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STOCHASTIC COOLING OF BUNCHED BEAMS*

J.J. Bisognano and S. Chattopadhyay
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract

Numerical simulation studies are presented for transverse and longitudinal stochastic cooling of bunched particle beams. Radio frequency buckets of various shapes (e.g. rectangular, parabolic well, single sinusoidal waveform) are used to investigate the enhancement of phase space cooling by nonlinearities of synchrotron motion. The connection between the notions of Landau damping for instabilities and mixing for stochastic cooling are discussed. In particular, the need for synchrotron frequency spread for both Landau damping and good mixing is seen to be comparable for bunched beams.

I. Introduction

Stochastic cooling of particle beams, the feedback damping of transverse oscillations and longitudinal momentum spread, is an essential feature of plans to produce antiproton beams of sufficient quality for $p\bar{p}$ storage ring physics.[1,2] These schemes call for cooling a coasting beam; the question arises whether it is possible to apply stochastic cooling directly to a bunch and obtain a reasonable amount of cooling. The success of this technique would ease the requirements for RF manipulations in accummulation, and opens the possibility of compensating for emittance growth due to diffusion caused by periodic beam-beam interactions in storage rings. Another motivation to study bunched beam cooling is its direct connection to bunched beam instability phenomena -- Schottky signal suppression being the anologue of coherent instabilities.

II. Particle Orbits

Single particle orbits in a coasting beam and bunched beam are topologically different. The cooling dynamics in the two cases differ nontrivially. While particles in a coasting beam are essentially freestreaming, particles in a bunch follow trapped oscillatory orbits, which repeat in phase space. Particles with different revolution frequencies in a coasting beam eventually separate; on the other hand, the separation between two particles in a bunch is bounded by the amplitudes of the synchrotron oscillations. During the cooling process, a particle is affected primarily by particles neighboring in frequency. For a bunch without synchrotron frequency spread (a parabolic potential well), all particles have the same Schottky frequencies.

In the simulation study we investigate three types of particle orbits -- orbits in a rectangular potential well, in a parabolic well, and in a sinusoidal RF bucket. In order to study directly the effect of a spread in synchrotron frequencies on the cooling rate, we have also considered the case of elliptical orbits as in the parabolic case but with an imposed amplitude dependence of the synchrotron frequency.

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III. Algorithm for Model Cooling System

In the simulation, the correction per step to the cooled variable was taken to be of the form

$$\Delta\vec{x}_i = -\sum_{k=1}^{N} g(\theta_i - \theta_k)\,\vec{x}_k$$

where $\vec{x}_k = (x_k, x'_k)$ is the cooled phase-space vector, transverse or longitudinal of k^{th} particle and $\theta_k(t)$ describes the longitudinal orbit of k^{th} particle (angle around the ring) as a function of time. We report results for cases where the transverse correction was applied to transverse betatron position (x_k) only, and the longitudinal correction was applied to the longitudinal velocity deviation from the synchronous particle, $x_k' = v_k - v_s$ only. In real cooling systems, corrections are applied impulsively in momentum; however, it is a matter of interpretation in a simulation experiment, since a correction in either position or velocity of the transverse motion will lead to a change in betatron oscillation energy in general.

The function $g(\theta)$ simulates the response function of the feedback system and determines the azimuthal distance of the effective interaction between particles. The rotational symmetry in the angle 'θ' implies $g(\theta)$ to be periodic in 'θ'. Thus we use a finite number of azimuthal harmonics to simulate a realistic $g(\theta)$:

$$g(\theta) = \sum_{\ell=0}^{m} a_\ell \,\mathrm{Cos}\,\ell\,\theta = \sum_{\ell=-m}^{+m} g_\ell\, e^{i\ell\theta}$$

We use up to m = 4 harmonics, implying a feedback system of effective angular extent $\theta_0 \sim 90°$. Since the bunch length has to be larger than the feedback system length electrically in order to have effective cooling, we expect good cooling only for bunches longer than 90° in angular extent.

Initial distributions of particles in phase space were constructed out of a random number generator to tailor to a desired amplitude profile. The transverse and longitudinal oscillation amplitudes were chosen according to $a_T = [1 - \sqrt{1 - R}]^{\frac{1}{2}}$ and $a_L = (1/2)\cdot F\cdot\sqrt{R}$ where 'R' is a random number between 0 and 1, and F is the fraction of the total length of the ring occupied by the bunch ($F = \theta_B/2\pi$, θ_B = angular extent of the bunch). In the particular case of the rectangular potential well, the revolution frequencies were chosen from a rectangular distribution.

