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Toward high precision position control using laser interferometry: main sources of error.

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INTRODUCTION

Position and displacement sensors are one of the key elements when designing a positioning system or a scanning device. In many applications, the performances of the instrument are dependent on the position sensor performances and on the way it is integrated into the machine.

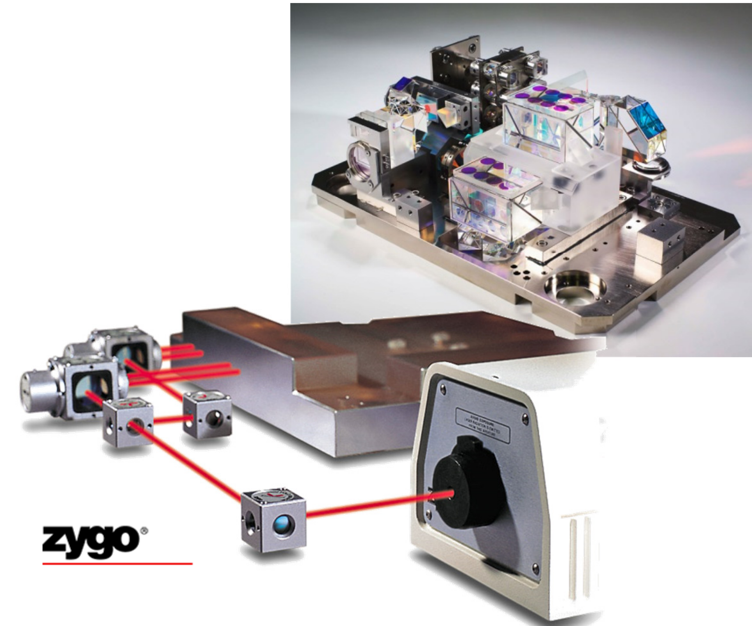
Many position sensors are available on the market, with many technologies. This is the reason why the choice is not easy to do and the best compromise not always easy to reach when designing a new setup.

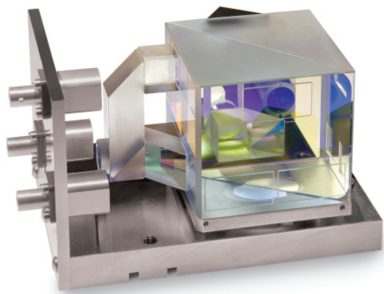
But when the development request severe constrains as for example:

- Accuracy in the nanometer range (<100 nm)
- High dynamic: sub-nanometer resolution over a long range of displacement (over 1 meter)
- Capable of measuring several degrees of freedom simultaneously
- Measurement traceable to the SI (International system of units)
- Long term stability
- Linearity of the response (< few tens of nm)
- High bandwidth (velocities up to few m/s, sampling frequency up to few MHz)

Then the number of eligible sensors is greatly reduced and attention generally focuses on optical interferometers.

MANY INTERFEROMETERS ON THE MARKET TO MEASURE LINEAR AND ANGULAR DISPLACEMENTS



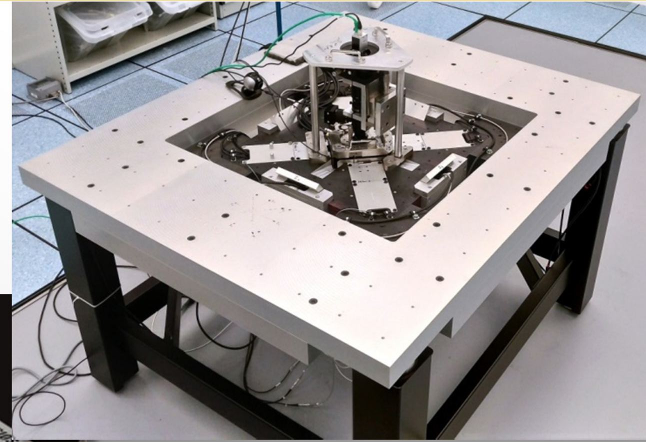


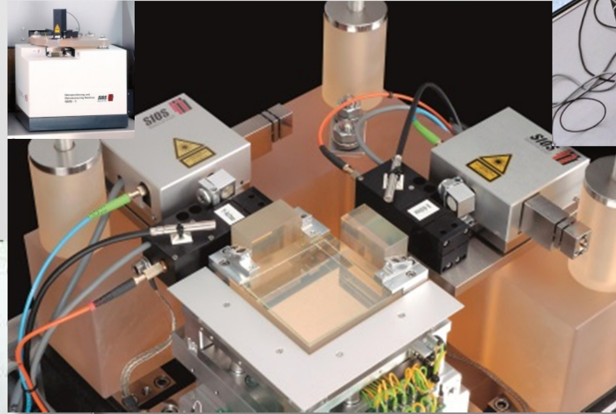
+ home made and custom made systems

INTEGRATION IN HIGH PRECISION MACHINE

Working volume ↘
Uncertainty ↘

metrological Atomic Force Microscope at LNE
Working volume : 60 µm x 60 µm x 15 µm
Uncertainties: less than 6 nm





NMM-1 Nanopositioning and Nanomeasuring Machine (SIOS)
Working volume : 25 mm x 25 mm x 5 mm
Uncertainties: less than 10 nm

Ultraprecision Micro CMM at METAS
Working volume: 90 mm x 90 mm x 38 mm
Uncertainties in the range of 50 nm for 3D probing

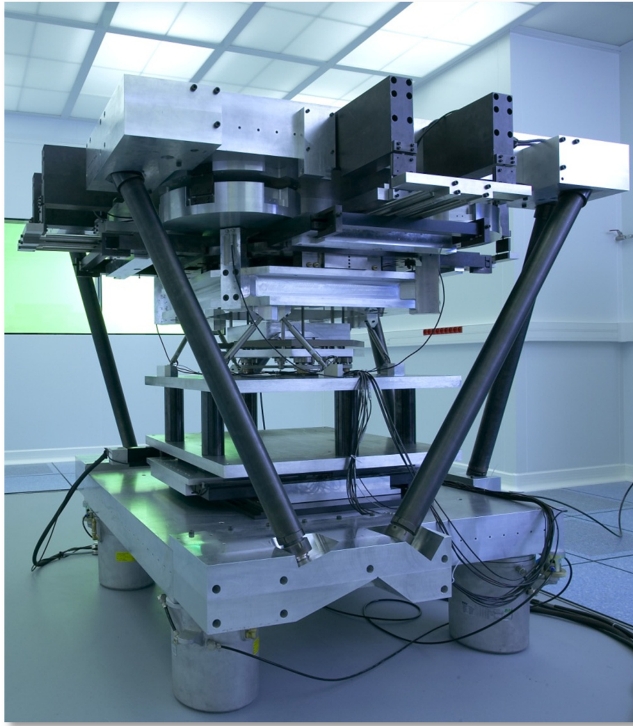
ISARA 400 ultraprecision coordinate measuring machine (CMM) from IBS Precision Engineering
Working volume: 400 mm x 400 mm x 100 mm
Uncertainties in the range of 100 nm (3d full stroke)

WHAT ARE THE LIMITING FACTORS TO POSITIONING ACCURACY USING INTERFEROMETRY ?

- It's not an easy question !
- How to evaluate an interferometer ?
- What are the limitation in term of displacement measurement ?
- What are the positioning uncertainties ?
- What are the main contributors in the measurement uncertainties ?
- Which precautions should be taken ?

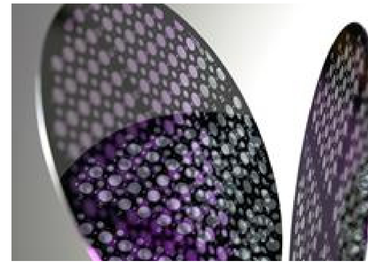
These are essential questions for an instrument designer that I will try to answer today !

PRESENTATION BASED ON TWO INSTRUMENTS DESIGNED AT LNE



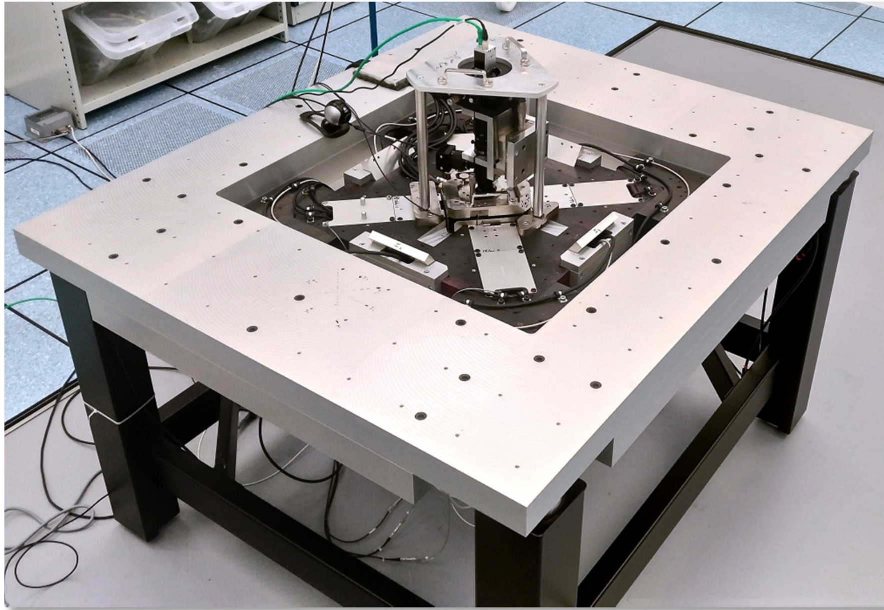
High precision coordinate measuring machine (hp-CMM)

- XY displacement range: 300 mm
- Z displacement range: 100 µm
- 6 DoF (in a sphere of 100 µm diameter)
- Maximum positioning uncertainty : about 50 nm
- Dedicated to the metrology of flat sample (i.e. wafer, masks, optics)



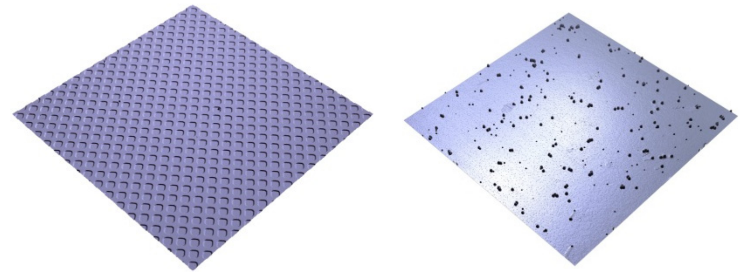
- 8 Renishaw RLD10 plane mirror interferometers for real-time XY displacement measurements and compensation of air index variation (4 refractometers).

PRESENTATION BASED ON TWO INSTRUMENTS DESIGNED AT LNE



metrological Atomic Force Microscope (mAFM)

- XY displacement range: 60 µm
- Z displacement range: 10 µm
- Positioning uncertainty: < 6 nm
- Dedicated to the calibration of standard used in scanning electron microscopy and scanning probe microscopy.



- 4 Renishaw RLD10 differential plane mirror interferometers for measurement in real-time of XYZ displacements.
- Air index compensation using a weather station.
- Characterization of unwanted parasitic rotations using one triple beam interferometer from SIOS (SP-TR series).

