IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, June 1981

HIGH CURRENT BETATRON*

G. Barak, A. Fisher, and H. Ishizuka
Physics Department
University of California
Irvine, California 92717

and

N. Rostoker
University of California, Irvine
and
Maxwell Laboratories, Inc.
San Diego, California

In the conventional Betatron the problems of electron injection, beam stability during acceleration and beam extraction were solved. The electron current was limited by the method of injection. We propose to inject electrons by the method of inductive charging. This involves a toroidal magnetic field in addition to the normal Betatron fields. It should then be possible to increase the electron current by a factor of $10^4 - 10^6$. The toroidal magnetic field is stabilizing for collective instabilities such as negative mass, but it introduces particle orbit resonances that are absent in a normal Betatron. We show how orbital instabilities can be eliminated. After accelerating the electrons the toroidal magnetic field is no longer necessary to control space charge. It can be allowed to decay so that the beam can be extracted with a conventional magnetic peeler.

Introduction

The largest conventional Betatron was completed about 1950.[1] The energy was about 300 MeV and the beam current about 100 milliamperes. The maximum beam current was limited by space charge during injection. Electrons from a 100 keV electron gun were injected into an orbit of radius $R \cong 1$ meter. This implies a Betatron magnetic field $B_y = 10.6$ gauss. The space charge limit is proportional to B^2; thus the limit is very small. After the electrons have been accelerated by increasing B_y the space charge limit is greatly increased. Thus the problem for high currents is at injection.

To eliminate the space charge problem the Plasma Betatron was proposed by Budker.[2] After many studies[3] were carried out, the maximum current reached about 10 Amperes, much less than expected. The current is probably instability limited. The precise instability has not been identified, but the negative mass instability is mentioned frequently.[4]

By increasing the injection energy and decreasing the orbit in a conventional Betatron, the magnetic field B_y can be increased and therefore the space charge limit is extended. Small "Ironless" Betatrons have been developed with an electron energy of 100 MeV and an electron current of about 90 Amperes.[5] This is the most successful method to increase the current to date.

We propose a new method of injection for Betatrons that we have demonstrated in our studies of the Collective Focusing Ion Accelerator.[6] The method has been called inductive charging and was first used in HIPAC.[7] Electrons are injected from thermionic injectors by means of a rising toroidal magnetic field. A Betatron with this type of injection is illustrated in Figure 1. The electrons move in an orbit of radius R because of the toroidal magnetic field B_z. There are no restrictions on B_z as there are on B_y in a conventional

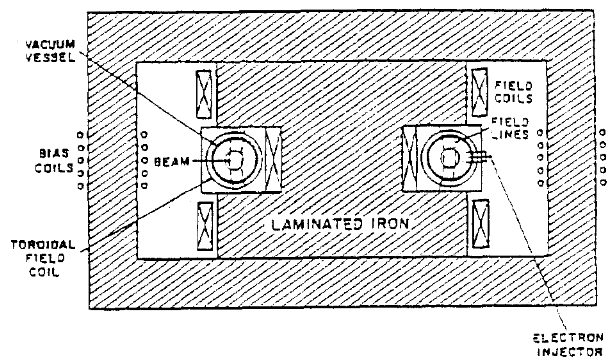

FIG. 1: SCHEMATIC DESIGN OF HIGH CURRENT BETATRON WITH INDUCTIVE CHARGING.

Betatron. In the largest "Kerst" Betatron the gyro-radius was $R \cong 1$ meter. With the toroidal magnetic field, the gyro-radius may be as small as 1 mm (the only restriction concerns the injector design; electrons after injection must subsequently miss the injector to be trapped). This involves an increase in B of a factor of 10^3 and therefore of 10^6 in the space charge limit.

Particle Orbits

In order to describe the particle orbits of a toroidal electron beam, consider the local coordinates (x,y) or (r, θ) as illustrated in Figure 2.

