IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, June 1981

A SYNCHROTRON FREQUENCY SPLITTING CAVITY - Q_s CAVITY - FOR THE S.R.S. STORAGE RING

by
T.E. Swain
Science Research Council, Daresbury Laboratory, Warrington WA4 4AD, UK.

Summary

An additional r.f. cavity powered at a frequency 3h/2 × the orbit frequency is to be installed in the SRS. This modifies the coupled-bunch mode spectrum and thus the guide vessel/beam interaction. In particular the coupled-bunch modes that interact with the r.f. cavity system and with the tune-splitting cavity are themselves coupled and the consequences of this upon the stability of these systems are considered.

Introduction

The SRS storage ring is designed to operate from 0.6 GeV to 2.0 GeV with a harmonic number h=160, orbit frequency f_o = 3.125 MHz, and a specified current of 1 A. Following reported experience with other machines, and theoretical studies[1], problems with coupled bunch instabilities were anticipated from the outset that appeared likely to limit the attainable current.

Two levels of "active" attack are possible to discourage coherent motion; a circumferential modulation of beam properties to upset bunch/ bunch coherency, and a within bunch variation of properties to discourage internal coherency. The former may be effected by an r.f. cavity driven at an integral multiple of the orbit frequency; - but not an r.f. harmonic - which will impart a bunch-to-bunch spread in synchrotron frequencies. This technique has been used at CEA, ADONE, DORIS and SPEAR.

Choice of Frequency

Ideally, this would be based on the nature of the impedance in the completed machine, with further considerations, that the effectiveness of such a device in terms of frequency split obtained for given r.f. power consumption increases with frequency, and also that the device must not drive an instability mode with its own impedance. The easiest way to be sure of the latter is to use an odd multiple of $hf_o/2$, and 750 MHz ($3hf_o/2$) rather than 250 MHz was chosen for the technical reasons of power source, and cavity size. This gives a rather special modulation of the bunches into two groups. The device may however be tuned and powered at $(3h/2 + 1)f_o$ in order to impart a sinusoidal modulation of frequencies, although this may drive an instability on its own account above a certain current threshold.

Theory of Operation

The incoherent particle motion in machines with multiple r.f. systems has been treated extensively in the literature; particularly for the specific case of frequency splitting by Allen et al[2]. This aspect will not be considered here, except to note that as quantum lifetime is over-voltage dependent, care must be taken that at high energy the lifetime of alternate bunches is not severely reduced since the present design of the SRS is rather critical in this respect. The theory of coherent motion in azimuthally non-uniform machines has only received limited attention in the literature; Mohl[3], and Sacherer[4] derived some fundamental results using a different approach to the following work.

Operation of an r.f. cavity powered at $3hf_o/2$ produces two families of bunches having incoherent particle frequencies, neglecting any within bunch frequency spreads.

$$\Omega_{s1,2} = \Omega_{s+,-} = f_o \sqrt{\frac{2\pi h\alpha p}{E_o} (V \cos \phi_s \pm m \, Vm \cos \phi_{sm})} \quad (1)$$

$$= f_o \sqrt{\frac{2\pi h\alpha p}{E_o} V'_{+,-}} \quad (2)$$

with m = 3/2, Vm the effective r.f. voltage on the Q_s splitting cavity, and V the main r.f. voltage.

For convenience it is intended to operate with ϕ_{sm}, the bunch crossing phase associated with Vm, equal alternately to 0 and π, which will require fairly precise phase control of V_m. With uniform bunch intensities around the circumference, there should be no 750 MHz stationary component, to upset the phasing; in practice the operation of the device itself, may produce a modulation in bunch intensities resulting in an induced voltage which will require compensation by detuning, as with the main r.f. system.

If we ignore circumferential non-uniformities - other than the bunch/bunch frequency split produced by the Q_s cavity - we observe that each family of h/2 identical bunches must oscillate coherently. With frequency ω_c, and a constant bunch-to-bunch phase shift in synchrotron motion, the spectrum of one such group may be readily deduced in terms of a reference δt_i, for the i^{th} group.

$$I(t) = \underset{\text{component}}{\text{stationary}} - j \frac{I\omega_o}{h} \delta t_i \sum_p p \, A_p \, e^{j(p\omega_o+\omega_c)t} X(p) \quad (3)$$

$$(\omega_o = 2\pi f_o)$$

$$\text{where bunch/bunch phase shift} = \frac{4\pi k}{h} \quad (4)$$

and

$$X(p) = \sum_{n=0}^{h/2-1} e^{-j(4\pi/h\,(p-k)n)} = \frac{h}{2} \text{ if } p = k \pm \frac{\lambda h}{2} \quad (5)$$

$$\lambda = 0, \pm 1, \pm 2 \ldots.$$

$$= 0 \text{ otherwise}$$

(I the d.c. total beam current)

k may be regarded as mode number, once chosen specifies the p to be used in summation $\sum_p$. $0 \leqslant k \leqslant h/2$.

