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A FREE ELECTRON LASER*

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Summary

This paper includes a brief review of the theory of stimulated bremsstrahlung by relativistic electrons in a periodic magnetic field, a discussion of the possibilities for a classical interpretation, and a description of the apparatus under construction in W. W. Hansen Laboratories of Physics.

Background

Radiation emitted by an electron beam in a periodic magnetic field was studied in a classical context by Motz and his collaborators [1-3] in a program initiated at Stanford and continued at Oxford University. The cited publications derive expressions for the distribution in frequency and the intensity of the spontaneously radiated power. Motz and Nakamura[4] also discussed the problem of amplification relying on an adaptation of Kompfner and Pierce's travelling wave mechanism although this appears to be distinct from stimulated bremsstrahlung. Finally, Pantell, Puthoff, and Soncini[5] -- also at Stanford -- have preposed the use of stimulated inverse Compton scattering by relativistic electrons in an intense microwave field. The analysis for stimulated Compton scattering has fundamental parallels in the bremsstrahlung problem.

Review of Theory

An analysis of the gain due to stimulated bremsstrahlung has previously been published.[6,7] Figure 1 indicates the problem reduced to its essentials. A relativistic electron beam (moving in the $+\hat{z}$ direction) passes through a region in which there is a spatially periodic transverse magnetic field. A certain amount of radiation will be emitted spontaneously in passage through the field. We are interested in the degree to which the rate for radiation emitted in the $+\hat{z}$ direction can be enhanced due to stimulated emission in the presence of a pre-existing radiation field.

In the laboratory frame the transverse component of the magnetic field can be represented as $B_o \exp i[(2\pi/\lambda_q)z]$ where λ_q is the period of the field. In the electron rest frame -- provided that B_o is not too large -- there is both an $\mathcal{E}$ and a B' field of the form

$$B'(z', t') = \gamma B_o \exp i[\gamma(2\pi/\lambda_q)(z'+\beta ct')]$$

$$\mathcal{E}'(z',t') = \beta r B_o \exp i[\gamma(2\pi/\lambda_q)(z'+\beta ct')]. \quad (1)$$

The longitudinal component of the field transforms without modification and becomes negligible in comparison to the transverse component at sufficiently high energy.

If β were equal to one, these fields would be identical to the fields due to a plane wave travelling in the $-\hat{z}$ direction. But β is equal to one to order $1/\gamma^2$ and, for $\gamma >> 1$, we can replace the physical magnetic field with the equivalent plane wave. The amplitude of the B-field in the equivalent wave is one half B_o while the wavelength is twice the magnet period λ_q.

Radiation emitted in the $+\hat{z}$ direction can then be considered as radiation backscattered from the equivalent plane wave. The wavelength of the backscattered radiation is reduced by the doppler shift to $\lambda_q/2\gamma^2$. The polarization of the backscattered radiation is determined by the symmetry of the physical magnetic field: a laboratory frame field with helical symmetry will produce circularly polarization while translational symmetry will result in elliptical or linear polarization. Stimulated emission is possible only if the stimulating radiation has the same polarization as the spontaneous radiation.

This approximation can be considered a variation of the Weizsacker-Williams Method. It is particularly useful since it permits analysis in terms of the well known formulae for Compton scattering.

The possibility of stimulated emission arises due to the presence of the creation operator a^* in the interaction Hamiltonian. Application of the interaction Halliltonion to the initial state introduces a multiplicative factor $\sqrt{n+1}$ in the matrix element and $(n+1)$ in the transition rate where n is the number of stimulating photons per unit volume.

Absorption must also be considered. In absorption an electron absorbs one of the high energy photons $\lambda_f = \lambda_q/2\gamma^2$ and radiates, a low energy photon into the equivalent plane wave. For this case the interaction Hamiltonian introduces a factor of $\sqrt{n}$ in the matrix element and (n) in the transition rate.

For $n >> 1$ the rates for absorption and emission become very nearly equal. The only thing that distinguishes the two rates for large n is kinematics. Both processes feed on the same population of electrons.

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As a consquence it can be shown that absorption tends to take place at a slightly higher frequency than stimulated emission.

