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THE IMPEDANCE OF RF-SHIELDING WIRES

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Abstract

We studied the electrostatic fields due to the longitudinal and transverse perturbations of a charged particle beam with a uniform distribution propagating inside an rf-shielding cage constructed from evenly-spaced conducting wires. The beam and the rf-cage are surrounded by a concentric conducting beam pipe. Simple formulae are derived for estimating the space-charge impedances. Numerical examples are given.

1 INTRODUCTION

An rf-shielding cage, or an rf-cage, used in an accelerator or storage ring is a cage-like structure made of conducting wires stretched in parallel to the direction of the circulating charged particle beam.[1] The conducting wires on the cage are arranged to surround the beam to create an electromagnetically shielded environment for the beam. This kind or the similar kinds of devices together with ceramic beam pipes have been implemented[1] and planned[2-4], or is being planned[5] in some high-intensity rapid cycling proton synchrotrons. There are two reasons for using the rf-cage instead of solid beam pipe. The first reason is to avoid excess eddy current that may be induced on the beam pipe by the fast-changing magnetic field. The second reason is that it is easier to vary the cross-section of an rf-cage to reduce the coupling impedance.

Although an rf-cage has been put in service for many years,[1] a serious study of the electromagnetic field of a charged particle beam propagating in an rf-cage has never been documented until recently.[6,7] In Refs. 6 and 7, a rigorous formalism was established to investigate the electrostatic field of a charged particle beam with a uniform distribution inside an rf-shielding cage constructed from evenly-spaced conducting wires. The purpose of the this work is to extend the previous study to include the effect of an external solid beam pipe. Simple formulae will be derived for the longitudinal and transverse coupling impedances in the long wavelength regime. Numerical examples will be given.

2 THE FIELD AND IMPEDANCE

The system considered here is shown in Fig. 1. A beam having a circular cross-section of radius r_b and a uniform charge distribution is propagating inside of an rf-cage composed of N conducting wires extended in the direction parallel to the beam. The beam and the rf-cage are surrounded by a conducting beam pipe with radius r_t. For simplicity, we shall limit our discussion to the geometry in which wires are evenly distributed over a circle; the surrounding pipe and the rf-cage is positioned concentric with the beam. The radius of the rf-cage, measured from the center of the cage to the centers of wires, is r_c. We assume that the pipe and wires are electrically grounded and all wires have the same circular cross-section of radius ρ_w. The discussions here will be restricted to the regime $\rho_w \ll r_c$ and $N \gg 1$.

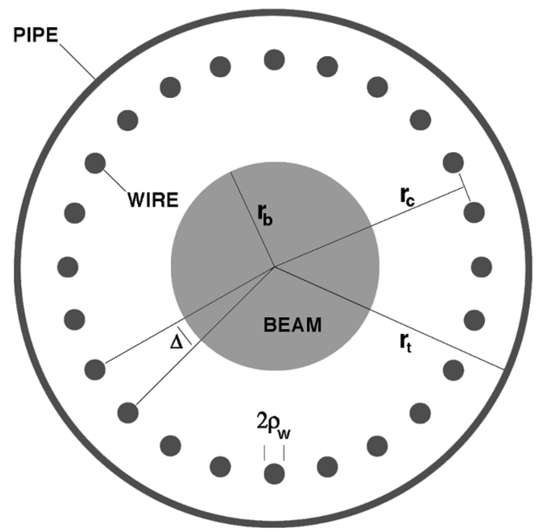

Fig. 1. Cross-sectional view of a beam inside an rf-cage and beam pipe. r_t, r_c, and r_b are the radii of the beam pipe, the rf-cage, and the beam, respectively. Δ is the angle subtended by two adjacent wires, and ρ_w is the radius of a wire.

Although one can estimate the impedance in the long wavelength regime by solving the two-dimensional electrostatic field using various techniques like the image method etc., we elect to use the three-dimensional treatment here. This approach allows one to examine the frequency dependence of the impedance in the low frequency domain, if needed.

We choose a cylindrical coordinate system (r, θ, z) such that the z-axis coincides with the central axis of the beam, and we shall call it the "global coordinate system". In order to conveniently describe the electric field near an individual wire, we shall also use another cylindrical coordinate system, be referred to as the "local coordinate system" in the following, (ρ, ψ, z) in which the z-axis coincides with the central axis of a wire as shown in Fig. 2.

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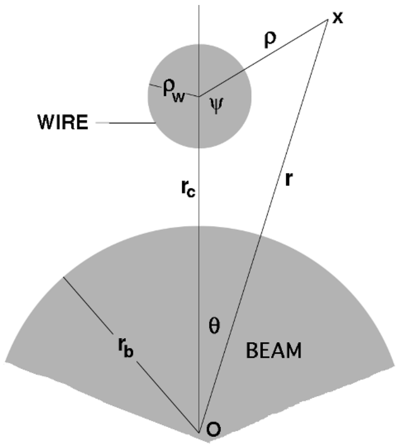

Fig. 2. The local and the global coordinates adopted in this study. The origins of the local and the global coordinates are located at the center of beam and the center of a wire, respectively.

