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A DESIGN OF A SLOT TYPE PICK-UP AND KICKER FOR STOCHASTIC COOLING OF A COASTING BEAM

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Summary

A novel analytical approach to a slot type pickup and kicker is presented. Expressions of the coupling field from the particle beam as well as other parameters are substantially different from that obtained by Faltin. This result shows that the design is competitive with other kinds of pickup or kickers.

A. Introduction

It becomes increasingly apparent that a decent design of a pickup or kicker for stochastic cooling of particle beams is essential to the Tevatron Colliding experiment here at Fermilab. Various schemes had been proposed and designed for this purpose.[1,2] However, it is either because of their bandwidth characteristic or the low energy requirement that they do not seem to be able to meet this challenge. Pickups using ferrite cores are also reported elsewhere[3] and they seem to perform well. Nevertheless, it requires a vast amount of summing circuits. Slot type pickup and kickers was proposed by Faltin.[4] However, due to its design, the induced signal onto the electrodes was found to be very weak and verticle cooling was almost impossible to carry out. In this paper, a similar design but suitable for any energy level for slot type pickup and kicker is introduced. A novel analytical approach deriving the power coupling between particle beam and the electrodes is also presented. Substantially different results compared with that of Faltin's are also obtained. Due to its simplicity in structure and its compactness in analysis, it is believed that this electrode design is competitive with others mentioned above.

B. Waveguide Field in the Beam Chamber

The physical structure under consideration is shown in Fig. 1. A ribbon beam of finite cross-section is flowing in the vacuum chamber with cross-sectional size axb. This vacuum chamber is coupled, on both sides of the broad side walls, to a rectangular TEM line through common slots. The cut-away structure in Fig. 1 shows the details. The TEM line consists of a retangular pipe with a center conductor surrounded with a dielectric medium. The dielectric constant in the medium is ε and is intended to match the electromagnetic wave propagation velocity in the TEM line with the beam velocity in the beam chamber. Dimensions of the TEM line is so designed that a characteristic impedance of 50 Ω is obtained.[5] Aside from the imperfection of the structure, it is assumed that this physical structure is infinitely extended and thus results in no reflections at both ends.

Since there is only one current component in the Z direction, the waveguide modal fields in the beam chamber is easily seen to be transverse magnetic fields (TM). To find these modal fields in a waveguide with such a longitudinal current, we shall take the following strategy. Instead of solving the wave equation in the presence of a beam current directly, we shall solve it by assuming the beam and the slots are absent. This thus becomes common waveguide fields and their expressions can be written down immediately.[6] Expressions for the same waveguide fields when the beam is present can be obtained by using these modal fields obtained previously as base functions and finding the corresponding Fourier series. Finally, the fields in a waveguide with longitudinal current can be expressed as[7]

$$\begin{cases} E_x = \dfrac{\Gamma}{\omega\varepsilon_o} \displaystyle\sum_{m,n=-\infty}^{\infty} \dfrac{J_{mn}\, m\pi}{(\Gamma^2 - \Gamma'^2_{mn})\, a} \left[\cos\left(\dfrac{m\pi x}{a}\right) \sin\left(\dfrac{n\pi y}{b}\right)\right] \\ E_y = \dfrac{-\Gamma}{\omega\varepsilon_o} \displaystyle\sum_{m,n=\infty}^{\infty} \dfrac{J_{mn}\, n\pi}{(\Gamma^2 - \Gamma'^2_{mn})\, b} \left[\sin\left(\dfrac{m\pi x}{a}\right) \cos\left(\dfrac{n\pi y}{b}\right)\right] \\ H_x = -\dfrac{\omega\varepsilon_o}{\Gamma} E_y, \quad H_y = \dfrac{\omega\varepsilon_o}{\Gamma} E_x \end{cases} \qquad (1)$$

