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PRESCRIPTION FOR A GAIN-ENHANCED FREE ELECTRON LASER WITH AN ELECTROMAGNETIC PUMP FIELD

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Summary

The equations of motion for a free electron laser with an electromagnetic pump field are presented. A scalar potential representing an external static axial electric field for gain enhancement has been included in the model. Equations governing gain and phase shift for each of the electromagnetic fields are given. The equation for the separatrix has been derived using the resonant particle concept and found to contain new terms. These expressions have been incorporated into a computer code which has been used to simulate several amplifier designs. The results of these designs and, in particular, the z-dependence of the accelerating voltage are discussed.

Introduction

Considerable interest has been generated recently in free electron lasers utilizing tapered wiggler magnets to provide the pump field (MW FEL),[1] and derivations of the equations of motion for the MW FEL assuming a classical, one-dimensional, single-particle model have been published.[1,2,3] In this paper, we list the equations of motion for the EM FEL with an external accelerating field, which we have presented in detail in an earlier paper.[4]

Gain Enhancement

It has been shown[1] that trapping electrons in "buckets" allows continuous extraction of energy from the trapped electrons. For the variable MW FEL, one maintains the electrons near resonance by decelerating the bucket with respect to the electrons. This is accomplished by variation with z of the periodic wavelength and/or magnetic field strength of the wiggler magnet in various combinations. For the EM FEL, acceleration of the electrons by means of an axial electric field to maintain them in resonance with a constant velocity ponderomotive wave appears to be the best method of enhancing laser gain.

Model Assumptions

The classical, one-dimensional, single-particle Hamiltonian framework is adopted here. It is assumed that the circularly polarized pump and laser fields have only transverse components, although the magnitudes are allowed to be functions of the axial coordinate, z. The scalar potential is assumed to be a function of z only. Electrostatic effects due to the Coulomb forces of the electrons are included in this potential. The evolution of the amplitudes and phases of the fields is determined from linearized forms of Maxwell's equations. The electron beam is assumed to be infinitely long, with an initially uniform phase-space density and energy spread, $\Delta\gamma$. We assume that the cross-sections of the optical and electron beams are equal (filling factor = 1).

Theory

The classical, one-dimensional, single particle Hamiltonian is written in the cgs system

$$H = mc^2\left[1 + \alpha^2 + \left(\frac{P_z}{mc}\right)^2\right]^{1/2} + e\,\Phi(z) \qquad (1)$$

where

$$\vec{\alpha} = \frac{e\vec{A}}{mc^2} \quad \text{and} \quad \vec{A} = \vec{A}_L + \vec{A}_P. \qquad (2)$$

The transverse momentum is a constant of the motion and it has been assumed that one can arrange the initial conditions such that both $P_x = 0$ and $P_y = 0$. The subscript L refers to the laser field and P, to the pump field.

The vector potentials are given by

$$\vec{A}_L = A_L(z)\left[\hat{x}\cos(\theta_L - \omega_L t + \phi_L) - \hat{y}\sin(\theta_L - \omega_L t + \phi_L)\right] \qquad (3)$$

$$\vec{A}_P = -A_P(z)\left[\hat{x}\cos(\theta_P + \omega_P t + \phi_P) + \hat{y}\sin(\theta_P + \omega_P t + \phi_P)\right] \qquad (4)$$

where $\hat{x}$, $\hat{y}$ are unit vectors, and θ_L, θ_P are defined by

$$\theta_L = \int_0^z k_L(z)dz \quad \text{and} \quad \theta_P = \int_0^z k_P(z)dz \qquad (5,6)$$

Using dimensionless variables:

$$\vec{a}_L \equiv \frac{e\vec{A}_L}{mc^2} \quad \text{and} \quad \vec{a}_P \equiv \frac{e\vec{A}_P}{mc^2}. \qquad (7,8)$$

Then

$$\alpha^2 = a_L^2 + a_P^2 - 2a_L a_P \cos\psi \qquad (9)$$

where

$$\psi = \int_0^z (k_L + k_P)dz - (\omega_L - \omega_P)t + \phi_L + \phi_P. \qquad (10)$$

