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EFFECT OF A SPECTROMETER MAGNET ON THE BEAM-BEAM INTERACTION*

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I. Introduction

The presence of experimental apparatus in the interaction regions of an intersecting beam accelerator changes the configuration of the crossing beams. This changes the space-charge forces with respect to the standard, magnet-free crossing. In ISABELLE, the standard collision is for the two beams to cross at an angle of 11.188 mrad in the horizontal plane (Fig. 1). There is strong interest in instrumenting one of the intersections at ISABELLE with a major spectrometer facility, which involves a large aperture dipole magnet located at the intersection of the beams.[1] Two possible configurations have been proposed. In the vertically deflecting spectrometer (Fig. 2) the colliding beams are deflected vertically by the magnets A and brought to the same height in the interaction region by the magnets B. In the horizontal plane, the beams cross according to the standard configuration

In the horizontally deflecting spectrometer, all the bending occurs in the horizontal plane (Fig. 3). For the sake of comparison, we have assumed that, both in the horizontal and vertical case, the spectrometers have the same magnetic length, 6m. The peak field, for a 400 GeV beam, may vary from 5 to 15 kG, and scales like the energy for different collision energies. The question we are faced with is the following: what is the maximum allowable perturbation caused by the spectrometer magnet that can be tolerated from the point of view of the beam dynamics? In this paper, we limit ourselves to the perturbations that the curved trajectories cause the beam-beam space charge nonlinearities. The magnetic perturbations due to the unavoidable nonlinear content of the field in the spectrometer magnet are not considered in this paper, although they will have, of course, to be taken into account. The criterion we have proposed to follow in determining the maximum allowable perturbation introduced by the experimental apparatus is the following: during the initial stage of ISABELLE operation, the strength of the perturbation must not be greater than the perturbation introduced by the effect of the unavoidable orbit errors at all the interaction points, as estimated from the machine tolerances. Eventually, as one learns more about the beam behavior and the effect of the beam-beam interaction, the above condition may be relaxed, and stronger perturbations may become tolerable.

The question has arisen of how one defines the strength of the perturbation. The most obvious quantity, at first sight, is the linear beam-beam tune shift, which is conventionally accepted as a measure of strength of all the nonlinear resonances. However, as we shall see, this parameter turns out not to be directly related to the strength of the nonlinear resonances for the crossing geometries as modified by a horizontal or vertical spectrometer magnet. Thus, the only solution is to compute the strength of the most important nonlinear resources. In what follows, we describe the computational method used in calculating these resonances, and compare them with those induced by random orbit errors.

In an earlier paper,[2] we considered the vertically deflecting spectrometer. Since, in the same horizontal plane, two parallel trajectories "seem" to experience the same integrated space charge field, we made the assumption that horizontal and coupling resonances are absent in this geometry. The present more detailed analysis, instead, shows that the asymmetry introduced by the vertical deflection results in considerable nonlinear coupling terms, which do not cancel out when integrated over the whole space-charge region. As a result, both the horizontal and vertical spectrometers introduce coupling resonances which would not be present even when orbit errors are taken into account. This seems to be the price one has to pay for the perturbation introduced by the spectrometer magnets. Following the criterion discussed earlier on, we have proposed that these coupling resonances be "small" compared to the one dimensional resonances excited by orbit errors.

II. Computation of Stop-bands

The stop-band width of the resonance line $n\nu_x + m\nu_y = q$ is given by[3]

$$\Delta\nu_{nm} = \frac{1}{2^{N-2}} \frac{1}{|n|!\,|m|!} (n^2+m^2)^{1/2} \varepsilon^{(N-2)/2} \cdot \qquad (2.1a)$$

$$\cdot \frac{1}{4\pi B\rho} \int ds \exp\left[i(n\psi_x + m\psi_y)\right] \beta_x^{|n|/2} \beta_y^{|m|/2} D_x^{|n|} D_y^{|m|} U,$$

where ε is the emittance divided by π, $N = |n| + |m|$, $D_x \equiv \partial/\partial x$, $D_y = \partial/\partial y$, and U is the potential function which is related to the force on the particle by

$$F_x/\beta e = \partial U/\partial x,$$
$$F_y/\beta e = \partial U/\partial y. \qquad (2.1b)$$

ε_x, ε_y are assumed to be equal in Eq. (2.1). $\Delta\nu m_{nm}$ is the half-width of the stop-band.