OUTLINE OF THE PRESENTATION

1. **Short introduction to Michelson interferometer**

2. **Errors due to the interferometer**
   - Wavelength
   - Dead path
   - Air turbulences
   - Resolution limit
   - Refractive index of air
   - Non symmetry of interferometer
   - Nonlinearity
   - Data age

3. **Errors due to the interferometer integration on the instrument (common to other sensors)**
   - Cosine error
   - Abbe error
   - Mirror flatness
   - Orthogonality of mirrors
   - Vibration
   - Metrology loop

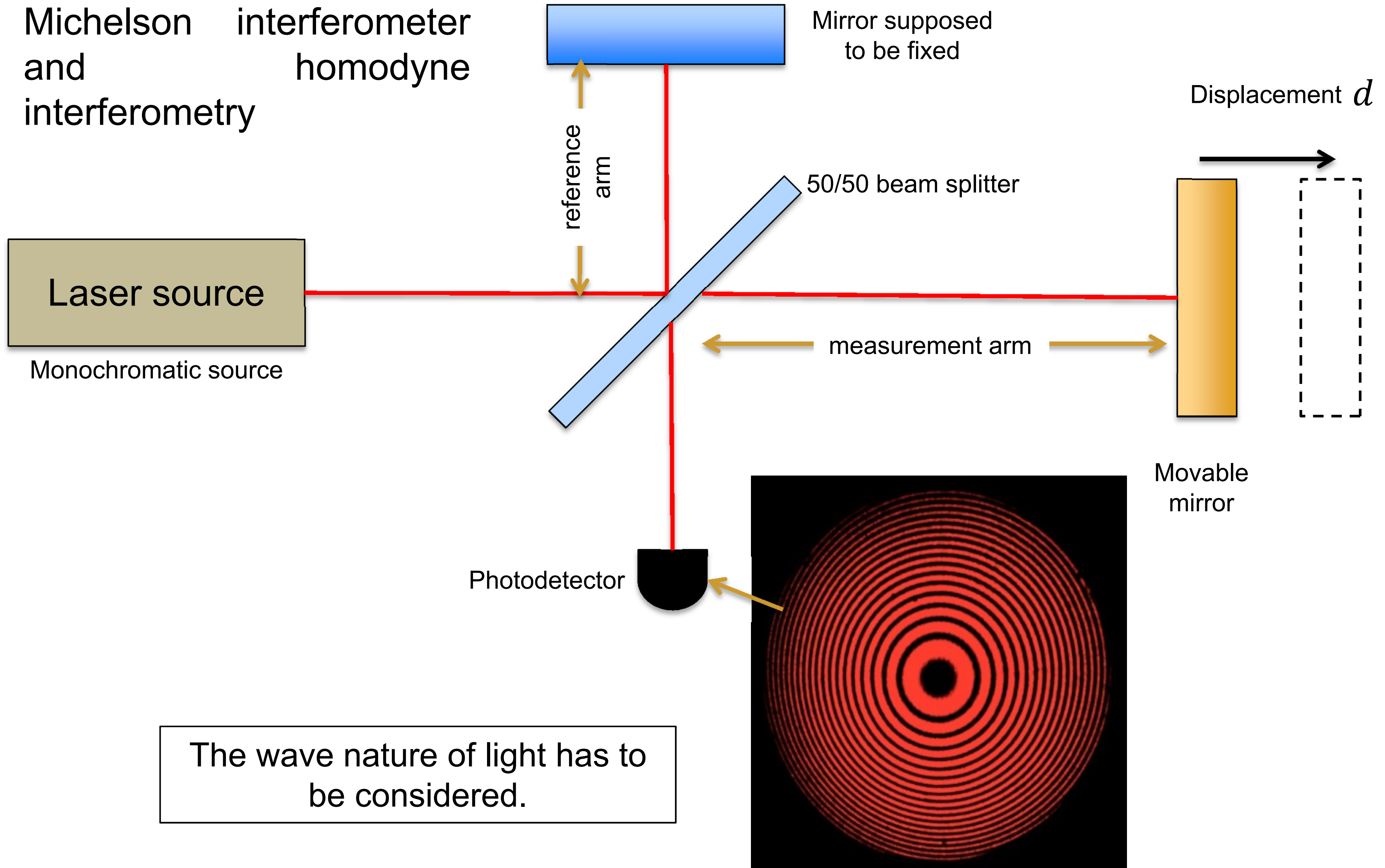
4. **Uncertainty budget and comparison for short and long range of displacements**

5. **Conclusion**

PRINCIPLE OF MICHELSON INTERFEROMETER

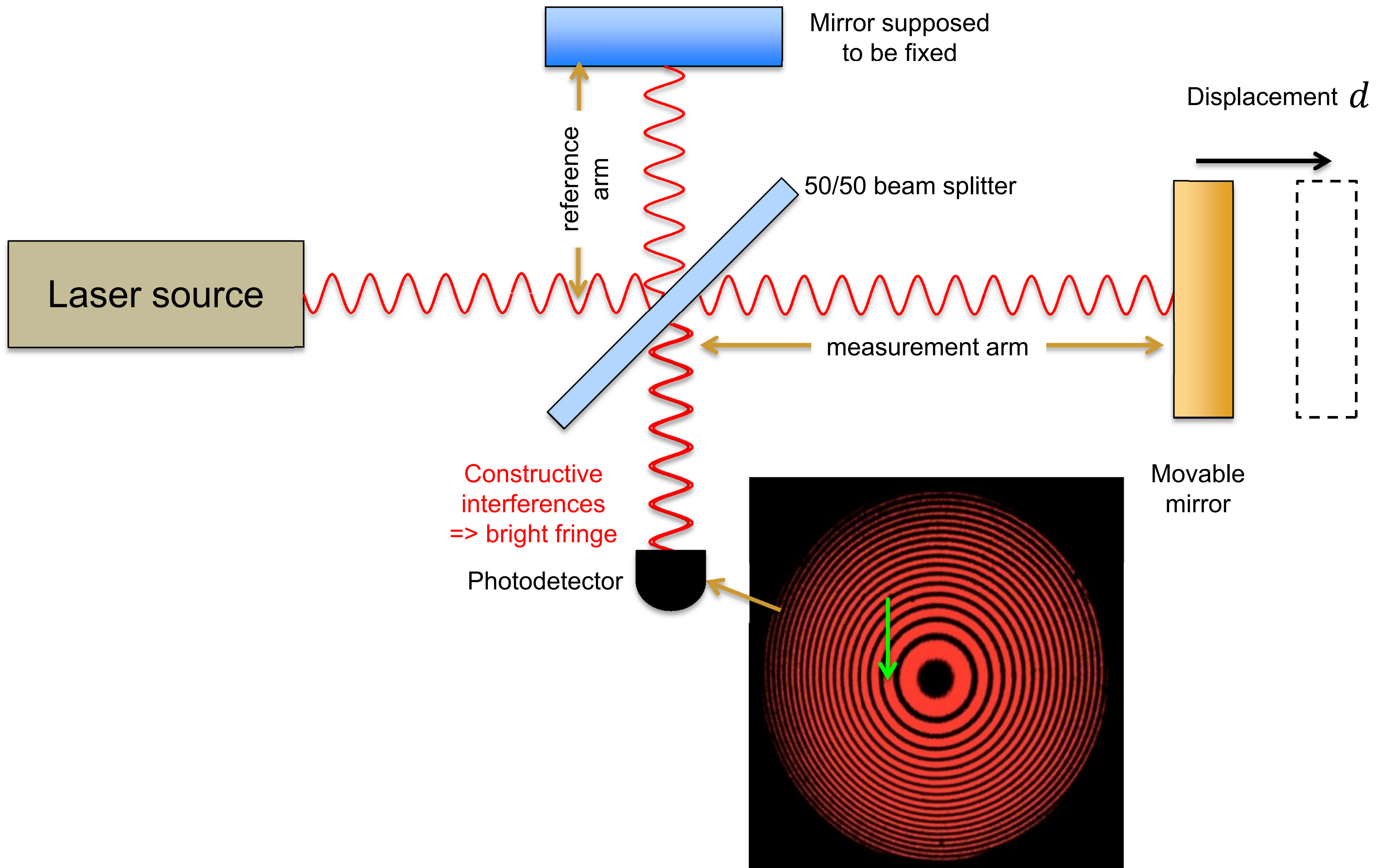
PRINCIPLE OF MICHELSON INTERFEROMETER

Michelson interferometer and homodyne interferometry

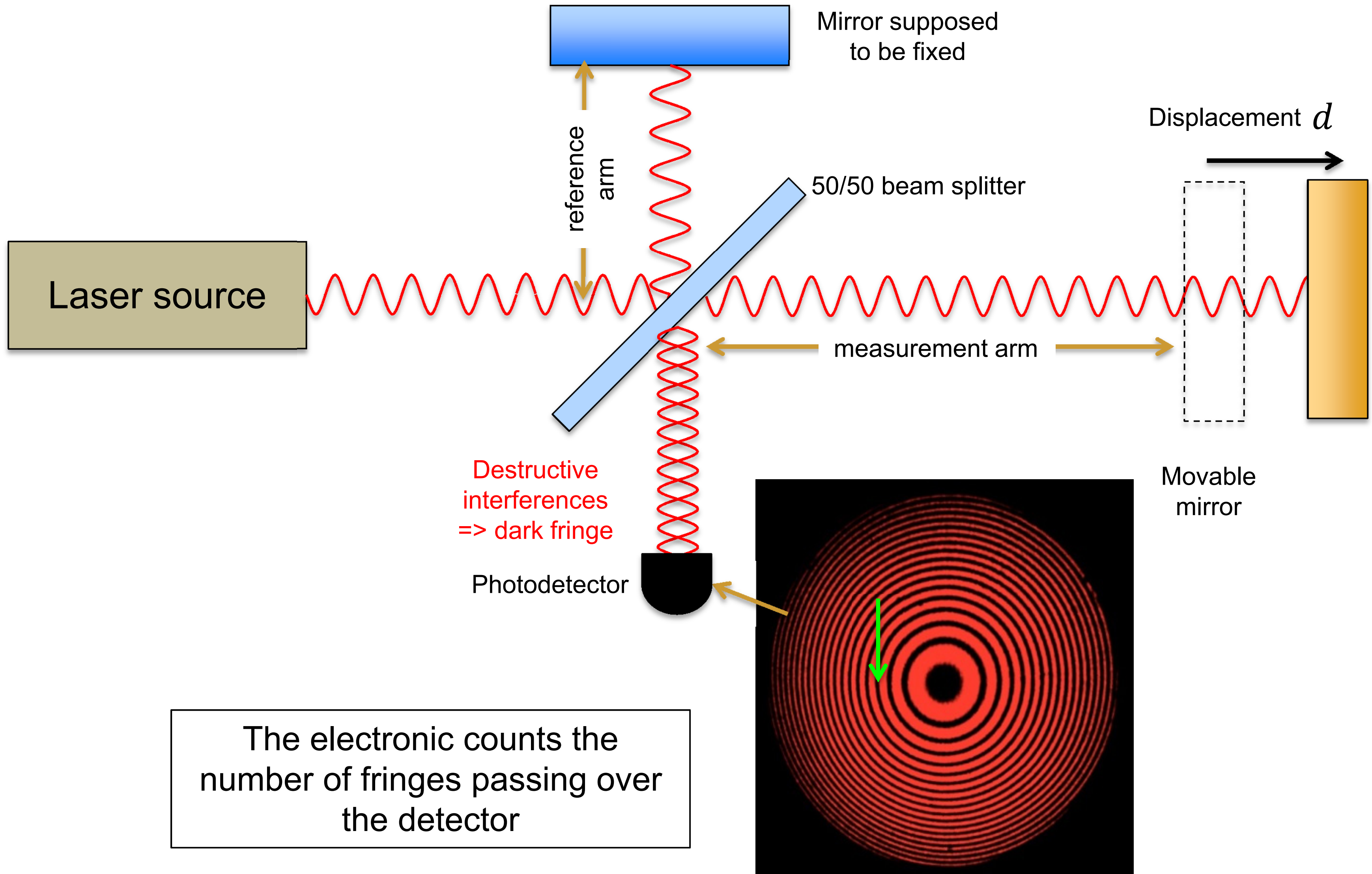


The wave nature of light has to be considered.

