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A NEW TYPE OF PULSED AIR-CORE MULTIPOLES OF SIMPLE CONSTRUCTION

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Summary

Because of the rising cost of electric energy, pulsed magnets for beam transport systems attract increasing interest. Especially pulsed air-core systems are advantageous because of their very low weight, small volume and the possibility to install them inside the vacuum system utilizing the good insulating properties of vacuum. This paper describes some general theoretical aspects of pulsed air-core systems as well as two given examples in form of a pulsed quadrupole and a kicker magnet operating at DESY. It is well known that magnetic multipole fields of the order of n may be generated by an axial current on the surface of a circular cylinder with a current distribution of the form $I_0\cos(n/2\cdot\varphi)$. Since for beam optical applications only the integral field over the system length counts, it is a straightforward method to use periodic filamentary conductors on the cylindrical surface with a contour which, on averages, gives a cosine distribution. Due to the skin-effect, thin conducting sheets of the same contour are equally suitable. A rather good approximation is already attainable with comb-like conducting sheets, as will be shown in detail. It is evident that this concept leads to air-core systems of extremely simple construction.

Some principal considerations on the approximation of magnetic multipoles by filamentary conductors

For an ideal multipole of the order of n one needs a distribution of axial current of the form $I_0\cdot\cos(n/2\cdot\varphi)$ on the surface of a circular cylinder [1]. It is evident that any geometrical arrangement of conductors with a cosinelike distribution of current on average - taken over the system length - is equally suitable for the approximation of a magnetic air-core multipole for beam optical applications, since only its integral field is relevant for its optical effect. A simple construction may be achieved with periodic filamentary conductors on the surface of a circular cylinder or on the surface of an elliptical cylinder. Such a solution may be applied to DC-operation as well as to pulsed operation. From Biot-Savart's law it may be shown[2] that for the periodic arrangement of fig. 1 the contribution of one conductor period to the integrated field length along the axis - as seen by a particle traversing in the z-direction at $x_p, y_p =$ const - is given by

$$L_o\cdot\bar{\vec{H}}_p = \frac{I}{2\pi}\int_{z=-\frac{Lo}{2}}^{z+\frac{Lo}{2}} \frac{\vec{k}\times\vec{r}_{t(z)}}{r_t^2(z)}\,dz \qquad (1)$$

with the definition of L_o for the period length of $\vec{k}$ for the unit vector in z-direction and of $\vec{r}_t$ for the projection of the vector r between source point Q and field point P on the x,y-plane. The important assumption of $x_p, y_p =$ const for the particle trajectory means that we are dealing with particles almost parallel to the system axis z, at least over a length equal to the period length L_o. Obviously in practice this is always achievable by a proper choice of the period length L_o. Returning to circular cylindrical systems, we are now able to deduce the special conductor geometry on the cylindrical surface, which gives a cosinelike distribution on the average and therefore the ideal field distribution, when averaging over the path length of the particle.

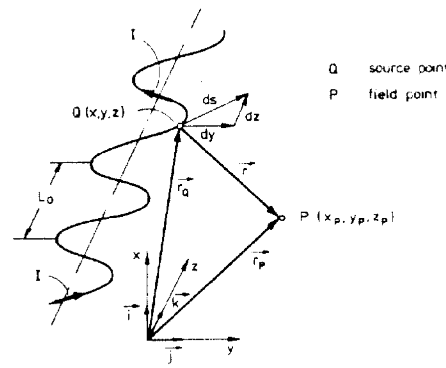

Fig. 1: Periodic filamentary conductor (current I) with symmetry axis parallel to z-axis

According to fig. 2, where we have a periodic conductor with the current I_o and the period length L_o taking into account eq. (1), we have to demand

$$2\,\frac{I_o}{L_o}\,\frac{dz(\varphi)}{r(\varphi)} = \frac{1}{2}\,\frac{I_o}{r(\varphi)}\,\cos(\tfrac{n}{2}\varphi)\,d\varphi \qquad (2)$$

which gives $$\varphi = \frac{2}{n}\arcsin\frac{2nz}{L_o} \qquad (3)$$

or in general $$R\varphi = \frac{2}{n}R\,\arcsin\frac{z}{L_o/4} \qquad (4)$$

with φ...azimuthal angle in cylindrical coordinates
R...system radius.

