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A COMPUTER SIMULATION OF INJECTION INTO A SUPERCONDUCTING SYNCHROTRON

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Summary

Persistent currents circulating in the windings of a superconducting synchrotron produce strong sextupole field imperfections. Apart from the chromaticity which results, and which may be corrected with other sextupoles, stop-bands are excited which limit the region of the working diagram in which the beam will survive at injection when the effect of persistent currents is most felt. In this computer simulation we studied how small closed orbit distortions and a residual chromaticity aggravate this situation using as a model a machine of 6.5 km circumference and including synchrotron motion.

The Lattice

The lattice used for this simulation was that of the superconducting proton synchrotron whose design is under study at DESY. This proton ring, together with an electron ring in the same tunnel form the HERA e-p project[1]. It is very similar in parameters to the Fermilab energy doubler/saver ring. The only difference likely to affect beam dynamics is the higher ν value, 26.6, of the HERA ring. This has been deliberately chosen to be higher than the Fermilab ring to minimise the chromaticity produced by sextupole fields. It was the aim of this study to discover whether with this higher ν it is possible to inject at an energy as low as 40 GeV. Table 1 lists the parameters of the lattice.

Table 1 : HERA Lattice Parameters

Energy	(GeV)	840
Circumference	(m)	6451.2
Number of superperiods		4
Lattice		FODO
Straight section length	(m)	360
Number of cells		80
Cell length	(m)	62.64
Number of cell dipoles		640
Number of quadrupoles		160
Magnetic length of dipole	(m)	5.6886
Bending field	(Tesla)	4.83563
Bending angle	(mrad)	9.8175
Bending radius	(m)	579.436
Magnetic length of quad.	(m)	1.8
Working point ν_x/ν_z		26.55/26.60
Nominal phase advance per cell	(°)	100
Cell quad. strength	(m^{-2})	- 0.02366
Max. cell beta function	(m)	111.57
Max. cell dispersion	(m)	4.29
Emittance $\varepsilon_x/\varepsilon_z$ *	(πmm.mrad)	0.4584/0.22925
Injection energy	(GeV)	40

* $\varepsilon = 4\sigma^2/\beta$ at injection energy of 40 GeV

The Effect of Persistent Currents

We take our information on measurements of sextupole field due to persistent current from the Fermilab Energy Doubler Design Report[2] assuming the same magnet aperture and filament diameter. Fig. 1 from Ref. 2 shows the measured persistent sextupole term as a function of excitation. The curve is displaced upward by a geometric sextupole which we remove from our calculations. We assume the magnet current is cycled below the injection level somewhat and then brought up to 200 A injection value to avoid a rapid transition from the upper to lower branch of the curve at the beginning of acceleration. The current of 200 A is 1/20th of the excitation peak and corresponds in the HERA case to 40 GeV/c. We find a sextupole error of $K' = B''/B\rho = -0.0107\ m^{-3}$. This represents a $B''/B\rho$ which is about 18 times that in an average CERN SPS magnet at injection[3], the SPS being a machine of comparable size and dynamical properties.