The expression Eq. (2.1a) for $\Delta\nu_{nm}$ differs somewhat from the usual expression which is expressed in terms of Δe, $\Delta e = n_x\nu_x + n_y\nu_y - q$. Eq. (2.1a) gives half the ν-width of the resonance line if one moves, in ν-space, along a direction perpendicular to the resonance line; ψ_x and ψ_y are the betatron phase functions for ν_x, ν_y on the resonance.

For the case of two intersecting beams with identical Gaussian elleptical cross sections, the potential function which gives the field due to one beam is given by

$$U = \frac{1+\beta^2}{\beta} V$$

$$V = -\frac{\lambda}{4\pi\varepsilon_o} \int_o^\infty 2u\,du \frac{1 - \exp\left[-x^2(2\sigma_x^2+u^2) - y^2(2\sigma_y^2+u^2)\right]}{(2\sigma_x^2+u^2)^{1/2}\,(2\sigma_y^2+u^2)^{1/2}} \qquad (2.2)$$

where σ_x and σ_y are the rms widths of the beam, x and y are coordinates perpendicular to the beam, and λ is the linear charge density.

In the integral over s in Eq. (2.1) β_x and β_y are functions of s and this dependence was assumed to be given by

$$\beta_x = \beta_x^* + s^2/\beta_x^*$$
$$\beta_y = \beta_y^* + s^2/\beta_y^* \qquad (2.3)$$

For the ISABELLE accelerator $\beta_x^* = 43$m, $\beta_y^* = 7.5$m, σ_x=1.8mm, σ_y=0.75 mm and ε=0.5 x 10^{-6} mrad at 30 GeV.

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In order to compute the derivatives of $V(x,y)$, $\partial^{n+m}V/\partial x^n \partial y^m$, V was computed on a mesh, $V(I,J)$. The mesh intervals, Δx, Δy have to be small compared to the beam size. For the ISABELLE case, $\Delta x = \Delta y = 0.1$ mm was used. The extent of the mesh in the x and y directions has to cover the region where the beams interact. Along the s-direction, the beams will continue to interact until each beam starts moving along its own vacuum chamber, which shield them from each other. This occurs for ISABELLE at about s = 8m from the crossing point where the beams are separated by 88 mm in the horizontal plane.

In order to reduce the number of mesh points at which the potential function is computed, $V(I,J)$ is computed only in an interval around the path of the particle. For each value of x, the horizontal separation of the beams, the test beam will be at $y(x)$ given by the vertical path of the test beam. The mesh $V(I,J)$ was computed at just 4 mesh points above, and 4 mesh points below the mesh point at $y(x)$. Four mesh points above and below the particle path were sufficient to compute up to the fifth order resonances for the test beam paths considered in this paper.

The potential $V(x,y)$ is computed at the mesh points using the integral formulation given by Eq. (2.2). This form of the integral gives rise to some numerical accuracy problems which can be alleviated by a transformation of variables. The integral $f(u)$ in Eq. (2.2) has a long tail; as $u \to \infty$, $f(u) \sim 1/u^3$. For equal intervals Δu, the area in each interval goes down like $\Delta u/u^3$. This indicates that a transformation to a new variable z such that $\Delta z \sim \Delta u/u^3$ as $u \to \infty$ would be helpful. The transformation used was

$$z = 2\sigma_y^2/(2\sigma_y^2 + u^2),$$

$$u = \left[2\sigma_y^2(1/z-1)\right]^{1/2}, \qquad (2.4)$$

$$dz/z = -(zu/\sigma_y^2).du,$$

$$V = \frac{-\lambda}{4\pi\varepsilon_o}\int_o^\infty \frac{dz}{z^2}\,\frac{2\sigma_y^2\{1-\exp[-x^2(2\sigma_x^2+u^2)-y^2(2\sigma_y^2+u^2)]\}}{(2\sigma_x^2+u^2)^{1/2}\,(2\sigma_y^2+u^2)^{1/2}}$$

Also, when $u \sim \sigma_y$, $z \sim 1$, and $dz \simeq du/\sigma_y$. Thus, the choice of the dz interval that corresponds to a du interval of about $du \simeq 0.1\,\sigma_y$ is $dz \simeq 0.1$. The interval $dz = 0.1$ was used in evaluating the integral in Eq. (2.4).