Equations of Motion

From eq. (1) and (10), the equations of motion are obtained:

$$\dot{\gamma} = -\frac{(\omega_L - \omega_P)}{\gamma} a_L a_P \sin\psi - \frac{e\dot{\Phi}}{mc^2} \qquad (11)$$

$$\dot{\psi} = (k_L + k_P)c\beta_z - (\omega_L - \omega_P) + \dot{\phi}_L + \dot{\phi}_P \qquad (12)$$

and

$$\frac{\dot{z}}{c} = \beta_z = \left[1 - \frac{1 + \alpha^2}{\gamma^2}\right]^{1/2}. \qquad (13)$$

These are the exact equations of motion in (γ,ψ) space and, with $\omega_P = 0$ and $\phi_P = 0$, apply equally to the MW FEL. It can be seen that with an accelerating potential, even a fixed-period, fixed-field MW FEL can be made to provide enhanced gain.

Relative Equations of Motion

Eqs. (11) and (12) may be written for the resonant particle and the phases and energies of the other particles relative to the resonant particle are then obtained by subtracting:

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$$\frac{d}{dt}(\gamma^2 - \gamma_R^2) = -2(\omega_L - \omega_p)a_L a_p(\sin\psi - \sin\psi_R) - \frac{2e\,\dot{\Phi}(\gamma - \gamma_R)}{mc^2}, \quad (14)$$

$$\frac{d}{dt}(\psi - \psi_R) = c(k_L + k_p)(\beta_z - \beta_{z_R}). \quad (15)$$

To obtain an approximate equation for the particle trajectories, the following assumptions are made:

$$\beta_z \cong 1 - \frac{1+\alpha^2}{2\gamma^2}, \text{ i.e. } \frac{1+\alpha^2}{\gamma^2} << 1 \quad (16)$$

$$(\gamma^2 - \gamma_R^2) \cong 2\gamma_R(\gamma - \gamma_R) \equiv 2\gamma_R P \quad (17)$$

and $\dot{\psi}_R$ very small, $\dot{\beta}_{z_R} = 0$. (18)

The latter assumption is the adiabatic assumption which requires the resonant phase to be varied slowly enough that the resonant particle still has the velocity of the ponderomotive wave, β_{z_R}. The approximate relative equations of motion become:

$$\dot{P} = -C(\sin\psi - \sin\psi_R) - DP \quad (19)$$

$$\dot{\psi} = AP + B(\sin\psi - \sin\psi_R) \quad (20)$$

where

$$A = \frac{c(k_L + k_p)(1+\alpha_R^2)}{\gamma_R^3}, \quad B = \frac{c(k_L + k_p)a_L a_p}{\gamma_R^2}$$

$$C = \frac{(\omega_L - \omega_p)a_L a_p}{\gamma_R} \text{ and } D = \frac{e\dot{\Phi}}{mc^2}. \quad (21\text{ a,b,c,d})$$

The two new terms are those with coefficients B and D. The D term arises from the scalar potential and the B term results from retaining the complete form of the magnitude of the vector potential. The orbit equation is then written:

$$\frac{dP}{d\psi} = -\frac{C(\sin\psi - \sin\psi_R) + DP}{B(\sin\psi - \sin\psi_R) + AP}. \quad (22)$$

The above equation is given for completeness and indicates possible distortion to the usual orbit shapes. For the cases considered here, the B and D terms are negligible and the orbit equation becomes that of Ref. (3) and is readily integrable, to give:

$$P = \left\{\frac{2C}{A}\left[\cos\psi + \cos\psi_R + (\psi + \psi_R - \pi)\sin\psi_R\right]\right\}^{1/2}. \quad (23)$$