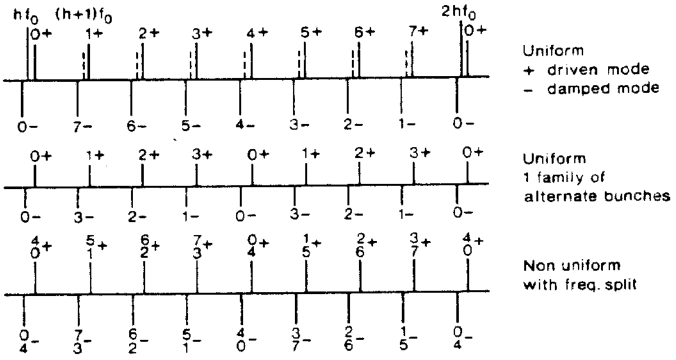

Fig.1 Mode Coupling h = 8

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This spectrum of upper and lower sidebands, repeating every $hf_o/2$ is added to that of the other group of bunches at phases depending on the synchrotron phase shift between bunches in the different groups. If all bunches are identical then all bunch/bunch phase shifts are identical, and two modes of addition are possible giving h possible modes, each having the familiar spectrum of 1 upper sideband and 1 lower sideband in each frequency interval hf_o as discussed in (5). See Fig.1.

Thus the total current spectrum may be generated by superposition, and the "effective" voltage Vk deduced from the overlap with the machine impedance $Z(\omega)$, subject to certain reservations. This leads to equations of motion for a representative bunch in each group, and a determinantal equation for the eigenfrequencies ω_c of the coupled system.

$$\begin{vmatrix} A_{11K}(\omega_c) & A_{12K}(\omega_c) \\ A_{21K}(\omega_c) & A_{22K}(\omega_c) \end{vmatrix} = 0 \qquad (6)$$

In the expressions for V_k there are scalar products of current (Fourier component A_p) and impedance $Z(\omega)$. This form of calculation of voltage is thus only strictly valid for steady Fourier components of constant amplitude, i.e. zero growth or damping, real ω_c. The consequential calculation of ω_c is thus only valid for stability limits, and will become increasingly inaccurate as growth rates become large comparable to time constants associated with $Z(\omega)$, and, indeed on account of the averaging, with the orbit period. Also ω_c appears explicitly in the impedance function $Z(\omega)$. For broad band impedance, ie low Q resonator, or resistive wall background, this local dependence may be suppressed as any real frequency shifts are likely to be small compared to the bandwidth, and $Z(p\omega_o+\omega_c)\approx Z(p\omega_o+\Omega_s)$ for given p where Ω_s is an average incoherent synchrotron frequency. For high Q systems such as the main r.f. cavities and Q_s splitting cavity this will not hold (for other than very small currents) and it is necessary to consider this dependence.

CASE A. Broadband (low Q) Impedance

Proceeding from the determinantal equation suppression of the ω_c dependence in the impedance term, we obtain a quartic equation for ω_c.

If the elements of the determinant are linearised the following expression is obtained for the coherent frequency ω_c for a specified k. $0 \leqslant k \leqslant h/2$

$$\omega_c = \frac{\Omega_{s1} + \Omega_{s2}}{2} + \frac{(\alpha + \beta)}{2} \pm \frac{1}{2}\sqrt{D} \qquad (7)$$

where $D = (\Omega_{s1} + \Omega_{s2} + \alpha + \beta)^2 - 4\big((\Omega_{s1} + \alpha)(\Omega_{s2} + \beta) - \gamma\big)$ (8)

$$\alpha = j\frac{\Omega_{s1}}{2V'_+}\sum_\lambda (g(\lambda)) \qquad (9)$$

$$\beta = j\frac{\Omega_{s2}}{2V'_-}\sum_\lambda (g(\lambda)) \qquad (10)$$

$$\gamma = j\frac{\Omega_{s1}\Omega_{s2}}{4V'_+V'_-}\sum_\lambda (g(\lambda)e^{-\lambda\pi})^2 \qquad (11)$$

where $p = k \pm \frac{\lambda h}{2}$ λ takes 0, ±1, ±2....