We now consider the electrostatic potential due to the charge-density perturbation that varies in the z-direction according to e^{ikz}, where k is the wave-number of the perturbation. The Poisson equation we want to solve is

$$\frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\partial\Phi}{\partial r}\Big)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\partial^2\Phi}{\partial z^2}=\begin{cases}0, & \text{if } r_t>r>r_b,\\ -\sigma\mathrm{e}^{ikz}/\epsilon_o, & \text{if } r\le r_b,\end{cases} \quad (1)$$

where σ is the volume charge density associated with the perturbation, and ϵ_o is the permittivity of free space. To solve Eq. (1) in the presence of wires, we first solve the equation for no wire, i.e., for the boundary condition of $\Phi=0$ at $r=r_t$ only. Then we assume that in the region of $r_t\ge r\ge r_c$ each wire induces a field which has the following multipole expansion in the local coordinate:

$$\phi_w=\sum_{n=-\infty}^{\infty}\big[A_nK_n(k\rho)+B_nI_n(k\rho)\big]\mathrm{e}^{in\psi}\mathrm{e}^{ikz}\,, \quad (2)$$

where $I_n(x)$ and $K_n(x)$ are the nth order modified Bessel functions of the first and the second kind, respectively; A_n and B_n are the unknowns to be determined. Applying the addition theorem of Bessel functions[8], ϕ_w can also be expressed in the global coordinate variables. Then applying the boundary condition of $\phi_w=0$ at $r=r_t$, one can solve B_n in terms of A_n. Because of the symmetry embodied in the system, one can study the field around wires by considering the electric potential around any individual wire. Thus, we call the wire under consideration the 0th wire and number all others by their relative locations with respect to the 0th wire. On the surface of each wire, the potential due to the induced charge should cancel that due to the beam plus that contributed by all other wires. This requirement leads to a complicated equation for A_n which appears to have no closed-form solution. In the regime $kr_t\ll 1$, it is possible to find a solution for A_n expressed in a power series of $h_n(k\rho_w)$. If the coupling among multipoles is neglected, the lowest order solution is

$$A_n\approx -b_{\parallel}h_n(k\rho_w)\Big[K_n(kr_c)-\frac{I_n(kr_c)}{h_0(kr_t)}\Big]\Big\{1+(-1)^n \times h_n(k\rho_w)\sum_{\mu=1}^{N-1}\mathrm{e}^{-in\mu\Delta}K_0(kd_\mu)-h_n(k\rho_w) \times\sum_{j=-\infty}^{\infty}\frac{\big[I_{n+j}(kr_c)\big]^2}{h_j(kr_t)}\Big[1+(-1)^n\sum_{\mu=1}^{N-1}\mathrm{e}^{ij\mu\Delta}\Big]\Big\}^{-1}, \quad (3)$$

where $b_{\parallel}=(\sigma r_b/k\epsilon_o)I_1(kr_b)$, $h_n(x)=I_n(x)/K_n(x)$, $\Delta=2\pi/N$ is the angular separation between two adjacent wires, and d_μ is the distance between the centers of the 0th and the μth wires.

Using the solution (3) and the addition theorem of Bessel functions, one can derive the total electric potential in the region of $r\le r_c$. Then taking the approximation by considering $n=0$ (the monopole solution) only, we find the total electric potential in the region of $r\le r_b$ as

$$\Phi=\frac{\sigma}{\epsilon_o k^2}\Big\{1-kr_bI_0(kr)\Big[\frac{I_1(kr_b)}{h_0(kr_t)}+K_1(kr_b)\Big]\Big\}\mathrm{e}^{ikz} + N\sum_{p=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}A_n\Big[K_{n+pN}(kr_c)-(-1)^n \times\frac{I_{n+pN}(kr_c)}{h_{pN}(kr_t)}\Big]I_{pN}(kr)\mathrm{e}^{ipN\theta}\mathrm{e}^{ikz}\,. \quad (4)$$

For $N\gg 1$ and $kr_t\ll 1$, we can apply the small argument expansions of Bessel functions to Eqs. (3) and (4) to yield

$$Z_{\parallel}\approx\frac{iLkZ_o}{4\beta\gamma^2}\Big[1+2\ln\Big(\frac{r_t}{r_b}\Big)+C_{\parallel}\Big]\,, \quad (5)$$

where

$$C_{\parallel}\approx\frac{-2N[\ln(r_t/r_c)]^2}{N\ln(r_t/r_c)-\ln(\pi f_w)+\ln\big[1-(r_c/r_t)^{2N}\big]}\,, \quad (6)$$

L is the length or the circumference of the machine, $Z_0=377\Omega$, and the *wire filling factor* f_w is defined as the ratio between the angle subtended by a wire in the global coordinate system θ_w, and Δ, i.e. $f_w=\theta_w/\Delta\approx N\rho_w/(\pi r_c)$.Note that $h_n(kr_t)\to\infty$ when $r_t\to\infty$. Thus, in the absence of the external beam pipe Eqs. (5) and (6) reduce to the previous result.[7]

