

# Recovery Process Stability Study in Energy Recovery Accelerator

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# Recovery Process Stability Study in Energy Recovery Accelerator

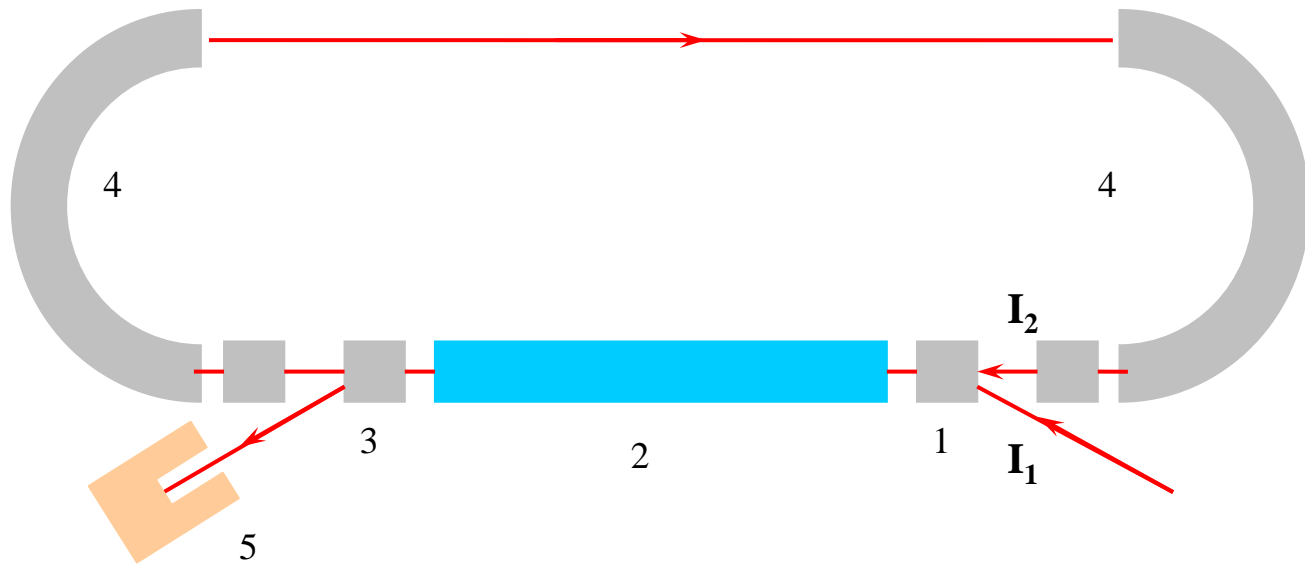


Fig.1. General layout of energy recovery linac. 1– magnet inflector, 2– main linac, 3 – magnet deflector, 4 – bending magnets, 5 – beam absorber.

