Thomas Jefferson National Accelerator Facility, Newport News, Virginia, USA
Managed by the Jefferson Science Associates, LLC for the U.S. Department of Energy Office of Science

Analysis of Microbunching Structures in Transverse and Longitudinal Phase Spaces

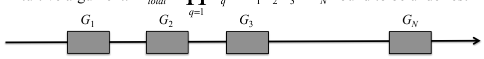
C. -Y. Tsai, *Virginia Tech, Blacksburg, VA 24061, USA*
R. Li, *Jefferson Lab, Newport News, VA 23606, USA*

ABSTRACT

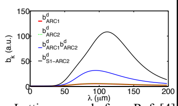
Microbunching instability (MBI) has been a challenging issue in high-brightness electron beam transport for modern accelerators. Our Vlasov analysis of MBI is based on single-pass configuration. For multi-pass recirculation or a long beamline, the intuitive argument of quantifying MBI by successive multiplication of individual MBI gains was found to underestimate the effect. More thorough analyses based on concatenation of gain matrices aimed to combine both density and energy modulations for a general beamline. Yet, quantification still focuses on characterizing longitudinal phase space; microbunching structures residing in (*x*,*z*) or (*x*',*z*) was observed in particle tracking simulation. Inclusion of such cross-plane microbunching structures in Vlasov analysis shall be a crucial step to systematically characterize MBI for a beamline complex in terms of concatenating individual beamline segments. We derived a semi-analytical formulation to include the microbunching structures in longitudinal and transverse phase spaces. Using these generalized formulas, we studied an example lattice and found the microbunching gains calculated from multiplication of concatenated gain matrices can be considered as upper limit to the start-to-end gains.

I. INTRODUCTION

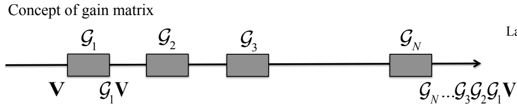
- Intuitive argument: $G_{total} = \prod_{q=1}^{N} G_q = G_1 G_2 G_3 \ldots G_N$ found to be under-estimated



- Start-to-end simulation is necessary but time consuming for particle tracking (also for Vlasov simulation). Possible middle-road solution might be desired
- Need more aspects of quantitative phase-space description
- Concept of gain matrix



Lattice example from Ref. [4]



II. THEORY

- Theoretical formulations follow Refs. [1, 2] and are continuation of Ref. [3].
- Consider the small phase-space perturbation

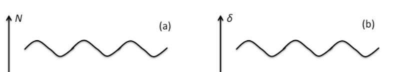
$$f_0(X_0) = \frac{n_0 + \Delta n_0(z_0)}{(2\pi)\varepsilon_{x0}\sqrt{2\pi}\sigma_\delta} e^{-\frac{(x_0+\Delta x_0(z_0))^2 + (\beta_{x0}(x_0'+\Delta x_0'(z_0)) + \alpha_{x0}(x_0+\Delta x_0(z_0)))^2}{2\varepsilon_{x0}\beta_{x0}} - \frac{(\delta_0 - hz_0 + \Delta\delta(z_0))^2}{2\sigma_\delta^2}}$$

- During collective-force kick

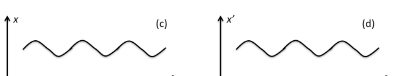
$$f(X;s) = f_0(X_0) - \int_0^s d\tau \frac{\partial f_0(X_\tau)}{\partial \delta_\tau}\frac{d\delta}{d\tau}$$

$$\frac{d\delta}{d\tau} = -\frac{Nr_e}{\gamma}\int \frac{dk_1}{2\pi} Z(k_1;\tau) b(k_1;\tau) e^{ik_1 z_\tau}$$

- Consider four types of microbunching



$$b(k_z;s) = \frac{1}{N}\int dX\, f(X;s) e^{-ik_z(s)z_s} \qquad p(k_z;s) = \frac{1}{N}\int dX\, (\delta_s - hz_s) f(X;s) e^{-ik_z(s)z_s}$$



$$a_x(k_z;s) = \frac{1}{N}\int dX\, (x_s) f(X;s) e^{-ik_z(s)z_s} \qquad a_{x'}(k_z;s) = \frac{1}{N}\int dX\, (x_s') f(X;s) e^{-ik_z(s)z_s}$$

