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THE PROBLEM OF COUPLING A 120-MHz RF CAVITY TO A 700-A, 6-MeV PULSED ELECTRON BEAM*

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Summary

A 120-MHz cavity and RF system were built to modulate the beam energy of the Astron electron accelerator by ±5%. The beam pulse has a rise time of 20 ns which initiates a transient in the cavity. We designed the cavity to limit the transient voltage in order to avoid sparking. The excitation problem was studied both experimentally and theoretically. We established design criteria that allowed us to couple an RF cavity to a high-current beam as easily as to the low-current beams of the usual research accelerators.

Introduction

The Beam Research Program at the Laboratory achieved a bunched high current beam from the Astron Accelerator with a frequency of 120 MHz. Since the beam particles were highly relativistic at 6 MeV, and the current was high (700 A) we had a unique opportunity to experimentally study the interaction between a beam and an RF cavity. A simplified sketch of the cavity with the critical dimensions is shown in Fig. 1. The cavity is a foreshortened quarter-wave TEM mode coaxial resonator. The beam pipe extends into the high voltage region of the cavity where momentum modulation takes place. The high current end of the cavity is movable and provides coarse tuning. Fine tuning is provided by a movable plunger near the shorted end of the cavity. It has a tuning range of 800 kHz. In normal operation the cavity is driven by a 250-kW RF amplifier.

Theory

Passing a beam of particles through an RF cavity excites electromagnetic fields in the cavity. The eigenmodes of the cavity offer a convenient method for investigating the particles effect theoretically. We will outline this method briefly.

Let the vector potential of an eigenmode by $\bar{A}_\ell(\bar{r})$, a vector function that satisfies the Helmholtz equation

$$\nabla^2\bar{A}_\ell(\bar{r}) + (\omega_\ell/c)^2\bar{A}_\ell(\bar{r}) = 0, \qquad (1)$$

where ω_ℓ is the eigenfrequency of the mode and c is the speed of light. We use Gaussian units for the calculation, but the results will be converted to practical units. The vectors $\bar{A}_\ell$ satisfy $\nabla\cdot\bar{A}_\ell = 0$ and the normalization condition

$$\int \bar{A}_\ell\cdot\bar{A}_{\ell'}\, dv = 4\pi c^2\delta_{\ell\ell'}. \qquad (2)$$

In Eq. (2) the integral extends over the volume of the cavity. The subscript ℓ in general represents the three mode numbers necessary to characterize the mode. For purposes of this calculation, we can ignore holes in the cavity, in particular the hole through which the beam passes. We may then consider the walls of the cavity to be perfectly conducting, so that the tangential component of $\bar{A}_\ell$ is zero on the walls. The effects of large but finite conductivity in the wall material will be included subsequently.

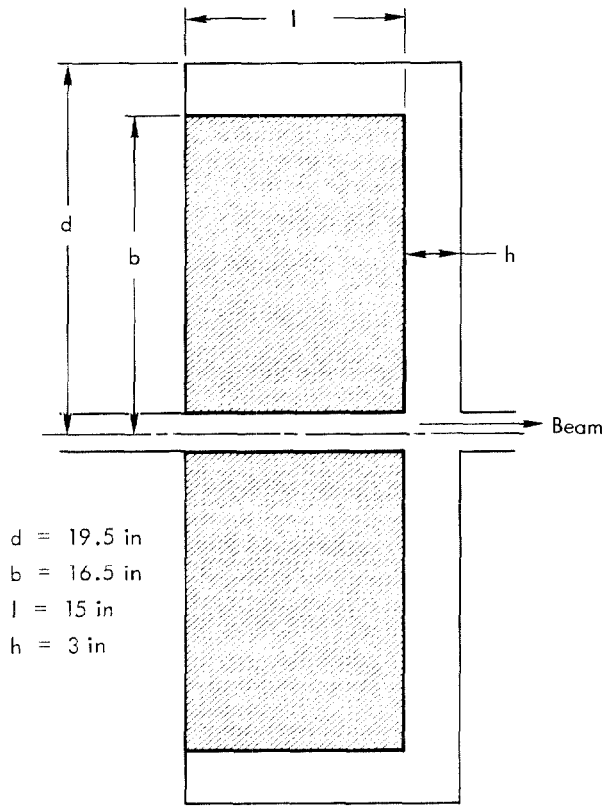


Fig. 1. Simplified representation of a 120-MHz cavity with critical dimensions noted.

