© 1981 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, June 1981

CYCLIC MOTION OF HEAVY IONS WITH CHARGE SPREAD

V.P.Belov, I.A.Shukeilo

D.V.Efremov Scientific Research Institute of Electrophysical Apparatus, Leningrad, 188631, USSR

Summary

A non-linear theory of multicharged ion cyclic motion is presented in the report, the charge q being considered a discretely changing parameter. Expressions for ion closed orbits are found to an accuracy of $\sim(\Delta q/q_0)^2$, where $|\Delta q|$ is the charge spread near the mean value q_0. Conditions of magnetic structure chromaticity compensations are also formulated to the same approximation. The study is made in an easy-to-use form, which may be directly applied to studying monocharged beam dynamics at high momentum spreads.

1. Introduction

Increasing the pulsed intensity of a heavy ion synchrotron may be achieved by the use of on intermediate storage ring [1, 2]. The main purpose of such a device is to transform low-intensity pulses of long duration into shorter and more intensive ones, which are adequate to the conditions of injection into a synchrotron.

Because the phase density of heavy ion beams is low, it is reasonable to use the method of charge-exchange ion injection into the storage ring. A method similar to this is widely used for H^- ion injection into proton synchrotrons, which results in an increase in phase density.

A feature of such heavy ion storage is that ions of different charge states simultaneously circulate in the ring. The equilibrium distribution of charge is close to a Gaussian one. Charge spread may be high enough, for example, d/q_0 =0.04 at an energy of 10 MeV/nucleon for uranium ions, where d is the dispersion of the charge distribution and q_0 is the average beam charge. This ratio exceeds the particle momentum spread in antiproton accumulators. Heavy ion storage may continue during several hundreds of turns, and to avoid beam losses the radial aperture of the magnetic ring in the above case should be high enough to confine ions with a charge spread of $|\Delta q|/q_0 \simeq 0.10$.

Another feature of heavy ion storage is that the ion charge changes from one turn to another, though it stays within an equilibrium distribution. The probability for an ion to be in a certain charge state depends upon the distribution function. Therefore, probability for an ion to be at the edges of the charge spectrum is rather low. This in turn means that ions rarely reach the distant parts of the radial aperture, so the field quality in these parts may not be as high as that in the center. Changes in charge states make resonance phenomena less dangerous provided that there is spread in betatron frequencies. The natural chromaticity of a magnetic lattice gives rise to excessive frequency spread, so there is a need for controlling chromaticity by means of introducing non-linear field components.

2. Equations of Transverse Motion

Let us consider radial and axial motion of ions in a magnetic system having a median plane. The ion charge is considered a discretely changing parameter. The equations of transverse motion describing the ion deviation from the chamber axis in x and y are:

$$x_s'' - \frac{1}{R_0}\left(1+\frac{x}{R_0}\right) = -\frac{q e B_y}{p}\left(1+\frac{2x}{R_0}\right) \qquad (1a)$$

$$y_s'' = \frac{q e B_x}{p}\left(1+\frac{2x}{R_0}\right) \qquad (1b)$$

where p = momentum, R_0 =curvature radius, B_x and B_y = field components. The terms of the order $\sim(x/R_0)^2$ as well as of $\sim x'^2$ and $\sim y'^2$ are not allowed for here because of their small contribution, but it is worth noticing that an analysis is needed of such a step in considering lattices with small R_0.

Let us further represent q, p, and B_y as

$$q = q_0\left(1+\frac{\Delta q}{q_0}\right),\quad p = p_0\left(1+\frac{\Delta p}{p_0}\right),\quad B_y = B_{y0}\left(1+\frac{\Delta B_y}{B_{y0}}\right)$$

where "0" means that a corresponding value is taken on the chamber axis.

Introducing

$$\varepsilon = \frac{\Delta p/p_0 - \Delta q/q_0}{1+\Delta p/p_0} \qquad (2)$$

we can rewrite Eqs.(1a) and (1b) as

$$x'' - \frac{1}{R_0^2}x = \frac{1}{R_0}\varepsilon - (1-\varepsilon)\left[\frac{\Delta B_y}{\mathcal{P}_0} + \frac{2x}{R_0^2} + \frac{2\Delta B_y}{\mathcal{P}_0}\cdot\frac{x}{R_0}\right], \qquad (3a)$$

$$y'' = (1-\varepsilon)\frac{B_x}{\mathcal{P}_0}\left(1+\frac{2x}{R_0}\right), \qquad (3b)$$

where $\mathcal{P}_0 = B_0 R_0$.