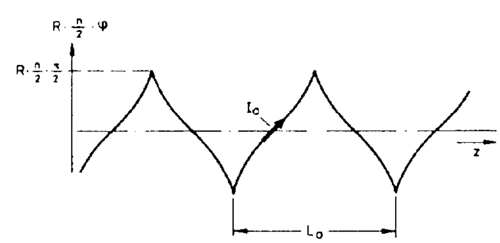

Fig. 2: Periodic filamentary conductor of a circular cylindrical multipole-system unwound in the Rφ, z-plane

Fig. 2 shows the function $R\cdot\varphi(z)$. Nothing will change if we take the mirror image of this shape with respect to the z-axis. When taking both, we arrive at the symmetrical structure of fig. 3, every half period of which looks like a wing. For a dipole e.g. we need two conductor systems of this shape, one extending from $-\pi/2$ to $+\pi/2$ and the other from $+\pi/2$ to $3/2\pi$ on the circular circumference. The two conductor systems of opposite current directions are not allowed to touch each other at the points $\pm\pi/2$, so that in practice the sharp tips have to be cut off a little, causing a certain error as compared to the ideal field. Another error source is introduced by the finite conductor dimensions.

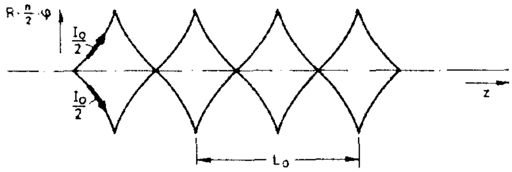

Fig. 3: Symmetrical wing-contour of periodic filamentary conductor

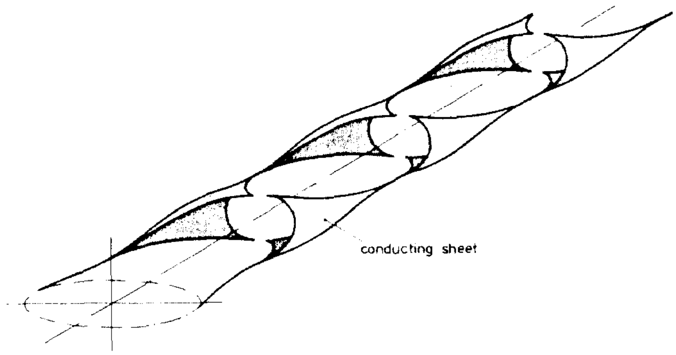

Fig. 4: Schematic view of an elliptical cylindrical dipole with wing-like conductor contours

Field calculation

For beam optical applications often only the field $\overline{B}$, averaged over the total system length including the fringe fields, is of practical interest. Since we have a cosinelike azimuthal distribution of axial current on the average, we may directly make use of the well-known field calculations, which have been treated by several authors [1,4] and which are commonly used for superconducting applications. The same applies to the elliptical case [4]. The corresponding field expressions become:

$$\text{dipole} \qquad \overline{B} = \frac{\mu_0}{4}\frac{I}{R} \qquad (5)$$

circular cylindr.

$$\text{quarupole} \qquad \overline{g} = \frac{\mu_0}{2}\frac{I}{R^2} \qquad (6)$$

$$\text{ellipt.cylindr.dipole} \qquad \overline{B} = \frac{\mu_0}{2}\frac{I}{(a+b)} \qquad (7)$$

with: B flux density; I current; R system radius; a,b half axes of the ellipse

The inductance L of these systems may be calculated from the total magnetic energy content W_m

$$L = \frac{2W_m}{I^2} \qquad (8)$$

which is twice the energy content of the inner systems [5].

Comb-like conductor contours

From fig.'s 2 and 3 we may conclude[5] that an approximation of the contour by a comb-like step function (fig. 5) will give an approximation of the ideal field configuration. It is obvious that in such a comb structure (fig.(5)) the influence of the azimuthal currents (φ-direction) on the average magnetic field is negligeable, since the currents of opposite directions on both edges of a "tooth" cancel each other. Then only the axial current components (z-direction) contribute to the field generation. For the purpose of field calculation the comb structure may therefore be substituted by 4 filamentary conductors (1...4) shown in fig. 5.

