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SPIRAL DESIGN AND BEAM DYNAMICS FOR A VARIABLE ENERGY CYCLOTRON

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Summary

Beam-orbit studies were performed for the conversion of the SREL synchrocyclotron magnet for use as a room temperature, multiparticle, isochronous cyclotron. Based on model magnet measurements of field profiles for 8 to 23 K gauss hill fields, a four sector spiral pole tip design has been realized which allows all isotope species of heavy ion beams to be accelerated to required final energies. The total spiral angle of 38° allows injection of the beams from the MP tandem into the cyclotron through a valley. The two valley RF system of 140 kV peak accelerates beams on harmonic numbers 2, 3, 4, 6 and 10 at 14 to 21 MHz. Computer calculations indicated acceptable ν_z, ν_r and phase space beam characteristics and passing of resonances for typical beams considered: ^{16}O at 8 and 150 MeV/amu, ^{60}Ni at 100 MeV/amu and ^{238}U at 2.5 and 16 MeV/amu. Single turn extraction is achieved with electrostatic deflection.

The Hill-Valley Contours

We have proposed conversion of the 250 cm radius SREL magnet into a variable energy, multiparticle spiral focused isochronous cyclotron containing 4 hills and 4 valleys with two dees in opposing valleys.[1] For the sake of the discussion to follow, we briefly state focussing characteristics of isochronous spiral-ridged cyclotrons. Approximate expressions for the axial and radial oscillation frequencies (per turn) of the beam at any radius are given by

$$\nu_z^2 = -k + \frac{N^2}{N^2-1} F^2(1 + 2\tan^2 \xi), \quad \nu_r^2 = 1 + k$$

where $k = \frac{R}{B}\frac{dB}{dR}$ is the radial field index, N is the number of sectors, ξ is the spiral angle between a hill edge and a ray from the center of the cyclotron, and F^2 is the flutter defined by

$$F^2 = \frac{\langle B^2\rangle - \langle B\rangle^2}{\langle B\rangle^2}$$

where B is the magnetic field as a function of θ at the given radius.

The isochronous condition, that the field rise radially to compensate the relativistic increase in mass with increasing energy, requires

$$k = \gamma^2 - 1 \qquad \gamma = \frac{m}{m_o}$$

creating an axially defocussing machine in the absence of the flutter term. The radial oscillation frequency for the isochronous condition is just γ which rises slowly from 1 at the center to 1.16 for the most relativistic ion, 150 MeV/nucleon. Since ν_r is to lowest order independent of flutter and spiral, it does not significantly constrain the design of these parameters.

The design of an appropriate spiral for a variable energy, variable ion species machine has to satisfy special constraints. The operating range of ν_z has to be considered for all beams desired. Clearly one must have a sufficient combination of flutter and spiral to focus the most energetic beam, in our case a 150 MeV/nucleon beam. Yet for low energy beams, ν_z must remain smaller than one, even though k decreases. A further constraint with this machine is the need to provide a minimum total integrated spiral angle of about 35° to allow injection through a valley. Finally, unlike a superconducting machine in which the iron is always in saturation, the proposed machine shows no simple variations in flutter with average field. The properties of the magnetic field are discussed elsewhere at this conference.[2] The measured flutters at 230 cm varied from .23 for 8 MeV/amu ^{16}O to .09 for 16 MeV/amu ^{238}U, corresponding to hill fields of 8 and 23 K gauss respectively.

The required parameters for beam studies were established in the following way. Field measurement data from an 18-inch model magnet were used to generate magnetic fields (as discussed in Ref. 2) for 8 cases spanning the range of performance of the cyclotron. To satisfy the injection constraint and focus all beams, a pole sector was realized which was radial out to 65 cm and then spiraled such that the value of ξ maintained ν_z = .36, as given by the approximate formula, out to ∿ 240 cm for the 150 MeV/amu ^{16}O beam. The total spiral angle is 38°, quite acceptable from the standpoint of injection and mechanical design. A more exact calculation for ν_z with the equilibrium orbit computer code TRIUMF[3] using this same spiral and model magnetic field gave a well behaved ν_z close to the approximate value of .36 (Fig. 1). Computer calculations of ν_z for four other cases using the spiral angle fixed from the ^{16}O beam and the appropriate magnetic field from the model measurements are also displayed in Fig. 1. All beams focus in the Z direction. Two high energy beams (150 MeV/amu ^{16}O and 100 MeV/amu ^{60}Ni) operate well below ν_z = .5 until the extraction region. A 51 MeV/A ^{127}I beam (not shown) behaves similarly. Three low energy beams (8 MeV/amu ^{16}O shown, as well as very similar 5 MeV/amu ^{60}Ni and 3.5 MeV/amu ^{127}I beams not shown) pass rapidly through ν_z = .5 during acceleration but remain below ν_z = 1 until after extraction. The two beams in an intermediate ν_z region (16 MeV/amu and 2.5 MeV/amu ^{238}U) also are well behaved in ν_z.