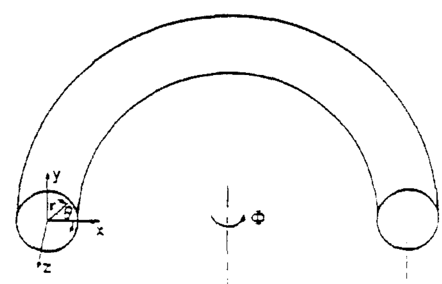

FIG. 2 COORDINATES FOR A TOROIDAL ELECTRON BEAM

The toroidal direction is indicated by z. The equations of motion are approximately

0018-9499/81/0600-3340$00.75© 1981 IEEE

$$\frac{d}{dt}(m\gamma v_x) + \frac{m\gamma v_z^2}{R-x} =$$

$$- e\left\{(E_r - \beta B_\theta)\frac{x}{r} + \frac{1}{c}(v_y B_z - v_z B_y)\right\} \quad (1)$$

$$\frac{d}{dt}(m\gamma v_y) =$$

$$- e\left\{(E_r - \beta B_\theta)\frac{y}{r} + \frac{1}{c}(v_z B_x - v_x B_z)\right\} \quad (2)$$

These equations contain the Betatron fields

$$B_y = B_o\left[\frac{R}{R-x}\right]^s \cong B_o\left[1 + \frac{sx}{R}\right]$$

$$B_x = s\frac{B_o}{R}y \quad ,$$

the toroidal magnetic field B_z and the self-electric and magnetic fields

$$E_r = -2Ner/a^2$$

$$B_\theta = \beta E_r \quad ,$$

where a is the minor radius of the electron beam and $\beta = v_z/c$. N is the line density of the beam. We have assumed a uniform beam for which

$$-e(E_r - \beta B_\theta)/m\gamma r = \omega_p^2/2\gamma^2$$

where $\omega_p^2 = 4\pi ne^2/\gamma m$ and $\gamma = (1-\beta^2)^{-\frac{1}{2}}$.

Equations (1) and (2) involve the following approximations:

(i) An expansion in the ratio a/R to the lowest significant order.
(ii) $v_z >> v_x, v_y$; $v_z = V[1 + O(a/R)^2 ..]$ provided that $V = R\Omega_y$ where $\Omega_y = eB_o/\gamma mc$.
(iii) The changes in γ are either small due to the electric field Er or slow due to an induced toroidal electric field E_z,

If we omit the toroidal field B_z we recover standard Betatron equations with self-fields.

$$\ddot{x} - \Omega^2 x + \Omega_y^2(1-s)x = 0 \quad , \quad (1.1)$$

$$\ddot{y} - \Omega^2 y + \Omega_y^2 sy = 0 \quad , \quad (2.1)$$

$\Omega^2 = \omega_p^2/2\gamma^2$, and $\Omega_y = eB_o/\gamma mc = V/R$. Assuming, for example, $s = \frac{1}{2}$, the condition for orbit stability is

$$\Omega_y^2/2 > \Omega^2 \quad \text{or} \quad n < \gamma B_o^2/4\pi mc^2 \quad (3)$$

After acceleration $B_o \sim 5$ kilogauss and $\gamma \sim 600$ so that $n < 1.5 \times 10^{16}\,\text{cm}^{-3}$ or the current density limit is $j_z < 7 \times 10^7$ amps/cm^2. However, during injection at 100 keV, R = 1 meter, $B_o = 10.6$ gauss, and $n < 1.1\ 10^7\,\text{cm}^{-3}$ is the space charge limit. After acceleration this corresponds to $j_z < 5.3 \times 10^{-2}$ amps/cm^2 which is consistent with observed currents in the conventional Betatron.[1] In the experiments of Pavlevskii et al.,[5] $V = 2.6 \times 10^{10}$cm/sec corresponding to 500 keV, and R = 3.9 cm. Thus, $B_o = 370$ Gauss corresponds to $j_z = 65$ amps/cm^2 which is consistent with the observations.

When both toroidal and Betatron fields are included the orbit equations are:

$$\ddot{x} + \frac{\dot{\gamma}}{\gamma}\dot{x} + \Omega_z\dot{y} - \Omega^2 x + \Omega_y^2(1-s)x = 0 \quad (1.2)$$

$$\ddot{y} + \frac{\dot{\gamma}}{\gamma}\dot{y} - \Omega_z\dot{x} - \Omega^2 y + \Omega_y^2 sy = 0 \quad (2.2)$$

where $\Omega_z = eB_z/\gamma mc$.