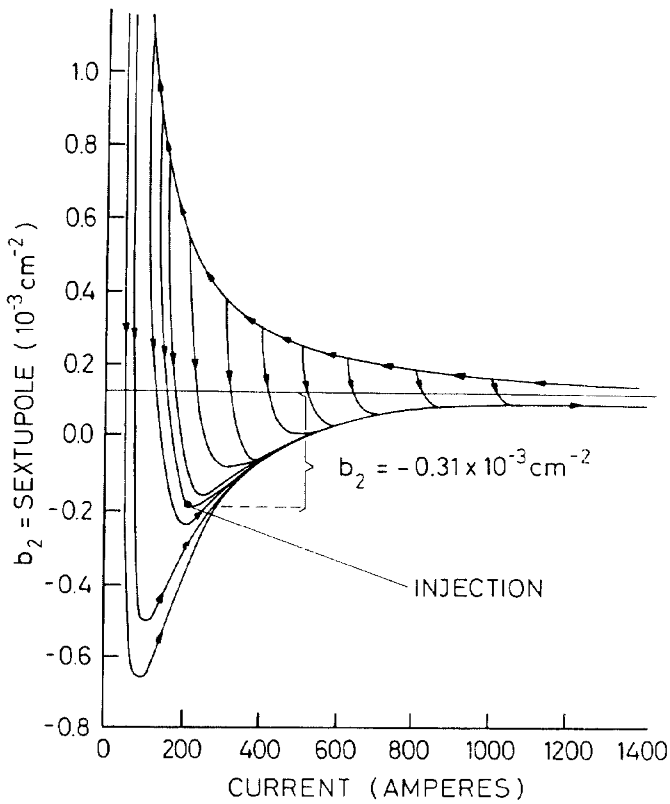

Fig. 1. Hysteresis cycle of persistent current sextupole effect in a dipole

We define the chromaticity :

$$\xi = \frac{p}{\nu}\frac{\partial\nu}{\partial p} = \frac{1}{4\pi\nu}\oint K'(s)\ \beta(s)\ D_x(s)\ ds \qquad (1)$$

It can be seen from this expression that machines which have a high ν, and hence also a low β and dispersion D_x, will have a small chromaticity for a given energy and sextupole error. This reduction in sensitivity to sextupole error is a strong argument for a high working point. We have chosen a Q value close to 27 for HERA in order to try to counterbalance the low 40 GeV injection energy.

We have used the program AGS to compute the chromaticity and the sextupole strength necessary to cancel

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these chromaticities with a simple pattern of sextupoles, one family close to F quadrupoles (SF) and one family close to D quadrupoles (SD) in normal cells. Each family has 80 sextupoles 50 cm long. Table 1 shows the results of these calculations including the combined effect of natural and persistent chromaticity at 40 GeV.

Table 1 : Chromaticity and Corrector Strength

	K' (m^{-3})		ξ_H	ξ_V
$B''/B\rho$	SF	SD		
-0.0107	0.0	0.0	16.58	-17.08
-0.0107	0.2633	0.63907	0.0	0.0

Particle Tracking with LIMATRA

The program LIMATRA[4] tracks single particle or groups of particles distributed in six dimensional phase space around a synchrotron noting when individual particles leave the vacuum chamber of the machine. At the end of a specified number of turns, particle coordinates or a phase space distributions are printed to see if surviving particles have departed from ideal linear motion.

The synchrotron model comprises a sequence of non-linear magnetic elements which in this simulation study were the sextupoles, one for each dipole of the HERA lattice, formed by persistent currents. Also included were the chromaticity correcting sextupoles, set to make the chromaticity zero in each plane.

Transport between these elements is described by a simple linear matrix whose parameters are determined by the betatron amplitudes and phases and dispersion associated with the non-linear elements. The matrix for off-momentum particle will differ from that of the synchronous particle only in phase advance. No attempt is made to include the chromatic dependence of other betatron quantities.

An unusual feature of LIMATRA is that the effect of closed orbit distortion at the non-linear elements may be studied. The user specifies an r.m.s. closed orbit amplitude x_{co} or y_{co} and before the tracking commences a Monte-Carlo routine assigns a displacement to each element from a Gaussian distribution with the specified r.m.s. width. Each time the particle encounters an element it receives a kick, calculated from its position in the non-linear field including the closed orbit displacement. It is thought that the random selection of closed orbit amplitude simulates well the situation in a machine where the orbit has been corrected but imperfectly.

Another facility in the program is a partial simulation of synchrotron motion. By specifying a synchrotron frequency one can force the $\Delta p/p$ of the particle to oscillate with this frequency. Its ν value the follows this oscillation and in this way at least the numerology of synchro-betatron motion may be included. The simulation excludes the non-linear momentum dependence of the beta functions, an approximation made deliberately to speed the tracking.

