

THE MOTION OF IONIC FLUX IN AN ELECTRON LAYER

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Abstract

An acceleration of ion flux in an electronic layer is studied. The layer is created by electrons that move in transverse electric field and are confined by self-generating magnetic field. It is shown that from such one can extract heavy ions with the velocities up to ion-sound velocity.

INTRODUCTION

The process of extraction of ions from plasma is very important in terms of experiment. A great number of theoretical studies are dedicated to this process. The paper [1] shows that ions leave plasma at velocities which are in excess of ionic sound velocities. Under conditions close to be actual environment when the temperature of electrons is above that of ions ($T_e \gg T_i$), the number of ions being accelerated turns out to be exponentially small. The paper [2] deals with the process of accelerating ions in a non-stationary problem. The paper [3] discloses that a transition layer in the plasma-vacuum system appears out to be infinitely large. The paper [4] considers equilibrium conditions with the presence of an electron flow that is directed in line of an ionic flux. This paper deals with equilibrium states when a non-zero flow of electrons is perpendicular to across an ionic flux.

IONIC FLUX STATES IN AN ELECTRON LAYER

Let us assume that at $x = 0$ there is given an ionic flux with a negligible ($\sim v_{Ti}$) initial velocity v_0 . We consider 1-D problem where physical quantities are independent of coordinates y and z . The magnetic field has only one component $B_z = -\frac{dA_y}{dx}$, where A_y is a component of a 4 - D potential. This a magnetic field is induced by the electron flow along the y axes. The electrons move in the x, y plane under the action of electric and magnetic fields with $v_z^{(e)} \equiv 0$. We describe ensemble of electrons by means of a collisionless kinetic equation. A solution of this equation may be form of an arbitrary function of motion integrals: energy H and generalized momenta $P_y = p_y - \frac{e}{c}A_y(x)$ and $P_z = p_z$. Here p_y, p_z - are components of an electron momentum. We take a distribution function in following form:

$$f(\vec{p}, \vec{r}) = \Psi(H, P_y, p_z) = \kappa \frac{\sigma(H_0 - H)}{\sqrt{H_0 - H}} \delta(P_y - p_0) \delta(p_z), \quad (1)$$

where $\sigma(x)$ - is Heaviside function: $\sigma(x) = 1, x > 0; \sigma = 0, x < 0$. The Hamiltonian $H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} - e\Phi$, m - is the mass of an electron, $-e$ - is a charge, $\Phi(x)$ - is

a potential of electric field. The distribution of type (1) leads to a compact description of the complicated physical situation.

The calculated density of electrons is as follows:

$$n_e = \kappa \int \frac{dH}{\sqrt{H_0 - H}} \frac{1}{\sqrt{H + e\Phi - \frac{1}{2m}(p_0 + \frac{e}{c}A_y(x))^2}}. \quad (2)$$

One can get:

$$n_e = n_0 \sigma(H_0 + e\Phi - \frac{1}{2m}(p_0 + \frac{e}{c}A_y)^2), \quad (3)$$

where $n_0 = \pi\kappa\sqrt{H_0}$ and the current density j_y takes the following form:

$$j_y = -\frac{e}{m} \int p_y f d\vec{p} = -\frac{e}{m}(p_0 + \frac{e}{c}A_y(x))n_e. \quad (4)$$

Let us introduce dimensionless variables:

$$\frac{e\Phi}{H_0} = \phi, \xi = \frac{x}{l_0}, l_0 = \sqrt{\frac{H_0}{4\pi n_0 e^2}}, a(x) = \frac{e}{p_0 c} A_y(x).$$

The density of ions specified by hydrodynamical equations: $n_i = \frac{\Gamma_i}{v_i}$ where Γ is density of ionic flux, v_i is velocity of ions. If M - is the mass of ions, v_0 - initial velocity then $n_i = \frac{\Gamma_i}{\sqrt{v_0^2 - \frac{2e\Phi}{M}}}$.