The vector potential $\bar{A}(\bar{r},t)$ inside the cavity may be expressed as

$$\bar{A}(\bar{r},t) = \sum_\ell q_\ell(t)\bar{A}_\ell(\bar{r}), \qquad (3)$$

where the sum extends over all three mode numbers. We represent the particle beam by a current density $\bar{j}(\bar{r},t)$. The time dependent coefficients $q_\ell(t)$ obey the equation

$$\frac{d^2q_\ell}{dt^2} + \omega_\ell^2 q_\ell = \frac{1}{c}\int \bar{j}(\bar{r},t)\cdot\bar{A}_\ell(\bar{r})\, dv. \qquad (4)$$

Provided only one mode of the cavity is excited appreciably, the finite conductivity of the walls may be included by adding a term $(\omega_\ell/Q)\, dq_\ell/dt$ to the left side of Eq. (4), where Q is the usual quality factor of the mode.

We now find an approximate expression for the lowest mode of the cavity shown in Fig. 1. This mode is azimuthally symmetric. In the region $0 < z < h$, $0 < r < b$, the electric field is in the axial (z)

*Work performed under the auspices of the U.S. Atomic Energy Commission.

direction and is a function of r only. In this region the field components of the lowest mode are given approximately by

$$E_z = C_1 J_0(\rho r), \quad B_\theta = iC_1 J_1(\rho r), \tag{5}$$

where J_0 and J_1 are the zero and first order Bessel functions and C_1 and ρ are constants. In writing Eq. (5) we have assumed a time dependence $e^{-i\omega t}$. In Gaussian units $\bar{E} = -(1/c)\,\partial\bar{A}/\partial t$. In the region $-\ell < z < 0$, $b < r < d$, the field components are given approximately by

$$E_r = (C_2/r)\sin\rho\,(z + \ell),$$

$$B_\theta = i(C_2/r)\cos\rho\,(z + \ell), \tag{6}$$

with C_2 another constant. We now do slight violence to Maxwell's equations by matching the wall current, or B_θ, and the voltage at $r = b$, $z = 0$. We have

$$C_1 J_1(\rho b) = (C_2/b)\cos\rho\ell, \tag{7a}$$

and

$$hC_1 J_0(\rho b) = C_2 \ln(d/b)\sin\rho\ell. \tag{7b}$$

The approximations inherent in Eqs. (7a) and (7b) completely neglect the details of the fields at the corner, but the approximation is quite good for a description of the lowest mode, provided $(d - b)/b \ll 1$, and $h/b \ll 1$.

The eigenfrequency ω_1 of the lowest mode is $c\rho$, where ρ is found by solving the equation

$$\frac{h}{b}\frac{J_0(\rho b)}{J_1(\rho b)} = \ln(d/b)\tan\rho\ell. \tag{8}$$

For the parameters of the 120-MHz cavity, we find $\rho b = 1.05$. Once the value of ρ is found, the ratio C_1/C_2 may be calculated from Eq. (7a). The vector function $\bar{A}_1(\bar{r})$ is then found to be

$$\bar{A}_1(\bar{r}) = N\begin{cases} J_0(\rho r)\hat{z}, & 0 < Z < h,\ 0 < r < b \\ \dfrac{b}{r}J_1(\rho b)\dfrac{\sin\rho(Z + \ell)}{\cos\rho\ell}\hat{r}, & -\ell < Z < 0, \\ & b < r < d. \end{cases} \tag{9}$$

The normalization constant N is found from Eq. (2), and is given by

$$N^2 = 4(c/b)^2\left\{h\left[J_0^2(\rho b) + J_1^2(\rho b)\right] + 2\ln(d/b)\left[\frac{J_1(\rho b)}{\cos\rho\ell}\right]^2\left(\frac{\ell}{2} - \frac{\sin 2\rho\ell}{4\rho}\right)\right\}^{-1}. \tag{10}$$

From the parameters in Fig. 1 we find

$$N^2 = 0.4(c/b)^2\ \text{cm}^{-1}\ \text{sec}^{-2}. \tag{11}$$

We take the current density $\bar{j}(\bar{r},t)$ to be a given function unaffected by the cavity voltage. This is an excellent approximation to the current density of the relativistic (5-MeV kinetic energy) electron beam. Furthermore, we can neglect the variation of $\bar{A}_1$ over the beam cross section and write

$$\bar{j}(\bar{r},t) = i(t)\,\delta(r)\,\hat{z}/\pi r, \tag{12}$$

where I(t) is the total time dependent beam current. Using Eqs. (9) and (12) in Eq. (4) we obtain

$$\ddot{q}_1 + \omega_1^2 q_1 = NhI(t)/c. \tag{13}$$

We define $t = 0$ to be the time at which the head of the beam enters the cavity. With the initial condition $q_1 = 0$ and $\dot{q}_1 = 0$ at $t = 0$, the solution to Eq. (13) is

$$q_1(t) = \frac{Nh}{\omega_1 c}\int_0^t I(t')\sin\omega_1(t - t')\,dt'. \tag{14}$$

Note that the finite Q of the mode has been neglected in Eqs. (13) and (14). This is a valid approximation for times such that $t \ll Q/\omega_1$, a condition well satisfied for times $t \sim 300$ nsec, which is the duration of the beam.

