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Simple Estimation of Focusing Properties of a Separated Sector Cyclotron with a Soft-edge and Non-Zero Gradient Field

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Introduction

Recently, separated sector cyclotrons have been constructed or proposed to accelerate light ions as well as heavy ions. Before detailed design studies magnet shape and orbit dynamics should be examined with simple calculation. General properties of separated sector cyclotron were studied by Gordon[1,2]. In the case of homogeneous field, transfer matrix method developed by Schatz[3] gives a good starting point. On the other hand a traditional analytical formula with Fourier analysis is still useful. In these cases hard edge approximations for field boundary have been used. Recently some people have been introduced soft edge effect. Jungwirth et al[4] have used a two-step edge field to introduce soft edge effect. Craddock et al.[5] have introduced effective edge angle to the matrix method. Miura et al.[6] similar modification of an incident angle has been adopted to modify 'Spyring code' originally developed by Gordon[2] though it is not an analytical calculation.

In this paper, a matrix method with non-homogeneous field including a soft edge effect and a Fourier method with a trapezoidal field are described and compared with numerical calculations of realistic fields.

Soft edge effect for matrix methods

We need modify Schatz's method[3] to apply to non-homogeneous fields for a variable energy cyclotron. In the case of N sector cyclotron the matrices corresponding to the magnetic sector ($M_{mr,z}$) and to the fieldfree sector (M_f) are written in the following expression using the notation of fig.1.

$$M_f = \begin{pmatrix} 1 & \ell/\rho \\ 0 & 1 \end{pmatrix} \qquad (1)$$

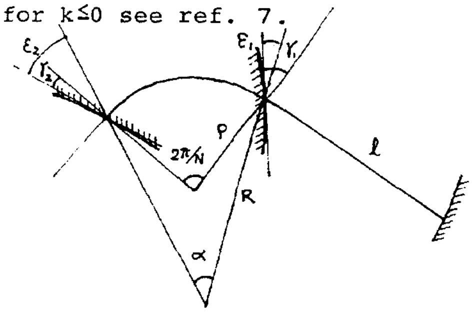
$$M_{mr} = \begin{pmatrix} \cos(\varepsilon_H \cdot 2\pi/N) + \varepsilon_H^{-1}\sin(\varepsilon_H \cdot 2\pi/N)\tan\gamma_1 & \varepsilon_H^{-1}\sin(\varepsilon_H\cdot 2\pi/N) \\ -\varepsilon_H \sin(\varepsilon_H\cdot 2\pi/N)(1+\varepsilon_H^{-1}\tan\gamma_1\tan\gamma_2) + \cos(\varepsilon_H\cdot 2\pi/N)(\tan\gamma_1 - \tan\gamma_2) & \cos(\varepsilon_H\cdot 2\pi/N) - \varepsilon_H^{-1}\sin(\varepsilon_H\cdot 2\pi/N)\tan\gamma_2 \end{pmatrix} \qquad (2)$$

where $\varepsilon_H = (1+k)^{1/2}$

$$M_{mz} = \begin{pmatrix} \cosh(\varepsilon_v\cdot 2\pi/N) - \varepsilon_v^{-1}\sinh(\varepsilon_v\cdot 2\pi/N)\tan\gamma_1 & \varepsilon_v^{-1}\sinh(\varepsilon_v\cdot 2\pi/N) \\ \varepsilon_v \sinh(\varepsilon_v\cdot 2\pi/N)(1 - \varepsilon_v^{-2}\tan\gamma_1\tan\gamma_2) - \cosh(\varepsilon_v\cdot 2\pi/N)(\tan\gamma_1 - \tan\gamma_2) & \cosh(\varepsilon_v\cdot 2\pi/N) + \varepsilon_v^{-1}\sinh(\varepsilon_v\cdot 2\pi/N)\tan\gamma_2 \end{pmatrix} \text{(for } k>0) \qquad (3)$$

where $\varepsilon_v = k^{1/2}$

for $k \le 0$ see ref. 7.



$$\ell/\rho = 2\sin(\pi/N)\cdot\sin(\pi/N-\alpha/2)/\sin(\alpha/2) \qquad (4)$$

$$\gamma_1 = \pi/N - \alpha/2 + \varepsilon_1 \qquad (5)$$

$$\gamma_2 = -\pi/N + \alpha/2 + \varepsilon_2 \qquad (6)$$

fig. 1. Orbit geometry for one period.

