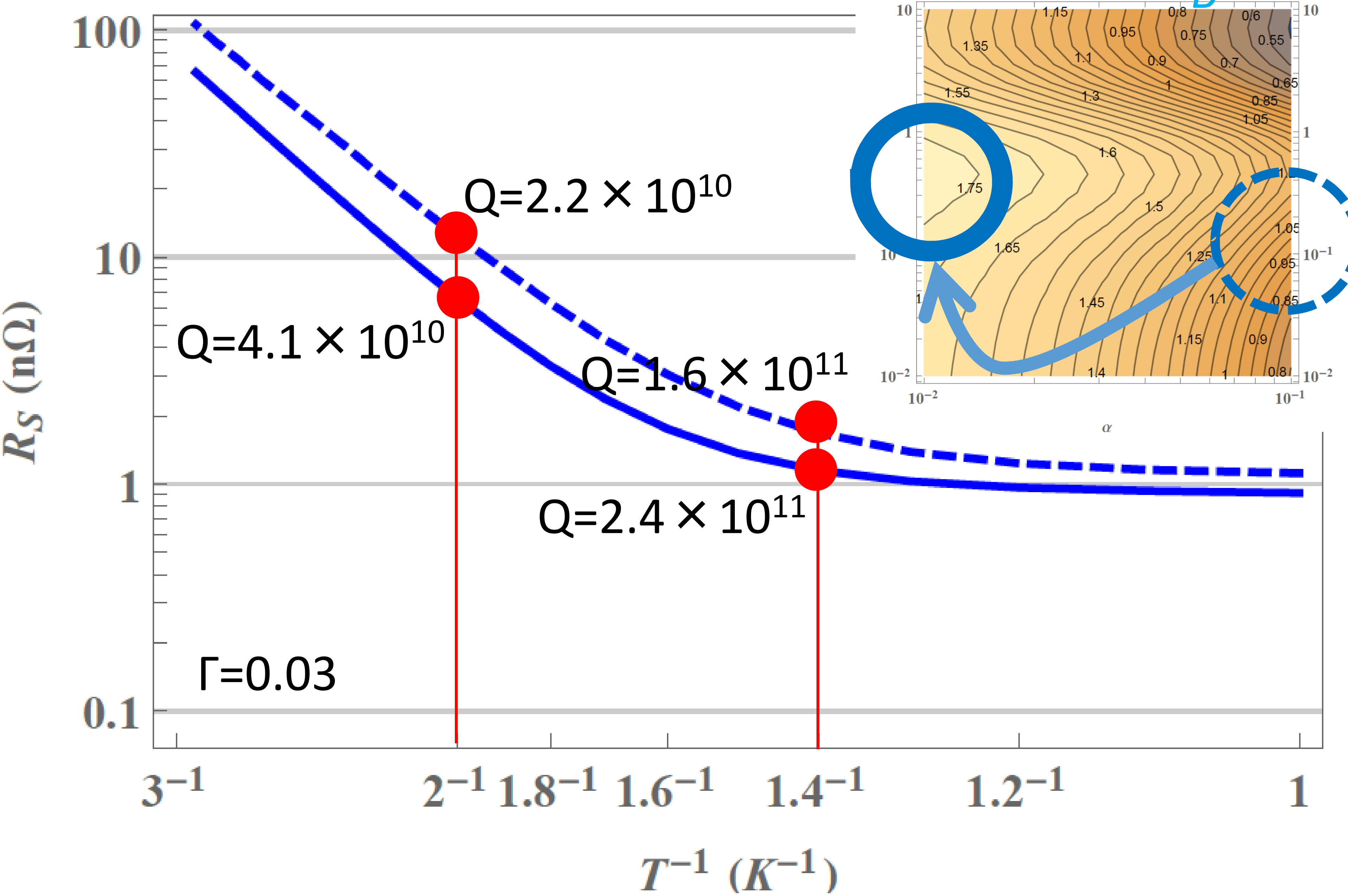


Q=2.2 × 10^{10}

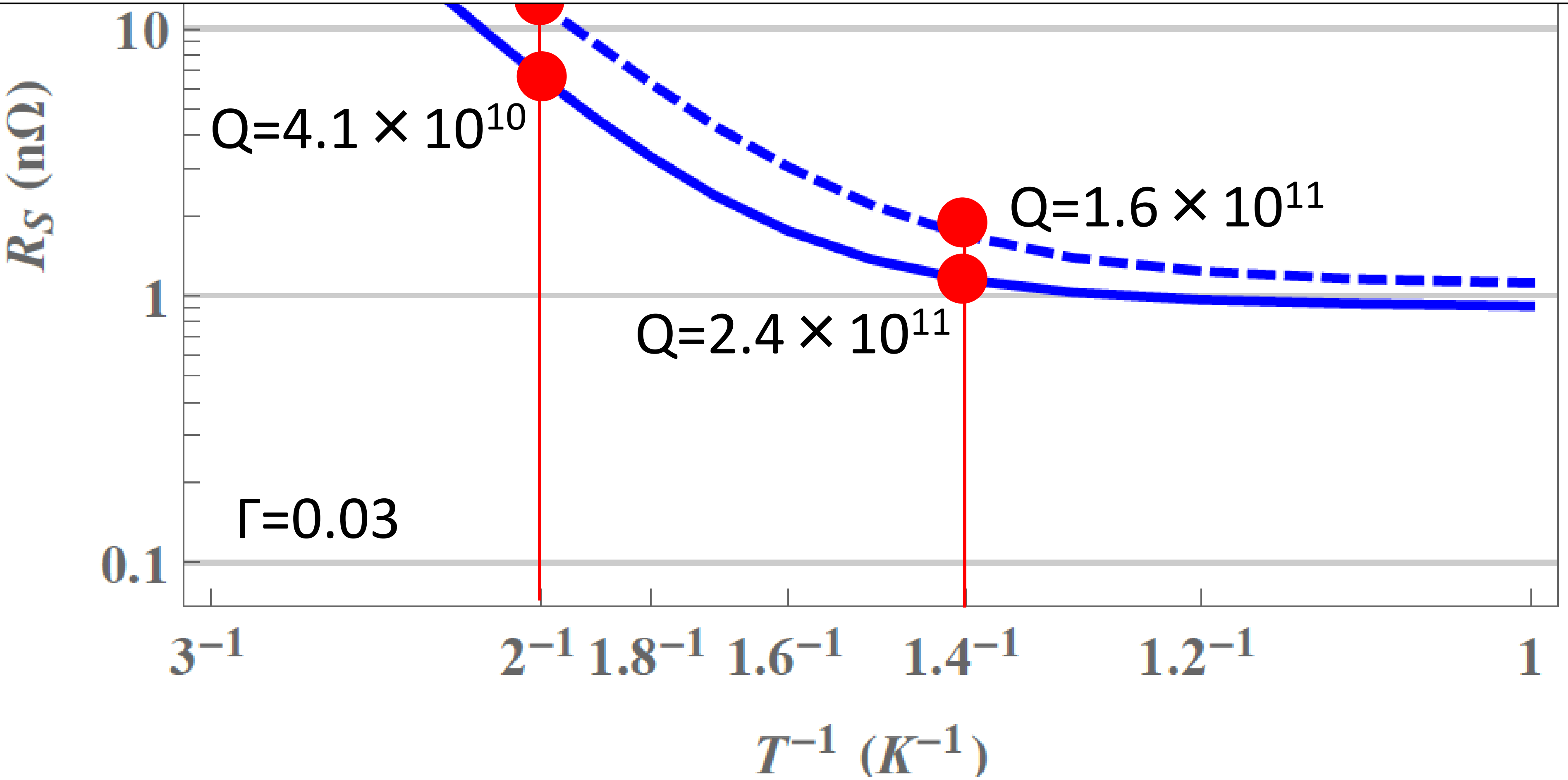
Q=1.6 × 10^{11}

Γ=0.03

Decreases N layer thickness *d* and increases the interface resistance R_B



The differences between the 2 curves are analogous to those between ILC recipe and nitrogen dope! The low field R_s for different surface processing might be explained by **the normal layer thickness and interface resistance**.



Summary

- *We developed a unified theory of surface resistance and residual resistance*
- *The theory incorporates the effects of the surface normal layer or damaged layer and mechanism which produce finite density of subgap states in the bulk.*
- *The theory incorporates both the conventional MB contribution and the residual resistance.*
- *The NS coupling affects not only residual resistance but also surface resistance at T~2K.*
- *We showed it is possible to tune R_s by optimizing the thickness of metallic suboxide layer and the interface resistance.*

Tune the surface!

Thanks!

Oct. 2016
@Norfolk