The equation for components of 4 - D potential will transform to the following form:

$$\frac{d^2\phi}{d\xi^2} = \sigma \left(1 + \phi(\xi) - \frac{p_0^2}{2mH_0} (1 + a(\xi))^2 \right) - \frac{\nu_i}{\sqrt{u_0^2 - 2\phi(\xi)}}, \quad (5)$$

where $\nu_i = \Gamma_i / (n_0 v_s)$, $v_s = \sqrt{H_0 / M}$, $u_0 = v_0 / v_s$,

$$\frac{d^2 a(\xi)}{d\xi^2} = \frac{H_0}{mc^2} (1 + a(\xi)) = \varrho^2 (1 + a(\xi)), \varrho = \sqrt{H_0 / mc^2}. \quad (6)$$

Let us examine the condition $1 + \phi - \frac{p_0^2}{2mH_0} (1 + a)^2$, that defines boundaries of the electron layer. Assume that $\phi(0) = 0$, then

$1 + \phi - \frac{p_0^2}{2mH_0} (1 + a)^2 < 0$ with $\xi < 0$. Let us have $\frac{p_0^2}{2mH_0} = 1$ and write a general solution of the equation (6) can be written in the following form: $a = -1 + \alpha \sinh(\varrho\xi) + \beta \cosh(\varrho\xi)$, where constants α and β are defined by physical conditions. If $\beta = 1$, then it turns out that electrons may be localized only at $\xi > 0$, i.e. $\xi = 0$ is a boundary of the layer. The value of H_0 fulfils a role of temperature of the electron ensemble. For a characteristic quantity of this temperature, 50eV may be taken. Then

the parameter $\rho = 0.01$. Let us write a condition that is to be met by the boundary value of $\xi = \xi_{max}$ for the electron layer: $1 + \phi(\xi_{max}) - (\cosh(\rho\xi_{max}) + \alpha \sinh(\rho\xi_{max}))^2 = 0$. This condition depends on the type function $\phi(\xi)$. If ions fall into a layer with zero initial energy, their maximum energy can not be greater than H_0 , that is, the ions may be accelerated only to energy that is less than or equal to the temperature of electrons. Let us consider a solution of equation (6) in the region of the layer, assuming that $\frac{p_0^2}{2mH_0} = 1$ and $\phi_0 = 0, u_0 = 0$. A solution to these values given in Fig.1.

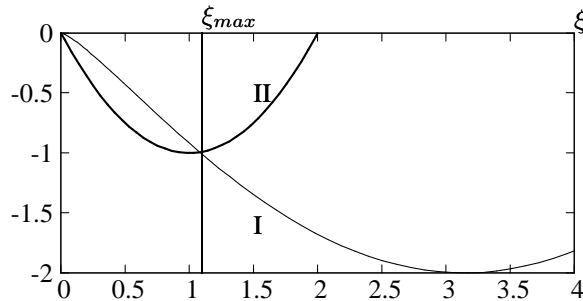


Fig.1 Curve I is a solution for potential $\phi(\xi)$ with $u_0 = 0, \phi'_0 = 0, \nu_i = 1$, curve II shows a relationship of the form $-1 + (\cosh(0.01\xi) - 100 \sinh(0.01\xi))^2$.

At these conditions ions are accelerated from the zero energy at $\xi = 0$ to the energy $W \sim H_0$ at $\xi = \xi_{max} \sim 1.1$. Then the ions are accelerated by a field defined by the potential an external electrode.

Let us next examine the situation when the initial energy of ions is equal to H_0 , with $u_0 = \sqrt{2}$. In this case the coefficient α can be of substantially lesser value, say $\alpha = 10$ A relevant solution is presented in Fig.2.