$$g(\lambda) = p(\lambda)\, A_p^2\, Z(p(\lambda)\omega_o + \Omega_s) \qquad (12)$$

Since k is restricted to h/2 values, for each value of k, two values of ω_c are created, giving a total of h modes for the composite system of h bunches. For the uniform machine, $\Omega_{s1} = \Omega_{s2}$, the summations give the line spectrum repeating every hf_o, rather than the more dense but less strong spectrum of the non-uniform machine.

One can thus briefly summarise how the Q_s splitting device works in terms of mode coupling and beam spectrum. In the unperturbed machine a mode is selectively driven and damped by the overlap of its spectrum with resistive impedance and spectral lines occurring adjacent to orbit frequency harmonics at $p = k \pm h, p > 0$, giving upper sidebands which cause growth (above transition), $p < 0$ being interpreted as lower sidebands causing damping. When Q_s splitting is applied these side bands are reduced in strength for a given ampli- amplitude of oscillation and thus the magnitude of the instability driving force is reduced for a given current × impedance product. Other sidebands are introduced at $p = k \pm \lambda h/2$ (where $\lambda = 1,3,5..$), but if these do not overlap any impedance, then there is a simple reduction in growth rate following the reduction in line strength. This is expressed in Fig.2 for a frequency split normalised to the frequency shift which would be produced by the instability driving force, and suggest that as these become comparable, a growth rate is halved^4.

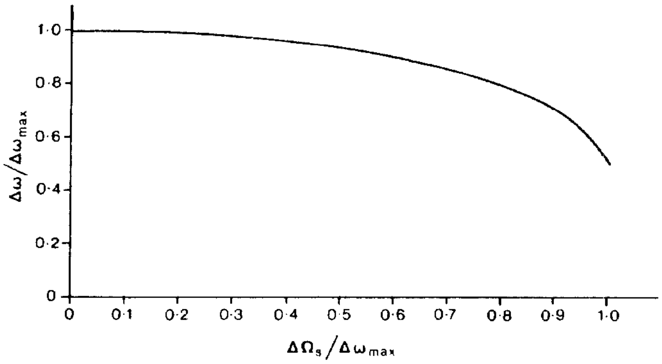

Fig.2 Calculated reduction in growth rate of instability v. frequency split for resonance overlapped by single spectral line.

If however these additional spectral lines do overlap an impedance, then damping could be reduced or enhanced depending whether an upper or lower sideband is overlapped. In the absence of a detailed knowledge of the frequency dependence of the guide-vessel impedance, the behaviour of the device becomes more speculative.

Thus in order to predict the performance of the device, a resonance chart of the guide vessel impedance is necessary since it is possible that anything from zero suppression to complete mode suppression will be obtained, depending on the initial growth rate of the k, k + h/2 modes (w.r.t. unperturbed machine) now coupled.

Case B. High Q impedance - The r.f. system and Q_s cavity interaction

The main r.f. system interacting with the k = 0 mode, and the Q_s cavity interacting with the h/2 mode, become coupled when the latter cavity is powered. It has been demonstrated theoretically, and observed in practice that the main r.f. system may become unstable during stacking unless suitable steps are taken, and and thus it was felt that this coupling of the two r.f.

systems requires examination: The starting point here is the determinantal equation with the explicit frequency dependence of $Z(\omega_c)$ not suppressed in the driving force terms, since large frequency shifts are to be expected in the interaction with the high Q, high impedance r.f. cavities. All the elements of the determinant are now functions of ω_c, and a characteristic equation may be derived.

This yields an equation of the eighth degree, implying four upper and lower sidebands about all integral multiples of $hf_o/2$. For $\Omega_{s1} = \Omega_{s2}$ (i.e. Q_s cavity unpowered but still present), the system resolves into two Quartics (Robinson) describing the now independent interactions with the r.f. and Q_s cavity systems. The Robinson Quartic for the main r.f. system and in particular associated stability criteria have been extensively discussed elsewhere[6] and will not be considered here except in relation to the perturbing effect of the driven Q_s cavity. The unperturbed spectrum consists of two lower and upper sidebands about multiples of hf_o. One frequency is related to the incoherent synchrotron frequency with both real part (frequency) and imaginary (damping) increasing with beam current for fixed operating voltage, the other is related at low current to the cavity detune frequency ($hf_o \tan \psi_L/2$) and damped $e^{-\alpha t}$, this damping falls off with increasing current in general, and changes sign (growth) when instability is promoted for instance by use of inadequate detune of the main r.f. cavities for a given situation. Use of the Q_s splitting cavity adds additional spectral lines related now to both incoherent frequencies, and cavity detune frequencies, which appear about multiples of $hf_o/2$.