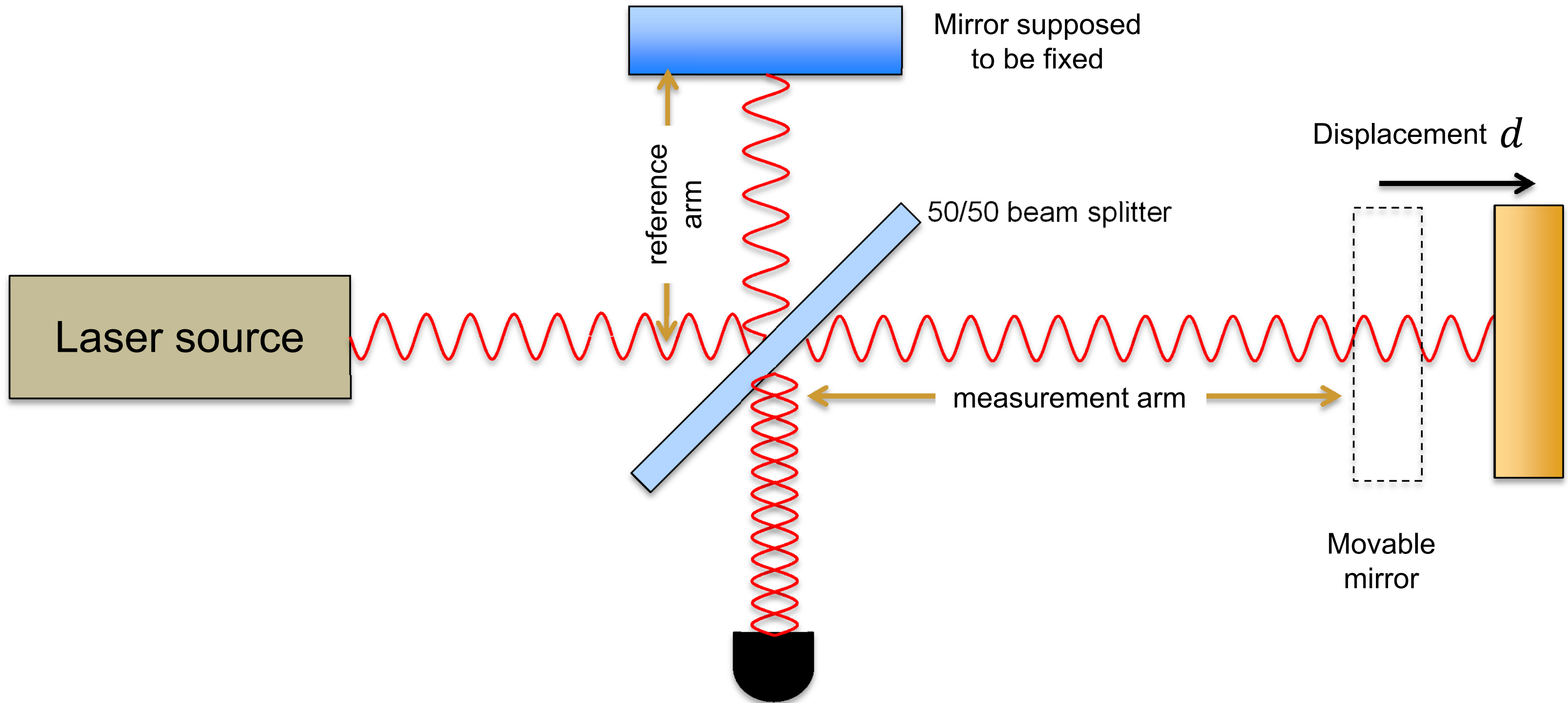
PRINCIPLE OF MICHELSON INTERFEROMETER



PRINCIPLE OF MICHELSON INTERFEROMETER



PRINCIPLE OF MICHELSON INTERFEROMETER



Electromagnetic wave propagating in the reference arm:

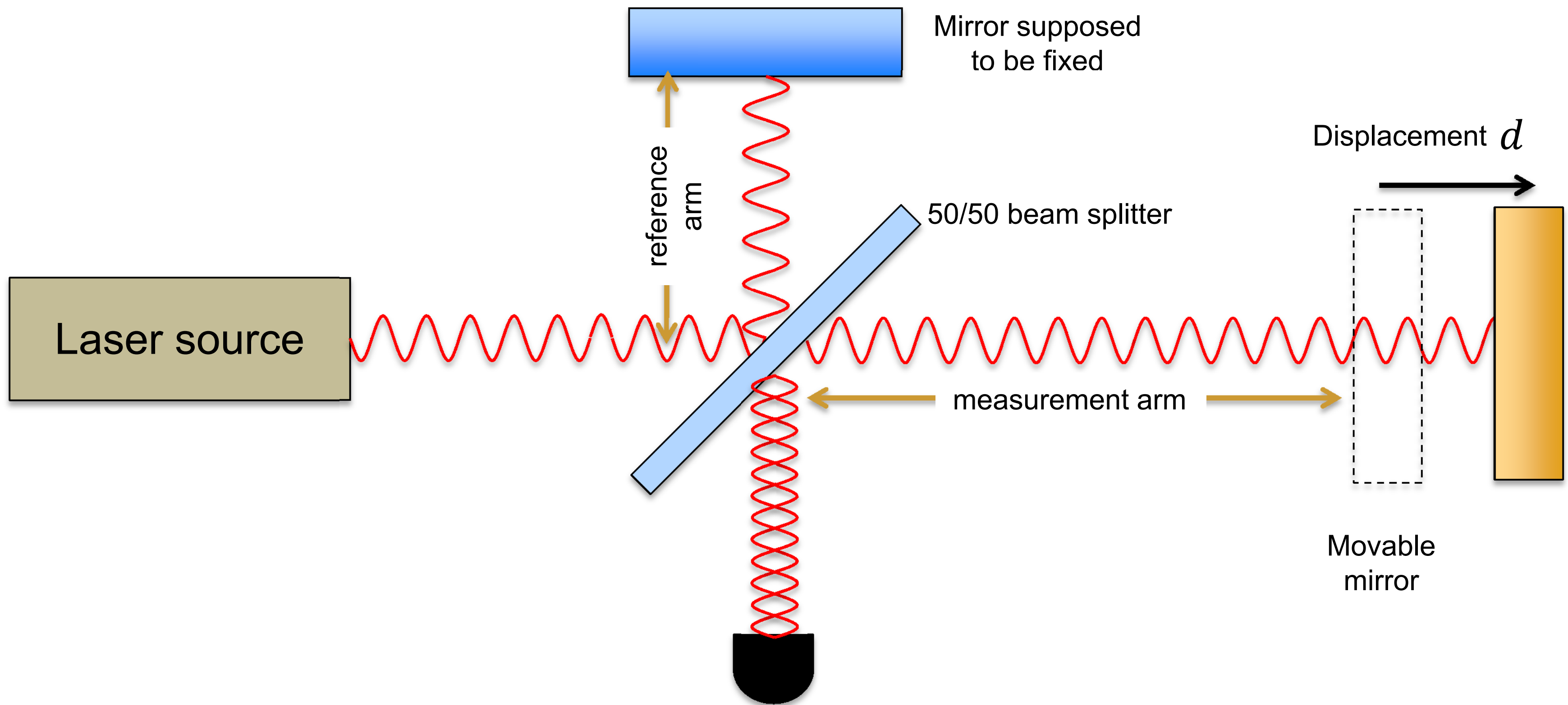
$$E_r(t, OPL_r) = E_0 e^{i\left(\omega t - \frac{2\pi}{\lambda_v} OPL_r\right)}$$

Electromagnetic wave propagating in the measurement arm:

$$E_m(t, OPL_m) = E_0 e^{i\left(\omega t - \frac{2\pi}{\lambda_v} OPL_m\right)}$$

E_0: electric field amplitude
ω: angular frequency
OPL: optical path length
λ_v: vacuum wavelength
t: time

PRINCIPLE OF MICHELSON INTERFEROMETER

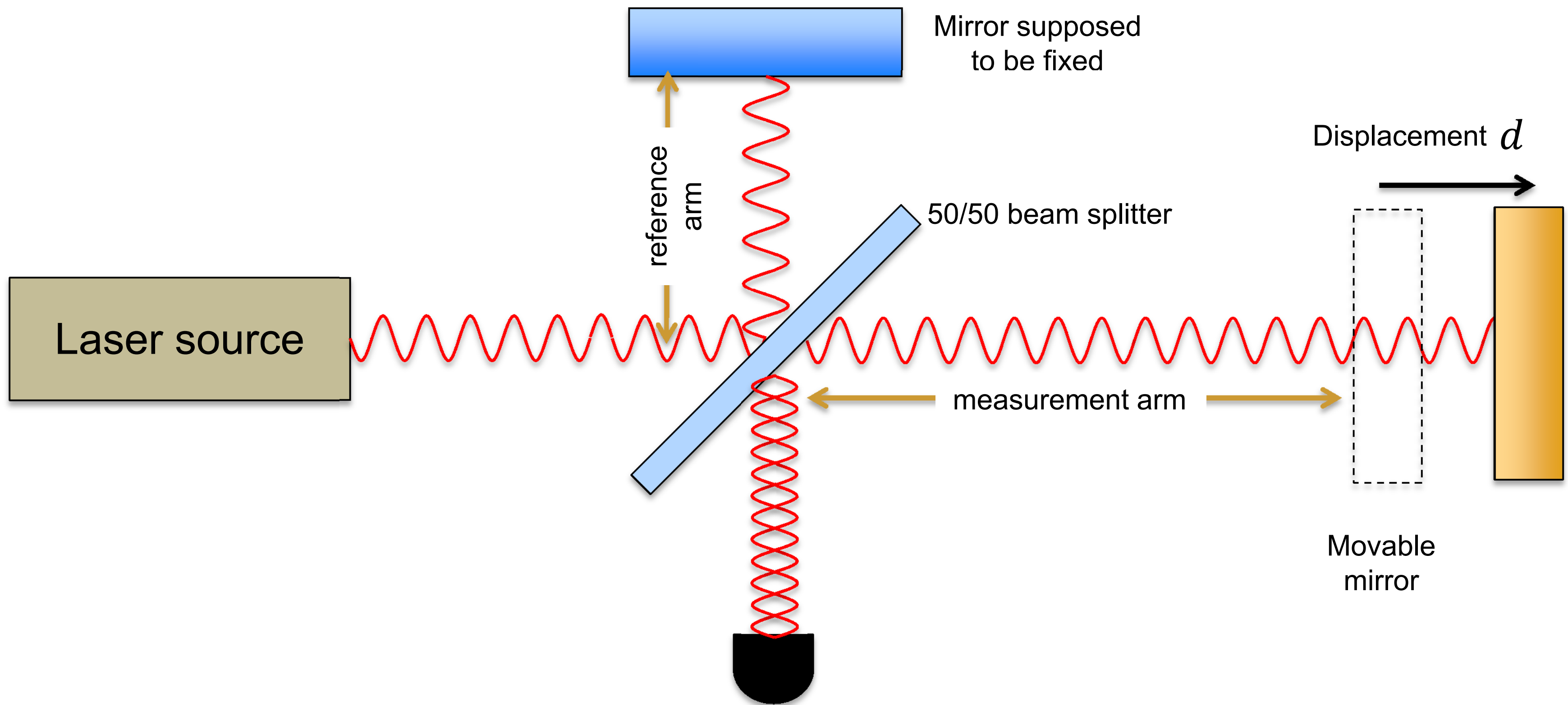


Intensity on the photodetector :

$$I = |E_r + E_m|^2$$

$$I = E_0 \left[e^{i\left(\omega t - \frac{2\pi}{\lambda_v} OPL_r\right)} + e^{i\left(\omega t - \frac{2\pi}{\lambda_v} OPL_m\right)} \right] \times \left[e^{-i\left(\omega t - \frac{2\pi}{\lambda_v} OPL_r\right)} + e^{-i\left(\omega t - \frac{2\pi}{\lambda_v} OPL_m\right)} \right]$$

PRINCIPLE OF MICHELSON INTERFEROMETER

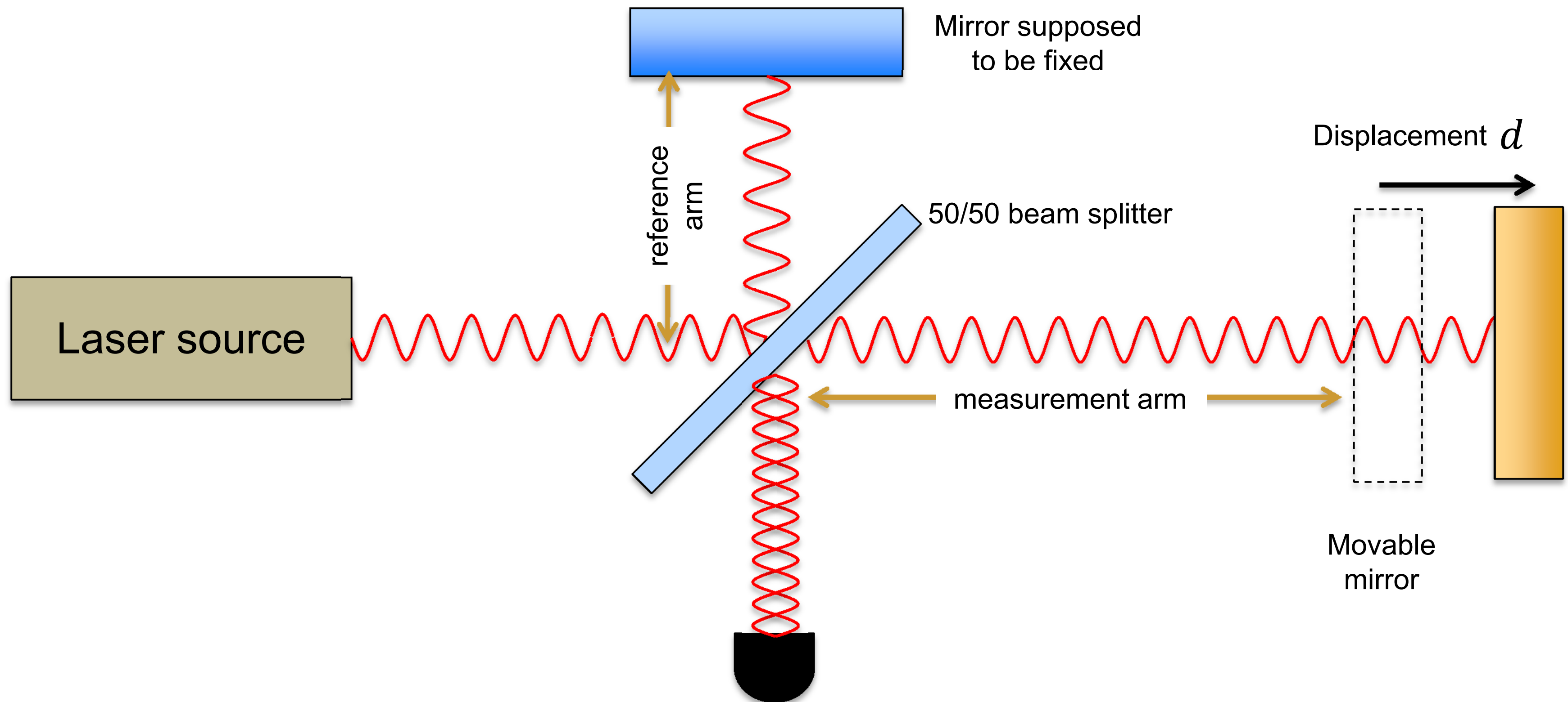


Intensity on the photodetector :

$$I = |E_r + E_m|^2$$

$$I = 2E_0{}^2 + 2E_0{}^2 \cos\left[\frac{2\pi}{\lambda_v}(OPL_m - OPL_r)\right]$$