- Governing equations

$$\boxed{b(k_z;s) = b_0(k_z;s) + \int_0^s d\tau K(\tau,s) b(k_z;\tau)}$$

$$K(\tau,s) = ik_z(s)\frac{I(\tau)}{\gamma I_A} R_{56}(\tau\to s) Z(k_z;\tau)\{\text{L.D.};\tau,s\}$$

$$\boxed{p(k_z;s) = p_0(k_z;s) + \int_0^s d\tau M(\tau,s) b(k_z;\tau)}$$

$$M(\tau,s) = \frac{I(\tau)}{\gamma I_A}\left\{ik_z^2(s)\sigma_\delta^2 R_{56}(\tau\to s)U(s,\tau) - 1\right\} Z(k(\tau);\tau)\{\text{L.D.};\tau,s\}$$

$$\boxed{a_x(k_z;s) = a_{x0}(k_z;s) + \int_0^s d\tau A(\tau,s) b(k_z;\tau)}$$

$$V(s,\tau) \equiv C(s)R_{51}(s) - C(\tau)R_{51}(\tau)$$
$$W(s,\tau) \equiv C(s)R_{52}(s) - C(\tau)R_{52}(\tau)$$
$$U(s,\tau) \equiv C(s)R_{56}(s) - C(\tau)R_{56}(\tau)$$

$$\boxed{a_{x'}(k_z;s) = a_{x'0}(k_z;s) + \int_0^s d\tau B(\tau,s) b(k_z;\tau)}$$

$$\{\text{L.D.};\tau,s\} \equiv e^{-\frac{k_z^2(s)\varepsilon_0\beta_0}{2}\left(V(s,\tau) - \frac{\alpha_0}{\beta_0}W(s,\tau)\right)^2 - \frac{k_z(s)\varepsilon_0}{2\beta_0}W^2(s,\tau) - \frac{k_z^2(s)\sigma_\delta^2}{2}U^2(s,\tau)}$$

$$A(\tau,s) = \frac{I_0}{\gamma I_A} Z(k(\tau);\tau)\left\{\begin{matrix} iR_{16}(\tau\to s) - k_z^2(s)R_{56}(\tau\to s)\times \\ \left[\begin{matrix} R_{11}(\tau\to s)\begin{bmatrix} R_{11}(\tau)\varepsilon_{x0}(W\alpha_{x0} - V\beta_{x0}) + \\ R_{12}(\tau)\varepsilon_{x0}(W\gamma_{x0} - V\alpha_{x0}) - \\ R_{16}(\tau)\sigma_\delta^2 U \end{bmatrix} + \\ R_{12}(\tau\to s)\begin{bmatrix} R_{21}(\tau)\varepsilon_{x0}(W\alpha_{x0} - V\beta_{x0}) + \\ R_{22}(\tau)\varepsilon_{x0}(W\gamma_{x0} - V\alpha_{x0}) - \\ R_{26}(\tau)\sigma_\delta^2 U \end{bmatrix} - R_{16}(\tau\to s)\sigma_\delta^2 U \end{matrix}\right] \end{matrix}\right\}\{L.D.;\tau,s\}$$

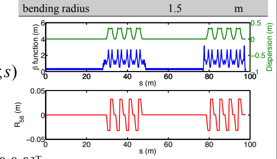
$$B(\tau,s) = \frac{I_0}{\gamma I_A} Z(k(\tau);\tau)\left\{\begin{matrix} iR_{26}(\tau\to s) - k_z^2(s)R_{56}(\tau\to s)\times \\ \left[\begin{matrix} R_{21}(\tau\to s)\begin{bmatrix} R_{11}(\tau)\varepsilon_{x0}(W\alpha_{x0} - V\beta_{x0}) + \\ R_{12}(\tau)\varepsilon_{x0}(W\gamma_{x0} - V\alpha_{x0}) - \\ R_{16}(\tau)\sigma_\delta^2 U \end{bmatrix} + \\ R_{22}(\tau\to s)\begin{bmatrix} R_{21}(\tau)\varepsilon_{x0}(W\alpha_{x0} - V\beta_{x0}) + \\ R_{22}(\tau)\varepsilon_{x0}(W\gamma_{x0} - V\alpha_{x0}) - \\ R_{26}(\tau)\sigma_\delta^2 U \end{bmatrix} - R_{26}(\tau\to s)\sigma_\delta^2 U \end{matrix}\right] \end{matrix}\right\}\{L.D.;\tau,s\}$$

III. EXAMPLE: Recirculating IBS ring [4]

Name	Example 3	Unit
beam energy	150	MeV
chirp	4	m^{-1}
bunch current (peak)	60	A
normalized emittance (H/V)	0.4/0.4	μm
relative rms energy spread	1.33×10^{-5}	
rms bunch length	~2.5	ps
bending radius	1.5	m