where

$$\begin{cases} \Gamma'^2_{mn} = \omega^2 \mu_o \varepsilon_o - \left(\dfrac{m\pi}{a}\right)^2 - \left(\dfrac{n\pi}{b}\right)^2, \quad m, n = \text{integers} \\ J_{mn} = \dfrac{4}{ab} \displaystyle\int_0^a \int_0^b J_z(x,y) \sin\left(\dfrac{m\pi x}{a}\right) \sin\left(\dfrac{m\pi y}{b}\right) dx\, dy \\ J_z(x,y) = \text{beam current distribution in the wave guide} \\ \Gamma = \text{wave propagation constant in the Z direction.} \end{cases} \qquad (2)$$

and a factor of $e^{j(wt-\Gamma Z)}$ is omitted in equation (1). It is observed that eq. (1) and (2) give a general form of a waveguide field for any given beam current distributions. It is also possible to calculate the power carried by these fields which are induced by various beam current distribution.[7]

C. Fields in the TEM Line

Although expressions for the characteristic impedance and the configurations for an infinite cross section TEM line have been found,[4] in order to find the coupling coefficient between the beam chamber and the TEM line, we need to find an exact solution for the field expressions in the TEM line. If the slots are absent and the conducting walls are perfect, referring to Figure 2, one can find the expressions for the potential in the entire cross-section by considering one quarter of the region. By symmetrical consideration, the potential in Regions 1 and 2 can be expressed as

$$\phi_1(x,y) = \sum_{m=1}^{\infty} A_m \cosh\frac{m\pi x}{c_2} \sin\frac{m\pi y}{c_2} + \frac{V_o y}{c_2} \quad \text{for } 0 \le x\ \ell,\ o \le Y \le c_2$$

$$\phi_2(x,y) = \sum_{n=\text{odd}}^{\infty} B_n \sinh\left[\frac{n\pi}{2b_2}(a_2 - x)\right] \sin\frac{n\pi y}{2b_2} \quad \text{for } \ell \le x \le a_2, \qquad (3)$$

$$o \le y \le b_2$$

The unknown constants A_m and B_n can be determined by making sure the potential and the electric fields are continuous across the boundary separting regions (1) and (2). It can be shown that[7]

$$B_n = \lim_{N\to\infty} \left\{ -V_o \left[n^2 - \left(\frac{2b_2}{c_2}\right)^2 \right] \right\} \Bigg/ \left\{ N \left(\frac{2b_2}{c_2}\right) \sin\left(\frac{n\pi c_2}{2b_2}\right) \left[n\cdot\coth\left(\frac{\pi\ell}{c_2}\right) \cosh\frac{n\pi}{2b_2}(a_2 - \ell) \right] + \frac{2b_2}{c_2} \sin\frac{n\pi}{2b_2}(a_2 - \ell) \right\} \qquad (4)$$

$$A_m = \operatorname{sech}\left(\frac{m\pi\ell}{c_2}\right) \frac{V_o}{\pi} (-1)^m \left\{ \frac{2}{m} - \sum_{n=\text{odd}}^{2N-1} 4m \left(b_2/c_2\right) \left[n^2 - \left(\frac{2b_2}{c_2}\right)^2 \right] \right.$$

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$$\Big/\left[n^2 - \left(\frac{2b_2}{c_2}\right)^2 m^2\right] N \left[n \cdot \coth\left(\frac{\pi \ell}{c_2}\right) \coth\left(\frac{n\pi}{2b_2}\right) \cdot (a_2 - \ell) + \left(\frac{2b_2}{c_2}\right)\right]\Bigg\} \qquad (5)$$

where V_o is the voltage induced in the center conductor of the TEM line by a beam current flowing in the beam chamber. The corresponding electric and magnetic field derived from these potentials can be calculated by using the following formulas.

$$\vec{E} = -\nabla\phi \qquad (6)$$
$$\vec{H} = \sqrt{\frac{\varepsilon}{\mu_0}}\, \hat{z} \times \vec{E}$$

Although fields in one-quarter of the region in the TEM line are calculated, fields in the other three regions can also be found in a similar fashion.