## Recovery Process Stability Study in Energy Recovery Accelerator

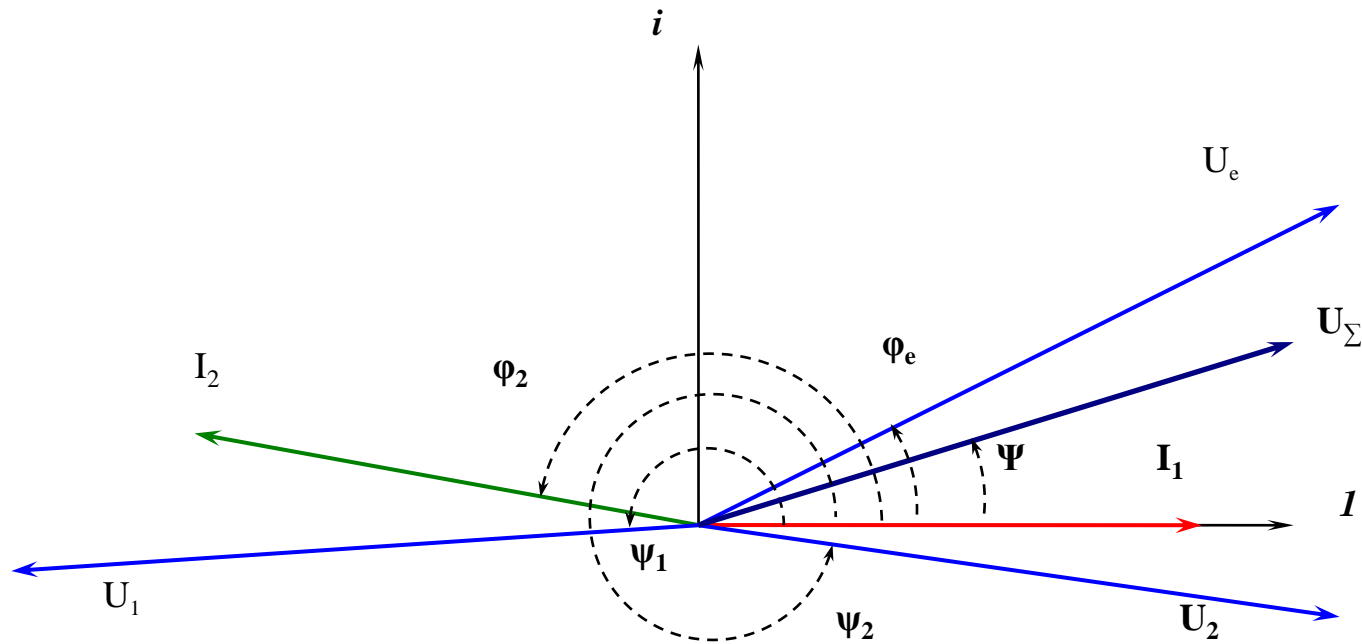


Fig.2. Voltages and currents on complex plane.  $I_1$ -accelerated beam,  $I_2$ -decelerated beam,  $U_0$ -the voltage induced in cavity by external rf generator,  $U_1$ -the voltage induced by accelerated beam,  $U_2$ -the voltage induced by decelerated beam,  $U_\Sigma$ -total voltage at the cavity gap.

## Recovery Process Stability Study in Energy Recovery Accelerator

$$\hat{U}_{\Sigma}(t) = \hat{U}_e(t) + \hat{U}_1(t) + \hat{U}_2(t)$$


$$E = eU_{\Sigma} \cos(\psi) = \text{Re}(e\hat{U}_{\Sigma})$$

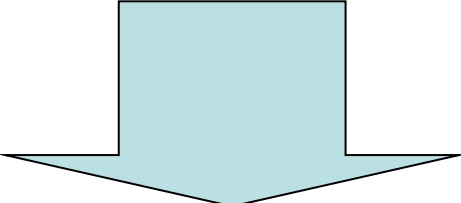
all voltages are understood as energies acquired by unit charge after cavity passage.

## Recovery Process Stability Study in Energy Recovery Accelerator

$$\frac{d^2 u}{dt^2} + \frac{\omega_0}{Q} \frac{du}{dt} + \omega_0^2 u = -\frac{\omega_0 R}{Q} \frac{dJ}{dt} \quad Q = \frac{Q_0}{1+\beta}, R = \frac{R_0}{1+\beta}$$

$$J(t) = I(t) \exp(i\omega t), u = U(t) \exp(i\omega t)$$

$I(t), U(t)$   Slow functions of time


$$\frac{dU}{dt} + \left(\frac{\omega_0}{2Q} + i\Delta\omega\right)U = -\frac{\omega_0 R}{2Q} I$$

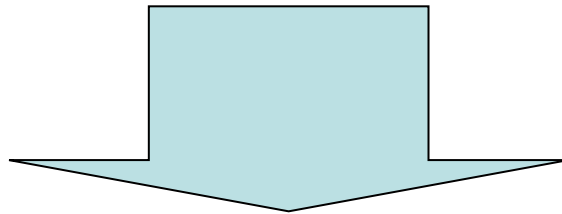
## Recovery Process Stability Study in Energy Recovery Accelerator

$$\frac{d\Delta\hat{U}}{dt} + \left(\frac{\omega_0}{2Q} + i\Delta\omega\right)\Delta\hat{U} = -\frac{\omega_0 R}{2Q}\Delta\hat{I}$$

$$\Delta\hat{I}_2 = \Delta(I_2 \exp(i\varphi_2)) = iI_2 \exp(i\varphi_2)\Delta\varphi_2$$

$$\Delta\varphi_2(t) = \frac{2\pi\alpha L}{\Lambda} \frac{\Delta E(t-T)}{E}$$

$$\Delta E = e \operatorname{Re} \Delta(\hat{U}_e + \hat{U}_1 + \hat{U}_2) = e \operatorname{Re}(\Delta\hat{U}_2)$$