PRINCIPLE OF MICHELSON INTERFEROMETER



Intensity on the photodetector :

$$I = 2{E_0}^2 \cos\left[\frac{2\pi}{\lambda_v}(OPL_m - OPL_r)\right]$$

As the interferometer counts some fringes, only this part as to be taken into account

PRINCIPLE OF MICHELSON INTERFEROMETER

Counting :

$$\frac{k}{2^i} = \frac{2}{\lambda_v}(OPL_m - OPL_r)$$

λ_v: vacuum wavelength
d: displacement
OPL: optical path length
k: counting
i: interpolation factor

1 - Interferometer are sensitive to the evolution of the optical path length difference between the two arms.

2 - It means that everything that will unbalance the difference between the two optical paths will produce a signal that will be interpreted as a displacement !!!

$$OPL = \int_s n(s)ds$$

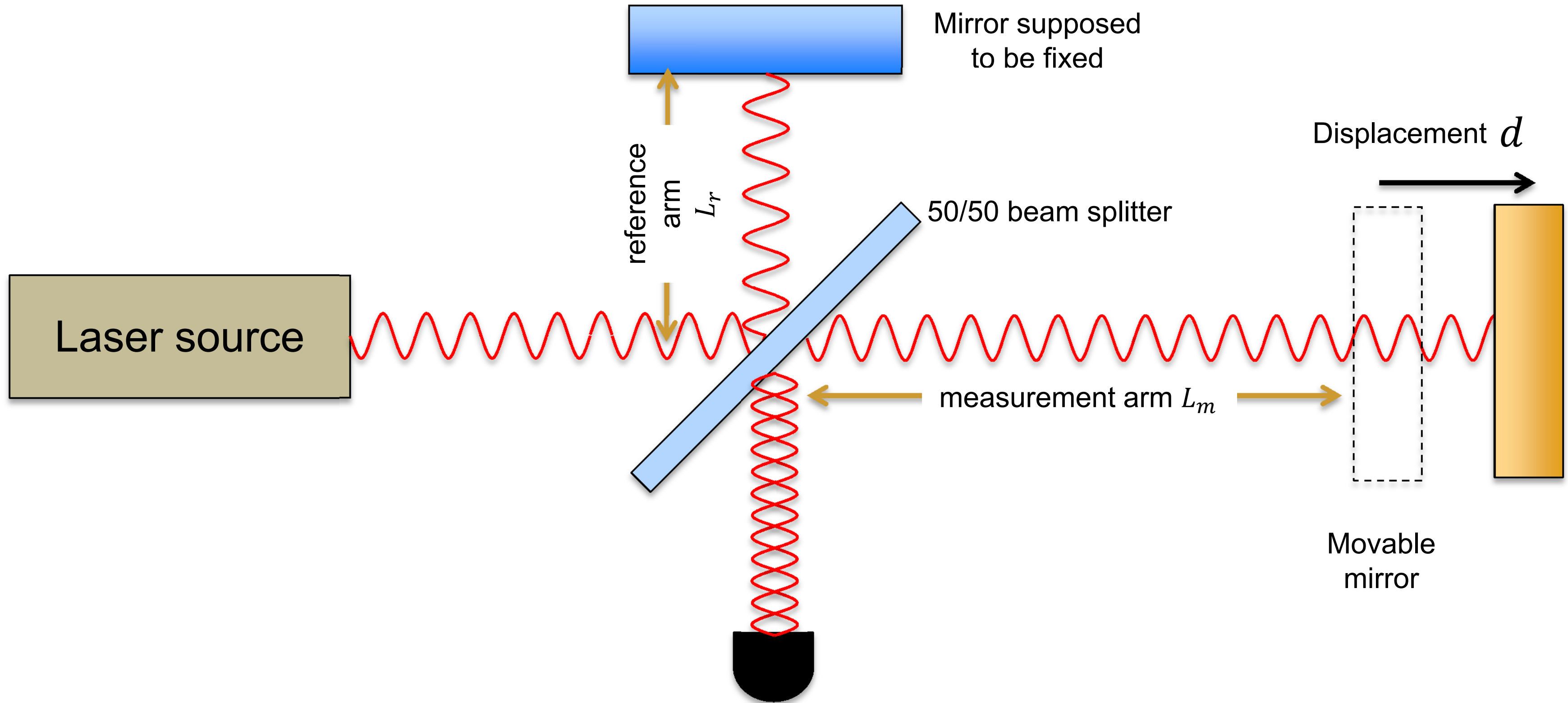
s: beam path
n: refractive index of air

Assumption 1: if the refractive index of air is constant along the beam path then:

$$OPL = n \times L_s$$

s: beam path
L_s: length of the beam path
n: refractive index of air

PRINCIPLE OF MICHELSON INTERFEROMETER



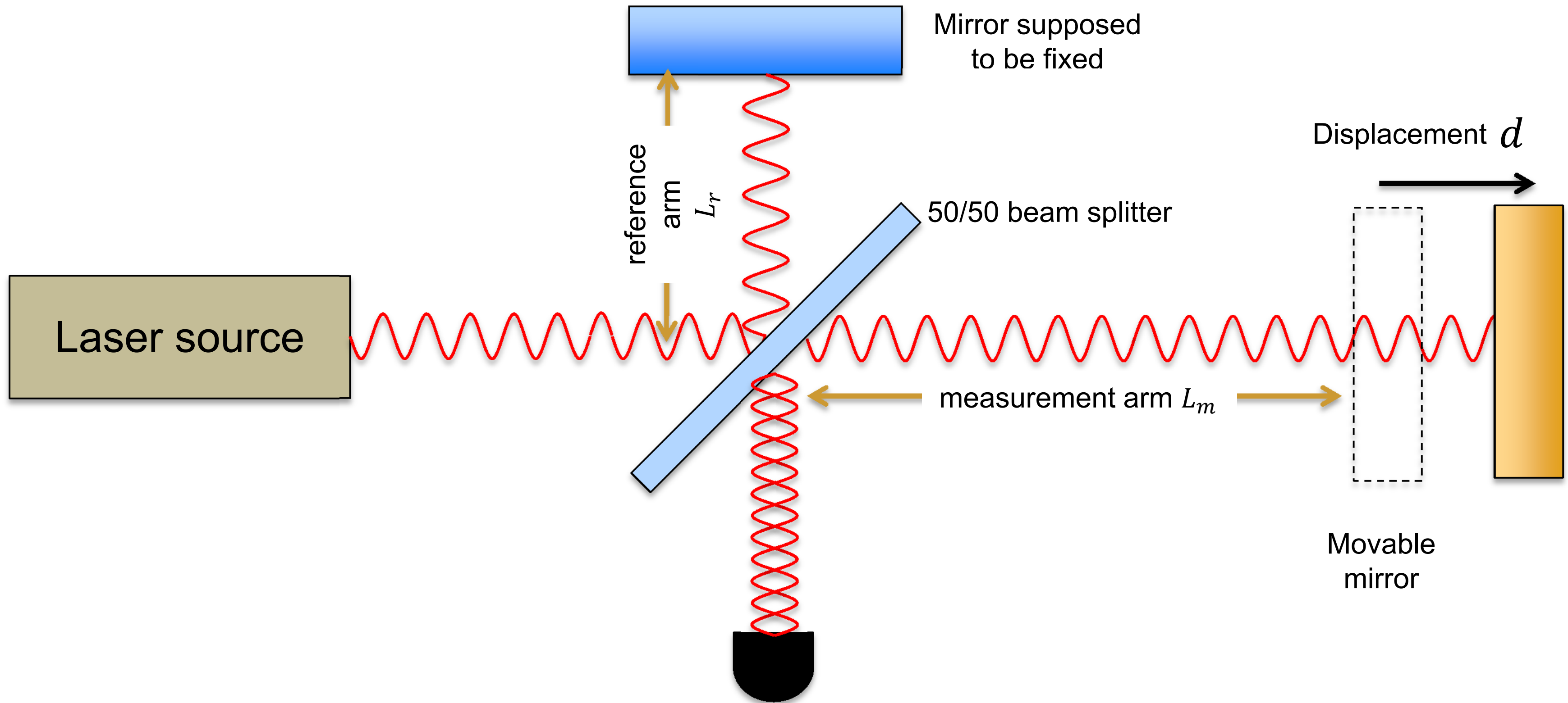
Counting:

$$\frac{k}{2^i} = \frac{2}{\lambda_v}(2\, n_m d + 2\, n_m L_m - 2 n_r L_r)$$

λ_v: vacuum wavelength
n_r or n_m: refractive index
L_r or L_m: physical length
d: displacement
k: counting
i: interpolation factor

Assumption 2: the refractive index of air in the two arms is homogeneous ($n_r = n_m = n$).

PRINCIPLE OF MICHELSON INTERFEROMETER



Counting:

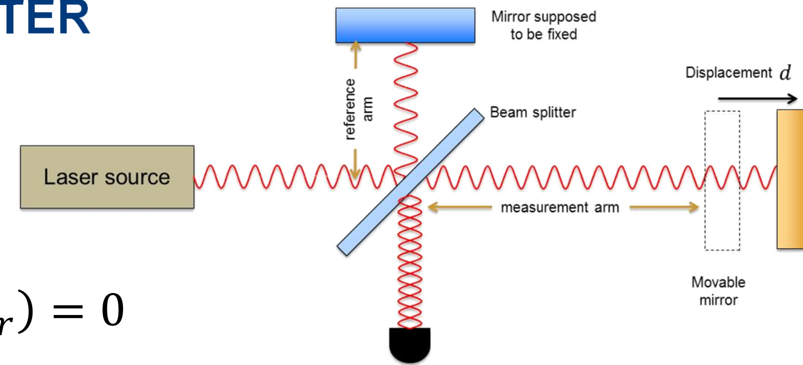
$$\frac{k}{2^i} = \frac{4n}{\lambda_v}(d + L_m - L_r)$$

λ_v: vacuum wavelength
n: refractive index
L_r or L_m: physical length
d: displacement
k: counting
i: interpolation factor

Interferometers measure a displacement, not an absolute position !!