To attempt to formulate stability criteria for the perturbed case it is desirable to make the assumption that the Q_s splitting cavity is tuned exactly to $3hf_o/2$. Under the approximation used in deriving the equations of motion, the upper and lower sideband effects cancel and the beam does not now "see" the cavity in the dynamic state. This tuning is of course, only strictly possible if there is no $3hf_o/2$ stationary component in the beam, i.e. the circumference fill is uniform. The characteristic equation is then simplified by omission of the Q_s splitting cavity parameters, and is reduced to the 6th degree, with absence of the frequency related to the Q_s splitting cavity detune.

$$\prod_{i=1,2} \Big((-\omega^2 + \Omega^2_{si})\big((j\omega_c + \alpha)^2 + \alpha^2\tan^2\psi_L\big) - \frac{\Omega^2_{si}}{V'_i}\, h\, \frac{I}{2}\, \alpha^2 \tan \psi_L Z_L \Big)$$

$$= -\frac{\Omega^2_{s1}\,\Omega^2_{s2}}{V'_1\,V'_2}\Big(h\,\frac{I}{2}\,\alpha^2 \tan \psi_L Z_L\Big)^2 \qquad (13)$$

where $V'_1 = V'_+$, $V'_2 = V'_-$

Z_L = r.f. cavity shunt impedance.

Descartes rule for signs yields the two following necessary criteria for a stable solution

$$\Omega^2_1\Omega^2_2 + \alpha^2(1+\tan^2\psi_L)(\Omega^2_1+\Omega^2_2) - V_L\,\alpha^2\,\frac{\tan \psi_L}{2}\left(\frac{\Omega^2_2}{V'_-} + \frac{\Omega^2_1}{V'_+}\right) > 0 \qquad (14)$$

$$1 + \tan^2\psi > V_L\,\frac{\tan \psi_L}{2}\left(\frac{1}{V'_-} + \frac{1}{V'_+}\right) \qquad (15)$$

(V_L beam induced voltage in resonant cavity)

the latter being related to the usual Robinson criteria[6], with a suitable "average" voltage gradient term in denominator on R.H.S. However these are only necessary conditions, Routh-Hurwitz gives a set of sufficient conditions, of increasing complexity and difficult to interpret in terms of operating conditions. As expected, we have

$$\tan \psi_L < 0, \qquad (16)$$

but in absence of a complete usable set of sufficient conditions recourse may be made to solution of the characteristic polynomial. The condition (26) has an interesting implication. For the unperturbed system, if the r.f. cavity has adequate detune to present a resistive match to the source, it is stable automatically. This now breaks down under such operating conditions that

$$Ib\,\delta V > \frac{V(1+\beta)}{Z_L} \qquad \text{where } \delta V = \frac{V_m}{V}$$

(β, cavity coupling factor)

a condition that will be attained in the SRS at injection energy, and necessitate excess detune. This theory requires extension to systems with feedback, by suitable definition of $Z(\omega_c)$. (In progress).

Parameters

The system will use a single cell cavity of conventional design, driven by a klystron amplifier and have the following parameters.

Shunt Impedance	6.5 MΩ
Frequency	750 - 753.125 MHz. Tunable (500 MHz main r.f.)
Q	29000
R.f. Power	10 kW. cw. at 750 MHz
Peak effective voltage	0.25 MV at 750 MHz
(Main r.f. voltage	0.6 MV at 600 MeV 1.9 MV at 2.0 GeV)
Q_s unperturbed	0.06
ΔQ_s (design)	0.012

Conclusions

This study has considered some aspects of a rather special form of Q_s splitting, and demonstrates that a gain of stored current by a factor of 2 may well be achieved if longitudinal coupled bunch instabilities are a limitation, at the expense of a slight interaction with the main r.f. system.

Acknowledgements

I wish to gratefully acknowledge discussions with G. Saxon of this establishment, also earlier discussion with F.Sacherer on a visit here in 1976.

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