INTRABEAM SCATTERING ON HALO FORMATION

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Abstract

The effect of the coulombian collision between particles of a beam (intra-beam scattering) is investigated. Starting from the basic two-bodies cross section formula in the centre of mass referential, the maximum energy gain that can be catched by a particle along one direction is calculated as a function of the equipartition factor. Then, assuming that particle trajectories are ellipses in (x, x') phase-space (linear force ~ no space charge forces), the intra-beam scattering halo magnitude is calculated and shown to be very small. These calculations are done with different beam distribution functions and equipartition factor. The effect of space-charge is then investigated.

1 INTRODUCTION

Numerical studies have shown that transporting a beam in equipartition conditions could be less halo-producing than in non equipartition ones [1]. The intrabeam scattering, even if badly simulated by space-charge routines, could be suspected to be responsible of this halo production. We have undertaken to specifically study the magnitude of this effect on halo formation in proton linac.

2 ASSUMPTIONS AND DEFINITIONS

Let f(x,y,z,x',y',z') be the distribution function of a beam in 6D phase-space. The projections of this function in 2D phase or real spaces are assumed to be elliptical.

Let $\mathbf{x_0}$, $\mathbf{y_0}$, $\mathbf{z_0}$, $\mathbf{x'_0}$, $\mathbf{y'_0}$ and $\mathbf{z'_0}$ be the maximum values of respectively x, y, z, x', y' and z' which can be reached by a beam-particle. We have :

$$\left(\frac{x}{x_0}\right)^2+\left(\frac{x'}{x'_0}\right)^2+\left(\frac{y}{y_0}\right)^2+\left(\frac{y'}{y'_0}\right)^2+\left(\frac{z}{z_0}\right)^2+\left(\frac{z'}{z'_0}\right)^2\leq 1. \quad (1)$$

We assume that $x'_0 = y'_0 = r'_0$, (same temperature in both transverse directions).

We define the equipartition factor χ as :

$$\chi=\frac{z'_0}{r'_0}, \quad (2)$$

which equals 1 if the beam is equipartitionned.

For each particle, we define its "x-emittance" along x-direction ε_x :

$$\varepsilon_x=\left(\frac{x}{x_0}\right)^2+\left(\frac{x'}{x'_0}\right)^2, \quad (3)$$

in the beam, $\varepsilon_x \leq 1$.

We will assume that ε_x is a constant of motion in the particle transport. This is the case in a linear confinement force (with no space-charge force). We will discuss later about this assumption in presence of high space-charge forces.

We will calculate the probability, per unit time, for a particle to scatter to an x-emittance ε_x such that $\varepsilon_x \in [\varepsilon, \varepsilon+\Delta\varepsilon]$.

3 HALO CALCULATION - NO SPACE CHARGE

2.1 Extent

Watching a binary collision in the centre of mass frame $\Re_c$, it can be shown [2] that the maximum energy (or slope) along x direction that can be reached by a particle is given by the collision of two particles (#1 and #2) such that:

$$y'_1 = y'_2 = 0, \text{ (not necessary if } \chi = 1),$$

$$x'_1 = x'_2 = \frac{1}{\sqrt{1+\chi^2}}\cdot x'_0,$$

$$z'_1 = -z'_2 = \frac{\chi^2}{\sqrt{1+\chi^2}}\cdot x'_0,$$

with a maximum energy transfer to one particle along x direction.

Then, this particle could reach a maximum slope $\mathbf{x'_{max}}$ given by :

$$x'_{max} = \sqrt{1+\chi^2}\cdot x'_0. \quad (4)$$

The extent of the halo of an equipartitionned beam is smaller than this of a non equipartitionned one. This is true for the halo extending in the direction with the smallest temperature, but not for the other one.

