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A SIMPLE MODEL FOR THE ENERGY LOSS OF A BUNCHED BEAM TRAVERSING A CAVITY

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Summary

Based on a simple model of the field pattern, an analytical formula for the energy loss of a relativistic bunched beam traversing a cavity is derived, for a range of parameters where the field lines do not spread and where reflections from the cavity walls are absent. The parameters of the formula are the beam tube radius (a), the gap length (g), the bunch length (σ), and if necessary, a nose cone angle (θ). The formula is applied to several cavities with various shapes, for which accurate data from computer programs are available. It is found that this model agrees with the data within an accuracy of 30 % for $0.2 < \sigma/a < 2.0$ when $g/\sigma > 2$ and σ is less that half the outer radius of the cavity.

1. The model

The exact computation of the transient electromagnetic field produced by a bunch of charged particles traversing an accelerating cavity at the speed of light is a very complicated matter. For designing an accelerator the particular shape of such fields is sometimes not as important as quantities like the total energy lost into the cavity.

Studying the transient behaviour of the field lines in a cavity, one sees that the field lines are often almost simply shaped. As an example we show the transient field of a Gaussian bunch (σ = 7 cm) traversing a 200 MHz cavity, see Fig.1.

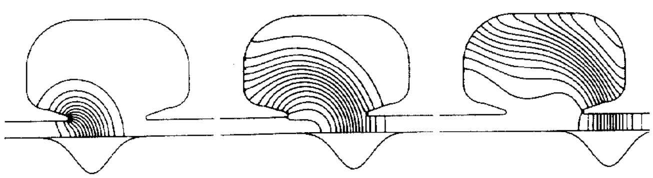

Fig. 1 The transient electromagnetic field of a Gaussian bunch (σ = 7.0 cm) passing a cavity with side tube of radius 7.0 cm.

These fields have been computed with the program BCI[1,2]. There seem to be no strong reflections when the bunch arrives at the left nose cone. When the bunch leaves the cavity, the field lines seem to be cut off by the right nose cone. The field lines for r less than the tube radius are unaffected by the cavity and are almost radial[3]. A second example shows that the influence of a beam tube with a small radius compared to the bunch-length can hardly be seen. If we increase the beam tube radius to the order of the bunch length, the field spreads out, see Fig.2.

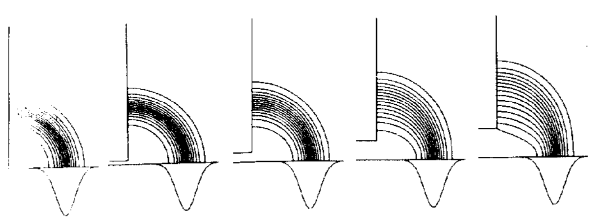

Fig. 2 The electric field lines of a Gaussian bunch (r.m.s. length σ) leaving a tube of radius a for several ratios of a/σ.

When the bunch length is much smaller than the beam tube radius the field becomes more and more complicated and strong toroidal components appear, see Fig. 3.

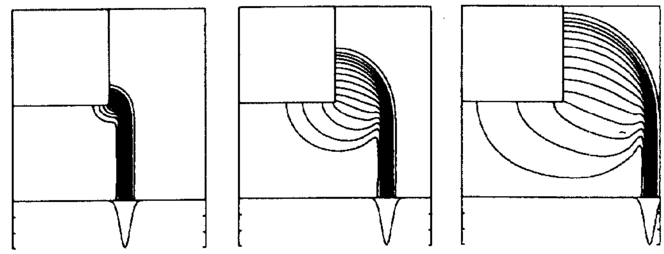

Fig. 3 The electric field lines of a Gaussian bunch leaving a tube. The bunch length σ is small compared to the tube radius.

Based on these results there seems to exist a medium range where the transient field lines keep a simple shape. We will consider now cases only where the bunch is not too short in order to avoid strong toroidal fields, and where the bunch is not too long, in order to avoid reflections from the radial cavity boundary and from the right end of the cavity returning to the beam.

The basic assumptions for the following model are:

1) Reflections at the first corner can be neglected.
2) The spread of the field lines is small.
3) The field lines are straight from the axis up to the beam tube radius and circles from there on.
4) The second corner acts as scissors cutting off the field lines at the tube radius.
5) The cavity radius is large enough that reflections from the outer wall cannot return to the beam.

Fig. 4 shows the idealized field pattern in a cavity with nose cones.

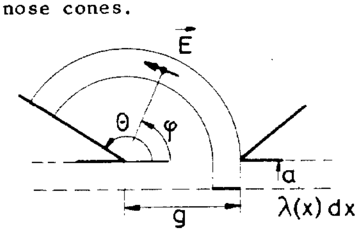

Fig. 4 The idealized field pattern of a bunch passing a cavity and the definition of the parameters a, g and θ.

