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EVALUATION OF FIELD QUALITY FOR ELLIPTICAL AND CURVED MAGNETS

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Abstract

The elliptical magnetic field zone is a useful tool for beam distribution homogenization and FFAG accelerators. Additionally, strongly curved magnets are studied for their application in beam transmission and nuclear fusion. However, traditional magnetic measurements, known as field harmonics, for straight magnets are not suitable for these two kinds of special magnets. In this article, advanced multipoles are used to characterize the fields of straight magnets with elliptical apertures and 2D axisymmetric enclosed curved magnets. The article provides a detailed analysis of magnetic fields using data from FEA or Biot-Savart law. Furthermore, the article discusses methods for characterizing the field quality of unclosed curved magnets used for gantry.

INTRODUCTION

Magnets are crucial components for beam transport, and their configuration depends on the specific usage scenario. In this article, we will focus on the evaluation of field quality in strongly curved and elliptical magnets. By utilizing advanced multipoles, we can determine the field in the central part based on the field data from reference curves [1].

CURVED MAGNETS

When evaluating the central magnetic field of an accelerator magnet, it is common to expand it in terms of circular multipoles, such as dipole, quadrupole and sextupole flux density distributions. The multipole coefficients, also known as field harmonics, are obtained using the Fourier series expansion of the magnetic field component along a circle. These coefficients can also represent the Taylor coefficients of a series expansion of the flux density at the horizontal or vertical axis. They are the transverse coordinates in the co-moving beam coordinate system. However, this approach is not suitable for strongly curved magnets. When particles traverse a curved orbit, they experience a magnetic field up to second order as follows.

$$\frac{ec}{\beta E}B_x = -\kappa_y - kx + ky - \frac{1}{2}m\left(x^2 - y^2\right) + mxy + \frac{1}{2}\left(-\kappa_y k + \kappa_x k + \kappa_y''\right)x^2$$

$$\frac{ec}{\beta E}B_y = +\kappa_x + ky + kx + \frac{1}{2}m\left(x^2 - y^2\right) + mxy - \frac{1}{2}\left(\kappa_x k + \kappa_y k + \kappa_x''\right)y^2$$

It is clear that different orders are mixed in the magnetic field if curvature is not zero. The Fourier coefficients from the reference circle deviate from the Taylor coefficients at the transverse coordinate axis. In beam dynamics, we are interested in the latter one. Assuming axial symmetry for a curved magnet, solutions of the vector Laplace equation can be found using bipolar coordinates in Fig. 1, where $k = \cosh\eta - \cos\xi$. Here are the expressions for the azimuthal magnetic vector potential inside and outside the current shell.

$$A_\phi^{in} = k^{1/2}\left[-b_n\cos(n\xi) + a_n\sin(n\xi)\right]Q_{n-1/2}^1(\cosh\eta) \quad (1)$$

$$A_\phi^{out} = k^{1/2}\left[-d_n\cos(n\xi) + c_n\sin(n\xi)\right]P_{n-1/2}^1(\cosh\eta) \quad (2)$$

One advantage of using bipolar coordinates is that the iso-η lines are circular, although their centres may differ slightly from the focuses. Typically, we require the good field region to be circular. Our goal is to reconstruct the magnetic field based on point data acquired along a reference circle.

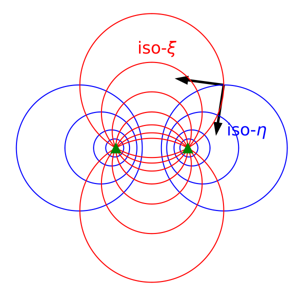

Figure 1: Bipolar coordinates.

The corresponding patterns of magnetic field, including normal and skew patterns, are shown in Fig. 2. It should be noted that the normal $n = 0$ pattern does exist. Associated Legendre functions with half-integer indexes are used, we can calculate the value from this paper [2]. With the increase of the value that bend radius divided by the bore radius, the focus approaches the bore centre, and traditional multipoles recover. Here are the expression of the magnetic field inside the current shell.