2.2 Magnitude

• The number of beam particle scattering, per unit time, to an emittance between ε and ε+dε is:

$$\frac{dN_\varepsilon}{d\varepsilon}\cdot d\varepsilon=\iiint\limits_{x,y,z}\iiint\limits_{x'_1,y'_1,z'_1}\left(\frac{dP'_\varepsilon}{d\varepsilon}\cdot d\varepsilon\right)\cdot f(\;)\,dz'_1\,dy'_1\,dx'_1\,dz\,dy\,dx. \quad (5)$$

f( ) = f(x,y,z,x'₁,y'₁,z'₁) is the distribution function of the beam..

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- $\frac{dP'_\varepsilon}{d\varepsilon}\cdot d\varepsilon$ is the probability, per unit time, for one particle at position (x,y,z,x'$_1$,y'$_1$,z'$_1$) in phase-space to scatter to a x-emittance between ε and ε+dε, it is given by:

$$\frac{dP'_\varepsilon}{d\varepsilon}\cdot d\varepsilon = \int_{x'_2}\int_{y'_2}\int_{z'_2}\frac{dP_\varepsilon}{d\varepsilon}\cdot d\varepsilon\cdot dx'_2\, dy'_2\, dz'_2\ . \qquad (6)$$

- $\frac{dP_\varepsilon}{d\varepsilon}\cdot d\varepsilon$ is the probability, per unit time, for one particle at (x,y,z,x'$_1$,y'$_1$,z'$_1$) to scatter 1) on a particle with slopes between x'$_2$ and x'$_2$+dx'$_2$, y'$_2$ and y'$_2$+dy'$_2$ and z'$_2$ and z'$_2$+dz'$_2$, 2) to a x-emittance between ε and ε+dε :

$$\frac{dP_\varepsilon}{d\varepsilon}\cdot d\varepsilon = \frac{dP_{\Delta X'}}{d\Delta X'}\left(\Delta X' = x'_0\sqrt{\varepsilon-(x/x_0)^2}-x'_1\right)\frac{x_0'^2}{2(x'_1+\Delta X')}\cdot d\varepsilon \qquad (7)$$

- $\frac{dP_{\Delta X'}}{d\Delta X'}\cdot d\Delta X'$ is the probability, per unit time, for one particle at position (x,y,z,x'$_1$,y'$_1$,z'$_1$) in the phase-space to scatter 1) on a particle with slopes between x'$_2$ and x'$_2$+dx'$_2$, y'$_2$ and y'$_2$+dy'$_2$ and z'$_2$ and z'$_2$+dz'$_2$, 2) with a x-angle ΔX' between ΔX' and ΔX'+dΔX' :

$$\frac{dP_{\Delta X'}}{d\Delta X'}\cdot d\Delta X' = \frac{-d\Delta X'}{2\pi W'_1}\cdot\int_{\theta_{min}}^{\theta_{max}}\frac{dP_\theta}{d\theta}\cdot\frac{d\theta}{\sin\theta\cdot\sin\left(\operatorname{acos}\left(\frac{\Delta X'-X'_1\cdot(\cos\theta-1)}{W'_1\cdot\sin\theta}\right)\right)} \qquad (8)$$

with:

$$\theta_{min} = \operatorname{asin}\left(\frac{X'_1+\Delta X'}{V}\right)-\theta_0\ ,\ \theta_{max} = \pi-2\cdot\theta_0-\theta_{min}\ ,$$

$$V=\sqrt{X_1'^2+Y_1'^2+Z_1'^2}\ ,\ W'_1=\sqrt{Y_1'^2+Z_1'^2}\ ,$$

$$X'_1=\frac{x'_1-x'_2}{2},\ Y'_1=\frac{y'_1-y'_2}{2},\ Z'_1=\frac{z'_1-z'_2}{2},$$

and $\tan\theta_0 = \frac{X'_1}{W'_1}$.