⇒ the initial positions of the mirrors are never known, only the relative change in position is determined

PRINCIPLE OF MICHELSON INTERFEROMETER



At $t = 0$, interferometer is reseted:

$$\frac{k_0}{2^i} = \frac{4n_0}{\lambda_v}(L_m - L_r) = 0$$

At t and with $n = n_0 + \Delta n$:

$$\frac{k}{2^i} = \frac{4(n_0 + \Delta n)}{\lambda_v}(d + L_m - L_r)$$

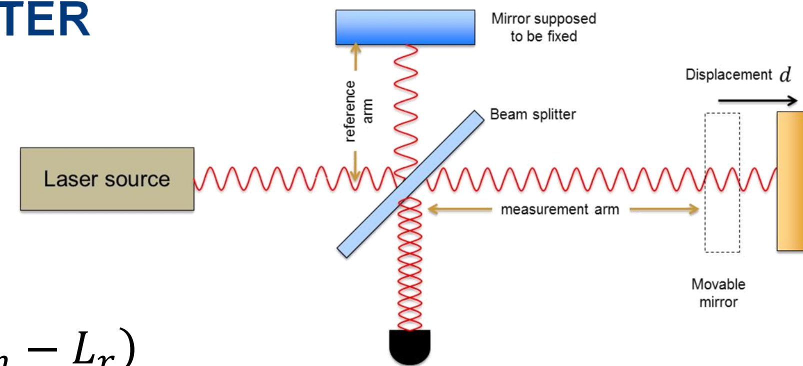
$$\frac{k}{2^i} = \underbrace{\frac{4n_0}{\lambda_v}(L_m - L_r)}_{=0} + \frac{4\Delta n}{\lambda_v}(L_m - L_r) + \frac{4(n_0 + \Delta n)d}{\lambda_v}$$

$$\frac{k}{2^i} = \frac{4(n_0 + \Delta n)\boxed{d}}{\lambda_v} + \frac{4\Delta n}{\lambda_v}(L_m - L_r)$$

$$d = \frac{k\lambda_v}{4 \times (n_0 + \Delta n) \times 2^i} - \frac{\Delta n(L_m - L_r)}{(n_0 + \Delta n)}$$

$$\boxed{d = \frac{k\lambda_v}{4n2^i} - \frac{\Delta n}{n}(L_m - L_r)}$$

PRINCIPLE OF MICHELSON INTERFEROMETER



$$d = \frac{k\lambda_v}{4n2^i} - \frac{\Delta n}{n}(L_m - L_r)$$

Displacement term *dead path term*

1 – the vacuum wavelength λ_v needs to be known (calibrated) to determine d

2 – in air, n and Δn also need to be determined and tracked to properly measure d

3 – for dead path term, difference of length between the measurement and reference arms $(L_m - L_r)$ need to be known

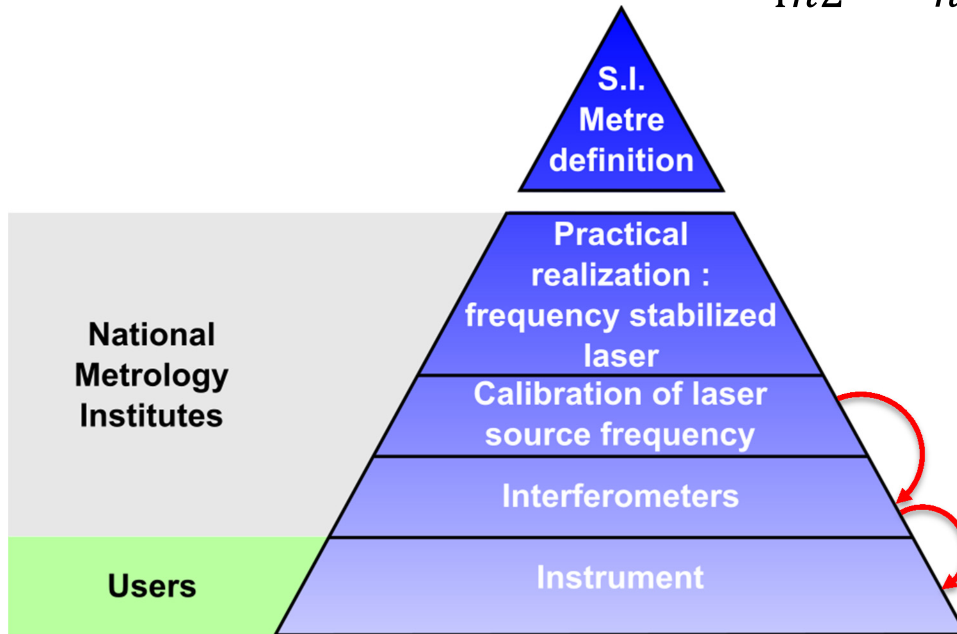
4 – in vacuum, $n = 1$ and $\Delta n = 0$: the measurement using interferometry is greatly simplified:

$$d = \frac{k\lambda_v}{4 \times 2^i}$$

WAVELENGTH CALIBRATION

VACUUM WAVELENGTH CALIBRATION AND TRACEABILITY TO SI METER

$$d = \frac{k\lambda_v}{4n2^i} - \frac{\Delta n}{n}(L_m - L_r)$$



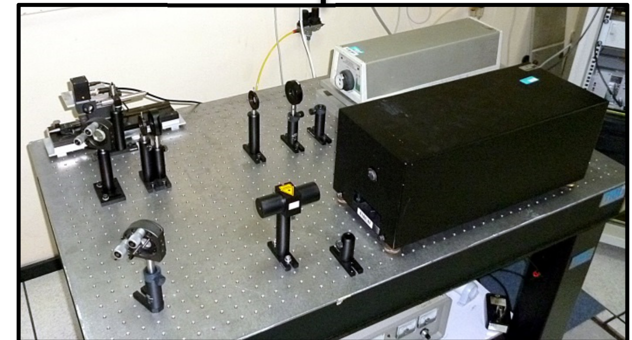
The laser source vacuum wavelength λ_{vac} need to be calibrated if measurement need to be comparable and traceable to meter definition (SI)

Laser wavelength

Displacement

Calibration of a laser frequency (wavelength) using a frequency beat comparison setup. The operation is generally directly realized by the supplier but can also be provided by a metrology institute (LNE, PTB, NIST, NMIJ, …)

It gives you λ_v with an associated uncertainty (most of time the wavelength stability of the laser source).



LNE's beat comparison setup

VACUUM WAVELENGTH UNCERTAINTY

Uncertainty in wavelength produces uncertainty in displacement measurement.

Uncertainty mainly originates from the laser stability in wavelength (or frequency).

λ = 632,8 nm for He-Ne sources with an uncertainty typically comprised between 1-60 femtometers (1 fm = 10^{-15} m). It represents an equivalent stability in part per billion: 1 to 100 ppb.

Depending on the laser source and the time frame

Laser source stability	*Displacement*					
	100 µm	***1 mm***	***10 mm***	***100 mm***	***400 mm***	***1000 mm***
1 ppb	0,0001 nm	0,001 nm	0,01 nm	0,1 nm	0,4 nm	1 nm
10 ppb	0,001 nm	0,01 nm	0,1 nm	1 nm	4 nm	10 nm
100 ppb	0,01 nm	0,1 nm	1 nm	10 nm	40 nm	100 nm

⇒ Need to check the laser source stability

⇒The higher the displacements are, the more stable the source must be !

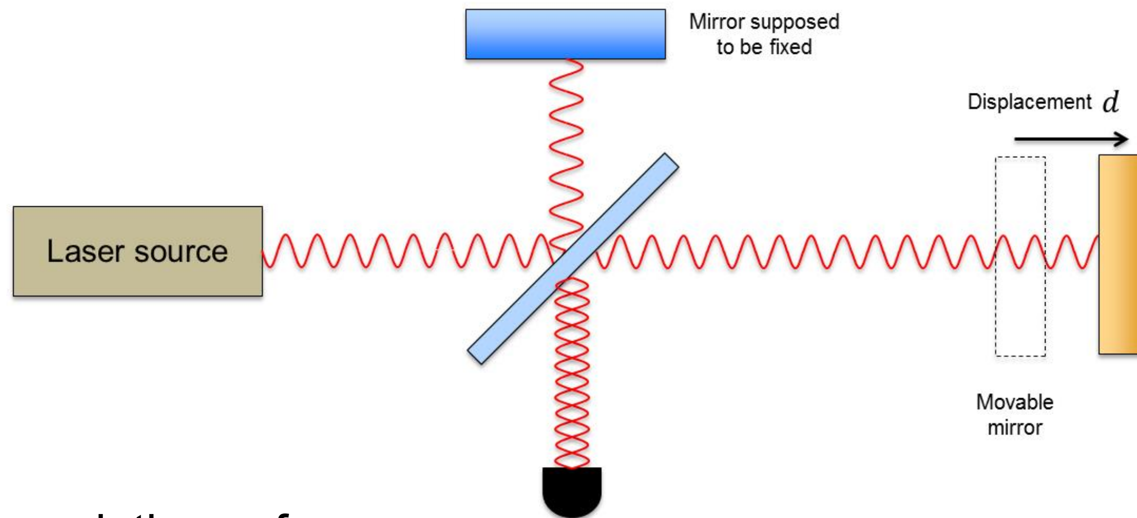
VARIATION OF REFRACTIVE INDEX OF AIR

VARIATION OF REFRACTIVE INDEX OF AIR

$$d = \frac{k\lambda_v}{4n2^i} - \frac{\Delta n}{n}(L_m - L_r)$$

n must be determined

Changes of the environmental condition during the measurement are the largest source of error in interferometry.



n is sensitive to variations of:

- temperature of air (T)
- pressure (P)
- relative humidity (%RH)
- sometime even exact gas composition (for very precise applications)

VARIATION OF REFRACTIVE INDEX OF AIR

Sensitivity of refractive air index to:

- Temperature (T)

$$K_T = \frac{\partial n}{\partial T} = -9.56 \times 10^{-7}\ K^{-1} \cong \underline{1\ ppm\ K^{-1}}$$

- Pressure (P)

$$K_P = \frac{\partial n}{\partial P} = 2.68 \times 10^{-7}\ hPa^{-1} \cong \underline{0.27\ ppm\ hPa^{-1}}$$

- Relative humidity (%RH)

$$K_{HR} = \frac{\partial n}{\partial RH} = -8.5 \times 10^{-9}\ \%RH^{-1} \cong \underline{0{,}01\ ppm\ \%\ RH^{-1}}$$

VARIATION OF REFRACTIVE INDEX OF AIR

Temperature:

Temperature variation	Displacement					
	100 µm	*1 mm*	*10 mm*	*100 mm*	*400 mm*	*1000 mm*
0.001 °C	0,0001 nm	0.001 nm	0.01 nm	0.1 nm	0.4 nm	1 nm
0.01 °C	0,001 nm	0.01 nm	0.1 nm	1 nm	4 nm	10 nm
0.1 °C	0,01 nm	0.1 nm	1 nm	10 nm	40 nm	100 nm
1 °C	0,1 nm	1 nm	10 nm	100 nm	400 nm	1000 nm

Pressure:

Pressure variation	Displacement					
	100 µm	*1 mm*	*10 mm*	*100 mm*	*400 mm*	*1000 mm*
0.01 hPa	0,00027 nm	0.0027 nm	0.027 nm	0.27 nm	1 nm	2,7 nm
0.1 hPa	0,0027 nm	0.027 nm	0.27 nm	2.7 nm	10,8 nm	27 nm
1 hPa	0,027 nm	0.27 nm	2.7 nm nm	27 nm	108 nm	270 nm
10 hPa	0,27 nm	2.7 nm nm	27 nm nm	270 nm	1080 nm	2700 nm

VARIATION OF REFRACTIVE INDEX OF AIR

Hygrometry:

	Displacement					
Hygrometry variation	***100 µm***	***1 mm***	***10 mm***	***100 mm***	***400 mm***	***1000 mm***
0,01 %RH	0,00001 nm	0.0001 nm	0.001 nm	0.01 nm	0.04 nm	0,1 nm
0,1 %RH	0,0001 nm	0.001 nm	0.01 nm	0.1 nm	0.4 nm	1 nm
1 %RH	0,001 nm	0.01 nm	0.1 nm	1 nm	4 nm	10 nm
10 %RH	0,01 nm	0.1 nm	1 nm	10 nm	4 0nm	100 nm

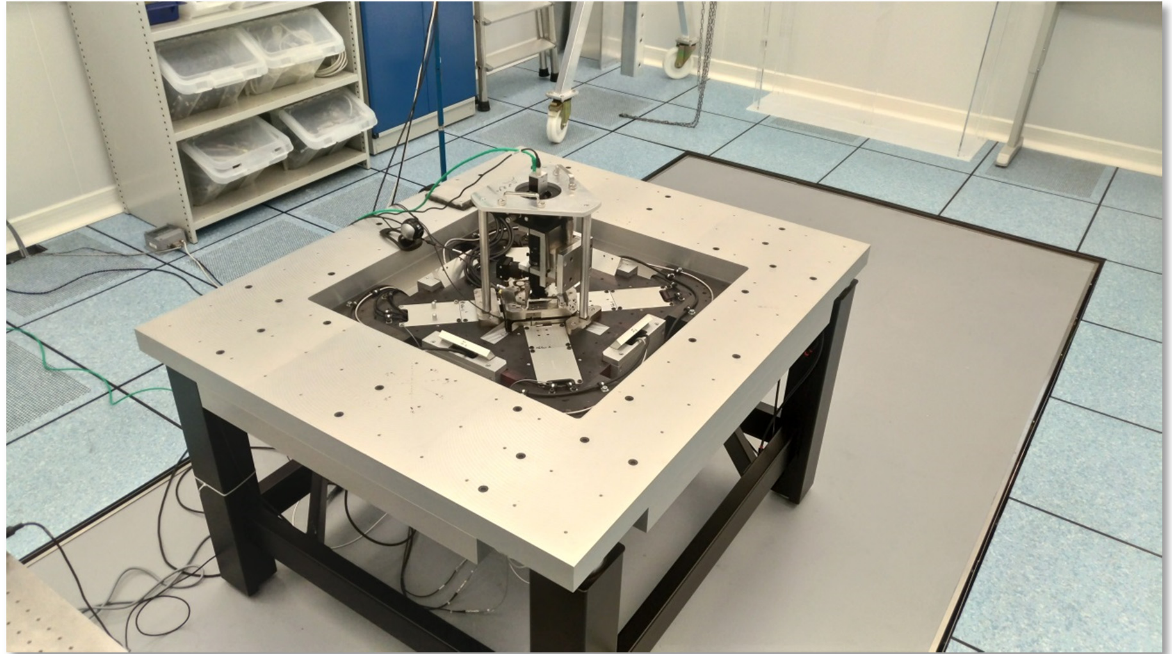
⇒ Very sensitive to T,P and HR (depending on the displacement range)
⇒ If possible, limit the variation of T, P and RH (enclosure around the instrument, control of the temperature and hygrometry of the room)