Expanding ΔB_y and B_x in Eqs.(3a) and (3b) into power series of deviation from the chamber axis:

0018-9499/81/0600-2540$00.75©1981 IEEE

$$\frac{\Delta B_y}{P_0} = g_1 x + g_2(x^2-y^2) + g_3(x^3-3xy^2) + \dots \quad (4a)$$

$$\frac{B_x}{P_0} = g_1 y + 2g_2 xy + g_3(3x^2y - y^3) + \dots \quad (4b)$$

where

$$g_m = \frac{1}{m!P_0}\left(\frac{\partial^m B_y}{\partial x^m}\right)_0 , \quad (5)$$

$$m = 1, 2, 3, \dots$$

The equations of transverse motion take the form convenient for studying the effects of the problem considered:

$$x'' + \left(\frac{1}{R_0^2} + g_1\right)x = \frac{1}{R_0}\varepsilon + \left(\frac{2}{R_0^2} + g_1\right)\varepsilon x - (1-\varepsilon)\left[\left(\frac{2}{R_0}g_1 + g_2\right)x^2 - g_2 y^2 + \left(\frac{2}{R_0}g_2 + g_3\right)x^3 - 3g_3 xy^2\right] + \dots \quad (6a)$$

$$y'' - g_1 y = -g_1\varepsilon y + (1-\varepsilon)\left[2\left(\frac{g_1}{R_0} + g_2\right)xy + \left(\frac{4}{R_0}g_2 + 3g_3\right)x^2 y - g_3 y^3\right] + \dots \quad (6b)$$

3. Closed Orbits

The solution of Eq.(6a) may be found in the form:

$$x(s) = \rho(s) + \xi(s) \quad (7)$$

where $\xi(s)$ describes betatron oscillations, and $\rho(s+L) = \rho(s)$, L being the length of the equilibrium orbit.

The periodic solution may be written as a series:

$$\rho(s) = \psi(s)\varepsilon + \psi_1(s)\varepsilon^2 + \dots , \quad (8)$$

where $\psi(s)$ is the dispersion function determining the closed orbits of particles as a linear approximation in $\Delta P/P_0$. This function satisfies the equation:

$$\psi'' + \left(\frac{1}{R_0^2} + g_1\right)\psi = \frac{1}{R_0} , \quad (9)$$

whose solution is well known:

$$\psi(s) = a(s)\varphi(s) + a^*(s)\varphi^*(s) ,$$

$$a(s) = \frac{1}{W}\frac{1}{1-e^{-i\mu M}}\int_s^{s+L}\frac{1}{R_0}\varphi^* ds , \quad (10)$$

where $W = \varphi\varphi^{*\prime} - \varphi^*\varphi'$ is the Wronskian of Floquet functions, μM is the phase shift of betatron oscillations per turn, an asterisk meaning complex conjugate.

If there is no momentum spread, then we have as a linear approximation in $\Delta q/q_0$:

$$\rho(s) = -\psi(s)\frac{\Delta q}{q_0} \quad (11)$$

This is practically enough to find out the radial aperture of the ring designed to confine a multicomponent beam. However, in order to determine betatron frequencies and lattice chromaticity there may be a need for the ρ-function in an ε^2 approximation. The second-order term may be found by solving an equation for ψ_1. Allowing for Eqs. (6a), (8) and (9) this equation is written as follows:

$$\psi_1'' + \left(\frac{1}{R_0^2} + g_1\right)\psi_1 = \left(\frac{2}{R_0^2} + g_1\right)\psi - \left(\frac{2}{R_0}g_1 + g_2\right)\psi^2 \equiv \frac{1}{R_1} \quad (12)$$

This equation appears to be similar to Eq. (9), but there is another function on the r.h.s. of it. This function may be expressed in terms of ψ, a periodic solution in a linear approximation, as well as of g_2, a quadratic non-linearity, which is determined from the conditions of chromaticity correction.

The solution of Eq.(12) is given by Eq.(10), where $1/R_0$ should be substituted by $1/R_1$.

4. Equations of Betatron Oscillations. Chromaticity Correction

Now assuming that the periodic solution (8) is found and allowing for Eqs. (6a), (6b), (9) and (12), we may obtain the equations of the betatron oscillations of ions near their closed orbits corresponding to their momentum and charge:

$$\xi'' + \left(\frac{1}{R_0^2} + g_1\right)\xi = \left\{\left[\frac{2}{R_0^2} + g_1 - 2\psi\left(\frac{2}{R_0}g_1 + g_2\right)\right]\varepsilon - \left[2(\psi_1 - \psi)\left(\frac{2}{R_0}g_1 + g_2\right) + 3\psi^2\left(\frac{2}{R_0}g_2 + g_3\right)\right]\varepsilon^2\right\}\xi - (1-\varepsilon)\left[\frac{2}{R_0}g_1 + g_2 + 3(\psi + \psi_1\varepsilon)\left(\frac{2}{R_0}g_2 + g_3\right)\varepsilon\right]\xi^2 + (1-\varepsilon)\left[g_2 + 3(\psi + \psi_1\varepsilon)g_3\varepsilon\right]y^2 + \dots \quad (13a)$$

$$y'' - g_1 y = \left\{\left[-g_1 + 2\psi\left(\frac{g_1}{R_0} + g_2\right)\right]\varepsilon + \left[2(\psi_1 - \psi)\left(\frac{g_1}{R_0} + g_2\right) + \psi^2\left(\frac{4}{R_0}g_2 + 3g_3\right)\right]\varepsilon^2\right\}y + 2(1-\varepsilon)\left[\frac{g_1}{R_0} + g_2 + (\psi + \varepsilon\psi_1)\left(\frac{4}{R_0}g_2 + 3g_3\right)\varepsilon\right]\xi y + \dots \quad (13b)$$