The situation is indicated schematically in Figure 2. The emission and absorption lines have the same shape with relative widths determined by the electron beam homogeneity ($\Delta\gamma/\gamma$) and the spectral purity of the equivalent plane wave ($\Delta\nu/\nu$). Attainable linewidths are likely to be of the order of 1 part in 10^3 or 10^4 .

Consideration of kinematics then shifts the absorption line up in frequency relative to the emission line center at ν_{fo} by a frequency $\nu_{fo}\,(h\nu_f/\gamma mc^2)$. While the shift is small it appears to be large enough to provide useful gain on the low frequency side of the emission line. If the energy distribution function for the electrons and the power spectral density of the equivalent plane wave are approximated by Gaussians, the net gain $\bar{G}$ in dB per meter can be shown to be given by equation (2) below. All quantities are in cgs .

$$\bar{G} = 15.8\left(\frac{r_o}{hc}\right)^2 \lambda_f^2 \lambda_q^2 B_o^2 \rho_e \left[\left(\frac{\Delta\gamma}{\gamma}\right)^2+\left(\frac{\Delta\nu}{2\nu}\right)^2\right]^{-\frac{1}{2}} \cdot \left\{ \exp\left[-\frac{\left(\frac{\delta\nu}{2\nu_f}\right)^2}{\left(\frac{\Delta\gamma}{\gamma_o}\right)^2+\left(\frac{\Delta\nu}{2\gamma}\right)^2}\right] - \exp\left[-\frac{\left(\frac{\delta\nu}{2\nu_f}-\frac{h\nu_f}{\gamma mc^2}\right)^2}{\left(\frac{\Delta\gamma}{\gamma}\right)^2+\left(\frac{\Delta\nu}{2\nu}\right)^2}\right]\right\} \cdot (\text{Filling Factor}) \qquad (2)$$

(units for $\bar{G}$: dB m^{-1})

where r_o = classical electron radius

ρ_e = electron density

λ_q = magnet period

γmc^2 = electron energy

λ_f, ν_f = operating wavelength and frequency

$\delta\nu$ = difference between the operating frequency and the emission line center.

The maximum value of $\bar{G}$ is obtained on the low frequency shoulder of the emission line shape and is given by:

$$\bar{G}_{max} \frac{36.8\left(\frac{r_o}{hc}\right)^2 \lambda_f^2 \lambda_q^2 B_o^2 \rho_{eo} \cdot \left(\frac{h\nu_f}{\gamma mc^2}\right)}{\left[\left(\frac{\Delta\gamma}{\gamma_o}\right)^2+\left(\frac{\Delta\nu}{2\nu_o}\right)^2\right]} \cdot (\text{Filling Factor}) \qquad (3)$$

(units for $\bar{G}_{max}$: dB m^{-1}) .

The filling factor is required to define the coupling of the electron beam to the stimulating radiation and can be approximated by the ratio of the area of the electron beam to the radiation beam. If the electron beam is larger than the radiation beam the filling factor is unity.

The derivation of these expressions relies on the assumption of small magnetic fields. It none the less appears that the form of these expressions should remain valid for large fields. For large magnetic fields, the expression for the wavelength acquires an additive term quadratic in the field amplitude:

$$\lambda_f = \frac{\lambda_q}{2\gamma^2}\left[1+\frac{1}{3}\left(\frac{\lambda_q}{4c}\right)^2\left(\frac{eB}{mc}\right)^2\right] \qquad (4)$$

The transition rate for spontaneous radiation in the $+\hat{z}$ direction continues to grow as B^2 provided that the electron beam energy is chosen to keep λ_f in equation (4) constant. The available gain then depends on the magnitude of the displacement of the absorption line relative to the emission line. The displacement and the gain decrease as γ grows. However, the required correction is built into the gain formulae. Since the relative displacement $(h\nu_f/\gamma mc^2)$ already appears in equations (3) and (4), no modifications ought to be necessary.

A Classical Interpretation?