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In the above expression field index $k = (\rho/B)(\partial B/\partial x)$, where $(\partial B/\partial x)$ is the component of the field gradient along the outward normal to the equilibrium orbit, is calculated by Gordon's expression[2]. In the case of constant sector angle, k is approximately expressed

$$k \approx (\beta\gamma)^2/[1 + \sin(\pi/N - \alpha/2)/\sin(\alpha/2)] \quad (7)$$

neglecting the angular dependence in the magnetic field.

On the other hand, a constant gap magnet is easy for fabrication. In this case the Schatz's magnet saves trim coil power if it is shaped isochronously corresponding to a median energy. In this case the magnet shape is obtained by the following condition[3]

$$\gamma_m(\gamma_m - 1)^{-1}[\cot(\alpha_0/2) - \cot(\alpha_m/2)] =$$

$$(\pi/N)[\sin(\pi/N)]^{-2} + \cot(\alpha_0/2) - \cot(\pi/N) \quad (8)$$

where γ_m corresponds to a median energy acceleration. Using this magnet and trim coils we produce an isochronous field for a given energy acceleration. The field index of this case is

$$k = (\eta^2-1)\ \frac{\gamma^2 m - 1}{\gamma_m{}^2(1-\eta^2)+\eta^2}$$

$$(1+\cot(\alpha_m/2)\frac{R}{2}\ \frac{d\alpha_m}{dR})/\lambda(\phi) \quad (9)$$

where $\lambda(\phi)$ is obtained by Gordon's expression[2] and roughly $\lambda \approx 2$. The η is cyclotron frequency ratio

$$\eta = (\frac{eB_0}{m_0})/(\frac{e_m B_0 m}{m_{0m}}) \quad (10)$$

where suffix m corresponds to the median energy acceleration and o represents central values.

The soft edge effect is introduced by modifying the angles γ_1 and γ_2. This effect is important for vertical focusing and obtained by Enge's expression as the following[7].

$$(\tan\gamma_1)_z \rightarrow \tan\gamma_1 - I_2(\frac{D}{\rho})\frac{(1 + \sin^2\gamma_1)}{\cos^3\gamma^1}$$

$$\times(1 - I_4(\frac{D}{\rho})\tan\gamma_1) \quad (11a)$$

$$(\tan\gamma_2)_z \rightarrow \tan\gamma_2 + I_2(\frac{D}{\rho})\frac{(1 + \sin^2\gamma_2)}{\cos^3\gamma_1}$$

$$\times(1 + I_4(\frac{D}{\rho})\tan\gamma_2) \quad (11b)$$

where D is magnet gap, and I_2, I_4 correspond to the edge shape (for example, $I_2 = 0$: hard edge, $I_2 = 0.708$: Rogowski edge).

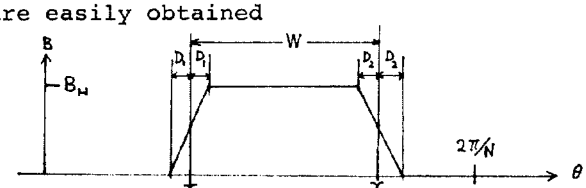
Using these expressions we can calculate the number of radial and axial betatron oscillation per turn ν_r, ν_z

$$\cos(\nu_{r,z}\ 2\pi/N) = \frac{1}{2}\ T_r\ (M_f \cdot M_{mr,z}) \quad (12)$$

Examples of results for straight edge magnets are shown in fig. 3. In fig. 3 numerical calculations were made for realistic fields using SIN program FIXPO with field synthesis of SIN model magnet data. Results of the matrix method corresponding to $I_2 = 0.708$ is fairly good to estimate beam qualities though the model magnet is not Rogowski type.

Simple edge field for Fourier method

Many analytical calculations of orbit properties are expressed using Fourier coefficients of a magnetic field. Therefore if we calculate the Fourier coefficients with ease a simple examination of a magnet shape is obtainable. A step function of magnet field is most simple but it does not contain the soft edge effect. Jain et al.[8] showed it was modified by limitation of the Fourier coefficients to some lowest order ones but it could not be sufficient to express a gap dependence. A trapezoidal function is good to introduce the soft edge effect and still easy to get the Fourier coefficients. Using the notation of fig. 2 the Fourier coefficients are easily obtained



$$D = \frac{I \times g}{R\cos\varepsilon} \quad (13)$$

fig. 2.