The electric field at $r = 0$ is $E_z = -\dot{q}_1 N/c$ and the voltage V_1 across the gap is hE_z, so that

$$V_1(t) = -\left(\frac{Nh}{c}\right)^2\int_0^t I(t')\cos\omega_1(t - t')\,dt'. \tag{15}$$

This expression is most easily evaluated for a step function current $I(t) = I_0 - (t)$, which yields the result

$$V_1(t) = -\left(\frac{Nh}{c}\right)^2\frac{I_0}{\omega_1}\sin\omega_1 t. \tag{16}$$

We introduce the surge impedance Z_0 of the mode by the definition

$$Z_0 \equiv \frac{1}{\omega_1}\left(\frac{Nh}{c}\right)^2. \tag{17}$$

Inserting Eq. (11) into Eq. (17) we find (in these units) $Z_0 = 1.75 \times 10^{-11}$ sec/cm, which is equivalent to 15.75 Ω. (Multiply the number of cgs units by 9×10^{11} to obtain the number of ohms.)

Equation (16) is valid as long as the rise time of the current is much less than a period of oscillation, $2\pi/\omega_1$. This condition is not satisfied in the practical circumstance, where the rise time of the current is about 10 ns. For a current rising linearly with time to a value I_0 at $t = \tau$ and constant in time for $t > \tau$, we find from Eq. (15) for $t > \tau$

$$V_1(t) = -\frac{2\sin(\omega\pi/2)}{\omega\tau}Z_0 I_0\sin\omega(t - \tau/2). \tag{18}$$

The passage of the tail of the beam will induce additional voltage, which may add constructively or destructively to that induced by the head of the beam, depending on the length of the pulse. Some voltage is left after the tail passes, and this voltage then decays with the usual time constant $2Q/\omega_r$.

Equation 18 shows that the transient voltage produced for the linear ramp is the same as that for the step function except that the former is multiplied by a sinx/x function. This has a magnitude between 0 and 1 depending upon the duration of the ramp with respect to the period of the resonant cavity. The computed value for the case of a step function with a 700-A beam is 10.8 kV. For a similar beam with a 10-ns ramp the voltage is 1.73 kV.

Another method of analysis, based on transmission line theory, yields a result which can be computed less laboriously. First, we calculate the equivalent inductance of the coaxial portion of the resonator. Second, we find the equivalent capacitance necessary to produce resonance at the desired frequency. Then we use the methods of circuit analysis to calculate the transient voltage at this point. The transient is then referred to the gap voltage by using the fact that the fields vary as $J_0(\beta r)$ in the radial portion of the resonator. To state this formally, we calculate the equivalent inductance of the coaxial portion of the resonator.

$$L_{eq} = \frac{Z_{01}}{\omega_1} \tan \beta\ell,$$

where $Z_{01} = 138 \log \frac{d}{b}$ and $\beta = \frac{\omega_1}{c}$. (19)

The equivalent shunt capacitance at this point is

$$C_{eq} = \frac{1}{\omega_1^2 L_{eq}}. \quad (20)$$

The surge impedance at this point is

$$Z_0 = \frac{L_{eq}}{C_{eq}} = \omega_1 L_{eq}. \quad (21)$$

The transient voltage developed across the inductor by the beam current, assuming a step function, can be obtained by equating the energy stored in the capacitance to that in the inductance:

$$\frac{1}{2} C_{eq} V_{1t}^2 = \frac{1}{2} L_{eq} I_b^2. \quad \text{If } Z_0 = \sqrt{\frac{L_{eq}}{C_{eq}}}$$

$$\text{then } V_{1t} = Z_0 I_b. \quad (22)$$

If the fields in the radial part of the resonator vary as $J_0(\beta r)$ then the transient voltage at the gap is

$$V_t = \frac{Z_0 I_b}{J_0\left[\beta \frac{(b+d)}{2}\right]} = \frac{I_b Z_{01} \tan \beta\ell}{J_0\left[\beta \frac{(b+d)}{2}\right]}. \quad (23)$$

For a 700-A step function beam, Eq. (23) gives a value of 14.1 kV.

