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ERROR ANALYSIS OF SURFACE RESISTANCE FITS TO EXPERIMENTAL DATA

ABSTRACT: Superconducting material properties such as energy gap, mean free path or residual resistance are commonly extracted by fitting experimental surface resistance data. Depending on the measurement setup, both, temperature range and the number of points are limited. In order to obtain significant results, systematic as well as statistical uncertainties have to be taken into account.
In this contribution we discuss the impact of systematic and statistical errors on BCS fit parameters. In particular, past measurements have yielded contradictory conclusions that, we believe, result from the use of insufficient data in the necessary temperature range. Furthermore, this study is applied to the boundary conditions of the Quadrupole Resonator and its measurement accuracy.

SYSTEMATIC ERRORS

ERRORS ON dB SCALE

RF power is typically measured in units of dBm.
The conversion from dBm to Watts if given by

$$P[\mathrm{W}] = 10^{\frac{P[\mathrm{dBm}]-30}{10}}$$

An offset of x dBm leads to

$$P_{\mathrm{err}}[\mathrm{W}] = 10^{\frac{P[\mathrm{dBm}]+x-30}{10}} = P[\mathrm{W}] \times 10^{\frac{x}{10}}$$

Looking at the relative error

$$\frac{P_{\mathrm{err}} - P}{P} = 10^{\frac{x}{10}} - 1$$

⇒ independent of the actual measurement.
Linear approximation for small values of x

$$\frac{\partial}{\partial x}\frac{P_{\mathrm{err}} - P}{P}\bigg|_{x=0} = \frac{\ln(10)}{10} \approx 23\,\%$$

⇒ **constant relative error: 2.3 % per 0.1 dB offset**

IMPACT ON BCS PARAMETERS

In General: constant relative error β

- ⇒ multiplation of R_s with β
- ⇒ **affects only a and R_{res}**
- ⇒ applicable to all multiplicative formulas
- ⇒ good approximation for QPR and cavity tests

STATISTICAL UNCERTAINTIES

1) Generate 10,000 surface resistance data sets

$$R_s(T) = \frac{a f^2}{T}\exp\left(-b\frac{T_c}{T}\right) + R_{\mathrm{res}}$$

f	a	b	T_c	R_{res}
1 GHz	$2\times10^4\,\frac{\mathrm{n\Omega K}}{\mathrm{GHz}^2}$	1.91	9.25 K	10 nΩ

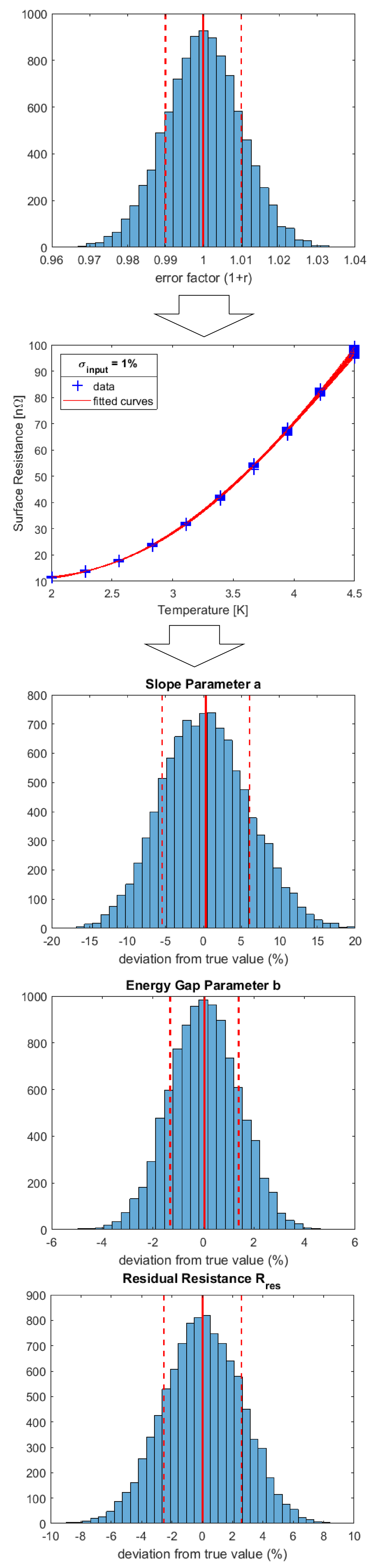
2) Multiply surface resistance with random error

$$R_s(T) \rightarrow R_s(T)\times(1+r)$$

3) compute BCS fit
⇒ statistical uncertainty of fit parameter from standard deviation