## Recovery Process Stability Study in Energy Recovery Accelerator

$$\frac{d\Delta u_{2x}}{dt} + \frac{\omega_0}{2Q} \Delta u_{2x} - \Delta\omega \Delta u_{2y} = A \sin \varphi_2 \Delta u_{2x} (t - T)$$

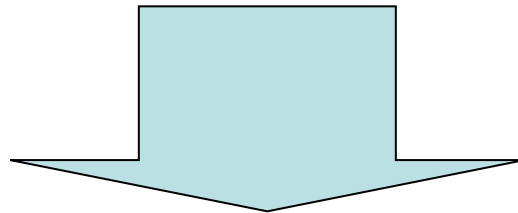
$$\frac{d\Delta u_{2y}}{dt} + \frac{\omega_0}{2Q} \Delta u_{2y} + \Delta\omega \Delta u_{2x} = -A \cos \varphi_2 \Delta u_{2x} (t - T)$$

$$A = \frac{\omega_0 R}{QU_\Sigma} I_2 \frac{\pi\alpha L}{\varepsilon\Lambda}$$

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$$\Delta u_{2x} = \exp(kt)$$

$$\Delta u_{2y} = a \exp(kt)$$

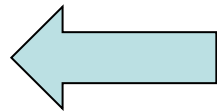


## Recovery Process Stability Study in Energy Recovery Accelerator

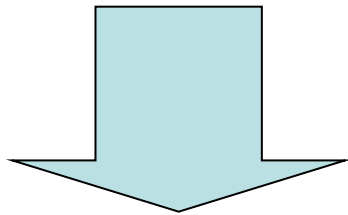
$$k + \frac{\omega_0}{2Q} - a\Delta\omega = A \exp(-kT) \sin \varphi_2$$

$$ak + a \frac{\omega_0}{2Q} + \Delta\omega = -A \exp(-kT) \cos \varphi_2$$

$$k = 0$$



Static instability



$$\frac{2\pi L}{\varepsilon\Lambda} \frac{I_2 R}{U_\Sigma} \alpha \left( \sin \varphi_2 - \frac{2Q\Delta\omega}{\omega_0} \cos \varphi_2 \right) = 1 + \left( \frac{2Q\Delta\omega}{\omega} \right)^2$$

## Recovery Process Stability Study in Energy Recovery Accelerator

$$\varphi_2 = \pi + \eta$$

$$\tan \eta > 2Q\Delta\omega / \omega_0 \quad \alpha > 0 \quad \leftarrow \text{Negative feed back}$$

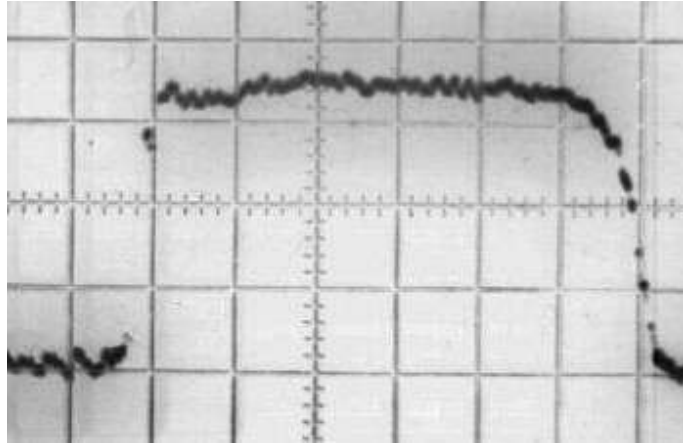
But it is well known from feed back systems theory that negative in static sense feed back may become positive at definite frequency range resulting in unstable state.