- $\frac{dP_\theta}{d\theta}\cdot d\theta$ is the probability, per unit time, for one particle at (x,y,z,x'$_1$,y'$_1$,z'$_1$) to scatter 1) on a particle with slopes between x'$_2$ and x'$_2$+dx'$_2$, y'$_2$ and y'$_2$+dy'$_2$ and z'$_2$ and z'$_2$+dz'$_2$, 2) to a collision angle between θ and θ+dθ, :

$$\frac{dP_\theta}{d\theta}\cdot d\theta = \frac{d\sigma}{d\theta}(\theta,V)\cdot v_1\cdot\beta c\cdot f(x,y,z,x'_2,y'_2,z'_2)\,dx'_2\,dy'_2\,dz'_2\,d\theta \qquad (9)$$

with $v_1=\sqrt{x_1'^2+y_1'^2+z_1'^2}$.

- $\frac{d\sigma}{d\theta}(\theta,V)$ is the cross section, differential to the scattering angle θ, of a coulombian collision in $\Re_c$ [2, Annexe 1] :

$$\frac{d\sigma}{d\theta}(\theta,V) = -\frac{2\pi\cdot r_0^2}{(2V\cdot\beta)^4}\cdot\frac{\cos\theta/2}{\sin^3\theta/2}\ , \qquad (10)$$

$r_0=\frac{q^2}{4\pi\varepsilon_0 mc^2}$ is the classical radius of proton,

ε_0 is the vacuum permitivity,

q and m are the charge and the mass of particle,

βc is the beam velocity in the lab frame,

V is the half relative slope of the 2 particles (slope of one particle in $\Re_c$).

4 RESULTS

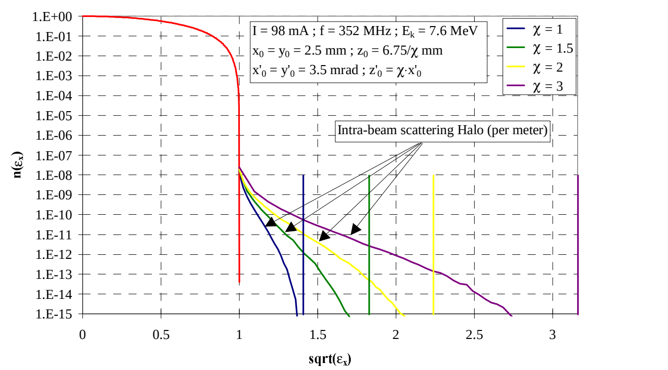
A program making the numerical integration needed to solve equation (5) with Gauss quadrature method has been written. Calculations have been done with a water-bag beam :

$$f(x,y,z,x',y',z')=\begin{cases} f_0 & \text{if } \left(\frac{x}{x_0}\right)^2+\left(\frac{x'}{x'_0}\right)^2+\left(\frac{y}{y_0}\right)^2+\left(\frac{y'}{y'_0}\right)^2+\left(\frac{z}{z_0}\right)^2+\left(\frac{z'}{z'_0}\right)^2\le 1 \\ 0 & \text{otherwise}\end{cases} \qquad (11)$$

If N is the number of particles in the bunch, we have :

$$f_0=\frac{6\cdot N}{\pi^3\cdot x_0\cdot y_0\cdot z_0\cdot x'_0\cdot y'_0\cdot z'_0}\ . \qquad (12)$$

On figure 1 have been represented the intrabeam scattering tails created per meter in different equipartition conditions at 2 beam energies (6.7 and 100 MeV). The beam is a typical APT beam. Vertical lines represent the theoretical halo extent obtained from equation (4).



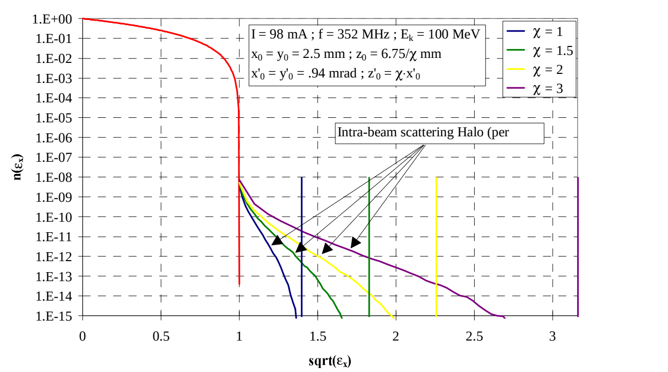


Figure 1 : Intrabeam scattering halo (per meter) of the typical APT beam at 6.7 MeV (up.) and 100 MeV (down) in (x/x$_0$, x'/x'$_0$).