It is interesting to note that h does not appear in the gain formula (3) due to cancellation of the h in the denominator by the h in the numerator arising from the approximation of the difference between the exponentials in equation (2). The absence of h from equation (3) raises the possibility of a classical interpretation of the amplification mechanism. The possibility is appealing because of the promise of a connection to the extensive literature on the interaction of travelling electromagnetic and space charge waves.

While the absence of Planck's constant is a necessary condition, it is not a sufficient condition for a classical interpretation. Comparison of the gain calculated due to stimulated bremsstrahlung and estimates of the gain for a travelling wave interaction indicates little similarity.

Several approximations are possible in analyzing the possible classical analogues. The amplitude of the transverse oscillations of the electron beam in the periodic magnetic field can be assumed fixed while the electromagnetic field rearranges the longitudinal charge distribution (longitudinal bunching). Alternatively, the longitudinal distribution can be fixed while allowing the electromagnetic field to modify the transverse amplitude (the transverse interaction). Longitudinal bunching was first discussed in this connection by Motz and Nakamura.[4] The formula derived by them yields a gain in dB m^{-1} given by equation (5) below. Units are cgs except for Z_o , I_o and V_o . The radiation beam is assumed to have a cross section A:

$$G = 434 \frac{(2\pi)^{1/3}}{4} \left(\frac{\lambda_q}{4c}\right)^{2/3} \left(\frac{eB}{mc}\right)^{2/3} \left[\frac{1}{\gamma^4} \cdot \frac{1}{\lambda_f A} \cdot \frac{Z_o I_o}{V_o}\right]^{1/3} \tag{5}$$

$$\propto \frac{\lambda_q^{2/3} B^{2/3} i^{1/3}}{\lambda_f^{1/3} \gamma^{5/3} A^{1/3}}$$

where Z_o = impedance of free space in ohms

I_o = electron beam current in amps

V_o = electron beam voltage in volts

The transverse interaction yields a gain represented approximately by equation (6) below. Units are again cgs except for Z_o, I_o, and V_o

$$G = 434 \left(\frac{\pi}{2}\right)\left(\frac{\lambda_q}{4c}\right)\left(\frac{eB}{mc}\right)\left[\frac{1}{A}\left(\frac{\lambda_f}{\lambda_q}\right) \frac{Z_o I_o}{V_o}\right]^{1/2} \tag{6}$$

$$\propto \frac{\lambda_f^{1/2} \lambda_q^{1/2} B i^{1/2}}{\gamma^{1/2} A^{1/2}} .$$

The frequency of operation is given for both cases by the same result as applies for stimulated bremsstrahlung, e.g., equation (4).

The distinguishing feature of the gain coefficients in equations (5) and (6) is their dependence on a fractional power of the electron beam current and the relatively weak dependence on the magnetic field. The coefficients are therefore quite distinct from the result for stimulated bremsstrahlung which is linear in the current and quadratic in B.

A more fundamental appraisal of the possibility of a classical interpretation is based on the properties of the electromagnetic field at "low" power levels. The operators for $\mathcal{E}$, B, and the photon density do not commute. The validity of the classical field formulae relay on the argument that for sufficiently large fields the commutator relations can safely be neglected. This corresponds to requiring that the photon density should be large in comparison to $(2\pi/\lambda)^3$.[8] Corresponding power densities for a travelling wave are 10^6 watts m^{-2} at 10μ and 10^{10} watts m^{-2} at 1μ. At lower power levels the equations for the classical travelling wave interactions must be viewed as operator equations. Classical electrodynamics will not necessarily yield reliable results at these power levels or below.

Apparatus

Figure 3 indicates schematically the operating parameters and the apparatus under construction in Hansen Laboratories. The system design was chosen to permit verification of the stimulated bremsstrahlung effect in an amplifier configuration and to allow exploration of the gain as a function of the radiation power level and operating frequency.

A helically wound superconducting solenoid 6 meters long with a period of 3.0 cm is designed to furnish fields up to 13 Kgauss on the axis. The magnet will be wound with multifilament niobium titanium wire on a machined aluminum helix. The bore diameter through which the electron beam will pass is 8 mm.