For s = 1/2 these equations can be combined and solved with the transformation

$$\zeta = \sqrt{\frac{\gamma}{\gamma_o}}(x+iy) \cdot \exp - \frac{i}{2}\int_o^t \Omega_z(t')dt' \quad (4)$$

Equations (1.2) and (2.2) are replaced by

$$\ddot{\zeta} + \frac{\omega^2}{4}\zeta = 0 \quad (5)$$

where $\omega^2 = \Omega_z^2 + 2\Omega_y^2 - 4\Omega^2 - \frac{1}{2}\frac{d}{dt}\frac{\dot{\gamma}}{\gamma} - \frac{1}{4}\left(\frac{\dot{\gamma}}{\gamma}\right)^2$.
If all quantities in ω^2 are slowly varying the terms involving $\dot{\gamma}$ can be omitted. The solution by the W.K.B. method is

$$\zeta = \frac{A}{\sqrt{\omega}}\exp i\int_o^t \omega(t')dt' \quad (6)$$

The space charge limit is $\omega^2 \geq 0$ or

$$n < \gamma\left[\frac{B_y^2}{4\pi mc^2} + \frac{B_z^2}{8\pi mc^2}\right] \quad (7)$$

Thus orbit stability can be maintained for combined toroidal and Betatron fields. If B_z is permitted to decay after acceleration, the orbits will be initially adiabatic in the large B_z and will become non-adiabatic as $B_z \to 0$. They remain stable through this transition as long as Eq. (7) is satisfied.

When electrons are accelerated the orbit radius changes according to

$$\left(\frac{R}{R_o}\right)^2 = \left|\frac{\zeta}{\zeta_o}\right|^2 = \frac{\gamma_o}{\gamma}\frac{\omega_o}{\sqrt{\Omega_z^2 + 2\Omega_y^2 - 4\Omega^2}} \quad (8)$$

γ increases by a large factor. $\Omega_z = eB_z/\gamma mc$ decreases; if the Betatron conditions are satisfied Ω_y stays nearly constant; Ω^2 decreases. The net result is compression or $R < R_o$ after acceleration.

According to Eq. (7), when $\gamma \sim 1$ and B_y is small the toroidal field B_z is necessary to handle the space charge. After acceleration γ and B_y are large; B_z is no longer needed for space charge and can be permitted to decay. Thus the final state will be that of a conventional Betatron where beam extraction has been done with a magnetic peeler.[1]

As a result of $B_z \to 0$ the orbit radius will grow according to Eq. (8).[2] The net change of radius from acceleration and letting $B_z \to 0$ will be small. In Figure 3 numerical calculations of Eq. (1.2) and (2.2) are shown for B_y = 7.5 k-Gauss, B_z = 25 k-Gauss initially, γ = 100 and s = .45. In the sequence $B_z \to 0$, the change from adiabatic to non-adiabatic orbits is apparent and the growth in orbit radius is slight.

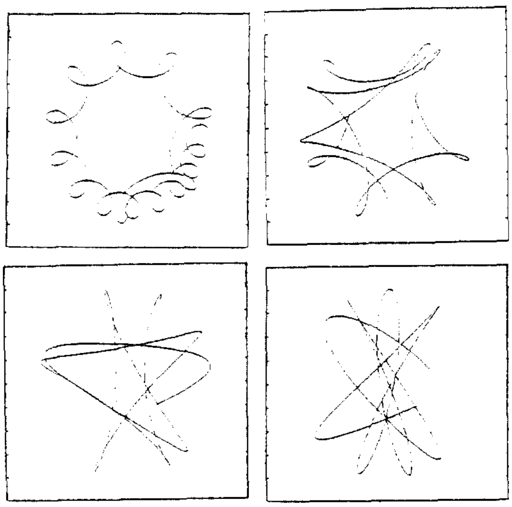

FIG. 3. NUMERICAL CALCULATIONS OF ELECTRON ORBITS IN THE HIGH CURRENT BETATRON.