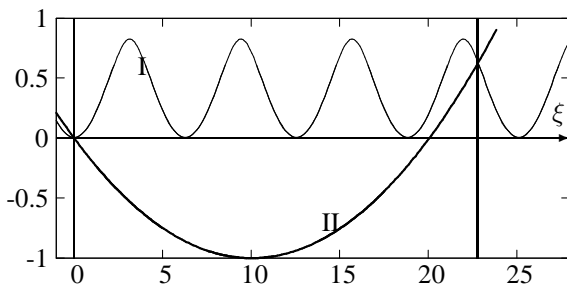


Fig.2 Curve I depicts the dependence of the potential on ξ , with $u_0 = \sqrt{2}, \nu_i = 1$; curve II depicts the dependence $-1 + (\cosh(0.01\xi) - 10 \sinh(0.01\xi))^2$.

The dependence of the potential on the coordinate is of an oscillative nature according to which the ions are at first decelerating from energy h_0 to energy $\sim 0.2H_0$ in the peak of potential.

The oscillations take place near points the relationship of quasineutrality $n_i = n_e$ is met. Upon achieving the point $\xi = \xi_{max} \approx 22.8$, the ions are accelerating in the electric field to the energy specified by an external electrode having

a potential that is dictated by a self-consistent solution for the field.

Thus, in the model being considered there are no restrictions on the extraction of ions having velocities that are less than ionic sonic velocities. This section deals with a situation when there is a non-zero flow of electrons. However, in contrast to paper [4] the direction of electronic flow here is not in line with the ionic flux and directed across this flux.

THE PLASMA-VACUUM MODEL WITH SHARP BOUNDARY

Let us consider a situation when electrons occupy a half-space $x < 0$. We take the solutions of equation (6) at $p_0 = 0$. These solutions are reasonable if they decrease $x \rightarrow -\infty$. Then $A_y(x) = C_0 \exp(2\kappa\xi)$. Let us assume that C_0 complies with the following relationship: $e^2 C_0^2 / 2mH_0 c^2 = 1$. The equation for the dimensionless potential $\phi(\xi)$ takes the following form:

$$\frac{d^2\phi(\xi)}{d\xi^2} = \sigma(1 + \phi(\xi) - \exp(2\kappa\xi)) - \frac{\nu_i}{\sqrt{u_0^2 - 2\phi(\xi)}} \quad (7)$$

Next we show a particular solution (7) at $\kappa = 0.01, \nu_i = 1, \phi(0) = 0, \phi'(0) = 0$. Upon achieving the point $\xi = \xi_{max} \approx 22.8$, the ions are accelerating in the electric field to the energy specified by an external electrode having a potential that is dictated by a self-consistent solution for the field.

In Fig.3, the dependencies of potential (curve I) and of ionic density (curve II) on the coordinate are shown. The electron density takes the form of a step:

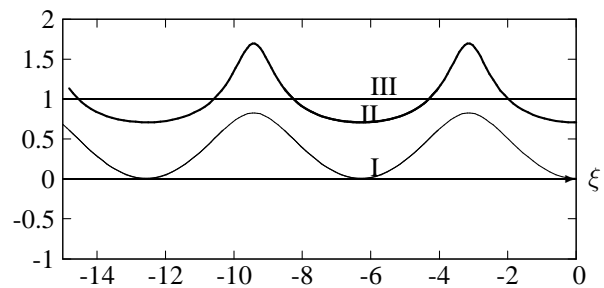


Fig.3. Curve I is dependence of the potential on the coordinate, curve II is dependence of ionic density on the coordinate. The electron density takes the form of a step: 1 with $\xi < 0$ and 0 in the region of $\xi > 0$ (III).

Thus, in the above considered cases - when there is an electron layer kept by a self-generated magnetic field and when electrons occupy a half-space - the ions with relatively low velocities (less than ionic sonic velocities) can leave the region of quasi-neutral plasma and obtain further acceleration in the field induced by the external electrode. It is essential to note that in the both above cases there is a non-zero flow of electrons that is perpendicular to an ionic flux.

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