A contour map of the magnetic field for 150 MeV/A ^{16}O is presented in Figure 2; it additionally illustrates the spiral shape.

Emittance at Injection and Beam Shape

In Reference 4 the emittance of the beams from the tandem is discussed along with the r.m.s. scattering angle induced by the cyclotron stripping foil. Injection studies have determined that a beam spot of axial full width ΔZ = 4 mm and radial full width Δr = 2 mm is realizable at the cyclotron stripping foil. Based on these considerations a list of emittances can be drawn up for representative cyclotron beams at injection after the stripping foil (Table 1).

Let us consider the 150 MeV/nucleon ^{16}O beam which has the largest emittance of the high energy beams. We have calculated the radial phase space envelope about an equilibrium orbit in the cyclotron for 2.8 mm^2 (or 2.75π mm-mr) phase space area just after the stripping foil and obtained Δp = 2 mm, Δr = 2 mm. The injected beam, with Δr = 2 mm, is well matched to this calculated shape. Likewise the phase space envelope after

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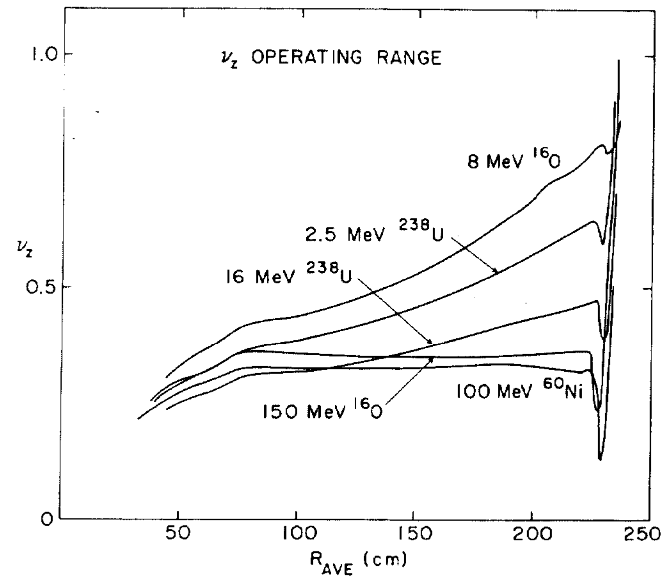

Figure 1. Operating range of representative beams in ν_Z. (Energies are per amu.)

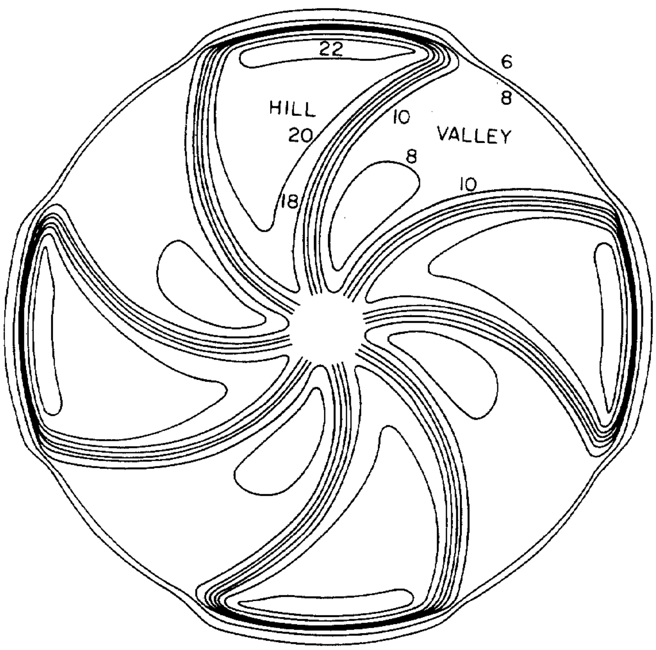

Figure 2. Hill and valley contours for the midplane magnetic field. Numerical values are in K gauss.

stripping for the axial dimension (Δp = 1 mm, ΔZ = 4 mm) corresponds to the injection full width of 4 mm. All other injected beams can be well matched to the cyclotron phase space at injection.

Resonances

An advantage of the four-sector geometry for the proposed cyclotron is the avoidance of low order essential resonances which must be passed during the acceleration process. The only important (order $\leq$ 4) essential resonance is the Walkinshaw coupling resonance ($\nu_r - 2\nu_Z = 0$) near the edge of the field at extraction. Error harmonics in the field will drive non-essential resonances of which the one-dimensional resonances $\nu_Z = 1/2$, $\nu_Z = 1$ and $\nu_r = 1$ are the only important ones, and care must be taken to reduce these harmonics in the main field.