Variation of the Resonant Phase

The prescription for gain enhancement may now be developed from the resonant particle concept. The acceleration or deceleration of the particles with respect to the ponderomotive wave can be related to the time variation of the resonant phase. Evaluating $\dot{\beta}_{z_R}$ from eq. (13), substituting $\dot{\gamma}_R$ in terms of $\dot{\Phi}$ from eq. (11), and setting the result equal to zero, one obtains:

$$\dot{\Phi} = -\left(\frac{mc^2}{e}\right)\frac{a_L a_p \sin\psi_R}{\gamma_R}\left[(\omega_L - \omega_p) + \frac{\dot{\psi}_R}{(1 - \beta_{z_R}^2)}\right]. \quad (24)$$

The first term in the above expression compensates exactly for the deceleration of the electrons by the bucket, and the second manifests the desired change in ψ_R. The approach then is to relate the programmed change desired for ψ_R which optimizes the FEL amplifier to the accelerating potential $\dot{\Phi}$ by means of eq. (24).

Phase Shifts and Energy Transfer

Maxwell's equations relating the changes in the vector potentials for the laser and pump fields to the transverse electron current have been linearized as in Ref. (5). The resulting equations have been used to determine the amplitudes and phases of the laser and pump fields with time. For the simulations performed in this study, the total phase shift, $\phi_L + \phi_p$, was only 10 mrad over 100 cm and could be neglected.

Coulomb Potential

The Coulomb potential is calculated from the E-field of the charge distribution. Only the derivative is needed, and the following expression is used:

$$E_c(z) = -\nabla\Phi_c = -\frac{1}{c\beta_z}\dot{\Phi}_c(z) \quad (25)$$

The space charge field $E_c(z)$, is inferred from the charge distribution by integration over the periodic electron density. Assuming the net change in the electric field over a beam segment one radiation wavelength long is zero, the difference field can be determined. For simulation purposes, $E_c(z)$ is determined at specified intervals within one wavelength, from which values for any desired point may be determined by interpolation. Then $\dot{\Phi}_c$ is calculated and included in eq. (11).

Computer Simulation

A computer program has been written to determine the trajectories of a number of electrons in (γ,ψ) space by forward integration of the differential equations of motion. A beam segment one ponderomotive wavelength long is considered and periodic boundary conditions are assumed. After each time step, the laser and pump field intensities and phases are updated. The fractional energy conversion (FEC) and gain are calculated from the following:

$$FEC(t) = \frac{1}{\gamma_R(0)}\int_0^t \langle\dot{\gamma}_L\rangle dt, \text{ and } Gain(t) = \frac{a_L^2(t)}{a_L^2(0)}$$

where $\langle\dot{\gamma}_L\rangle$ represents the average over all of the electrons and t = 0 for the initial conditions. The simulations studied here are all for the same laser and pump wavelengths and intensities. The three cases differ only in the programmed variation of ψ_R. It should be noted that for the parameters considered here, there is no bucket growth, and therefore complete trapping is not possible. The results for these cases are shown in Figures 1 - 4.

References

1. "Free Electron Generators of Coherent Radiation," Physics of Quantum Electronics, Vol. 7, Stephen F. Jacobs, ed. Addison Wesley (1980).
2. "Novel Sources of Coherent Radiation," Physics of Quantum Electronics, Vol. 5, Stephen F. Jacobs, ed. Addison Wesley (1978).
3. N. M. Kroll, P. Morton, and M. N. Rosenbluth, "Free Electron Lasers with Variable Parameter Wigglers," JASON Tech. Report JSR-79-01, 1980.
4. H. R. Hiddleston and S. B. Segall, "Equations of Motion for a Gain-Enhanced Free Electron Laser with an Electromagnetic Pump Field," submitted to IEEE J. Quantum Electronics.
5. W. B. Colson and S. K. Ride, Physics of Quantum Electronics, Vol. 7, Stephen F. Jacobs, ed., p. 377. Addison Wesley (1980).