$$k = \text{Re } k + i \text{Im } k = 0 + i\Omega$$

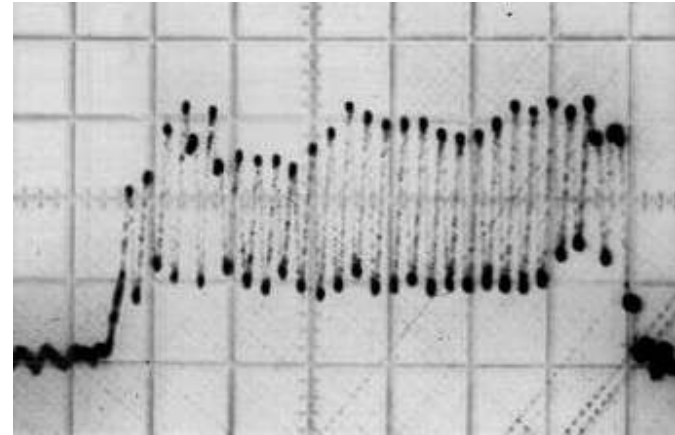
$$\frac{\omega_0}{2Q} = A \sin \varphi_2 \cos \Omega T, \quad \Omega = -A \sin \varphi_2 \sin \Omega T$$

$$\Omega T \approx \frac{\pi}{2}, \quad -A \sin \varphi_2 \approx \frac{\pi}{2T} \quad \frac{2\pi L I_2 R}{\epsilon \Lambda U_\Sigma} \alpha \sin \varphi_2 \approx \frac{\pi \tau}{2 T}$$

## Recovery Process Stability Study in Energy Recovery Accelerator



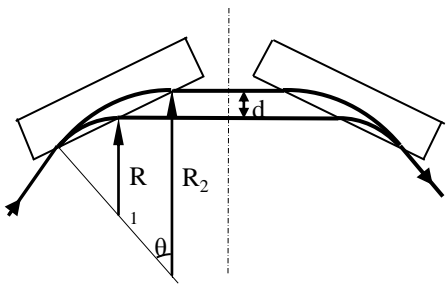
1



2

Beam current oscillograms of stable (1) and unstable (2) operation modes of racetrack microtron.