Because of magnetic field imperfections ω^2 in Eq. (5) is a periodic function of time with period T = $2\pi R/V$. This leads to integer and half integer resonances. For a conventional Betatron the standard Betatron conditions $V = R\Omega_y$ and $\langle B_y \rangle = 2B_y$ guarantee that all resonances are avoided. With a toroidal field the resonance condition is

$$\omega = \sqrt{\Omega_z^2 + 2\Omega_y^2 - 4\Omega^2} = \frac{n\pi}{T} \quad \text{where } n = 1,2,3\ldots\text{etc.} \tag{9}$$

During acceleration V changes until it reaches c, then γ changes; after acceleration we plan to let $B_z \to 0$. Therefore it is impossible to avoid these resonances. We have investigated a model in which

$$\Omega_z^{(1)} = \Omega_z(1+P) \quad 0 \le t \le a \qquad \Omega_z^{(2)} = \Omega_z \quad a \le t \le T$$

$$\omega^{(1)} = \omega(1+Q) \qquad\qquad \omega^{(2)} = \omega$$

The results are that if only the toroidal field (Ω_z) is perturbed the resonances do not lead to growth. If Ω_y is perturbed there are narrow bands of growth. For sufficiently large P, the growth and band width vanish; i.e., the toroidal field can be designed to control the resonances.

Betatron Experiment

We are developing an experiment at U. C. Irvine with the objective of accelerating 1 k-amp of electrons to 5 MeV . The torus has a 40 cm major radius and a 5 cm minor radius. The toroidal magnetic field rises to 12 k-Gauss in about 100 μsec. and decays in about 600 μsec. The Betatron field rises to 1 k-Gauss at the major radius in 1 millisec. The field index can be varied from .2 to .8. The method of injection is illustrated in Figure 4. It is directly based on the

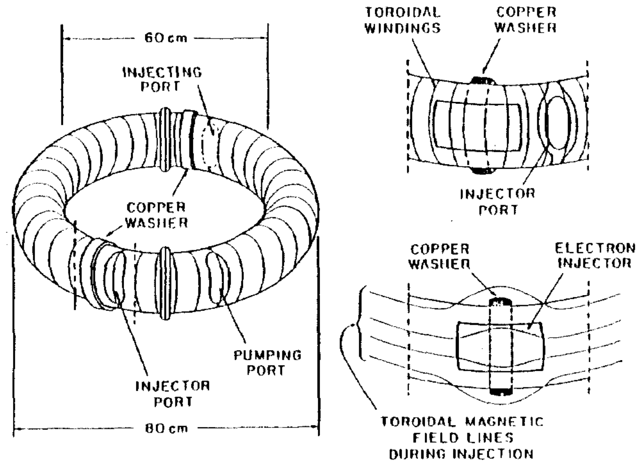

FIG. 4. BETATRON EXPERIMENT

experience with CFIA.[6] A local mirror is created into which injection and trapping takes place. The diamagnetic current in the copper washer decays on a short time scale compared to the toroidal field. The electrons trapped in the mirror are ejected along the field lines as the mirror collapses. Based on previous laboratory experiments we expect to trap enough electrons to produce 1 k-ampere after acceleration.

*Work supported by the Defense Advanced Research Project Agency and the Office of Naval Research.

References

1. D. W. Kerst, G. D. Adams, H. W. Koch, and C. S. Robinson, Phys. Rev. 78, 297 (1950).
2. G. I. Budker, CERN Symposium 1, 68 (1956); 1, 76 (1956).
3. K. C. Rogers, D. Finkelstein, L. Ferrari, D. Caulfield, L. Mansfield, and G. Brucker, Proc. Int. Conf. on Accel. CERN, Geneva (1959); P. Reynolds and H. M. Skarsgard, J. Nucl. Energy C1, 36 (1960).
4. I. M. Samoilov and A. A. Sokolov, Soviet Phys. JETP 12, 185 (1961); R. W. Landau and V. K. Neil, Phys. Fluids 9, 2412 (1966).
5. A. I. Pavlevskii, G. D. Kuleshov, G. V. Sklizkov, Y. A. Zysin, and A. I. Gerasimov, Soviet Physics Doklady 10, 30 (1965); Soviet Physics, Tech. Phys. 22, 210 (1977).
6. A. Fisher, P. Gilad, F. Goldin, and N. Rostoker, Appl. Phys. Lett. 36, 264 (1980).
7. J. D. Daugherty, J. Eninger, and G. S. Janes, AVCO Everett Research Report 375, Oct. 1971; G. S. Janes, R. H. Levy, H. A. Bethe, and B. T. Feld, Phys. Rev. 145, 1925 (1966).