Initial operation is planned for 10μ at a $\gamma \sim 10^2$. The experiment will utilize the beam from the superconducting accelerator. In the pulsed mode, the superconducting accelerator can supply a beam current of 0.5 mA and the calculated time-averaged gain is 0.015 dB per pass. Gain for radiation within the electron bunches will be larger by a factor in excess of 10^2 and may be sufficient to sustain oscillation if attenuation and diffraction losses can be minimized.

A pair of 45° mirrors bored to permit the passage of the electron beam will be used to inject and extract the optical radiation from the region of interaction of the electron beam and the periodic field.

The output of the CO_2 laser will be monitored by a beam splitter and a pyroelectric effect detector and the output of this detector (channel B) will be used to normalize the output of the signal channel (channel A) on a pulse to pulse basis.

Since amplification occurs only for one component of polarization, a polarization modulator consisting of a rotating quarter wave plate can be used to switch the stimulated scattering effect in and out on alternate pulses of the CO_2 laser. Phase sensitive detection can then be used for gain measurements down to the level of 0.001 dB per pass. Measurements at a power level of 10^9 watts m^{-2} will be possible using the CO_2 laser at its full capabilities.

Conclusions

Stimulated emission of bremsstrahlung in a periodic magnetic field appears to constitute a new mechanism for the amplification of radiation. Apparatus to verify the existence of this phenomenon is presently under construction. Initial measurements will be made at a wavelength of 10μ.

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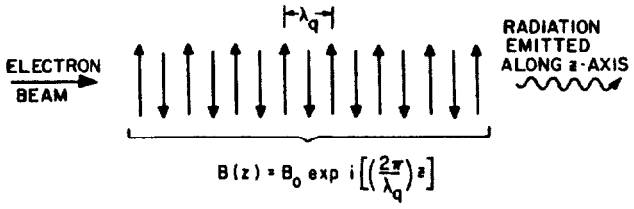


Fig. 1. Radiation by a relativistic electron beam in a periodic transverse magnetic field.

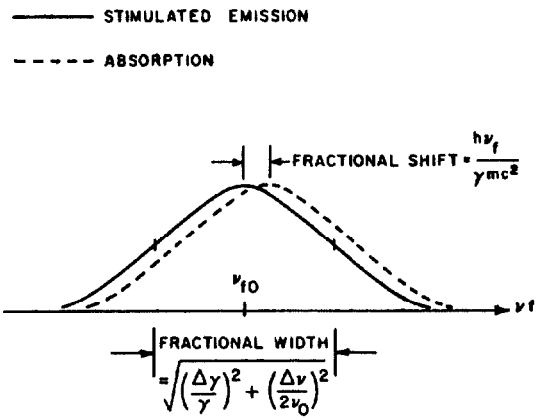


Fig. 2. Lineshapes for stimulated emission and absorption.

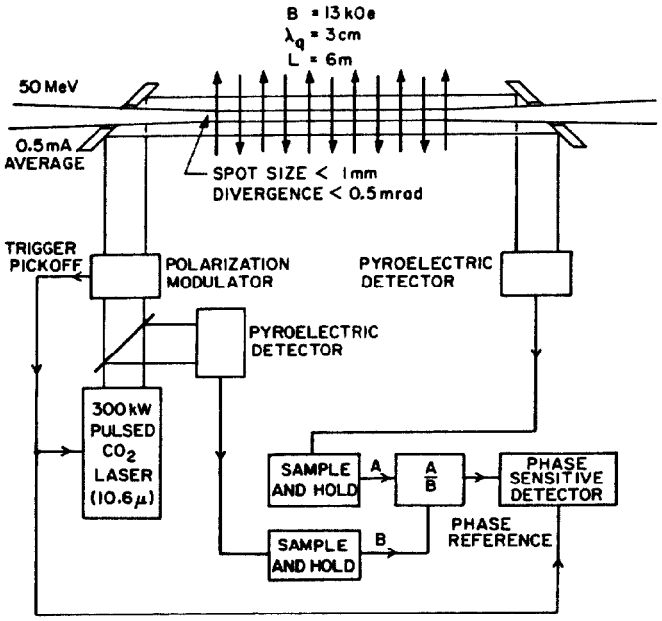


Fig. 3. Schematic of experimental apparatus.