$$\begin{cases} B_\xi^{in} = \frac{-b_n k^{3/2}}{a}\left[\frac{n+\frac{1}{2}}{\tanh\eta}Q_{n-1/2}^1 - \frac{1}{2}\sinh\eta k^{-1}Q_{n-1/2}^1 - \frac{n+\frac{1}{2}}{\sinh\eta}Q_{n-3/2}^1\right](-\cos n\xi) \\ B_\xi^{in} = \frac{-a_n k^{3/2}}{a}\left[\frac{n+\frac{1}{2}}{\tanh\eta}Q_{n-1/2}^1 - \frac{1}{2}\sinh\eta k^{-1}Q_{n-1/2}^1 - \frac{n+\frac{1}{2}}{\sinh\eta}Q_{n-3/2}^1\right]\sin n\xi \end{cases}$$

$$\begin{cases} B_\eta^{in} = \frac{-b_n k^{3/2}}{a}\left[-n\sin n\xi - \frac{1}{2}k^{-1}\sin\xi\cos n\xi\right]Q_{n-1/2}^1(\cosh\eta) \\ B_\eta^{in} = \frac{-a_n k^{3/2}}{a}\left[-n\cos n\xi + \frac{1}{2}k^{-1}\sin\xi\sin n\xi\right]Q_{n-1/2}^1(\cosh\eta) \end{cases}$$

If we obtain magnetic field data at several points sampled along a reference circle, the Fourier coefficients of the radial field B_ρ directly represent the multipole strength for straight

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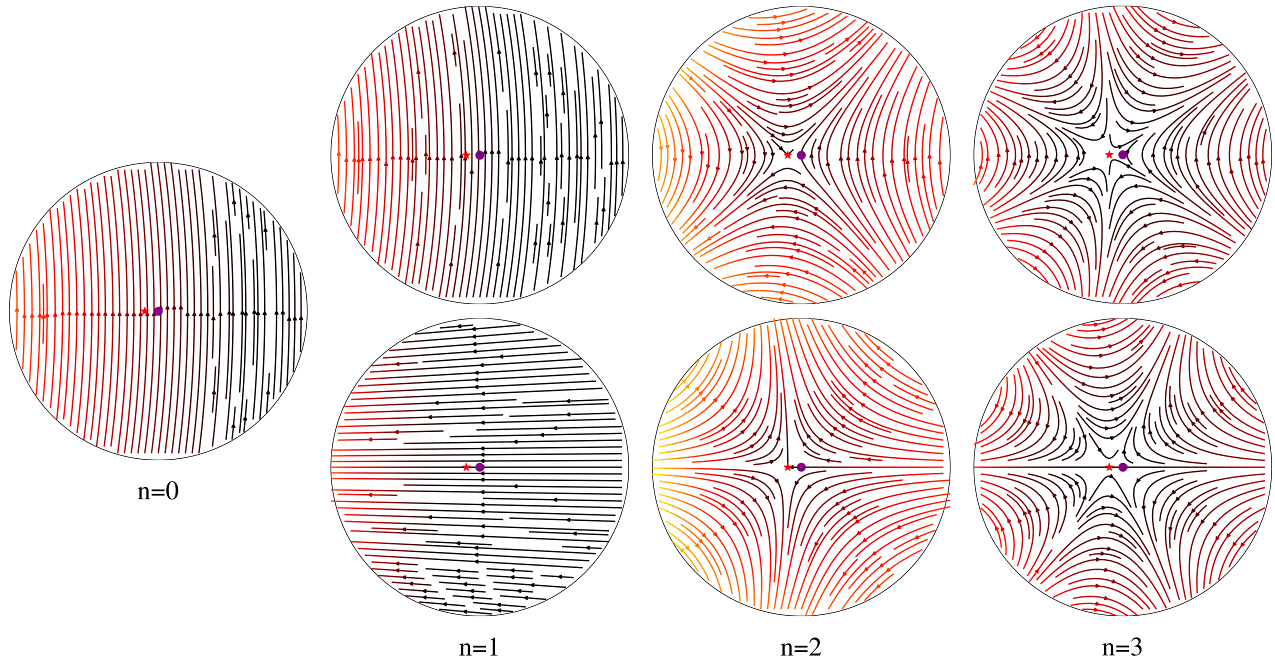

Figure 2: Toroidal harmonics.

magnets. The discrete Fourier series can also be used as a fitting function for the magnetic field along the reference circle. The toroidal harmonics are suitable for field description in a curved system. Decomposing the real data into toroidal harmonics is challenging due to the complex nature of the function $f(\xi)$ in the toroidal harmonics. However, if we calculate the Fourier series for the first several toroidal harmonics and combine them with the real discrete Fourier series, we can determine the strengths of the toroidal harmonics by using linear algebra. High-order pattern accounts little for the real field. This algorithm may not be precise, but it can work well.