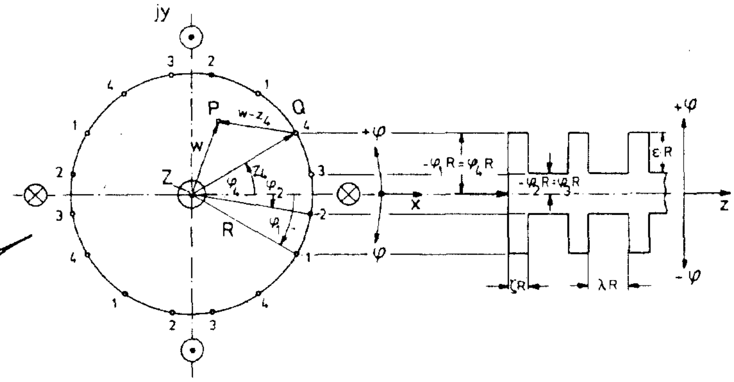

Fig. 5: Quadrupole geometry in complex plane with comb-like conductor

The locations of these conductors on the periphery of the cylinder are determined by the tooth dimensions like this is shown in fig. 5. The resulting distribution of axial current components allows the definition of an equivalent current for each of the four conductors, which may be calculated by averaging over the tooth period:

$$I_1 = I_4 = \frac{\zeta}{2(\lambda+\zeta)} I \qquad (9)$$

$$I_2 = I_3 = \frac{\lambda}{2(\lambda+\zeta)} I$$

where ζR is the tooth length and λR is the gap length between neighbouring teeth. Taking a quadrupole as an example, in the complex plane of fig. 5 the field $\overline{B}_p$ in the point P may be expressed[5] as an infinite series, the terms of which correspond to increasing multipole orders:

$$\overline{B}_p = 2j\,\frac{\mu_0}{\pi}\sum_{\nu=1}^{4}\sum_{k=0}^{\infty}\frac{\overline{w}^{4k+1}}{\overline{z}_\nu^{4k+2}}\,I_\nu \qquad (10)$$

where $\overline{w}$ and $\overline{z}$ denote the complex conjugate of w and z respectively. Then by a proper choice of the parameters $\varphi_\nu, \lambda, \zeta$ the comb structure allows to make at least two terms of this series disappear. Identifying the term B_{p0} (for k=0) as the principal quadrupole term it is then possible to cancel the terms B_{p1} and B_{p2}, representing the 12- and the 20-pole [5]. So the next non-vanishing term would represent the 28-pole. Since the amplitudes of the terms of the series expansion (eq. (10)), taken at the radius r (r<R) decrease as

$$\left(\frac{\overline{w}}{R}\right)^{4l+1} = \left(\frac{r}{R}\right)^{4k+1}$$

we may expect a reasonable good approximation already with this simple comb-structure. For a quadrupole a field accuracy of better than 1 % is expected inside up to 67 % of the aperture [5]. As was pointed out earlier, the operation of this kind of systems with filamentary conductors is possible with DC as well as with pulses. However it is interesting to notice that in the pulsed mode of operation with short pulses of a few μs, one may apply comb-like conducting sheets making use of the skin-effect, which leads to a current flow on the outer contour of the sheets only, so that one has a quasi filamentary conductor. This concept allows a very simple construction of the total system and has been realized at DESY for a quadrupole (s. fig.'s 6 and 7) and for a kicker magnet with the values given in the following table. The copper comb-sheets - 1.5 mm thick - are directly installed inside the vacuum-system and the feed-throughs serve the purpose of support, radial adjustment and electrical connections. So the total system has a weight, which is hardly more than that of the vacuum tube, which is necessary anyway.