III. Beam-Beam Stop-Band Results for Orbit Misalignment Errors

The criterion used in this paper is to compare the beam-beam stop bands introduced by the experimental devices with the beam-beam stop-bands introduced by the vertical closed orbit misalignments which remain after the closed orbit has been corrected with the closed orbit correction system. The stop bands due to closed orbit errors can be computed analytically with certain approximations.[4] They can be computed numerically and more accurately using the procedure described in Section II.

In Table I, the numerically computed stop bands due to a $\Delta y_{rms} = 0.05$ mm random separation of the ISABELLE beams are compared with the analytical results. The error in the analytical results appears to grow larger for higher order resonances. The analytical result appears to be off by a factor of 2 for the fifth order resonance. In Table I, stop bands are given in units of $\Delta\bar{\nu}_y$, the linear vertical beam-beam ν-shift for the unperturbed accelerator.

Table I. Comparison of the numerically computed stop bands with analytical results for ISABELLE at 30 GeV. The stop bands are excited by a $\Delta y_{rms} = 0.05$ mm vertical separation of the beams. $\Delta\nu_{nm}$ is in units of $\Delta\bar{\nu}_y$ the linear ν-shift for the unperturbed machine at 30 GeV. $\Delta\nu_y = 0.0057$. The $\Delta\nu_{nm}$ omitted have zero widths.

N	Resonance	Computed	Analytical
2	$\Delta\nu_y$	-0.0075	-0.008
	$\Delta\nu_x$	0	0
3	$\Delta\nu_{03}$	0.16	0.13
	$\Delta\nu_{21}$	0.0002	0
4	$\Delta\nu_{04}$	0.0095	0.0060
	$\Delta\nu_{22}$	0.0003	0
5	$\Delta\nu_{05}$	0.040	0.020
	$\Delta\nu_{23}$	0.0001	0

IV. Six Meter Spectrometer Results

This section presents the results for the computed beam-beam stop bands introduced by a six meter spectrometrer at one of the six crossing points in ISABELLE. The stop bands, $\Delta\nu_{nm}$, are computed from Eq. (2.1) as described in Section II. The integral over s in Eq. (2.1) is cut off as the point where the beams no longer interact. When no spectrometer is present, the two beams will separate into their separate vacuum tanks when they are about 7.86 meters from the crossing point, where they are separated by 88 mm for a crossing angle of $\alpha = 0.0112$. The vacuum tanks have a radius of 44 mm.

Table II lists the computed stop bands at 30 GeV for two 6 m spectrometers, one which bends the beams in the horizontal plane, and one that bends the beams in the vertical plane. The stop bands are given at three field levels corresponding to radii of curvature of 2700, 1350 and 900 m. At 400 GeV the spectrometer field would be 5, 10, and 15 kG. The results for $\Delta\nu_{nm}$ are given in units of $\Delta\bar{\nu}_y$, the unperturbed linear ν-shift. $\Delta\bar{\nu}_y = 0.0057$ at 30 GeV for ISABELLE.

One sees from Table II, that the odd N resonances are driven more strongly than the even N resonances, which is also true for the resonances derived by vertical orbit misalignments. Relatively strong coupling resonances are introduced, whereas vertical orbit misalignments do not introduce coupling resonances. The strongest resonance excited in Table II is the 3rd order coupling resonance $\Delta\nu_{21} = 0.159$ in units of $\Delta\bar{\nu}_y = 0.0057$, for the vertical bend spectrometer at $\rho = 900$ m. This can be compared with the stop band $\Delta\nu_3 = 0.16$ found for vertical misalignments in Section 2.

In Fig. 4, the strongest stop band for each order, N, at 30 GeV is plotted against the order of the resonance, N. The stop bands due to vertical orbit misalignments are indicated by large crosses, X. For beam curvatures $\rho \geq 900$ m the stop bands introduced by 96 m spectrometer with either vertical or horizontal bending are smaller than or of the same magnitude as the stop bands expected from vertical orbit misalignments of 0.05 mm rms at the crossing points.