D. Coupling Between TEM Lines and the Main Beam Chamber

Fields found in the previous two sections were derived under the assumption that they are independent of each other. However, it is our goal to determine an induced voltage in the center conductor, or the power carried by the TEM line for a given particle beam current in the beam chamber. To this end, let us now introduce slots to the common walls between the TEM line and the beam chamber. These slots are introduced so that the field configuration in the main beam chamber is not significantly altered due to the presence of the scattered slot field. To determine V_O, one could match the boundary condition at the slot by requiring that the electric flux and the tangential component of the magnetic field be continuous across the slot. If this is done, it can be shown that[7]

$$V_o = -\frac{C_2}{2\ell} \sum_{mn} \frac{J_{mn}(-1)^n}{k^2 - \Gamma'^2_{mn}} \left(\frac{n}{m}\right)\left(\frac{a}{b}\right)\sqrt{\frac{\mu_o}{\varepsilon}} \left[\cos\frac{m\pi}{a}\left(\frac{a}{2} + a_3 + x_3\right) - \cos\frac{m\pi}{a}\left(\frac{a}{2} - a_3 + x_3\right)\right] \Big/ (1 + H) \qquad (7)$$

where

$$H = \sum_{m=1}^{\infty} \frac{C_2}{\ell} \frac{(-1)^m}{\pi} \Bigg\{ \frac{2}{m} - \sum_{n=odd}^{2N-1} 4m\left(\frac{b_2}{C_2}\right)\left[n^2 - \left(\frac{2b_2}{C_2}\right)^2\right] \Big/ \left[n^2 - \left(\frac{2b_2}{C_2}\right)^2 \cdot m^2\right] . N . \left[n \coth\left(\frac{\pi\ell}{C_2}\right) \coth\frac{n\pi}{2b_2}\left(a_2 - \ell\right) + \frac{2b_2}{C_2}\right]\Bigg\} - \frac{C_2}{2\ell} \sum_{n=odd}^{\infty} \Bigg[n^2 - \left(\frac{2b_2}{C_2}\right)^2\Bigg]\Bigg[2\cosh\frac{n\pi}{2b_2}\left(a_2 - \ell\right) - \cosh\frac{n\pi}{2b_2}\left(a_2 + x_3 - a_3\right) - \cosh\frac{n\pi}{2b_2}\left(a_2 - x_3 - a_3\right)\Bigg] \Big/ \Bigg\{ N\left(\frac{2b_2}{C_2}\right) \sin\left(\frac{n\pi C_2}{2b_2}\right)\left[n \coth\left(\frac{\pi\ell}{C_2}\right) \cosh\frac{n\pi}{2b_2} \cdot \left(a_2 - \ell\right) + \left(\frac{2b_2}{C_2}\right) \cdot \sinh\frac{n\pi}{2b_2}\left(a_2 - \ell\right)\right]\Bigg\} \qquad (8)$$

and x_3 is the displacement of the center of the slot from the center of beam chamber. All other parameters refer back to Figure 1. It is seen that this induced voltage depends only upon the beam current density as well as the dimensions of the beam chamber and the TEM line. It is interesting to observe that this coupling voltage does not depend on the width of the slot. This apparent paradox is due to the fact that small signal coupling was assumed at the start, and the way the boundary conditions were matched at the slot. It is also interesting to calculate the power coupling and its transfer function between the beam chamber and the TEM line. To this end, we shall use a classical approximation method.[6] If the dimensions of the slots are small compared with the wavelength in question, the effect of the slot to the system is equivalent to an ideal electric ($\vec{P}$) and a magnetic ($\vec{M}$) dipole moment located at the center of the slot with the slot closed. Hence

$$\begin{cases} \vec{P} = -\alpha_e\, \varepsilon_o\, (\hat{n} \cdot \vec{E})\, \hat{n} \\ \vec{M} = -\alpha_m \vec{H}_t \end{cases} \qquad (9)$$

where $\hat{n}$ ($\hat{t}$) is the unit normal (tangent) to the plane containing the slot and α_e and α_m are polarizabilities for the electric and magnetic dipole moment in the slot and $\vec{E}$ and $\vec{H}$ are the EM fields of Eq. (1) evaluated at the center of the slot. It was shown[6,8] that for a rectangular slot with dimensions shown in Fig. 3

$$\alpha_e = -\pi\, a_3 z_3^2\, \pi/8$$
$$\alpha_m = 0.364\, a_3^2\, z_3 + 0.352\, a_3^3 \qquad (10)$$