	kicker	quad.
length	400 mm	400 mm
diameter	56 mm	50 mm
inductance	0.22 μH	0.4 μH
voltage, max.	10 kV	10 kV
current, max.	5 kA	5 kA
pulse length	1.1 μs	5 μs
field/current	0.011 T/kA	0.935 T/m/kA
repet. rate		50/12 1/s

table: Data of an air-core kicker and a quadrupole realized at DESY

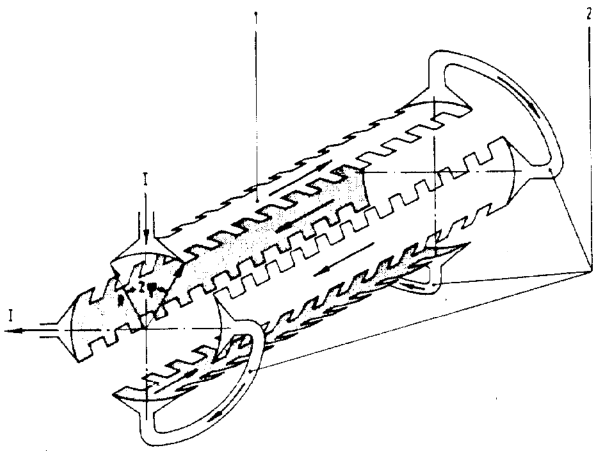

Fig. 6: Schematic view of pulsed quadrupole with comb-like conducting sheets
1 comb-sheet, 2 series connections

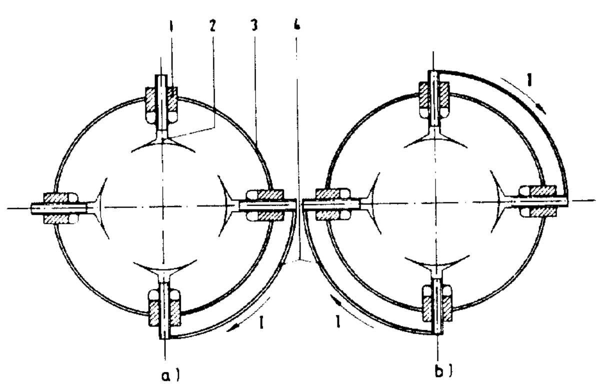

Fig. 7: Cross-sectional view of a quadrupole with comb-like sheets inside a vacuum tube on front a) and back b) side (schematic)
1 feed through, 2 conducting sheets,
3 vacuum tubes, 4 series connections

Technical aspects

It has been shown in this report that it is principally possible, to find a filamentary conductor geometry giving a cosinelike distribution of axial current on the surface of a circular or elliptical cylinder on the average. With this concept it should be possible to construct multipoles of high accuracy without ferromagnetic materials. For the practical realization of such systems we have suggested two solutions: The wire-like and the sheet-like conductor. For both techniques the influence of finite conductor dimensions on the accuracy has to be investigated in the future. In the case of a wire-like conductor a circular tube-conductor would be preferred with the advantage of an effective utilization of the conductor cross-section due to the skin-effect on the one hand and the possibility of direct watercooling on the other hand. Although the average Ohmic losses are expected to be in the order of a few hundred Watts only at pulse current amplitudes of the order of 10 kA and at duty factors around 10^{-3} as they normally occur, a watercooling is generally necessary, since the conductor-systems are to be installed inside the vacuum system. Concerning conductor heating two effects have to be taken care of: Adiabatic pulse heating and average heating. For both the max. allowable current density may be calculated as a function of pulse length and duty factor[6]. For a directly watercooled circular tube conductor of copper with an outer diameter of 4 mm as an example the allowable current amplitude is 23 kA at a pulse width of 120 μs and a rep. rate of 50/6 Hz. The adiabatic temperature rise per pulse in this case would be well below 50 K. These values may be interesting for high field dipoles. In the case of conducting sheets with wing- or comb-contours the exact current distribution over the sheet cross-section has to be further studied, especially its dependence on the pulse length. For high accuracy multipoles the assumption of quasi-filamentary currents on the sheet-contours may be too rough an approximation so that a modification of the ideal filamentary conductor geometry as derived in this paper would become necessary. However the principal idea of periodic sheet-contours is not effected by this. Obviously the sheet-solution allows a more rigid construction even when using very thin sheets, which makes it superior to the wire-solution in small-aperture-applications. At DESY prototype dipoles of the elliptical type are under construction now, which will be used for principal studies.

Acknowledgement

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