Specifically, we can use the following equations to calculate the toroidal strengths a_n & b_n: $\langle a_n \mid A_m \rangle = \frac{1}{\pi}\int_0^{2\pi} B_\eta^{in}(n, skew) \cdot \cos(m\theta)\, d\theta$ (where if $m = 0$, and the result is divided by 2), and $\langle b_n \mid B_m \rangle = \frac{1}{\pi}\int_0^{2\pi} B_\eta^{in}(n, normal) \cdot \sin(m\theta)\, d\theta$. We used COMSOL to construct a closed curved magnet with $R = 1$ m, $r = 0.3$ m, $r_{ref} = 0.18$ m, and a current density proportional to $J_\phi \sim 2\cos 2\theta + \sin 2\theta$. We sampled 20 points uniformly along the reference circle, and then calculated Fourier series $A_n(n = 0, \cdots, 10)$, $B_n(n = 1, \cdots, 9)$ of B_ρ, as shown in Fig. 3. If 21 points, then $A_n(n = 0, \cdots, 10)$ and $B_n(n = 1, \cdots, 10)$. The number of data is conserved anyway. Regarding the toroidal harmonics, $b_n(n = 0, 1, \cdots)$ only contributes to $B_n(n = 1, 2, \cdots)$ items, while $a_n(n = 1, 2, \cdots)$ only contributes to $A_n(n = 0, 1, \cdots)$ items. We used singular value decomposition (SVD) to obtain $a_n(n = 1, 2, \cdots, 10)$ even if we lost one data point. We made the assumption that $B_{10} = 0$, and then used linear algebra to obtain $b_n(0, \cdots, 9)$.

The magnetic field along the local horizontal axis can be represented by its derivatives, which approximate the coefficients of traditional Fourier series along the reference circle. In toroidal harmonics, we can fit the magnetic field of the analytical solution to get derivatives along horizontal axis. Finally, we compare the derivatives of the magnetic field along the local horizontal axis obtained from the traditional Fourier series and toroidal harmonics. The results show that toroidal harmonics provide a more precise characterization of the field as shown in Fig. 4. We also constructed an enclosed CCT coil to evaluate its feasibility, as shown in Fig. 5. We found that the relative error increases as the order of the derivative increases. However, even though the high order derivatives have a larger relative error, values are small enough to make little difference to the total magnetic field.

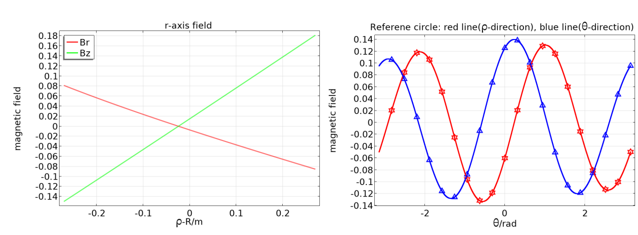

Figure 3: Field data at the horizontal axis and along a circle.

Unclosed curved magnets are often used for beam transmission to make it more compact, as shown in Fig. 6. But it is important to note that when magnets curve, the inner part of the magnet will be compressed while the outer part will be stretched. As a result, particles in the outer part will travel a longer distance, which is equivalent to passing through an enhanced magnetic field. To compensate for this effect, we can use $\frac{r}{R}\vec{B}$ as the new magnetic field. Here, r represents the local bend radius, and R represents the average bend radius. In Maxwell's equation, $\frac{r}{R}\vec{B}$ will violate

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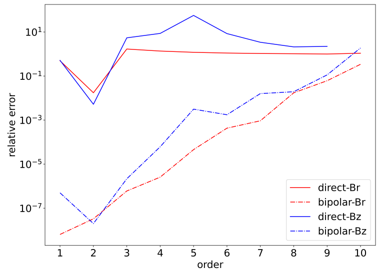


Figure 4: Field error from COMSOL FEA.

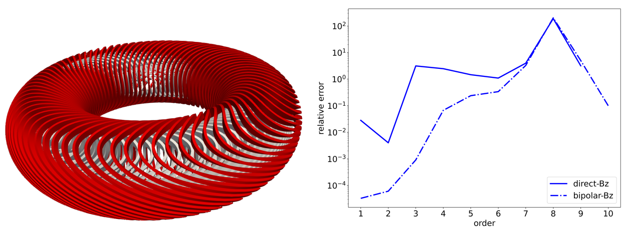


Figure 5: Field error in enclosed CCT coils.