VARIATION OF REFRACTIVE INDEX OF AIR

Clean room:

- temperature 20 ± 0.05 °C
- hygrometry 50 ± 5%RH

Instruments use large amount of matter to improve thermal inertia and then stability



Thermal stability achieved in the instruments for the interferometer measurement:

- hp-CMM: about 50 mK per 20 hours (impact due to ΔT : $15\ nm$)
- mAFM: about 1 mK per hour (impact due to ΔT : negligible)

VARIATION OF REFRACTIVE INDEX OF AIR

Hygrometry:

Hygrometry variation	*Displacement*					
	100 µm	*1 mm*	*10 mm*	*100 mm*	*400 mm*	*1000 mm*
0,01 %RH	0,00001 nm	0.0001 nm	0.001 nm	0.01 nm	0.04 nm	0,1 nm
0,1 %RH	0,0001 nm	0.001 nm	0.01 nm	0.1 nm	0.4 nm	1 nm
1 %RH	0,001 nm	0.01 nm	0.1 nm	1 nm	4 nm	10 nm
10 %RH	0,01 nm	0.1 nm	1 nm	10 nm	4 0nm	100 nm

⇒ Very sensitive to T,P and HR (depending on the displacement range)

⇒ If possible, limit the variation of T, P and RH (enclosure around the instrument, thermal and hygrometry control of the room)

⇒ Implement some corrections to compensate for refractive air index variations when displacement is larger than 1 mm

Weather station :

Measurement of temperature (T), pressure (P) and relative humidity (RH) using a calibrated weather station then correction using modified Edlén equation (empirical equation).

Edlen formulae uncertainty: 2×10^{-8}

Uncertainty for Edlen correction:

$$u(n) = \sqrt{(2 \times 10^{-8})^2 + (K_T u(T))^2 + (K_P u(P))^2 + (K_{RH} u(RH))^2}$$

formulae uncertainty

T, P and RH measurement uncertainties

HOW TO APPLY CORRECTIONS

$$u(n) = \sqrt{(2 \times 10^{-8})^2 + (K_T u(T))^2 + (K_T u(P))^2 + (K_T u(RH))^2}$$

Numerical application:

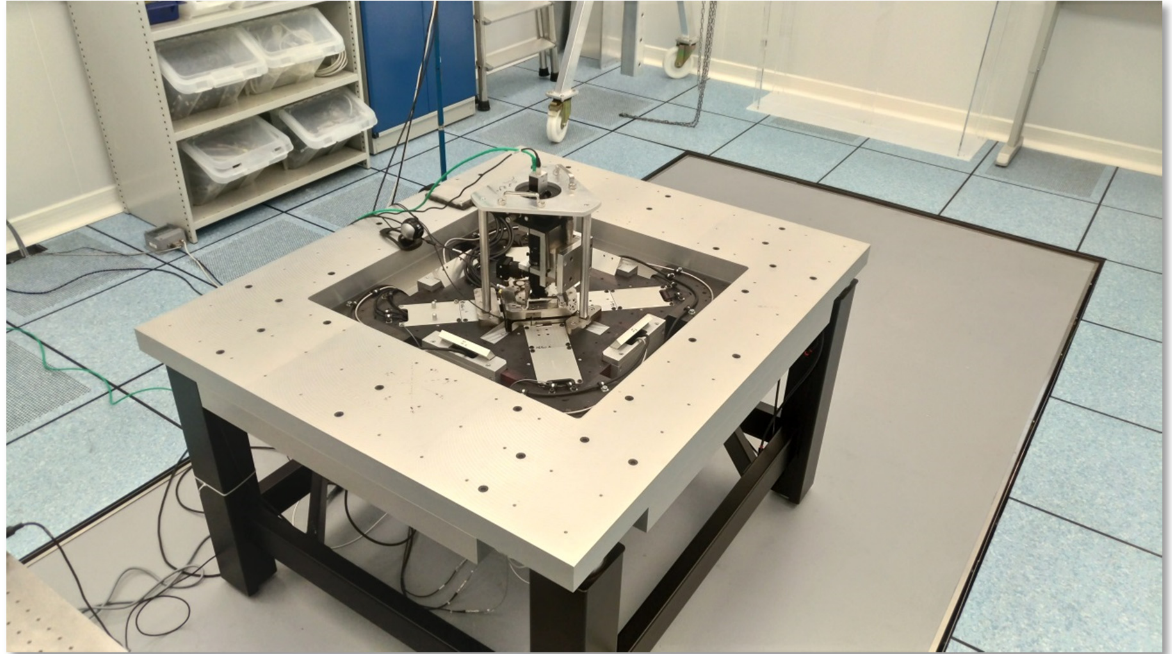
with $u(T) = 0.1\ K$, $u(P) = 0.01 hPa$ and $u(RH) = 1\ \% \Rightarrow\ u(n) = 10^{-7}$

hp-CMM

$d = 300\ mm$

$u(d)_n = 30\ nm$

mAFM



$d = 60\ \mu m$

$u(d)_n = 0{,}006\ nm$

Weather station :

Measurement of temperature (T), pressure (P) and relative humidity (RH) using a calibrated weather station then correction using modified Edlén equation (empirical equation).

- Easy to implement

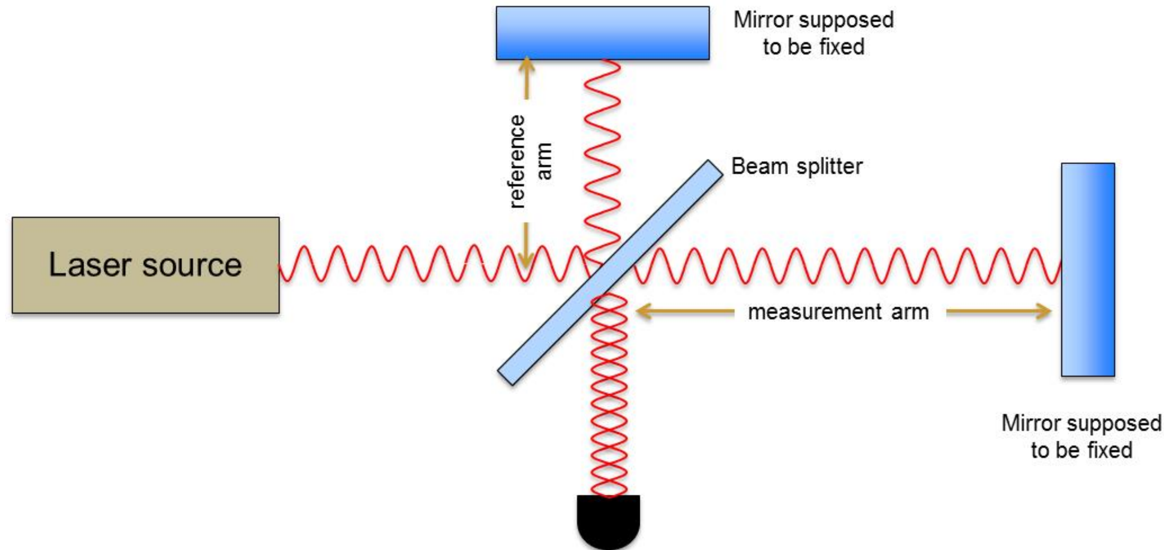
- Not really real-time (measurement of P, T, %HR are slow due to the low bandwidth of the sensor).
- P, T, %HR need to be calibrated
- Measurement are not really representative of the interferometer conditions because not realized at the same place.
- Does not take in account n changes due gas composition.
- T, P and RH must be measured with high precaution for high precision measurement

HOW TO APPLY CORRECTIONS

Wavelength tracker

It tracks changes in the refractive index from the initial value ($t = 0$) by using an interferometer with fixed physical path length.

Principle :



$$\Delta OPL = OPL_t - OPL_{t0} = 2n_t \times (L_{r_t} - L_{m_t}) - 2n_{t0} \times (L_{r_{t0}} - L_{m_{t0}})$$

If L_r and L_m are fixed (constant in time)

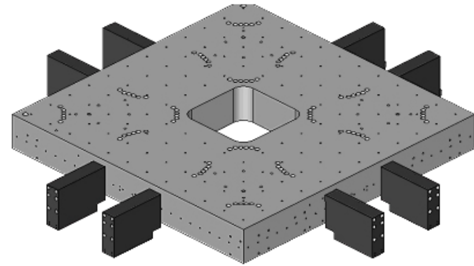
$$\Delta OPL \propto 2 \times \Delta n \times (L_r - L_m)$$