We correct the cooled phase space variable of a particle at a fixed kicker position at every nominal revolution period of the bunch. Once a central reference particle has arrived at the kicker, all particles are kicked irrespective of their angular position in the ring. The basic code is a modification of a transverse coasting beam simulation developed by Laslett.[3] In the coasting beam context, the results were well-described by stochastic cooling theory including signal suppression.[4] For this agreement, however, it was found necessary to introduce a small, random frequency variation to destroy very small frequency

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differences between particle pairs. Physically, this corresponds to energy variations induced by stray fields, noise, etc., which do not change the gross frequency distribution. This feature was retained in the bunch beam code, although it is unnecessary for the longitudianl simulation where Schottky noise effects provide sufficient "wiggle" of frequency.

IV. Coasting Beam Results

For transverse stochastic cooling, the rate s_ω for error σ^2 for particles of revolution frequency ω is (in a continuous correction limit)

$$s_\omega = -\sum_{\substack{n\\ \pm\nu}} \left[\frac{g_n}{\varepsilon_{-n\pm\nu}(\omega)} - \frac{1}{2}\frac{N\pi}{|n\pm\nu|} \frac{|g_n|^2}{\left|\varepsilon_{-n\pm\nu}(\omega)\right|^2} f(\omega) \right] \quad (1)$$

(amplifier noise is not included here) where ν is the tune, $f(\omega)$ is the angular frequency distribution normalized to unity, N is the number of particles, and $\varepsilon_{-n\pm\nu}(\omega)$ are feedback signal suppression factors.[4] Equation (1) may be rewritten as

$$s_\omega = -\sum_{\substack{n\\ \pm\nu}} \frac{g_n}{\left|\varepsilon_{n\pm\nu}(\omega)\right|^2} \quad (2)$$

by using cancellations arising in the expressions

$$\varepsilon_{(n\pm\nu)} = 1 + \frac{Ng_{\mp n}}{|n\pm\nu|} \int_{\eta\to 0} d\omega' \, \frac{f(\omega')}{\eta \pm i(\omega-\omega')} \quad (3)$$

In the simulation, it was found that the δ-function part of the singular integral describes well the signal suppression effects for a rectangular frequency distribution of half-width Δ (all times are measured in units of revolution period); i.e.,

$$\varepsilon_{(n\pm\nu)} \approx 1 + \frac{g_\ell N}{8\Delta\ |n\pm\nu|} \quad (4)$$

The factor $\varepsilon_n \geqslant 1$, and from (2) it is seen that the cooling rate monotonically decreases with decreasing Δ; at no time does the Schottky noise dominate over the coherent cooling rates. Coasting beam simulations were performed with 90 and 180 particles cooled for 1000 correction steps. Averages over 25 cases agree within 5% to theory, with case to case variation of ±10%. The ε_n factor ranged between 1 to 5. Some growth was seen for large g_ℓ which is attributable to the discrete nature of corrections, and was insensitive to Δ.

For g_ℓ of the "wrong" sign, the condition $\varepsilon_\eta = 0$ gives the condition for coherent instability. Thus, the condition

$$\left| \frac{g_\ell N}{8\Delta\ (n\pm\nu)} \right| \gtrsim 1 \quad (5)$$

gives both a condition on the sufficiency of Landau damping for instability, and a criteria for feedback and/or Schottky noise effects to be important in stochastic cooling. We note that $|Ng_\ell|$ is the magnitude of the coherent growth rate for instabilities or the damping time of Schottky signals in the zero frequency spread limit.

Analogous to the coasting beam situation, approximate criteria for bunch instabilities are

$$\delta s_{\|} \leqslant \frac{4}{\sqrt{m}} \left| \Delta\omega_{\|}\ mm \right| \quad (6)$$

or

$$\delta s_{\perp} < \left| \Delta\omega_{\perp}\ mm \right| \quad (7)$$

for longitudinal and transverse[5], respectively where the $\Delta\omega$ are coherent growth rates, and $\delta s_{\|}$ and $\delta s_\perp$ are the spreads in synchrotron oscillation and betatron frequencies, respectively. If this condition is applied to a stochastic cooling system, we arrive at the condition that the synchrotron frequency spread must exceed the coherent damping time of the Schottky signals with no mixing.

V. Bunched Beam Cooling

The two main distinguishing features of bunched beam versus coasting beam stochastic cooling are:

1) The frequency variation which provides mixing is now determined (as discussed in the previous paragraph) by the spread of synchrotron frequency rather than revolution frequency or momentum spread. The amplitude dependence of synchrotron frequency, which depends critically on the bucket shape, becomes crucial to cooling.

2) Because of the finite length of the bunch, the Schottky signals at different harmonics become correlated. This effect manifests itself in enhancing beam heating and coupling the signal suppression in different Schottky bands.[6]

One set of runs was performed with a parabolic potential energy bucket; i.e., the synchrotron oscillation is linear with frequency amplitude independent. The cooling system used harmonics 1 to 4 with equal weighting, 90 particles, and the gain would provide a perfect cooling rate of .009/turn. The synchrotron frequency $f_s = .05$ and the bunch length is .5 of the ring circumference. No significant cooling occurs after 1 synchrotron oscillation. Similar runs for longitudinal corrections also yields no cooling, and varying synchroton frequency and gain had no appreciable effect.