The electric field for r greater than the beam tube radius a may now easily be written as

$$E = \frac{1}{2\pi\,\varepsilon_0} \cdot \frac{1}{a+g\,\sin\phi} \cdot \lambda(x) \quad , \qquad (1)$$

a = beam tube radius ,
g = gap length ,
$\lambda(x)$ = line charge density ,

by using Gauss' law.

Hence the field energy due to a bunch slice dx in the region r > a becomes :

$$dU = \frac{1}{4\pi\,\varepsilon_0} \cdot \frac{g\,d\phi}{a+g\,\sin\phi} \cdot \lambda^2\,dx \quad . \qquad (2)$$

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Thus the total energy lost by the bunch is given by :

$$U = \frac{1}{4\pi\ \varepsilon_o} \int_0^{\theta} \frac{g\ d\phi}{a+g\ \sin\ \phi} \int_{-\infty}^{+\infty} \lambda^2\ (x)\ dx \qquad (3)$$

Using the abbreviations :

$$f(a/g,\ \theta) = \int_0^{\theta} \frac{g\ d\phi}{a+g\ \sin\ \phi} \qquad (4)$$

$$L(\lambda) = \left[\int_{-\infty}^{+\infty} \lambda\ (x)\ dx\right]^2 \Bigg/ \int_{-\infty}^{+\infty} \lambda^2\ (x)\ dx \qquad (5)$$

We can rewrite Equ.(3) for the commonly used loss parameter as :

$$k_{tot}\ (L) = \frac{U}{Q^2} = \frac{1}{4\pi\ \varepsilon_o\ L} \cdot f(a/g,\theta) \qquad (6)$$

with Q as the total charge in the bunch. For a Gaussian bunch the effective length $L(\lambda)$ becomes :

$$L_G = 2\sqrt{\pi}\ \sigma . \qquad (7)$$

Thus the total loss into the cavity goes like $1/\sigma$.

The function f may be written as :

$f(a/g,\theta)$ =

$$\frac{g}{y}\ \ell n \left[\frac{a\cdot\tan\frac{\theta}{2}+g-y}{a\cdot\tan\frac{\theta}{2}+g+y} \cdot \frac{g+y}{g-y}\right];\ y = \sqrt{g^2-a^2} \qquad ;\ g > a$$

$$1 - \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) \qquad ;\ g = a$$

$$\frac{2g}{y} \left\{ \tan^{-1}\left[\frac{a.\tan\frac{\theta}{2}+g}{y}\right] - \tan^{-1}\left[\frac{g}{y}\right]\right\};\ y = \sqrt{a^2-g^2};\ g < a$$

It can be seen that the loss predicted by this model goes to infinity logarithmically as g/a goes to infinity. This is a result of the assumed shape of the field lines at the second nose cone. In reality, reflections of the field from the bunch head interfere with the field from the bunch tail and make the infinity disappear, as in the case of a closed pill-box cavity. This approximation implies also that the inclination angle of the second nose cone is unimportant. In the present model these effects are ignored since they are not significant for all cases studied.

2. Comparison with accurate data

In order to check if the model gives predictions with a reasonably accuracy, we compared Equ.(6) with data for eight different cavities of various shapes. The data were computed with the programs BCI[1,2], KN7C[6] and SUPERFISH[7] :

A) A 200 MHz cavity with nose cones[4]
B) The LEP 353 MHz cavity with nose cones.
C) The 200 MHz PS cavity[4]
D) A simple cavity without nose cones, radius 1 m, gap length 1 m and beam tube radius 0.125 m
E) A simple cavity like cavity D but with a beam tube radius of 0.25 m
F) A cavity with the radius 0.3 m, gap length 0.05 m and tube radius 0.1 m
G) Same as cavity F but with a larger cavity radius of 0.6 m
H) The SLAC linac cavity[8], No. 45

Fig. 5 shows the total loss parameter for these cavities as a function of σ (Gaussian bunch). It is seen that for the range

$$1/5 \le \sigma/a \le 2$$
$$g/\sigma \ge 2$$
$$R/\sigma \ge 2$$

the model is accurate within ± 30 %.

In this range the loss is proportional to $1/\sigma_{r.m.s}$.