Table 1

Emittance at Injection after the Stripping Foil

Phase space in mm^2 is tabulated including factor π. Injected beam is assumed to have Δr = 2 mm and Δz = 4 mm for calculation of the effect of scattering in the foil on the emittance.

Ion	E/amu (MeV)	Vertical Emittance mm-mr	Vertical Emittance (mm^2)	Horizontal Emittance mm-mr	Horizontal Emittance (mm^2)
^{16}O	150	2.75π	(2.80)	3.12π	(3.18)
^{16}O	8	4.67π	(8.87)	5.19π	(9.85)
^{60}Ni	100	1.61π	(1.59)	2.13π	(2.10)
^{60}Ni	5	3.38π	(6.27)	3.92π	(7.27)
^{127}I	51	1.08π	(1.26)	1.43π	(1.67)
^{127}I	3.5	2.89π	(4.39)	3.25π	(4.95)
^{238}U	16	1.13π	(1.63)	1.62π	(2.33)
^{238}U	2.5	2.53π	(3.51)	2.90π	(4.02)

The introduction of a first harmonic field component for extraction will necessarily drive the $\nu_Z = 1/2$ resonance (by the gradient). The constraints on the axial motion are particularly severe since the vertical excursions of the beam must not exceed the dee aperture of ± 1.2 cm from the median plane. Joho[5] gives approximate expressions for the maximum fractional growth per turn of Z amplitude $\pi R \frac{g_1}{B_o}$ and stop band half-width

$$\Delta\nu_z = R\, g_1/2B_o$$

where g_1 is the radial derivative of the first harmonic bump.

For the low energy beams ν_Z = .5 is crossed well before extraction. The case of 2.5 MeV/amu ^{238}U will be taken as an example because the beam crosses ν_Z = .5 at R = 170 cm, closest to the center of the first harmonic bump that would be introduced for extraction. If we require that the Z motion of the beam not increase by more than 10% going through this resonance, then the simple formula for the linear growth per turn, combined with the number of contributing turns

$$n_{eff} = \left|\frac{d\nu_Z}{dn}\right|^{-1} \approx 16.9$$

taken from TRIUMF equilibrium orbit calculations, implies a limit on the gradient (g_1) of the first harmonic bump of 17 gauss/m. The stop band width for this gradient corresponds to about 1/8 cycle of the axial motion. These limits on stability of Z motion seem satisfactory and the limit on the first harmonic gradient is realizable. Calculations with first harmonic bumps of large enough magnitude to generate turn separation required to extract all beams indicate that gradients less than 17 gauss/m are sufficient for this purpose. In fact computer calculations (see below) are at least as optimistic as these formulas on the stability of axial motion.

In Fig. 3 we show representative plots of ν_Z vs. ν_r for two extreme cases, 150 MeV/A ^{16}O and 2.5 MeV/A ^{238}U. Clearly, the only other one-dimensional resonance is $\nu_Z = 1$. As mentioned earlier this resonance is crossed just after extraction, and is expected to cause no difficulties.

A third resonance of importance for Z motion is the two-dimensional Walkinshaw coupling resonance $\nu_r - 2\nu_Z = 0$ which depends strongly on radial beam quality. For the high energy beams, 150 MeV/amu ^{16}O, 100 MeV/amu ^{60}Ni and 16 MeV/amu ^{238}U this resonance is

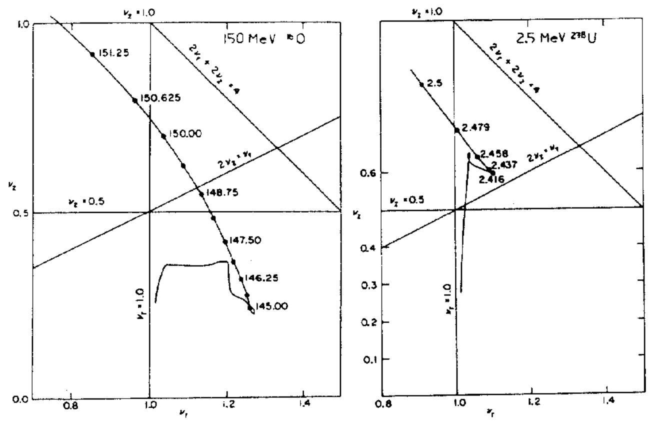

Figure 3. ν_Z, ν_r operating range for 150 MeV/nucleon ^{16}O and 2.5 MeV/amu ^{238}U.