The program was modified to include random fluctuations in the strength of the persistent current sextupole fields among the magnets of the lattice.

Results of Simulation

Table 2 shows the widths of stop-bands found by single particle tracking with LIMATRA. The momentum error of the particle was zero and the ν stepped to scan along a line normal to the stop-band. We assume that the particle is outside the stop-band if it survives 300 turns at a given ν value. The four resonances tabulated are those to be expected from a systematic pattern of sextupoles with four fold symmetry and the half integer stop-bands from the combined effect of closed orbit error within the sextupole pattern.

We see from Table 2 that LIMATRA demonstrates the expected independence of the third order stop-bands on closed orbit amplitude while the half integer stop-bands show the expected linear dependence.

The pure horizontal ($3\nu_H = 80$) stop-band is encouragingly narrow for a structure resonance but the skew resonance $\nu_H + 2\nu_V = 80$, turns out to be a nasty surprise being about 0.05 wide. One could try to choose a ν value remote from such a structure resonance but we persist in our study of the dynamics close to this resonance following the principle of clinging to the "devil we know" rather than falling under the influence of the "devil we don't". With a superperioditiy of 4 it is difficult not to be near a systematic stop-band of some sort.

Table 2 : Resonance Widths Found by Simulation (see Table 1 for emittances, $\Delta p/p = 0$)

Order		Closed orbit $x_{co} = z_{co}$ (mm)						Fourier Analysis
		0	1	2	4	6	8	$x_{co} = z_{co} = 4$
$3\nu_H = 80$	$\Delta\nu_H$	0.9×10^{-2}	0.8×10^{-2}	0.8×10^{-2}	0.8×10^{-2}	0.6×10^{-2}		1.97×10^{-2}
$2\nu_V+\nu_H = 80$	$\Delta\nu_H$	4.6×10^{-2}	4.6×10^{-2}	4.6×10^{-2}	4.2×10^{-2}	4.2×10^{-2}	4.2×10^{-2}	7.16×10^{-2}
	$\Delta\nu_V$	9.2×10^{-2}	9.2×10^{-2}	9.2×10^{-2}	8.4×10^{-2}	8.4×10^{-2}	8.4×10^{-2}	14.3×10^{-2}
$2\nu_H = 53$	$\Delta\nu_H$		0.2×10^{-2}	0.8×10^{-2}	1.7×10^{-2}	2.4×10^{-2}	3.2×10^{-2}	4.76×10^{-2}
$2\nu_V = 53$	$\Delta\nu_V$				3.0×10^{-2}	5.5×10^{-2}	8.0×10^{-2}	3.14×10^{-2}

We did depart briefly from this working point to check the behaviour of a non-systematic third order resonance $3\nu_H = 79$ driven by 10% random fluctuations in persistent currents around the bending magnets. LIMATRA had difficulty in finding the narrow resonance until the zero harmonic persistent current sextupole was artificially suppressed to minimise the non-linear contributions to ν spread. In this case the resonances appear from analysis and LIMATRA simulation to be between 10^{-3} and 3×10^{-3} width and we conclude that the fluctuations of 10% in persistent current sextupole component among bending magnets will not seriously effect the dynamics of the machine.

The next part of the simulation study was aimed at exploring two regions of the working diagram which have proved in other large synchrotrons [4,5] to be places where one can expect particle loss due to resonances to be minimal. We show the regions in Fig. 2 and label them A and B.