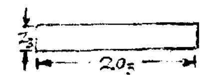

Fig.3 slot dimensions

and the fields in the TEM are

$$\vec{E}^+ = (e_1 + e_3)\, \vec{E}_c^+$$
$$\vec{H}^+ = (e_1 + e_3)\, \vec{H}_c^+ \qquad (11)$$

where the superscript + (−) indicates the forward (backward) propagating waves and $\vec{E}_c$ and $\vec{H}_c$ are given in Eq. (6) and

$$e_1 = -j\frac{\omega}{2p}\, \vec{P} \cdot \vec{E}_c^-$$
$$e_3 = j\frac{\omega}{2p}\, \vec{M} \cdot \vec{H}_c^- \qquad (12)$$

P is the power in one quarter of Fig. 2. Here, we only evaluate the forward propagating wave, it is undoubtly that backward wave is also induced in the TEM line by the beam current. Thus, it is desirable to minimize this backward wave by some means. To find the transfer function of the system, one can evaluate the power carried in the TEM line divided by the total current carried in the beam, that is, it is a transfer resistance. It is shown that this transfer resistance is

$$R_T = \frac{4k^2}{P I_{Z0}^2} \left[0.432 a_3^2 Z_3 + 0.2244\, a_3 Z_3^2 + 0.176\, a_3^2\right]^2 \left[\sum_{mn} \frac{J_{mn}\, n\pi\, (-1)^n}{(k^2 - \Gamma'^2_{mn})\, b} \sin\frac{m\pi}{a}\left(\frac{a}{2} + x_3\right)\right]^2 \begin{Bmatrix} \sum_{m=1}^{\infty} A_m \left(\frac{m\pi}{C_2}\right) \cosh\left(\frac{m\pi x_3}{C_2}\right) + V_o / C_2 & \text{for } x_3 \leq \ell \\ \sum_{n=odd}^{2N-1} B_n \left(\frac{n\pi}{2b_2}\right) \sinh\left[\frac{n\pi}{2b_2}\left(a_2 - x_3\right)\right] & \text{for } x_3 > \ell \end{Bmatrix}^2 \qquad (13)$$

It is interesting, at this juncture, to evaluate the ratio of power flow in the TEM line to the power carried by the beam. It is found that, indeed, this ratio is small or the assumption of weak coupling between the TEM line and the beam chamber is justified at the start.

E. The Kicker

In the Kicker, we want to determine the change of momentum in the Particle beam for an induced voltage V_O in the center conductor of the TEM line, assuming a similar physical structure in the kicker as in the pickup. However, it has proven to be useful if dimensions of the kicker is so designed so that dominant transverse electric (TE) mode is prevailed in the beam chamber waveguide. Starting from Eq. (5) and (6), the field in the slot can be found by evaluating Eq. (6) at the slot, thus determining the equivalent dipoles.

Waveguide fields in the beam chamber of the kicker can be expressed as

$$\begin{cases} \vec{E}^{TE} = \sum_{mn} A_{mn}^{TE} \vec{E}_{mn}^{+TE} + \sum_{mn} B_{mn}^{TE} \vec{E}_{mn}^{-TE} \\ \vec{H}^{TE} = \sum_{mn} A_{mn}^{TE} \vec{H}_{mn}^{+TE} + \sum_{mn} B_{mn}^{TE} \vec{H}_{mn}^{-TE} \end{cases} \quad (14)$$

where

$$\begin{cases} A_{mn}^{TE} \\ B_{mn}^{TE} \end{cases} = \frac{j\omega}{2P_{TE}} \left(\mu_o \vec{H}_{mn}^{\mp TE} \cdot \vec{M} - \vec{E}_{mn}^{\mp TE} \cdot \vec{P} \right) \quad (15)$$

and $\vec{H}_{mn}^{\pm TE}$ and $\vec{E}_{mn}^{\pm TE}$ are forward and backward propagating waveguide fields.[6]

The momentum of the beam carriers under the influence of these electromagnetic fields is

$$\frac{d\vec{P}}{dt} = q\,(\vec{E} + \vec{v} \times \mu_o \vec{H}) \quad (16)$$

Eq. (16) can be solved exactly by integrating both sides with respect to time while relativistic effect must also be taken into account. Nonetheless, transverse momentum in the y direction is easily seen to be affected under the TE_{10} waveguide field consideration. This thus contributes to a transverse momentum cooling. It is, however, possible to cool longitudinal momentum of the particle beam if higher order modal field is considered. It is noted that the physical mechanism in the pickup is not the same as in the kicker in this case, thus reciprocity theorem is not explicitly applied in the kicker.