Error amplification factor α

$$\alpha = \frac{\sigma \text{ of fit parameter / true value of fit parameter}}{\sigma \text{ of input random error}}$$



MEASUREMENT SCENARIOS

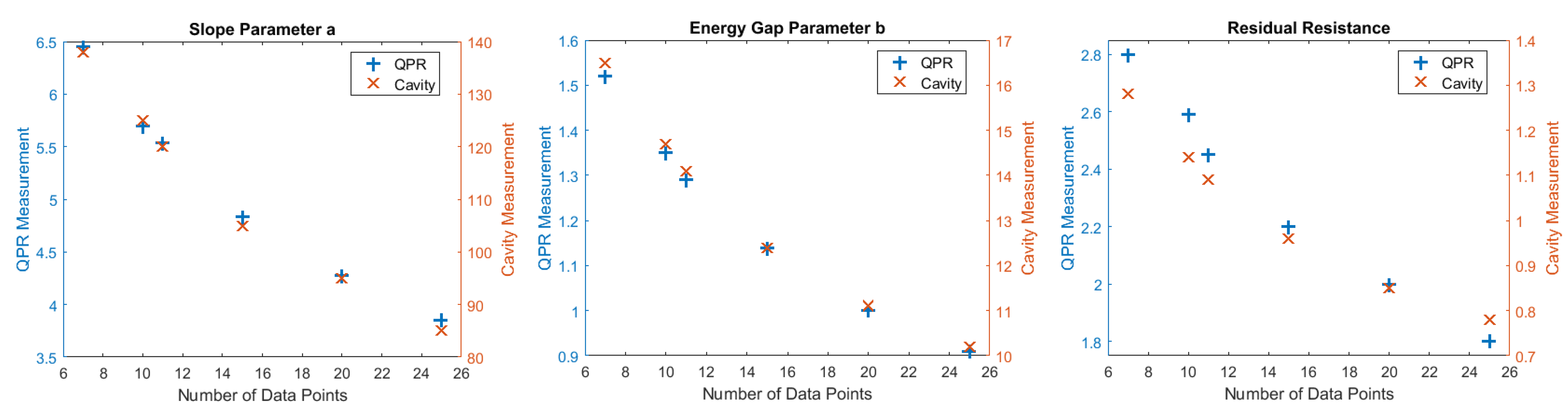
Number of meas. points	10
RMS error surface resistance	1 %

	Cavity	QPR
Temperature range	1.5 – 2.1 K	2.0 – 4.5 K

	a	b	R_{res}	
α_{QPR}	5.6	1.3	2.5	QPR
α_{Cavity}	125	14.7	1.14	Cavity

- ⇒ **Error amplification factor α > 1**
- ⇒ **Temperature range below 2.1 K not sufficient**

NUMBER OF DATA POINTS



⇒ **approx. linear decrease of α**

CONCLUSIONS

Systematic errors

- ⇒ on dB level ⇒ constant relative error 2.3 % per 0.1 dB
- ⇒ constant relative errors only affect parameters a and R_{res}

Statistical uncertainties

- ⇒ Impact depends on temperature range of fit
- ⇒ Error amplification factor α independent of input error
- ⇒ Linear decrease of α with increasing number of data points
- ⇒ 1.5 – 2.1 K ⇒ only R_{res} has sufficient accuracy
- ⇒ 2.0 – 4.5 K ⇒ α > 1 but acceptable for all fit parameters

References

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