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DISCUSSION

HIGH-INTENSITY EFFECTS IN THE LONGITUDINAL MOTION OF STORED PARTICLE BEAMS

B. Zotter: I would like to make a statement about a nonlinear effect we see in the blow-up of the bunches. The blow-up seems to fulfill a criterion given ten years ago by Dory, which is that the product of the final and initial momentum spreads equals the square of the critical momentum spread necessary for stability. In addition, there is the related phenomenon which is a sort of hysteresis, that if you turn up the coupling impedance of the cavity, then we get instability, and when we turn down the coupling impedance it disappears at a much lower value of coupling impedance.

Sessler: Pellegrini and I have been working on this problem and have a heuristic theory of nonlinear phenomena, and in this theory we did not get the Dory formula. We got a different formula, and it would be interesting for me to see the data, because in this theory what was involved was how many modes are unstable. Suppose we made a number of modes unstable by a cavity. Then, in this theory, one has the energy distributed over the unstable modes. The final energy spread arrived at was a function of over how many modes the initial energy is spread.

Zotter: We think that we have a single unstable mode. At least we tried to kill all higher modes in the cavity by damping.

Sessler: And in that case the final energy spread is greater than that you expect for threshold?

Zotter: Yes.

J. R. Rees (SLAC): This morning we heard Frank Sacherer talk about stability of bunched beams, and I was struck by the fact that the frequency spread required to stabilize bunched-beam instabilities varies as the square root of the mode number, while for continuous beams the required spread varies linearly Could someone explain physically why that is?

F. J. Sacherer (CERN): The criterion can be written as

$$S > \frac{1}{\sqrt{m}} \left[\Delta W_m\right]$$

but remember that the frequency shift ΔW_m also depends on m (m = 1) for dipole modes, m = 2 for quadrupole modes, etc.). For space charge $\Delta W_m \sim \sqrt{m}$ and the required spread is the same for all modes, just as for coasting beams. For cavities $\Delta W_m \sim 1/\sqrt{m}$ and $S \sim 1/m$, which again is analogous to the coasting beam case. The difference between coasting beams and bunched beams is not the dependence on mode number, but that frequency spread rather than energy spread is important, and that ΔW_m depends on bunched-beam parameters like bunching factor, RF voltage, etc.

Sessler: Let me remark that there are two different types of modes. Let us talk about just the dipole mode with B bunches; then there are B modes in which the particles move in rigid-bunch motion with different phase shifts between bunches. It is this mode which goes over into the continuous-beam case mode of order n and Z_n/n comes into the stability criterion. There is a different type of motion where you have quadrupole motion, etc., within one bunch, and I don't know how this goes over into the continuous-beam case. So you see these two types of modes are orthogonal and in one case you have the $\sqrt{n}$ in the criteria and in the other case you still get the Z_n/n.