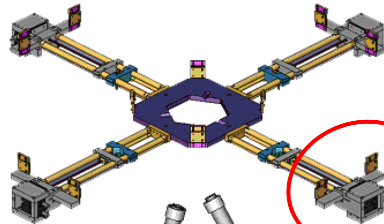
If $L_r - L_m$ is known, Δn is known

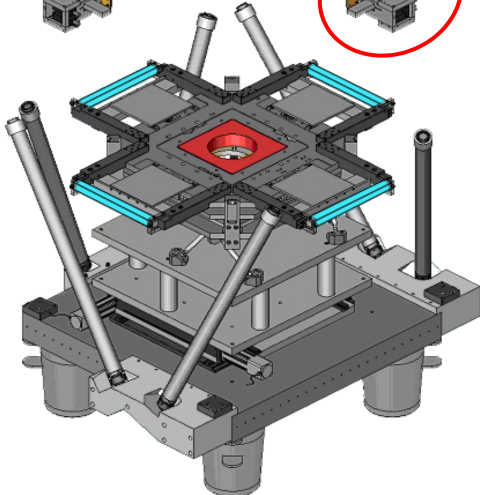
HOW TO APPLY CORRECTIONS

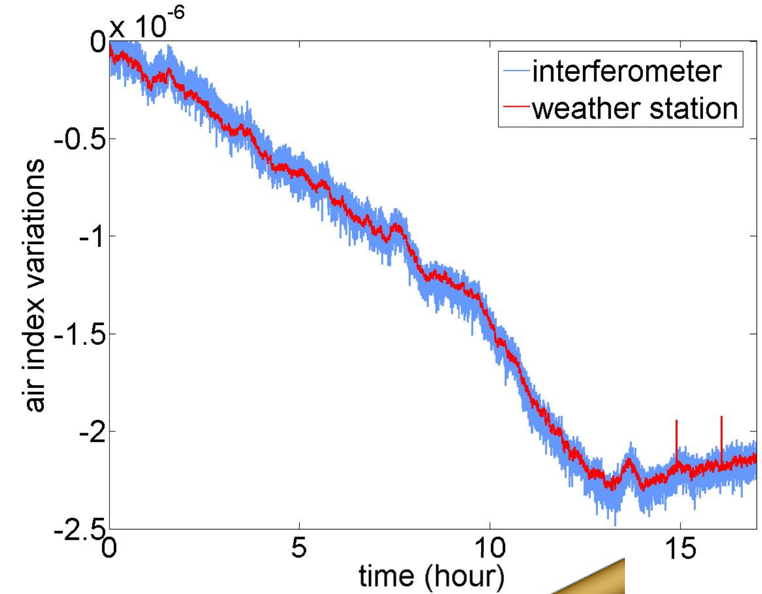
Wavelength tracker

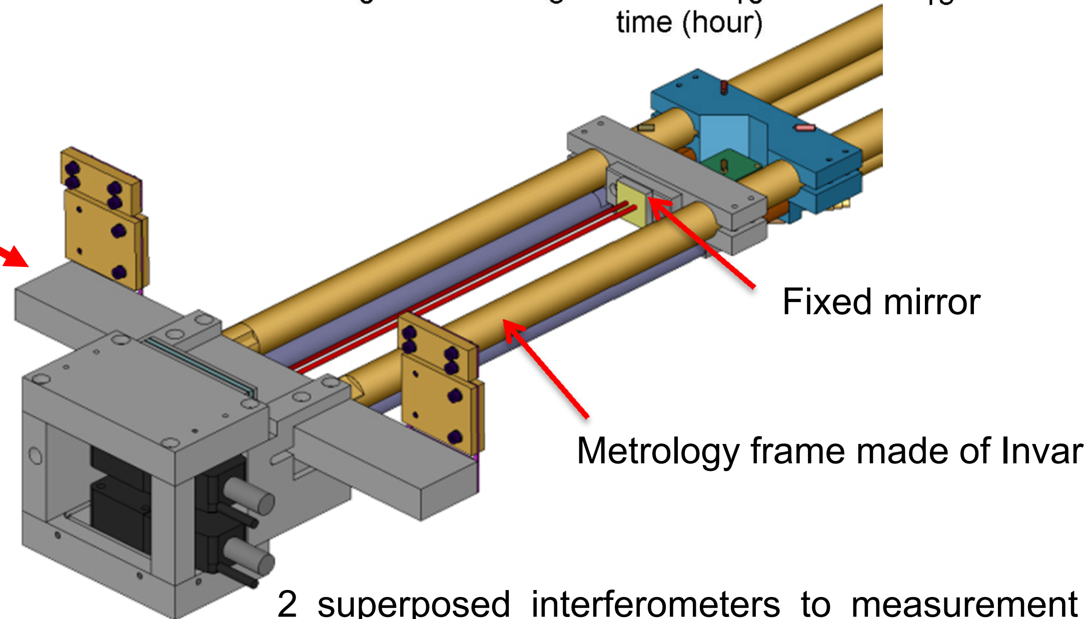
Example of implementation on the hp-CMM









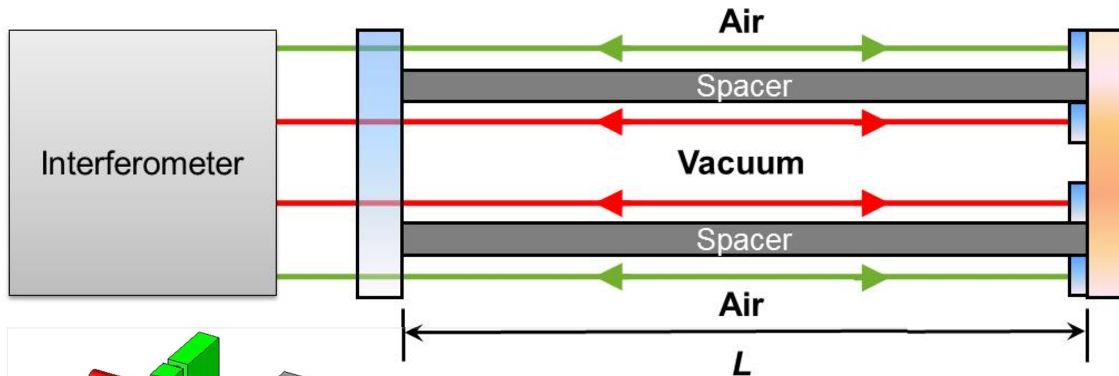


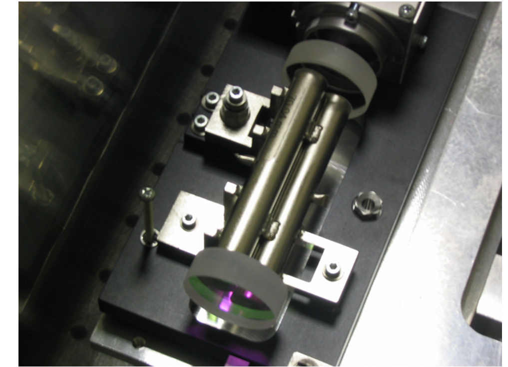
2 superposed interferometers to measurement one displacement, 1 for measurement of displacement, 1 for wavelength tracking.

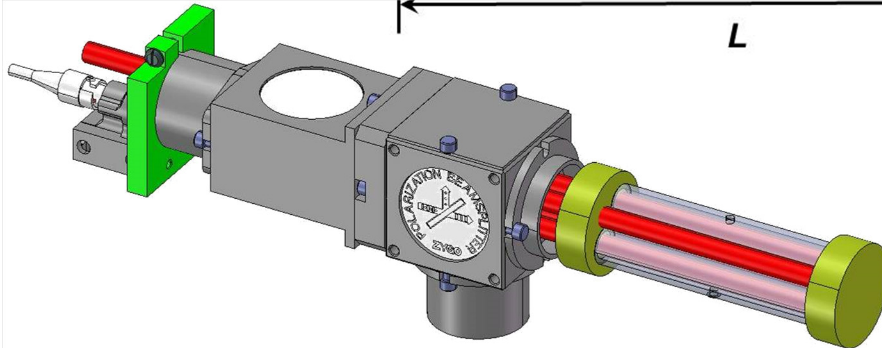
HOW TO APPLY CORRECTIONS

Wavelength tracker

Other solution proposed by Zygo : measurement beam passes through air, reference beam passes through a vacuum cell







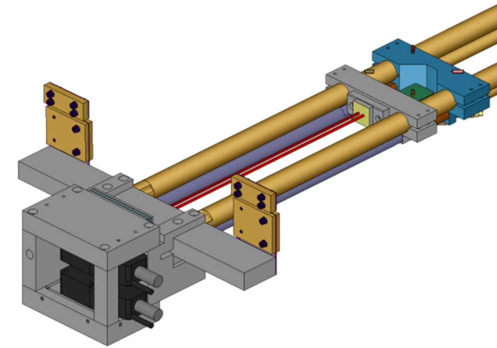
$$\Delta OPL = OPL_t - OPL_{t0} = 4L \times (n_{t0} - 1) - 4L \times (n - 1)$$

$$\Delta OPL = 4L \times \Delta \mathrm{n}$$

HOW TO APPLY CORRECTIONS

Wavelength tracker

It tracks changes in the refractive index from the initial value ($t = 0$) by using an interferometer with fixed physical path length.



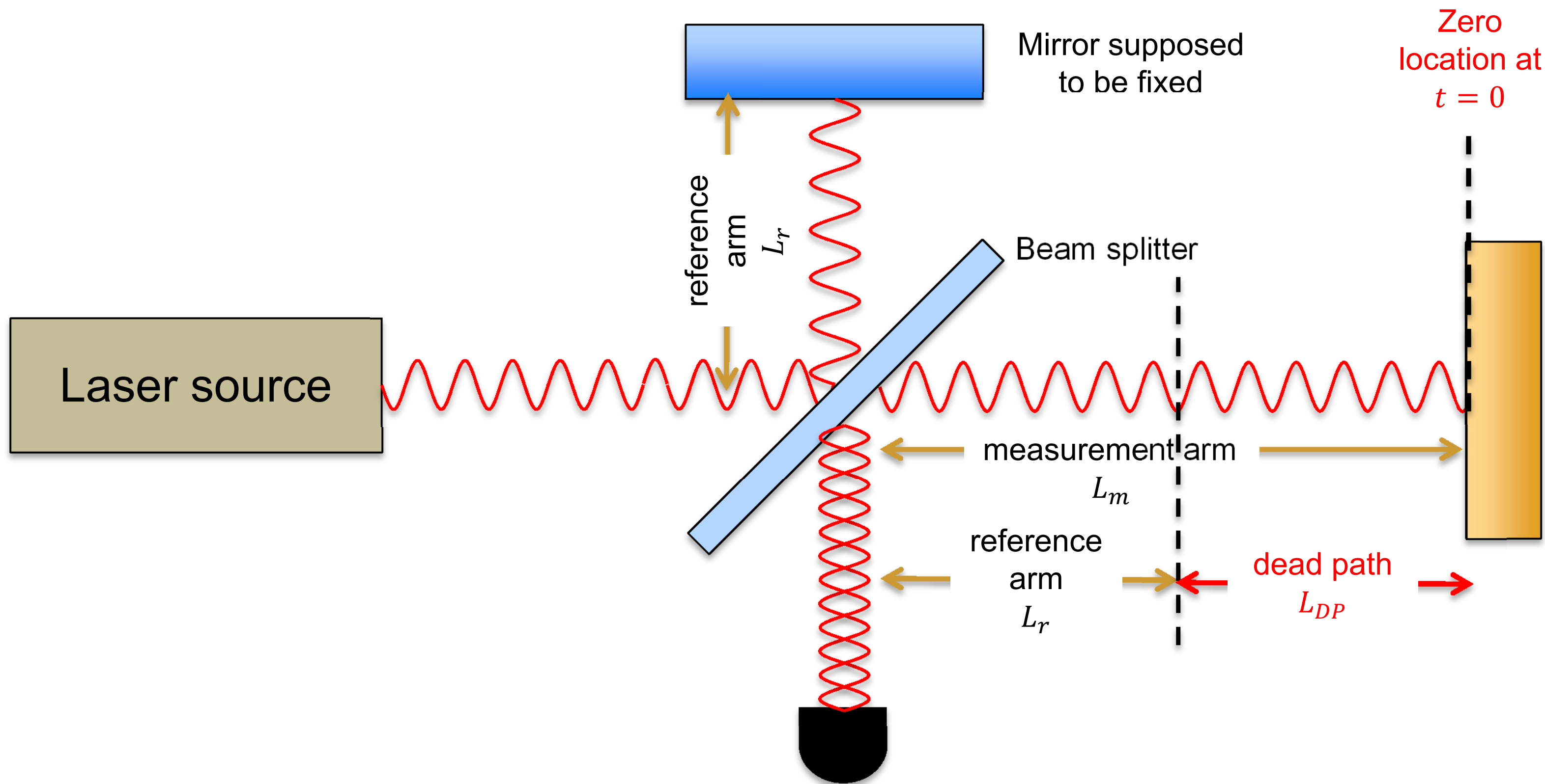
- High bandwidth compare to weather station

- Need an additional interferometer with a good stability.
- Need to determine the physical length of the optical path.
- Based on the assumption that interferometer and wavelength tracker detect the same index changes ! Not true in perturbed environment.
- Need to be placed very close from the interferometer to compensate.
- Does not measure an absolute index.
- Need to be completed with a weather station to fix n at $t = 0$.
- Uncertainty is limited by the uncertainty in the determination of n at $t = 0$ by the weather station.