g: magnet gap

I: soft edge factor, $I \approx 2.5$

$$a_{Nn} = \frac{B_H}{\pi}\ \frac{1}{Nn^2}\ \{\frac{1}{D_2}\ \sin(Nnx_2)\sin(NnD_2)$$

$$- \frac{1}{D_1}\ \sin(Nnx_1)\sin(NnD_1)\} \quad (14a)$$

$$b_{Nn} = \frac{B_H}{\pi}\ \frac{1}{Nn^2}\ \{\frac{1}{D_1}\ \cos(Nnx_1)\sin(NnD_1)$$

$$- \frac{1}{D_2}\ \cos(Nnx_2)\sin(NnD_2)\} \quad (14b)$$

$$a_0 = \frac{B_H}{2\pi/N}\ (x_2 - x_1) \quad (14c)$$

We use simple analytical formula derived by Hagedoorm et al.[9] to compare with the numerical calculations mentioned above.

$$\nu_r = 1 + \frac{1}{2}\,\overline{\mu^T}_{rel} + \sum_{n=1}^{10} \frac{3(Nn)^2}{4\{(Nn)^2-1\}\{(Nn)^2-4\}}(a_{Nn}^2 + b_{Nn}^2) + \frac{3(Nn)^2}{4\{(Nn)^2-1\}\{(Nn)^2-4\}}(a_{Nn}\,a'_{Nn} + b_{Nn}\,b'_{Nn}) + \frac{3}{4\{(Nn)^2-1\}\{(Nn)^2-4\}}(a'^2_{Nn} + b'^2_{Nn}) \quad (15a)$$

$$\nu_z^2 = -\overline{\mu^T}_{rel} + \sum_{n=1}^{10} \frac{(Nn)^2}{2\{(Nn)^2-1\}}(a_{Nn}^2 + b_{Nn}^2) + \frac{1}{2\{(Nn)^2-1\}}(a_{Nn}\,a'_{Nn} + b_{Nn}\,b'_{Nn}) + \frac{2(Nn)^2-1}{2(Nn)^2\{(Nn)^2-1\}}(a'^2_{Nn} + b'^2_{Nn}) \quad (15b)$$

where $\overline{\mu^T}_{rel} = \beta^2/(1-\beta^2)$ and $a'_{Nn} = R(da_{Nn}/dR)$.

Results of the calculation using these expressions are shown in fig. 3. The results reproduce again the soft edge effect well though the ν_r is slightly smaller at the high energy side. This small value of ν_r at the high energy is caused by a rough approximation of eq. (15a). A more precise expression for ν_r derived Parzen[10] for example may be better but it may be rather complicated.

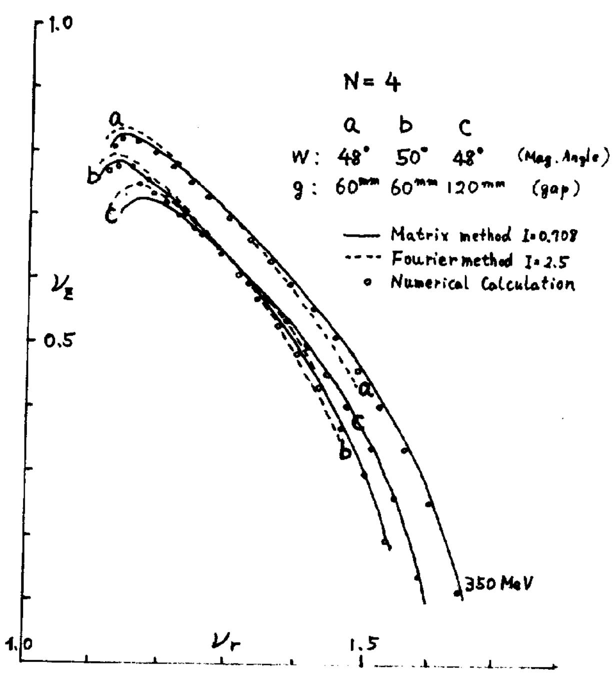


fig. 3. Comparison of calculations.

Conclusion and discussion

The matrix method with the modified incident and exit angles for vertical stability and the Fourier method with the trapezoidal approximation for the magnetic field shape are both sufficient to examine the separated sector cyclotron particularly at low energy region. In the case of a higher energy cyclotron, for example eight sectors and large spiral angles, it is difficult to get same value of ν_z with these methods and the numerical results, because each term of the expression becomes large but the resultunt ν_z is small. But these methods give still good starting points of magnet design.

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