Experimental Results

Figure 2a shows an oscilloscope trace of the transient produced in the cavity by a 340-A beam. It has an amplitude of 4.75 kV which scales to 9.8 kV at 700 A. This compares very well with 10.8 kV, the result calculated from field theory for a step function beam.

Figure 2b shows the leading edge of the actual beam current. It is clear from this figure that the linear ramp approximation is not good for this particular current waveform so far as the frequency spectral components are concerned. If it is approximated by a linear ramp of 20-ns duration, we find that the sinx/x function is 0.126, a value which is much too low to account for the experimental observation. These spectral components exciting the cavity must be determined primarily by the initial steep rise shown in Fig. 2b. It is clear that the excitation of the cavity depends, in an important way, upon the exact character of the leading edge of the beam pulse and that it is not easily approximated. However, it will always be less than that produced by a step function beam. Of course, a similar transient is produced by the trailing edge of the beam and exactly the same situation applies. The proper multiplying function could be obtained by a computer analysis using points obtained from Fig. 2b, but this would only be of academic interest because the character of the leading edge of the beam varies because of the frequent tuning of the accelerator.

The trailing edge of the beam is shown in Fig. 2c and the decay of the transients after the beam pulse in Fig. 2d. The latter simply shows that the transients decay in accordance with the Q of the resonator.

Conclusion

The voltage transient produced by a step function beam current has been derived by field theory and by transmission line theory. The results differ in the two cases because of the different approximations that were made in the derivations. If the beam has a linearly rising leading edge, a coefficient is introduced of the form sinx/x in which $x = \omega_1 \tau/2$, where τ is the rise time of the leading edge. Our experimental measurements indicate that the leading and trailing beam edges are not well represented by linear ramp approximations. In general, the linear ramp produces an estimate of the transient which is much too low. In the Astron induction linear accelerator the characteristic of the leading and trailing beam edges is a function of the way the accelerator is tuned. We do know that the size of the transient will lie between that given by the linear ramp approximation as a minimum and that indicated by the step function as a maximum. For many cases this is sufficient.

In the particular application for which this cavity was designed, it was desirable to minimize the RF transient so that the momentum modulation of the beam would be independent of the beam current. There are other cases where excitation of the cavity by the beam might be desired. For example, an accelerator has been described[1] in which the RF cavities are excited by the leading edge of the beam, and the fields thus produced are used to accelerate the following electrons.

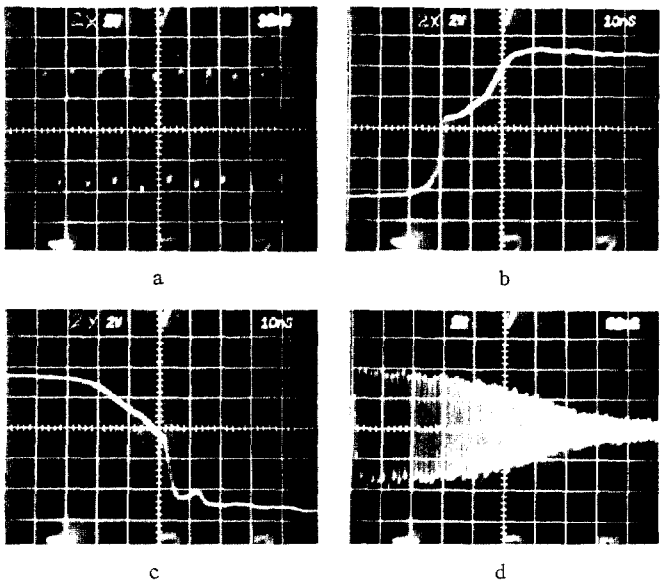


Fig. 2. Oscilloscope traces of beam current and RF transients. (a) A 120-MHz transient produced by a 340-A beam. The ordinate is 2.38 kV per division, the abscissa 10 ns per division. (b) The leading edge of the beam pulse producing the transient shown in Fig. 2a. The ordinate is 75.5 A per division, the abscissa 10 ns per division. (c) The trailing edge of the beam pulse producing the transient shown in Fig. 2a. The scales are the same as for Fig. 2b. (d) The decay of the transient following the beam pulse. The ordinate is about 1.9 kV per division, the abscissa 50 ns per division. The transient decays according to the resonator Q prediction.

Reference

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