V. Energy Dependence of Beam-Beam Stop Bands

This section treats the question of what happens to the stop bands as the energy is increased, e.g. in ISABELLE where the energy goes from 30 GeV to 400 GeV. For the case of the beam-beam stop bands due to vertical orbit errors, one can show that the odd order stop bands do not change with energy, if the orbit errors do not change with energy.

The odd order stop-bands due to vertical orbit errors are given by

$$\Delta\nu_{oN} = \Delta\bar{\nu}_y\, f(N)\Delta y_{rms}/\sigma_y \qquad (5.1)$$

Table II. The beam-beam stop bands, $\Delta\nu_{nm}$, at 30 GeV introduced by a 6 m spectrometer with either vertical or horizontal bending. α=0.0112, σ_y=0.75 mm, β_y^*=7.5 m and I=8A. The $\Delta\nu_{nm}$ are in units of $\Delta\bar{\nu}_y$=0.0057, the unperturbed linear ν-shift for ISABELLE at 30 GeV.

		Vertical Bend			Horizontal Bend		
		ρ = 2700	ρ = 1350	ρ=900	ρ=2700	ρ=1350	ρ=900
N=2	$\Delta\nu_y$	-0.0013	-0.0051	-0.0107	0.0014	0.0060	0.0157
	$\Delta\nu_x$	0.0051	0.0217	0.0460	-0.0088	-0.0330	-0.0790
N=3	$\Delta\nu_{03}$	0.0069	0.0234	0.0423	0	0	0
	$\Delta\nu_{12}$	0	0	0	0.0213	0.0523	0.0770
	$\Delta\nu_{21}$	0.0478	0.1083	0.1592	0	0	0
	$\Delta\nu_{30}$	0	0	0	0.0121	0.0018	0.0077
N=4	$\Delta\nu_{04}$	0.0004	0.0018	0.0041	0.0001	0.0005	0.0010
	$\Delta\nu_{13}$	0	0	0	0	0	0
	$\Delta\nu_{22}$	0.0011	0.0080	0.0186	0.0023	0.0039	0.0072
	$\Delta\nu_{31}$	0	0	0	0	0	0
	$\Delta\nu_{40}$	0.0008	0.0025	0.0056	0.0004	0.0001	0.0005
N=5	$\Delta\nu_{05}$	0.0010	0.0045	0.0085	0	0	0
	$\Delta\nu_{14}$	0	0	0	0.0030	0.0078	0.0121
	$\Delta\nu_{23}$	0.0119	0.0307	0.0460	0	0	0
	$\Delta\nu_{32}$	0	0	0	0.0059	0.0013	0.0003
	$\Delta\nu_{41}$	0.0052	0.0012	0.0023	0	0	0
	$\Delta\nu_{50}$	0	0	0	0.0045	0.0027	0.0001

where $\Delta\bar{\nu}_y$ is the unperturbed linear beam-beam ν-shift, Δy_{rms} is the rms orbit separation error as the crossing points, and f(N) is the some function of N, the order of the resonance. Now σ_y decreases with energy like $1/E^{1/2}$, and $\Delta\bar{\nu}_y$, also decreases with energy like $1/E^{1/2}$. Thus $\Delta\nu_{0N}$ due to vertical orbit errors is independent of energy for a constant orbit error, Δy_{rms}.

Numerical computations show that the beam-beam stop bands, $\Delta\nu_{nm}$, introduced by the spectrometer decrease with energy somewhat faster than 1/E. Although these numerical results were found for the case of the ISABELLE accelerator, it probably will hold for other situations provided the dispersion is zero at the beam crossing points and σ_x and σ_y both decrease like $1/E^{1/2}$.

This 1/E dependence may be understood as follows. In the case of the vertical bend spectrometer, the vertical separation of the beams is given by $\Delta y = s^2/2\rho$. The beam-beam interaction extends over an s-distance which is proportional to σ_x. Thus the average beam separation will be proportional to σ_x^2 and will decrease with energy like 1/E. The above mentioned result that the stop-bands due to a constant orbit error beam separation are independent of energy indicates that the spectrometer stop bands should decrease like 1/E.