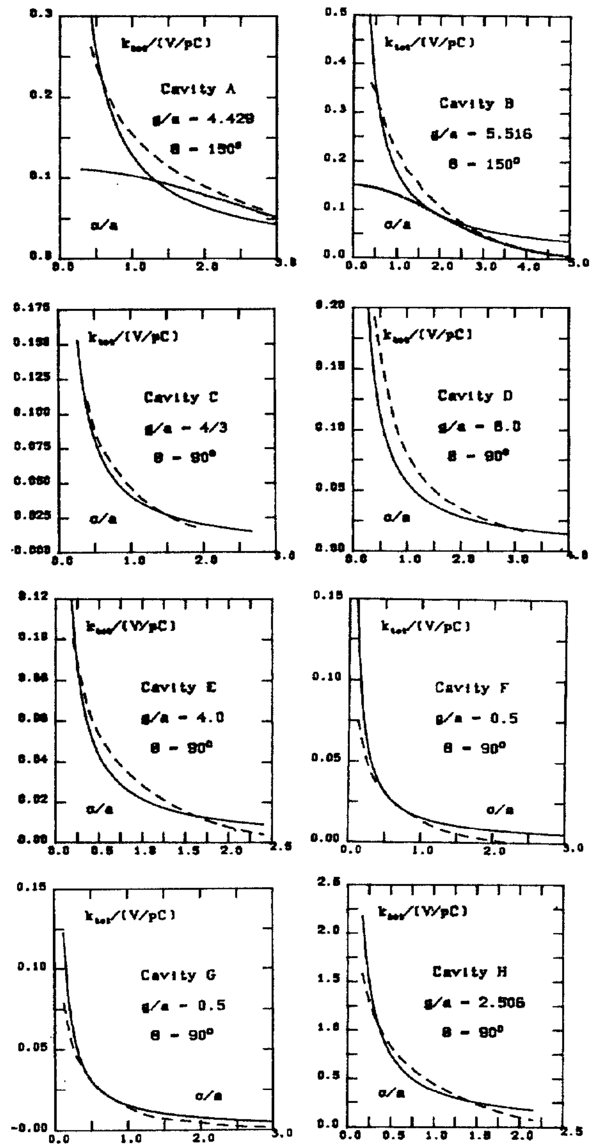

Fig. 5 The total loss parameter k_{tot} for eight different cavities. g = gap length, a = tube radius, σ = r.m.s bunch length, θ = nose cone angle.
—— k_{tot} for the idealized model.
---- results from other computer programs.
.... loss into the fundamental mode.

For the first two examples the fundamental loss parameter has been computed and the loss into the fundamental mode has been plotted also.

Since this model has a 1/L dependence, it is useful to plot the quantity $L(\lambda)\ k_{tot}$ as a function of the parameters g/a and θ.

Fig. 6 shows a loss table for different nose cone angles within the range of validity mentioned above. Finally, Fig. 7 shows the θ dependence for several values of g/a. These two figures cover the parameters of most realistic cavities for medium sized bunches.

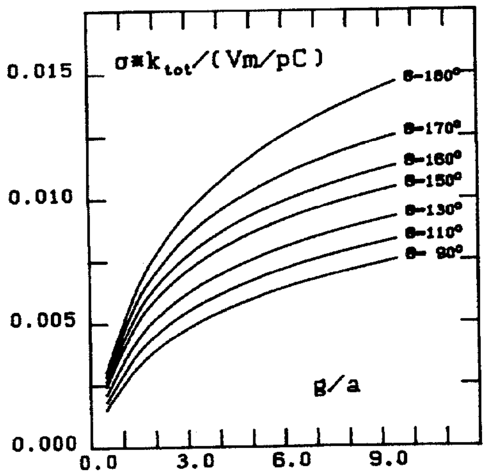

Fig. 6 $\sigma \cdot k_{tot}$ predicted by the model as a function of gap length/tube radius for several nose cone angles θ. These curves are valid for $0.2 \leq \sigma/a \leq 2$.

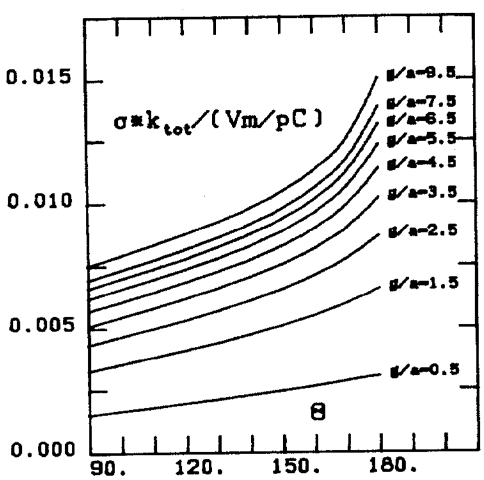

Fig. 7 $\sigma \cdot k_{tot}$ predicted by the model as a function of the nose cone angle θ for several ratios of gap length/tube radius. These curves are valid for $0.2 \leq \sigma/a \leq 2$.

3. Conclusions

We have derived a simple analytical model for the total energy loss of a relativistic bunch traversing a cavity for a large range of cavity parameters and a middle range of bunch lengths. It has been shown that this model is accurate within ± 30 % for various shaped cavities.

This model and a knowledge of the fundamental mode loss should be sufficient to estimate the total loss for most realistic cavities and for normal bunch lengths, but the model is not supposed to replace the computer programs mentioned above, which give results with much higher accuracy.

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