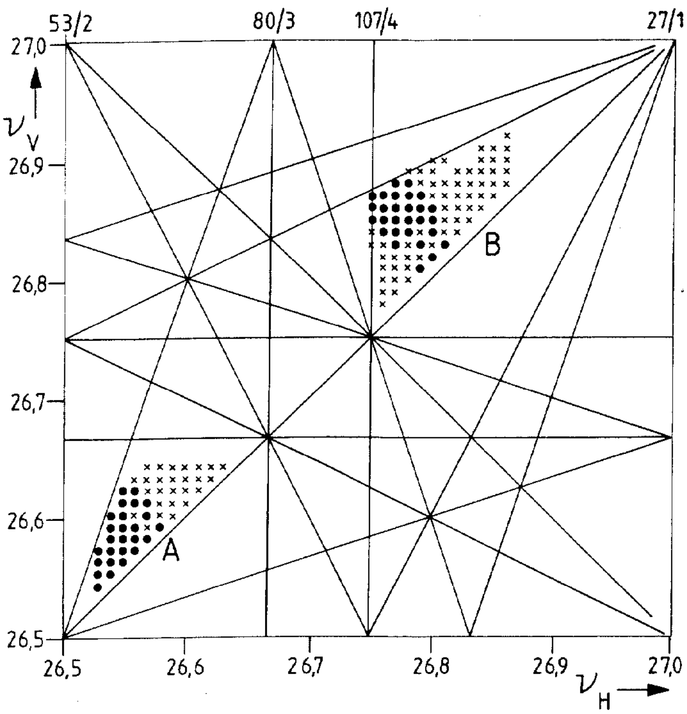

Fig. 2. Working diagram survey with no momentum spread (a dot indicates the particle survived 1000 turns. a cross shows it was lost).

Single particles were tracked for 1000 turns at each of the mesh points. The linear chromaticity imposed was zero and there were no synchrotron oscillations. Survival for 1000 turns with no sign of growth is indicated by a dot. A cross indicates that the particle became unstable and left the "machine".

Although the two regions appear to be of comparable extent we should say that the closed orbit distortion assumed in the B region is 12 mm in each plane while in the A region it is only 5 mm. This adjustment was made to ensure a finite and comparable region of stability in both cases and tends to reflect the difficulty of orbit correction near the integer.

The two regions are bounded by the resonances which we assumed are excited and by the coupling resonance. Region B makes less stringent demands on closed orbit correction and is likely to be less critical from the tuning point of view.

Finally, having established that region B is superior we made a more realistic simulation at its centre. This included not only closed orbit distortion but momentum spread, a finite residual chromaticity and modulation of momentum spread at the synchrotron frequency. One expects the continual crossing of stop-bands promoted by these ingredient to extract some of the particles which survived in the earlier simulation. The study was extended from 100 particle for 100 turns to 30 particles for 1000 turns for the last four entries in Table 3.

Table 3 : Tracking 100 Particles as a Function of Chromaticity and $\Delta p/p$

Momentum spread $\frac{\Delta p}{p}$	Closed Orbit (mm)		Chromaticity $\xi = \frac{\Delta\nu/\nu}{\Delta p/p}$	Number of particles lost in 100 turns
	x_{co}	z_{co}		
1.6×10^{-3}	0	0	0.4	0
			0.5	0
			0.6	1
1.6×10^{-3}	1	1	0.2	0
			0.3	0
			0.4	1
1.6×10^{-3}	2	2	0.001	0
			0.01	0
			0.025	2
1.0×10^{-3}	2	2	0.1	0
1.2×10^{-3}	2	2	0.1	0
1.4×10^{-3}	2	2	0.1	1
1.6×10^{-3}	2	2	0.1	2

Table 3 shows the results of various combinations of these quantities showing clearly that the condition that none of the particles are lost is jointly dependent on all these quantities. Practical limits might be :

$$\Delta p/p = \pm10^{-3},\ x_{co} = y_{co} = 2\ \text{mm (rms)},\ \xi = 0.1$$

We tracked a particle with these parameters for 500 000 turns, 10 seconds of machine time, without loss.

Conclusions

Clearly it is possible to inject particles at 40 GeV into a superconducting synchrotron of this size, but very careful control of ν value chromaticity and closed orbit distortion is needed. Bearing in mind the sensitivity of superconducting synchrotrons to particle loss it would be wise to find some way of reducing or backing off the persistent current effect at injection while the ring is being filled.

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