Dispersion terms linear and quadratic in ε are allowed for in these equations as well as terms of a power series of deviation from closed orbits including quadratic ones.

The most important feature of betatron motion is that the frequencies of betatron oscillations in linear magnetic field depend upon ε. Because ε in the problem considered is rather high ($\varepsilon \simeq 0.1$), special measures should be taken to make the frequency range narrower.

The procedure of eliminating the dependence of the betatron oscillation frequencies upon momentum is developed well enough and used in accelerators and storage rings. The principle of this procedure is based upon correcting the lattice chromaticity by means of quadratic non-linearity. The presence of charge spread does not in principle affect the matter, but the correction in this case should be more thorough. In order to compensate ε^2 - effects at a large charge spread there might be a need for the introduction of not only quadratic but also cubic non-linearity.

The conditions of zero chromaticity in an ε linear approximation according to Eqs. (13a) and (13b) are given by

$$\begin{cases} \int_0^L \left[\frac{2}{R_0^2} + \left(1 - 4\frac{\psi}{R_0}\right) g_1 - 2\psi g_2\right] |\varphi_x|^2 ds = 0 \\ \int_0^L \left[\left(1 - 2\frac{\psi}{R_0}\right) g_1 - 2\psi g_2\right] |\varphi_y|^2 ds = 0 \end{cases} \quad (14)$$

These conditions are ambiguous in correcting the fields of conventional accelerators, because the g_2 azimuthal dependence may be quite different. In the problem considered ε is large, so it is reasonable that non-linearity be introduced into those magnet elements in which basic inconsistency occurs between the ion charge and focusing/defocusing field. In particular, non-linearity should better be introduced into all the quadrupoles of a separate function lattice. Such a method for correcting chromaticity does not effect any substantial envelope distortion. The lattice periodicity stays the same, and there are no reasons for the occurrence of additional resonances.

The introduction of quadratic field non-linearity effects the dependence of frequencies upon the squares of the amplitudes of vertical and radial oscillations. This effect occurs in the second order, and at emittances natural to a beam in a synchrotron appears to be relatively small [3]. If the equilibrium orbit is distorted due to random errors in dipole field, non-linear components may cause additional resonances [4]. The distortions of the equilibrium orbit should not exceed several millimeters.

Lattice chromaticity correction in an ε^2 approximation is performed, as it follows from Eqs. (13a) and (13b), by means of cubic non-linearity. It is reasonable that this non-linearity be introduced into lattice elements. Because quadratic corrections to frequencies are naturally much smaller than linear ones, it is enough to allow for only the main terms in integrands similar to that in Eq.(14). Then the condition of eliminating undesired quadratic deviations in oscillation frequencies is given by

$$\int_0^L \left[2(\psi_1 - \psi) g_2 + 3\psi^2 g_3\right] |\varphi_{x,y}|^2 ds = 0 \quad (15)$$

The integrand of this expression contains g_2, which was determined earlier on the basis of the condition of compensating frequency spread in a linear approximation in ε.

In conclusion, let us now specify the procedure of calculating a closed orbit and correction fields in an ε^2 approximation:

1. According to Eq.(10) one finds the dispersion function ψ, which determines closed orbits in a linear approximation.
2. From Eq.(14) $g_2(s)$, a quadratic non-linearity, is calculated, which is needed to compensate chromaticity in a linear approximation.
3. Integrating Eq.(12) one finds the ψ_1 function and, allowing for Eq.(8), closed orbits in an ε^2 approximation.
4. From Eq.(15) g_3, a cubic non-linearity, is calculated, which makes it possible to compensate chromaticity in an ε^2 approximation.

References

1. I.A.Shukeilo. IEEE Trans. on Nucl. Sci., NS-26, No.3, 4281, 1979.
2. M.G.Nagaenko et al. Proc. 11th Intern. Conf. on High-Energy Acceler. p.283, Birkhäuser Verlag, Basel, 1980.
3. V.P.Belov, B.V.Vasilishin et al. Soobscheniye OIYaI (JINR Report), 9-11650, Dubna, 1978.
4. T.Suzuki. CERN/ISR/TH-77/64.