VI. Conclusions

The object of our study has been to analyze the effects of a spectrometer magnet, horizontally or vertically deflecting, on the beam-beam space charge force induced resources. For the model we have considered, the strengths of the perturbations, as measured by the widths of the nonlinear resonances, are smaller than those excited by unavoidable orbit errors, for a field up to 15 kG at 400 GeV and with a magnetic length of 6 meters. However, nonlinear coupling resonances are present, which would be absent in a conventional crossing geometry. The stop band widths are larger at low energy, and decrease with energy somewhat faster than linearly. Since the resonances excited by orbit errors are constant with energy (if the perturbations which cause them are constant), and if they are acceptable at low energy according to the criterion discussed in Section I, they are certainly acceptable at higher energy.

References

1. S. Ozaki, private communication.
2. M. Cornacchia, G. Parzen, BNL Report 51103.
3. G. Guignard, CERN Report, CERN 77-10 (1977).
4. G. Parzen, BNL Report 51154 (1979).

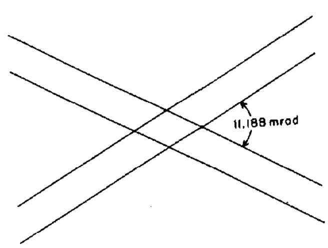

Fig. 1. Standard collision geometry in ISABELLE.

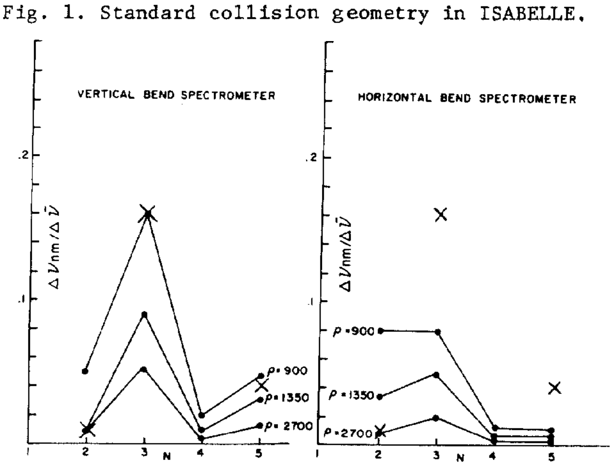

Fig. 4. Widths of stop-bands as a function of the order of the stop-band. For each order, the maximum width amongst the possible ones (horizontal, vertical or skew) is plotted. The widths are normalized, the normalization factor being the linear vertical beam-beam tune shift of the standard geometry. The large crosses are the stop-band widths due to random orbit errors.

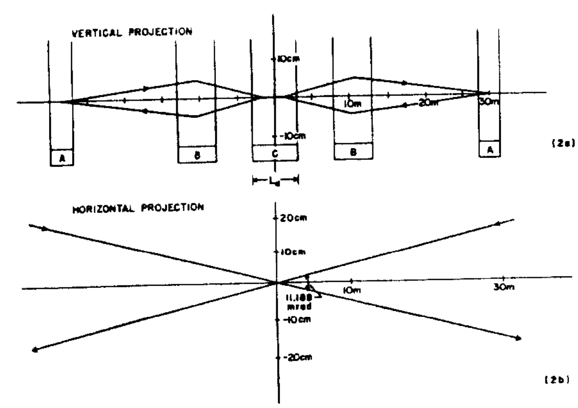

Fig. 2. Collision geometry of a vertically deflecting spectrometer.

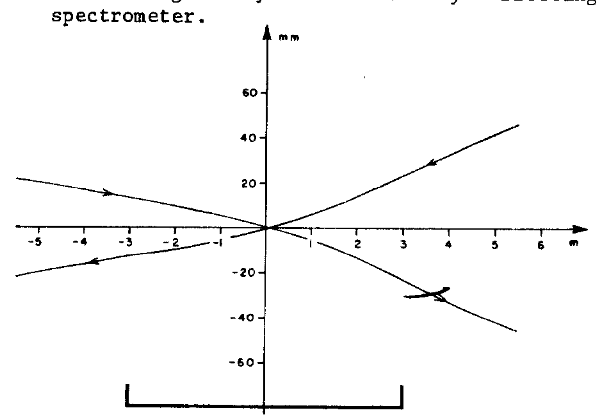

Fig. 3. Collision geometry of a horizontally deflecting spectrometer.