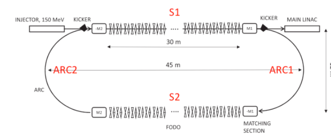
- Consider $\mathbf{V}(s) \equiv \left[b(k_z;s)\ p(k_z;s)\ a_x(k_z;s)\ a_{x'}(k_z;s)\right]^T$

$$b(k_z;s) = b^{(z)}(k_z;s) + b^{(\delta,z)}(k_z;s) + b^{(x,z)}(k_z;s) + b^{(x',z)}(k_z;s)$$
$$p(k_z;s) = p^{(z)}(k_z;s) + p^{(\delta,z)}(k_z;s) + p^{(x,z)}(k_z;s) + p^{(x',z)}(k_z;s)$$
$$a_x(k_z;s) = a_x^{(z)}(k_z;s) + a_x^{(\delta,z)}(k_z;s) + a_x^{(x,z)}(k_z;s) + a_x^{(x',z)}(k_z;s)$$
$$a_{x'}(k_z;s) = a_{x'}^{(z)}(k_z;s) + a_{x'}^{(\delta,z)}(k_z;s) + a_{x'}^{(x,z)}(k_z;s) + a_{x'}^{(x',z)}(k_z;s)$$



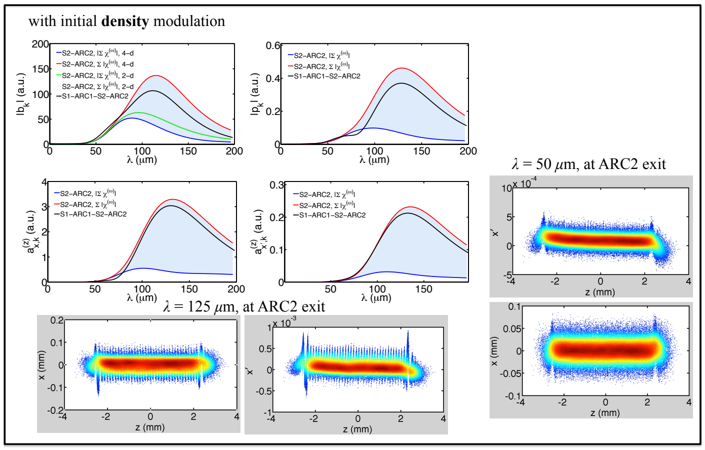
- Compare microbunching gains for the following cases:
  (i) start-to-end case, with initial **density** modulation, $\mathbf{V}(0) = [1\ 0\ 0\ 0]^T$;
  (ii) start-to-end case, with initial **energy** modulation, $\mathbf{V}(0) = [0\ 1\ 0\ 0]^T$;
  (iii) mid-to-end case, the initial condition to S2 takes the value at the exit of ARC1, $\mathbf{V}(S2) = [b(k)\ p(k)\ a_x(k)\ a_{x'}(k)]^T$ for case (i) and (ii), based on the present **4-d** theory;
  (iv) mid-to-end case, the initial condition to S2 takes the value at the exit of ARC1, $\mathbf{V}(S2) = [b(k)\ p(k)]^T$ for case (i) and (ii), based on **2-d** theory;

- Define $G_\chi = \left|\sum \chi^{(\omega)}\right| \qquad \chi = b, p, a_x, a_{x'}$
  $G_\chi^{\text{sup}} = \sum \left|\chi^{(\omega)}\right| \qquad \omega = (z), (\delta,z), (x,z), (x',z)$

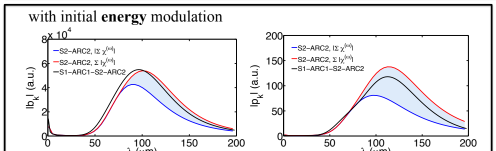


IV. RESULTS

with initial **density** modulation



with initial **energy** modulation



REFERENCES

[1] S. Heifets, G. Stupakov, and S. Krinsky, Phys. Rev. ST Accel. Beams 5, 064401 (2002)
[2] Z. Huang and K. Kim, Phys. Rev. ST Accel. Beams 5, 074401 (2002)
[3] C. -Y. Tsai and R. Li, IPAC'16 (TUPOR020)
[4] S. Di Mitri, Phys. Rev. ST Accel. Beams 17, 074401 (2014)

V. SUMMARY

In this poster we derived a set of governing equations for microbunching in different dimensions, including density, energy, and transverse-longitudinal modulations, and apply to an example of recirculating machine. The Vlasov solutions and tracking simulations agree qualitatively with each other. Although the Vlasov results from concatenated sections do not match well with those obtained directly from very beginning (start-to-end), $G_\chi^{\text{sup}} = \sum\left|\chi^{(\omega)}\right|$ gives **upper limit** for the modulation spectra. In addition, the extended formulations can give us further insights on how upstream beamline sections can accumulate density, energy modulation, and/or transverse-longitudinal microbunching, when the full-ring lattice is not provided.

*Authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177. The U.S. Government retains a non-exclusive, paid-up, irrevocable, world-wide license to publish or reproduce this manuscript for U.S. Government purposes.