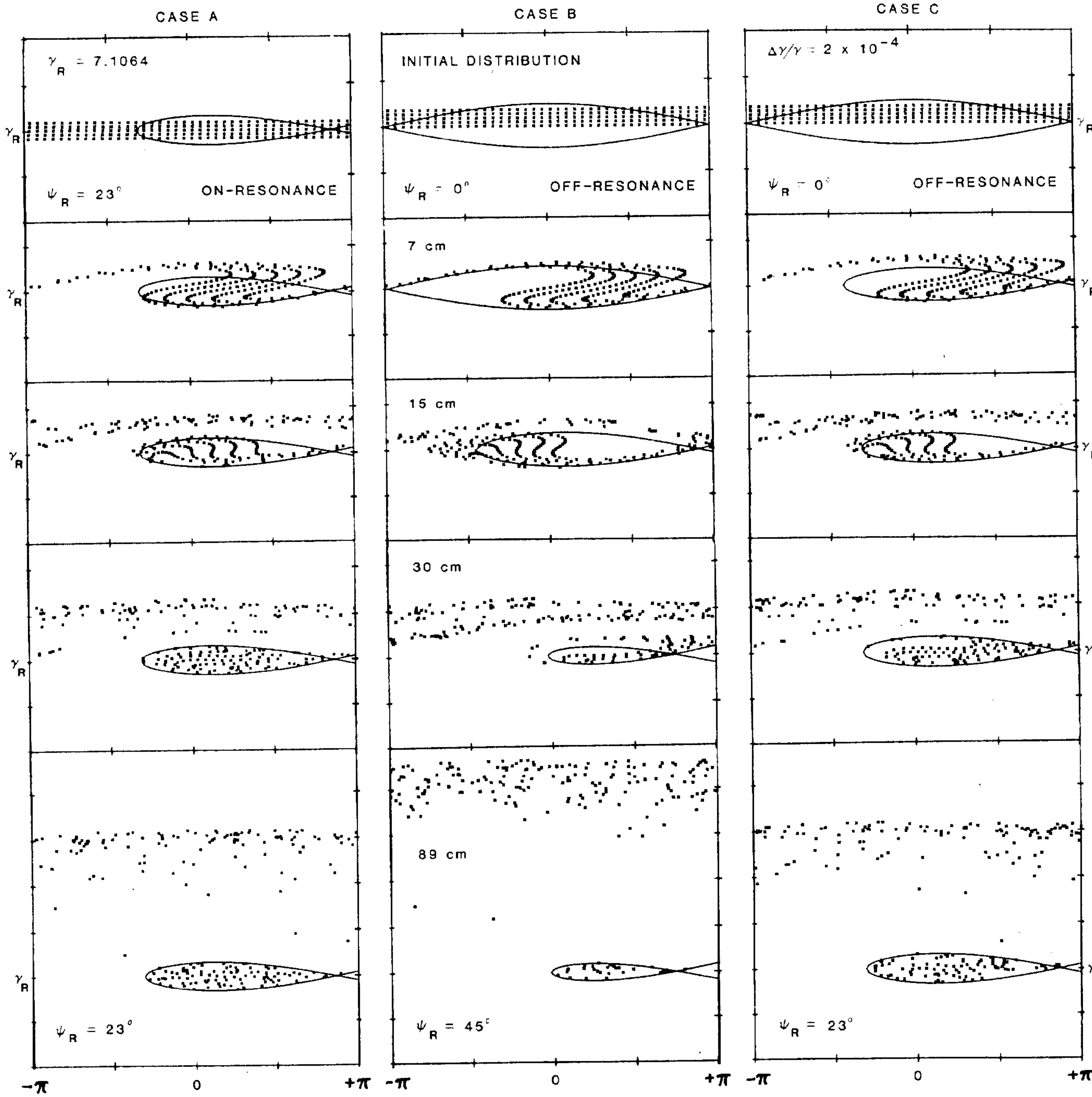

Fig. 1. Phase-space evolution of particles for an FEL amplifier with an electromagnetic pump field and enhanced gain provided by an axial accelerating field. The three cases shown differ in their initial conditions and in the z dependence of the programmed resonant phase.

I_L = 1 GW/cm^2 I_p = 1 GW/cm^2

λ_L = 0.5 μm λ_p = 100 μm

E = 3.1204 MeV N_e = 2 x 10^{11}/cm^3