Calculations with other beam distributions have been done. They give nearly the same results (see figure 2).

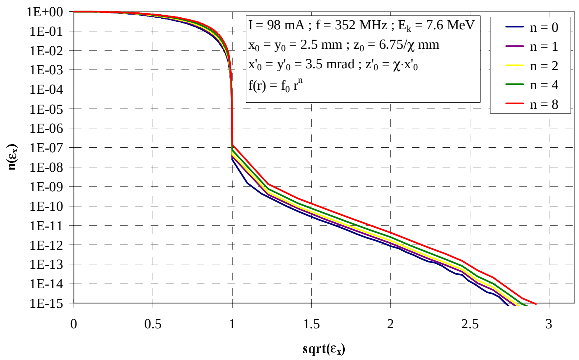


Figure 2 : Intrabeam scattering halo (per meter) of the typical APT beam at 6.7 MeV for different particle distribution functions.

5 INFLUENCE OF SPACE-CHARGE

Let's η be the depress tune factor in the beam. We have developed a model describing the trajectories of particles around a space-charge driven beam [3]. Using this model, the particle amplitude distribution around a space-charge driven beam can be deduced from the one obtained without space-charge. The particle amplitude distribution then obtained for different tune depressions is presented on figure3. The equipartition factor is χ=3.

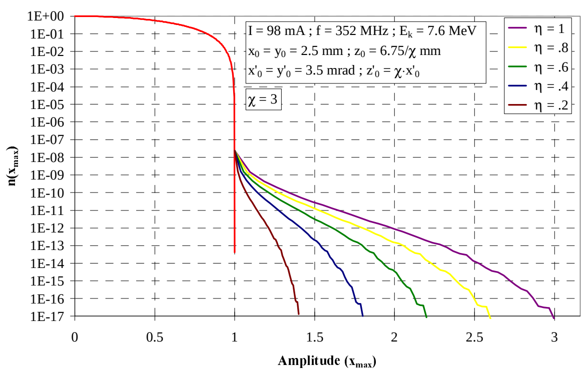


Figure 3 : Influence of space-charge on particle amplitude distribution.

This model gives a maximum amplitude :

$$x_{max}/x_0 = 1+\eta\left(\sqrt{1+\chi^2}-1\right). \qquad (13)$$

This shows that, with constant beam dimensions (x_0 and x'_0), the more space-charge dominated the beam is, the lower the extent and the magnitude of the halo are. This can be easily understood : In order to keep a beam with given dimensions (x_0, x'_0), the confinement potential should be deeper when η is small. This means that, in order to reach a given amplitude, a particle needs a higher transverse energy with a small η than with a bigger one. For a given energy (or maximum slope), the corresponding amplitude is then smaller. An equivalent result has been independently obtained by R. Glückstern and A. Fedotov in ref. [4].

6 CONCLUSION

The influence of the intrabeam scattering on halo formation seems to be negligible as well in extension as in density. Moreover, the space-charge reduces it a lot. R. Glückstern and A. Fedotov got the same conclusion, using an other model, in their paper presented in ref. [4]. The equipartition conditions are not made necessary by the intrabeam scattering phenomenon. The largest emittance growth and halo formation of non-equipartitionned beam observed in simulations can not be justified by intrabeam scattering. Are other physical effects (coupling resonance ?...) or spurious space-charge model effects [5] explaining these observations ?

7 ACKNOWLEDGMENT

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8 REFERENCES

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