# Recovery Process Stability Study in Energy Recovery Accelerator

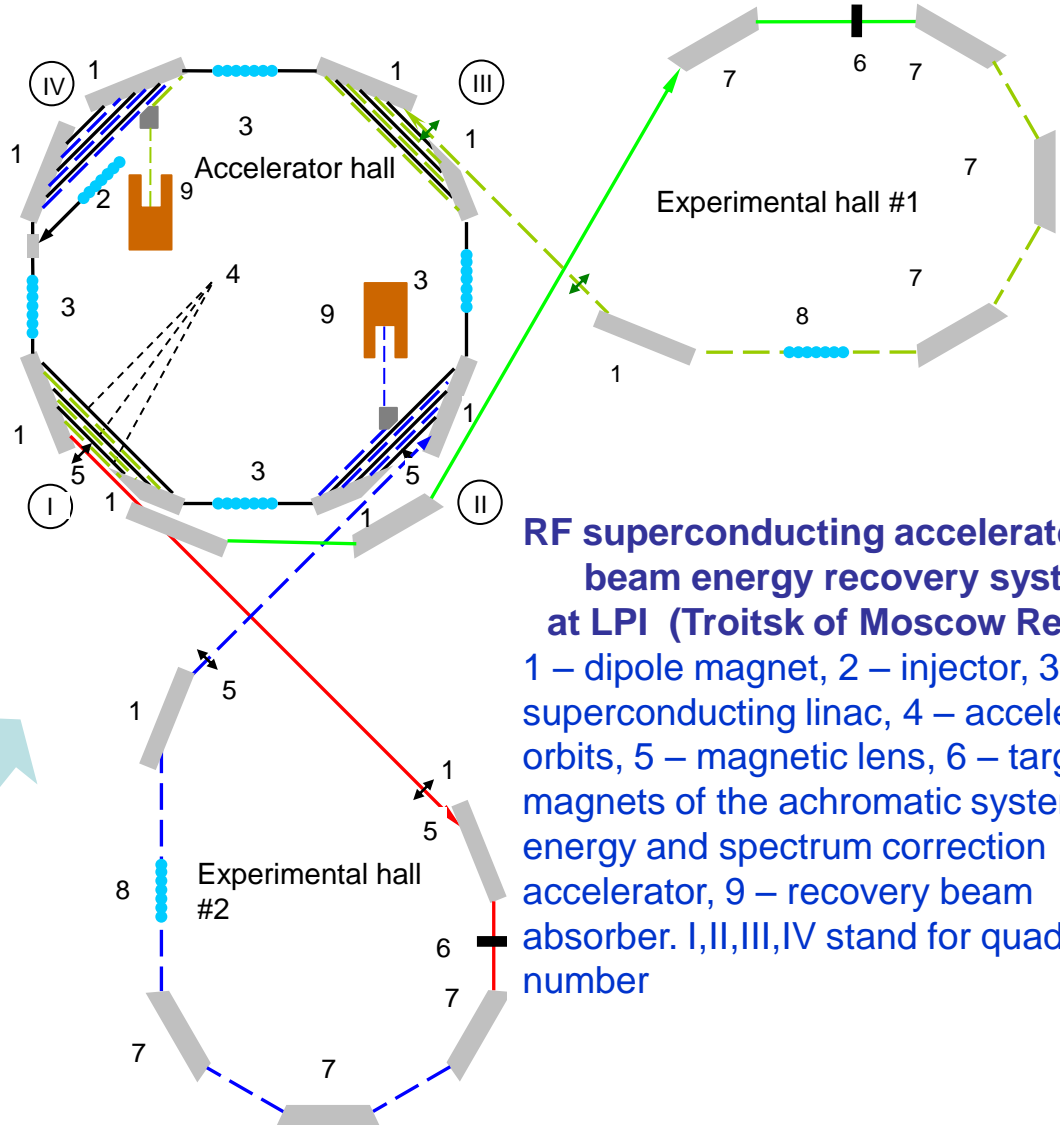


Electron trajectories in polytron

$$\Delta E(\text{MeV}) = 1.5B(T)g\lambda \frac{1}{\vartheta - \sin \vartheta}$$

$$d = \frac{g\lambda}{2} \frac{1 - \cos \vartheta}{\vartheta - \sin \vartheta}$$

**New accelerator in existing environment**



**RF superconducting accelerator with beam energy recovery system at LPI (Troitsk of Moscow Region)**

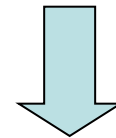
1 – dipole magnet, 2 – injector, 3 – superconducting linac, 4 – accelerator orbits, 5 – magnetic lens, 6 – target, 7 – magnets of the achromatic system, 8 – energy and spectrum correction accelerator, 9 – recovery beam absorber. I, II, III, IV stand for quadrant number

# Recovery Process Stability Study in Energy Recovery Accelerator

<i>Total energy</i>	<i>2050 MeV</i>
<i>Energy gain per turn</i>	<i>600 MeV</i>
<i>Energy gain in one linac</i>	<i>150 MeV</i>
<i>Injection energy</i>	<i>100 MeV</i>
<i>Number of turns</i>	<i>3</i>
<i>Magnetic field in dipoles</i>	<i>1.36 T</i>
<i>Dipole pole dimensions</i>	<i>0.4 m*3.6 m</i>
<i>Minimum/maximum radius of electron trajectory</i>	<i>0.6 m / 4.7 m</i>
<i>Linac frequency</i>	<i>1.3 GHz</i>
<i>Cavity Quality factor and shunt impedance <math>R/Q = V_{ac}^2 / P_{dis} * Q</math> at 2K</i>	<i><math>5 * 10^9</math> <math>1 \text{ k}\Omega</math></i>
<i>Accelerating gradient (MV/m)</i>	<i>25</i>



**TESLA type RF cavity  
Cryogenic module**



## Recovery Process Stability Study in Energy Recovery Accelerator

The instability takes place for any sign of recovery angle and longitudinal dispersion.

If cavity detuning is zero:

$D^* \sin(\varphi) > 0$  – static instability

$D^* \sin(\varphi) < 0$  – dynamical instability

$D^* \sin(\varphi) = 0$  – the instability does not take place

THANK YOU!