Next, we consider the electrostatic potential due to a transverse perturbation in a beam. The model of the perturbation to be studied here is a shell with surface charge density varying according to $\mathrm{e}^{ikz}\cos\theta$. The Poisson equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\partial\Phi}{\partial r}\Big)+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\partial^2\Phi}{\partial z^2}=\frac{-\sigma\bar{d}}{\epsilon_o}\delta(r-r_b)\mathrm{e}^{ikz}\cos\theta\,. \quad (7)$$

We again start the analysis by solving the Poisson equation in the absence of the rf-cage. Then the field due to the induced charge on wires is considered. In contrast to the case of longitudinal perturbation, the system now is not axisymmetric. Therefore, the multipole expansion coefficients of the field due to the induced charges on each wire depend on the angular location of the wire. Other than that, the analysis procedures and the boundary conditions are the same as in treating the longitudinal perturbation.

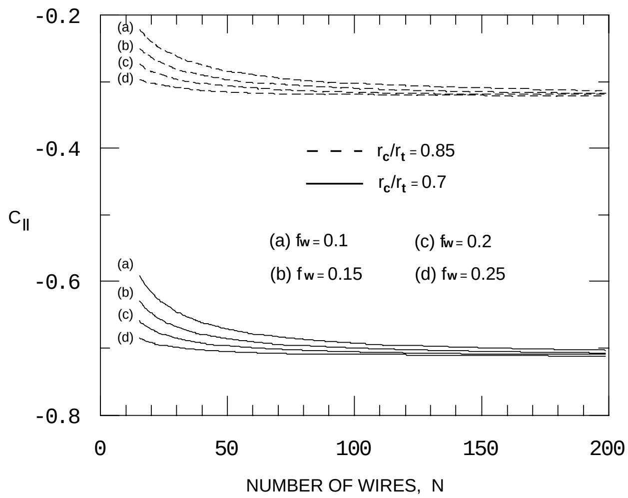

Fig. 3. A numerical example of $C_{\|}$ as a function of the total number of wires N. Where $C_{\|}$ is calculated using Eq. (6).

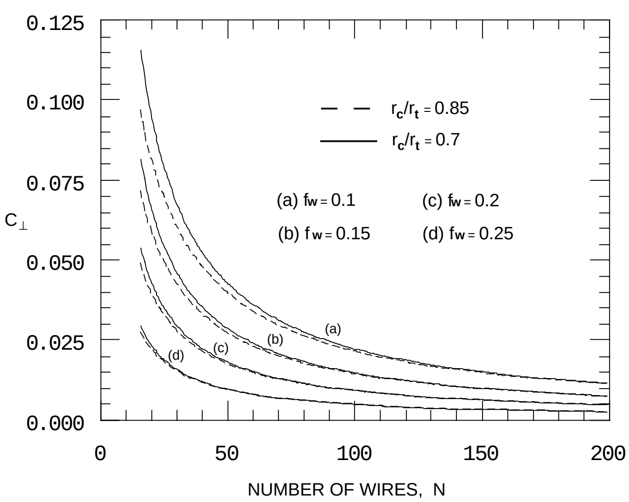

Fig. 4. A numerical example of $C_{\perp}$ as a function of the total number of wires N. Where $C_{\perp}$ is calculated using Eq. (9).

After obtaining the approximate perturbed field by retaining only the contribution from the monopoles due to the wires, all the Bessel functions are expanded to derive the following transverse impedance in the long wavelength regime:

$$Z_{\perp} \approx \frac{iLZ_o}{2\pi\beta^2\gamma^2}\left(\frac{1}{r_b^2} - \frac{1-C_{\perp}}{r_c^2}\right), \qquad (8)$$

where

$$C_{\perp} \approx \left[1-(r_c/r_t)^2\right]\Big\{\left[(r_c/r_t)^2+(r_t/r_c)^2\right]\ln\left[1-(r_c/r_t)^{2N}\right] - 2\ln(\pi f_w)\Big\}\Big\{N\left[1-(r_c/r_t)^2\right] - 2\ln(\pi f_w) + \left[(r_c/r_t)^2+(r_t/r_c)^2\right]\ln\left[1-(r_c/r_t)^{2N}\right]\Big\}^{-1}. \qquad (9)$$

When $r_t \to \infty$, $Z_{\perp}$ reduces to the limits of no external beam pipe obtained before.[7]

3 CONCLUSIONS

For a charged particle beam propagating inside of a beam pipe and an rf-shielding cage made of evenly-spaced conducting wires, the electrostatic fields due to sinusoidal longitudinal and dipole-mode transverse perturbations have been solved analytically for the case that the cage and the wires all have circular cross sections. It was assumed that the beam has a uniform charge distribution and the unperturbed system is azimuthally symmetric. We have derived simple formulae for the coupling impedances in the long wavelength regime. Numerical examples were presented to show the shielding effects of the rf-cage.

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