crossed just before extraction. For the low energy beams this resonance is crossed well inside the machine in a few turns as is $\nu_Z = 1/2$. Again following Joho,[5] one finds the maximum growth per turn of Z amplitude equal to $(2\pi \Delta r \Delta Z) \times d\nu_r/dR$. For the 8 MeV/amu ^{16}O (Fig. 1) it is crossed at R = 143 cm with $d\nu_r/dR$ about 2×10^{-4} cm^{-1}. Here the growth in Z amplitude is very small since the beam phase space is very small (Δr and ΔZ are a few mm at most), and the beam can be well centered by proper injection or by centering coils at small radius. The 2.5 MeV/nucleon ^{238}U requires little more consideration with $2\nu_Z = \nu_r$ being passed at R = 176 cm with $d\nu_r/dR$ equal to about 4×10^{-4} cm^{-1}. Computer calculations with the orbit tracing code GOBLIN[6] indicate that the combined effect of the Walkinshaw resonance and the $\nu_Z = 1/2$ resonance (with a first harmonic bump of 20 gauss/m and Δr = .9 cm) on the maximum Z excursion is less than 5% for the 2.5 MeV/amu ^{238}U case.

Acceleration and Beam Extraction

Studies of the beam extraction system have been made using the measured magnetic fields and the code GOBLIN. These calculations indicate that adequate turn separation can be obtained for electrostatic deflection of the beam, and that the quality of the beam can be preserved during extraction.[1] As is noted elsewhere at this conference[4] the time bunching of the beam is ± 1° in RF phase corresponding to in energy resolution of 1.5×10^{-4}.

Acceleration parameters are presented in Table 2. The turn separation due to acceleration is large enough (about 1 cm) for certain of the low energy beams to allow an electrostatic septum to be located between adjacent turns. For other beams this separation is not adequate, and magnetic field perturbations will be required to enhance the turn spacing so adjacent turns can clear the septum. A first harmonic field bump in the presence of the $\nu_r = 1$ resonance can be used to achieve the increased turn separation. For cases where the turn separation is marginally too small (the uranium beams), the extra separation can be obtained by using a first harmonic bump large enough to shift the beam into the extraction channel just before $\nu_r = 1$ (brute force extraction). For the high energy, low mass beams (typically taken as ^{16}O) the field bump required to obtain the necessary turn separation by brute force translation becomes quite large, and the presence of resonances (such as $\nu_Z = 1/2$ and $\nu_Z = 1$) driven by harmonics and gradients of the bump require that some other scheme be used.

Figure 4 presents calculations for the precessional extraction of the high energy ^{16}O beam made with the code GOBLIN. Acceleration through the $\nu_r = 1$ resonance in the presence of a first harmonic bump of 5 gauss occurs at turn 749, and the beam then precesses (and separates) as ν_r decreases to 0.78 at turn 755. Because of the large energy gain per turn and the rapid fall of the fringe field, the phase slip at turn 755 is only 9°. After turn 753 the turn spacing has become large enough to insert a septum for electrostatic deflection. An electrostatic deflector subtending 40° in azimuth placed between turns 753 and 754 can bring the beam out of the field in 146 degrees from the entrance to the deflector.

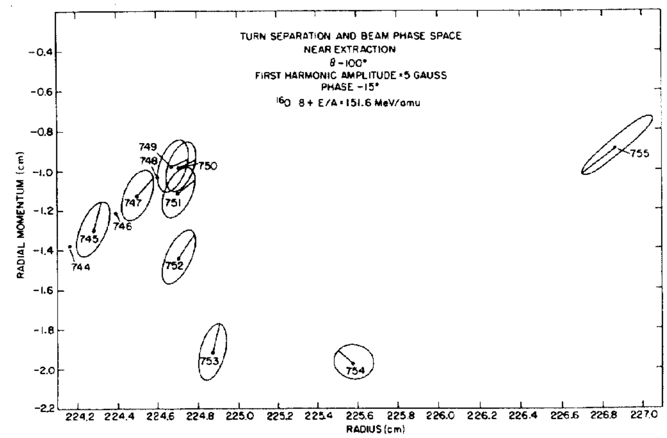

Figure 4. Turn separation and radial phase space envelopes for 150 MeV ^{16}O beam near extraction.

Table 2

Acceleration Parameters

The peak RF voltage is 140 MeV. Actual voltages will be reduced due to the geometry of the dees by 30% at worst. Reductions due to transit time effects are negligible.

Ion	E/amu (MeV)	N	V_{eff} (MV)	Q	E/amu-turn (MeV)	# turns
^{16}O	150	2	.099	8	.198	747
^{16}O	8	6	.099	5	.124	60
^{60}Ni	100	2	.099	21	.139	709
^{60}Ni	5	6	.099	10	.066	71
^{127}I	51	3	.129	31	.126	396
^{127}I	3.5	10	.099	11	.034	99
^{238}U	16	4	.140	33	.078	198
^{238}U	2.5	10	.099	13	.022	110

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