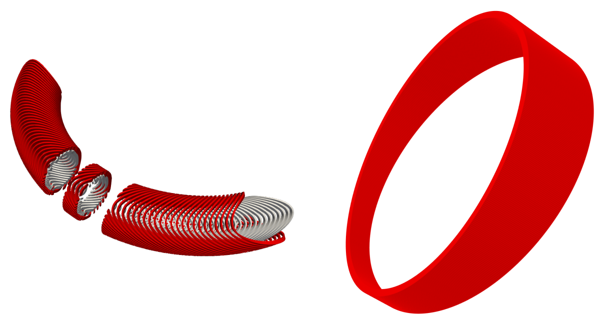


Figure 6: Unclosed curved magnets.

$\nabla \times \vec{B} = 0$ formally, but they satisfy $\nabla \cdot \vec{B} = 0$, which is different from traditional harmonics. Following equations are in cylindrical coordinates.

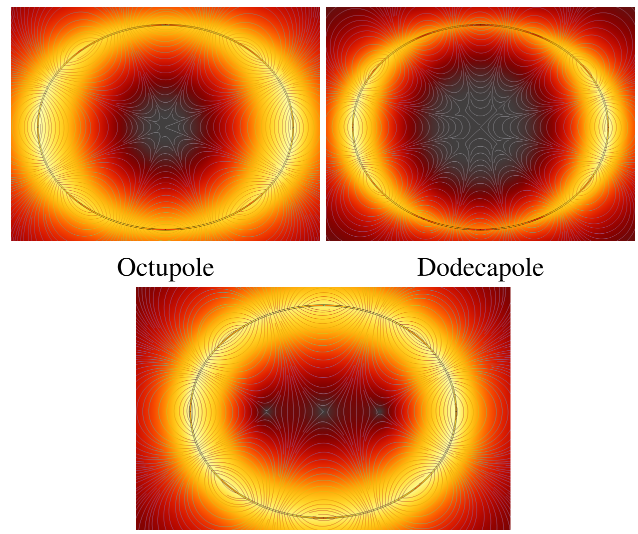
$$\nabla \cdot \vec{B} = \frac{1}{r}\frac{\partial (rB_r)}{\partial r} + \frac{\partial B_z}{\partial z} = 0 \Longrightarrow \frac{\partial (rB_r)}{\partial r} + \frac{\partial (rB_z)}{\partial z} = 0$$

$$\nabla \times \vec{B} = \vec{e}_\phi \left[\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right] = 0 \Longrightarrow \frac{\partial (rB_r)}{\partial z} - \frac{\partial (rB_z)}{\partial r} = -B_z$$

The field of beam physics places more importance on low-order harmonics. Due to the relatively weak curvature of the magnets used in beam physics, the difference in magnitude between harmonics and the field Taylor series is slight. In terms of integrated harmonics, the traditional harmonics correspond to integration over angle, while the compensation case is over distance. The integration path is a series of concentric circles. As for magnet engineers, real particle tracking is more reliable for field quality control. Particles have the same bending radius and the integration path is a series of eccentric circles with the same radii, and then integration over distance. We can not guarantee which measurement method is better.

ELLIPTICAL MAGNETS

In the context of beam distribution homogenization and FFAG accelerators, a magnetic field requires a narrow and elongated area [3]. A narrow beam profile can effectively decouple the two transverse spaces in beam distribution homogenization. Elliptical magnets are a proper choice, octupole and dodecapole magnetic fields are shown in Fig. 7. By measuring the magnetic fields along the elliptical boundary, we can employ cylindrical elliptical multipoles to reconstruct the magnetic field inside. Notably, there exists a linear transformation between cylindrical elliptical multipoles and cylindrical circular multipoles. This process is similar to that used for enclosed curved magnets.



Octupole Dodecapole

Cylindrical elliptical multipoles, n=4

Figure 7: Elliptical field zone.

CONCLUSION

This article focuses on the evaluation of field quality in curved and elliptical magnets. We introduce toroidal harmonics as a means to reconstruct the magnetic field in 2D axisymmetric enclosed curved magnets. Additionally, we provide suggestions for characterizing field quality in unclosed curved magnets. In an elliptical magnet, we can utilize cylindrical elliptical multipoles to reconstruct the magnetic field within the elliptical good field zone. All of the demonstrations are carried out in the two-dimensional case, either with rotational axis symmetry or in the two-dimensional plane. We hope this work can enlighten the use of curved and elliptical magnets.

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