To investigate the effect of synchrotron frequency spread in detail runs were made with a "square" bucket and a sinusoidal "RF" bucket. For the square bucket particles were assigned a range of revolution frequencies from a rectangular distribution. The particles were advanced in azimuthal angle as in a coasting beam until they reached the end of the bunch. At the ends, particles are elastically reflected with only their angular velocities changing sign. The motivation of this bucket shape was the hope that it would most closely resemble a coasting beam for analysis and offer in some sense a maximal degree of nonlinearity. The sinusoidal bucket corresponded to a harmonic 2 system. The phase orbits were determined by the first 5 terms of an expansion of the orbits in terms of elliptic integrals. In Table 1 results of a number of 90 particle runs are tabulated for transverse and longitudinal cooling. Table 2 gives coasting beam rates from theory. The bunching factor gives the fraction of the ring circumference occupied by the bunch. The results in Table 1 are for single cases with 90 particles. Within each category the same seed was used to initialize the random loading to lessen statistical variation as parameters were changed.

Table 1

Square Bucket - Transverse - BF = 0.5

	Δ	s
$g_4 = .0022$	.1	.0036
$g_3 = .0022$	.01	.0012
$g_4 = .0022$	.01	.0015
$g_3 = g_4 = .0022$	.01	.0020
$g_1 = g_2 = g_3 = g_4 = .0022$	.1	.0090
$g_1 = g_2 = g_3 = g_4 = .0022$	.01	.0021

Sinusoidal Bucket - Transverse

	Δf_s	BF	s
$g_4 = .0022$	.015	.7	.0022
$g_4 = .0022$	.025	.9	.0029
$g_4 = .0022$	.0015	.7	.00051
$g_1 = g_2 = g_3 = g_4 = .0022$	.015	.7	.003
$g_1 = g_2 = g_3 = g_4 = .0022$	.025	.9	.0045
$g_1 = g_2 = g_3 = g_4 = .0022$	.0015	.7	.00065

Sinusoidal-Like Bucket - Longitudinal - BF = .7

	Δf_s	s_{200}	s_{1000}
$g_1 = g_2 = g_3 = g_4 = .0022$	.15	.0096	.0065
$g_1 = g_2 = g_3 = g_4 = .0022$	.015	.0054	.0043

Table 2

Coasting Beam Theory - Transverse

	Δ	s
$g_3 = .0022$	.1	.0038
	.05	.0033
	.01	.0013
	.005	.00062
$g_4 = .0022$	.1	.004
	.05	.0035
	.025	.0028
	.015	.0022
	.01	.0017
	.005	.00088
$g_1 = g_2 = g_3 = g_4$	.1	.014
	.05	.011
$= .0022$	.025	.0084
	.015	.0059
	.01	.0042
	.005	.002
$g_3 = g_4 = .002$ effective	.01	.0022
$g_1 = g_2 = g_3 = g_4$ effective	.01	.0028

For the square bucket, single harmonic ($\ell = 3,4$) rates compare remarkably well with coasting beam theory. However, when both harmonics are present, the cooling rate is significantly different from that of coasting beam theory, where rates for each harmonic are simply added. For a square bucket, Schottky signals ℓ and m are coupled with a weighting

$$\left[\frac{\sin(\ell - m)\theta_0}{(\ell - m)\theta_0}\right]^2 \tag{8}$$

where θ_0 is the half length of the bunch. The last entries in Table 2 give rates calculated with coasting beam theory, using an effective gain

$$g_{\ell\,eff} = \sum_m g_m \left[\frac{\sin(\ell - m)\theta_0}{(\ell - m)\theta_0}\right]^2 \tag{9}$$

to evaluate the ε_ℓ. Agreement is good. These results clearly demonstrate the interference of neighboring harmonics; but for a long bunch, this interference does not totally cancel the effects of using neighboring harmonics in a cooling system.

For transverse cooling in a sinusoidal RF bucket, cooling rates for a synchrotron oscillation spread Δf_s are comparable to coasting beam rates with $\Delta = \Delta f_s$. Again, with several harmonics simultaneously acting, there is a degradation of coasting beam rates by a factor of 2. It should be noted that the longitudinal random load provides a uniform distribution in phase space.

Finally, the last entries in Table 1 are for longitudinal runs. Effective cooling rates/step are given after 200 and 1000 correction steps are given. The phase space orbits are elliptical with amplitude variation of synchrotron frequency. Cooling rates degraded as mixing lessens with higher phase space density.

Conclusions

Synchrotron frequency spread provides the necessary mixing mechanism for bunched beam cooling. In addition, it appears that the natural nonlinearities of a single, long full RF bucket can provide mixing comparable to a coasting beam for harmonics of higher frequency than those associated with the gross bunch structure. However, as the bunch length decreases degradation of cooling occurs as the mixing mechanism couples neighboring Schottky bands.[6]

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