F. Results and Conclusion

As an illustration for this study, the induced voltage in the center conductor of the TEM line is one of the most interesting parameter to us. It is plotted in Fig. 4. This induced voltage V_o normalized to the total beam current is plotted against the slot sized relative to the dimension of broadside wall of the beam chamber. It is seen that it has a value in the order of 1 micro-volt for a beam current of 1 micro-ampere, which is a strong signal in this study. The transfer resistance (Eq. (13)) is the next parameter to be plotted. It is shown in Fig. 5, in that, the transfer resistance is plotted as a function of the center position of the slot moving along the broadside wall of the chamber. It is seen that it attains a modest value of 5 ohms and as high as 100 ohms. This power coupling was shown to be linear,[7] hence, a 150-ohm transfer resistance is obtained for the system if 30 slots are used in each electrode. The bandwidth of this type pickup or kicker depends on the conducting losses of the structure, size of the slot, dielectric constant and the electrical connector to the system, etc. Results, such as field expressions, power transfer between beam current and the pickup, are significantly different from that obtained by Faltin.[4]

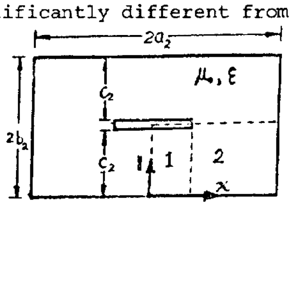

Fig. 2. Cross section of the TEM line.

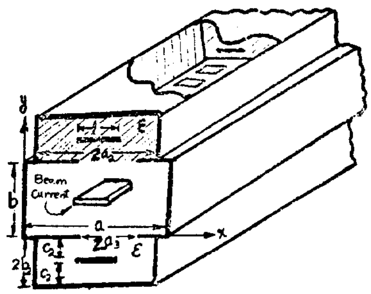

Fig. 1. The physical structure of the pick-up or kicker. Two TEM lines are coupled to the beam chamber.

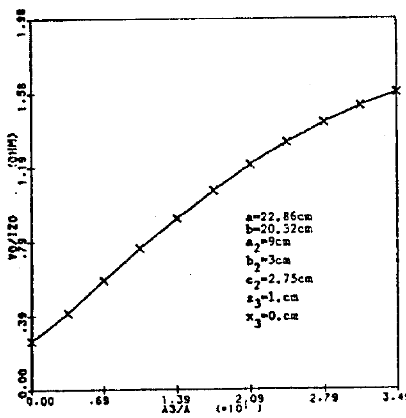

Fig. 4. Voltage, V_o, induced on the center conductor of the TEM line, normalized to the magnitude of the beam current, I_{zo}, as a function of the size of the slot, a_3, normalied to a.

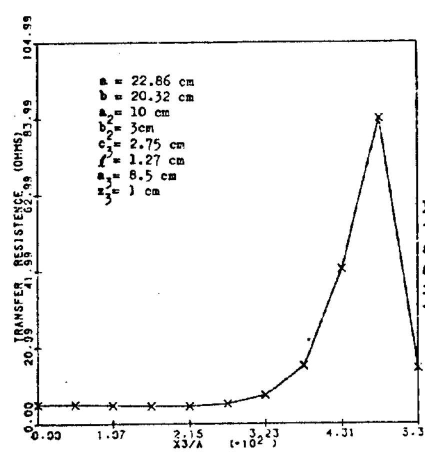

Fig. 5. Equivalent transfer resistance R_T as a function of the slot center x_3 away from the center line of the beam chamber.

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