DEAD PATH ERROR

DEAD PATH ERROR

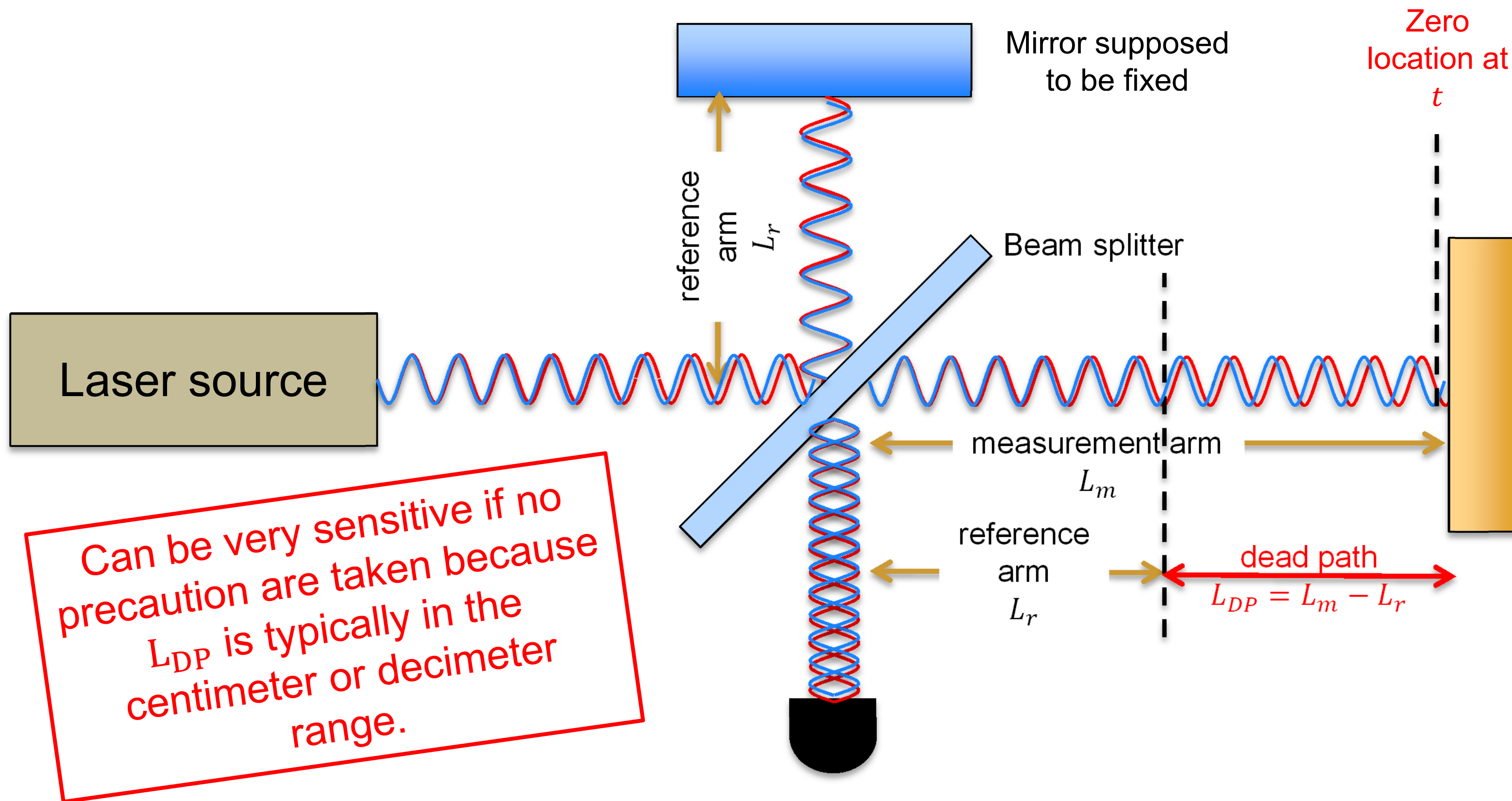
$$d = \frac{k\lambda_v}{4n2^i} - \frac{\Delta n}{n}(L_m - L_r)$$



The dead path is the difference in optical path length between the reference and measurement beams when the interferometer electronic is zeroed.

DEAD PATH ERROR

$$d = \frac{k\lambda_v}{4n2^i} - \frac{\Delta n}{n}(L_m - L_r)$$



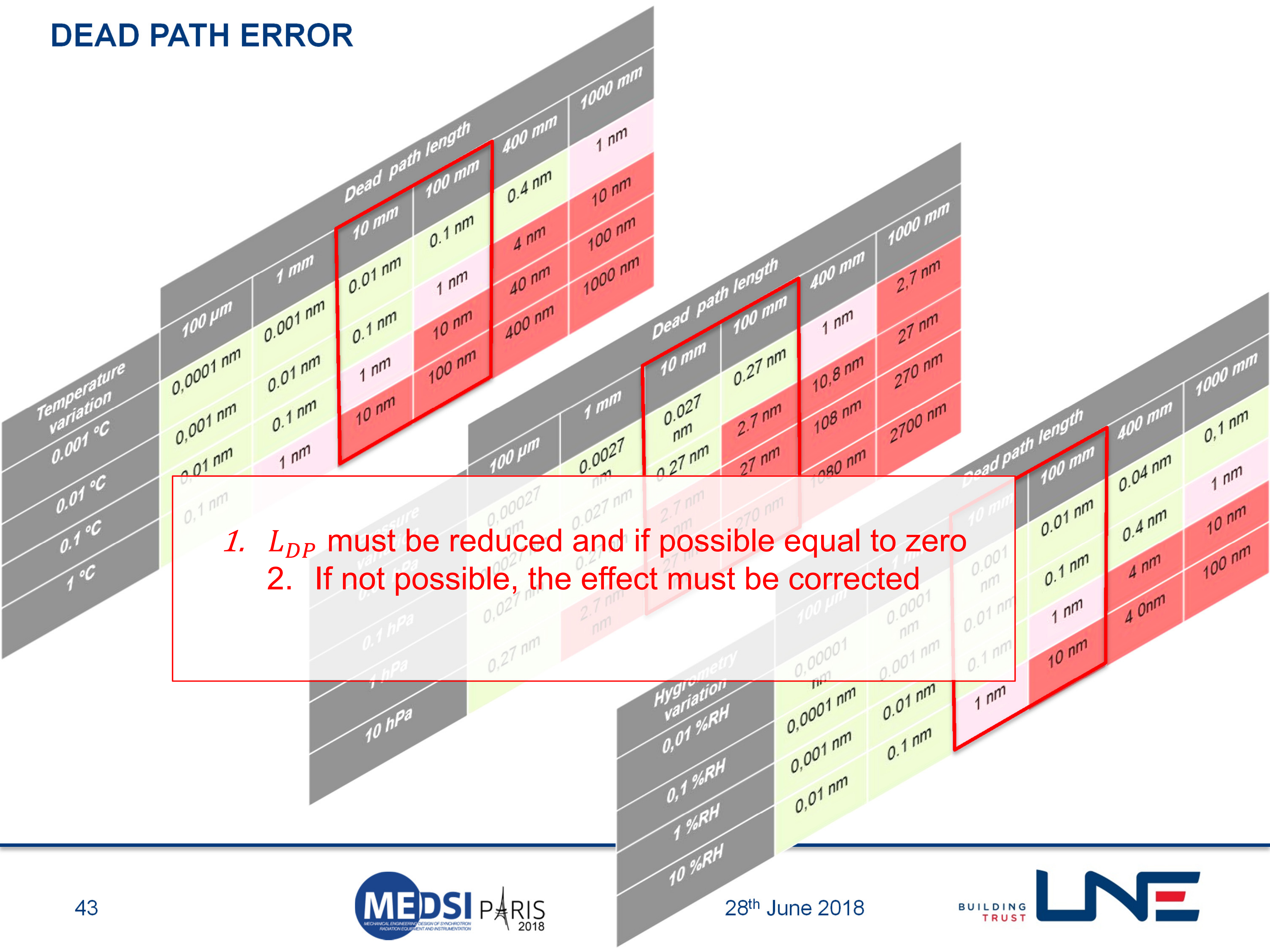
The unbalance between the two arms (non symmetry) will produce a sensitivity to both changes in wavelength $\Delta\lambda$ or in air index Δn that will shift in space the zero location. The signal produced is then interpreted as a displacement even though the mirrors are immobile.

DEAD PATH ERROR

Temperature variation	Dead path length: 100 µm	1 mm	10 mm	100 mm	400 mm	1000 mm
0.001 °C	0,0001 nm	0.001 nm	0.01 nm	0.1 nm	0.4 nm	1 nm
0.01 °C	0,001 nm	0.01 nm	0.1 nm	1 nm	4 nm	10 nm
0.1 °C	0,01 nm	0.1 nm	1 nm	10 nm	40 nm	100 nm
1 °C	0,1 nm	1 nm	10 nm	100 nm	400 nm	1000 nm

Pressure variation	Dead path length: 100 µm	1 mm	10 mm	100 mm	400 mm	1000 mm
0.01 hPa	0,00027 nm	0.0027 nm	0.027 nm	0.27 nm	1 nm	2,7 nm
0.1 hPa	0,0027 nm	0.027 nm	0.27 nm	2.7 nm	10,8 nm	27 nm
1 hPa	0,027 nm	0.27 nm	2.7 nm	27 nm	108 nm	270 nm
10 hPa	0,27 nm	2.7 nm	27 nm	270 nm	1080 nm	2700 nm

Hygrometry variation	Dead path length: 100 µm	1 mm	10 mm	100 mm	400 mm	1000 mm
0,01 %RH	0,00001 nm	0.0001 nm	0.001 nm	0.01 nm	0.04 nm	0,1 nm
0,1 %RH	0,0001 nm	0.001 nm	0.01 nm	0.1 nm	0.4 nm	1 nm
1 %RH	0,001 nm	0.01 nm	0.1 nm	1 nm	4 nm	10 nm
10 %RH	0,01 nm	0.1 nm	1 nm	10 nm	4 0nm	100 nm


DEAD PATH ERROR
Temperature variation
0.001 °C
0.01 °C
0.1 °C
1 °C
Dead path length
100 µm
1 mm
10 mm
100 mm
400 mm
1000 mm
0,0001 nm
0.001 nm
0.01 nm
0.1 nm
0.4 nm
1 nm
0,001 nm
0.01 nm
0.1 nm
1 nm
4 nm
10 nm
0.1 nm
1 nm
10 nm
40 nm
100 nm
1 nm
10 nm
100 nm
400 nm
1000 nm
Dead path length
100 µm
1 mm
10 mm
100 mm
400 mm
1000 mm
0.027 nm
0.27 nm
1 nm
2,7 nm
2.7 nm
10,8 nm
27 nm
0.27 nm
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108 nm
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Dead path length
100 mm
400 mm
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0.01 nm
0.04 nm
0,1 nm
0.1 nm
0.4 nm
1 nm
1 nm
4 nm
10 nm
10 nm
4 0nm
100 nm
Hygrometry variation
0,01 %RH
0,1 %RH
1 %RH
10 %RH
0,0001 nm
0,001 nm
0,01 nm
0.01 nm
0.1 nm
1 nm
1. L_{DP} must be reduced and if possible equal to zero
2. If not possible, the effect must be corrected
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28th June 2018
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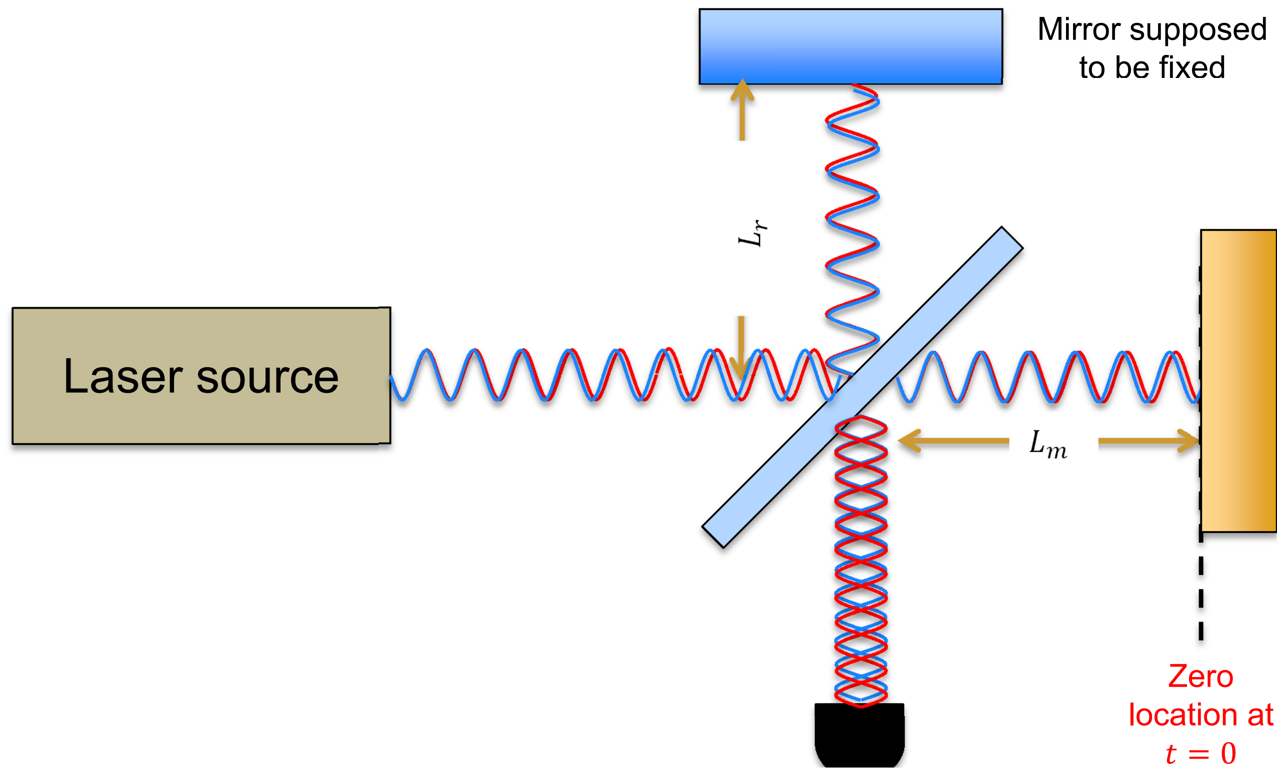
REDUCTION OF DEAD PATH ERROR



The first solution to reduce the dead path error is to balance the optical path lengths of the two arms (symmetry) at $t = 0$

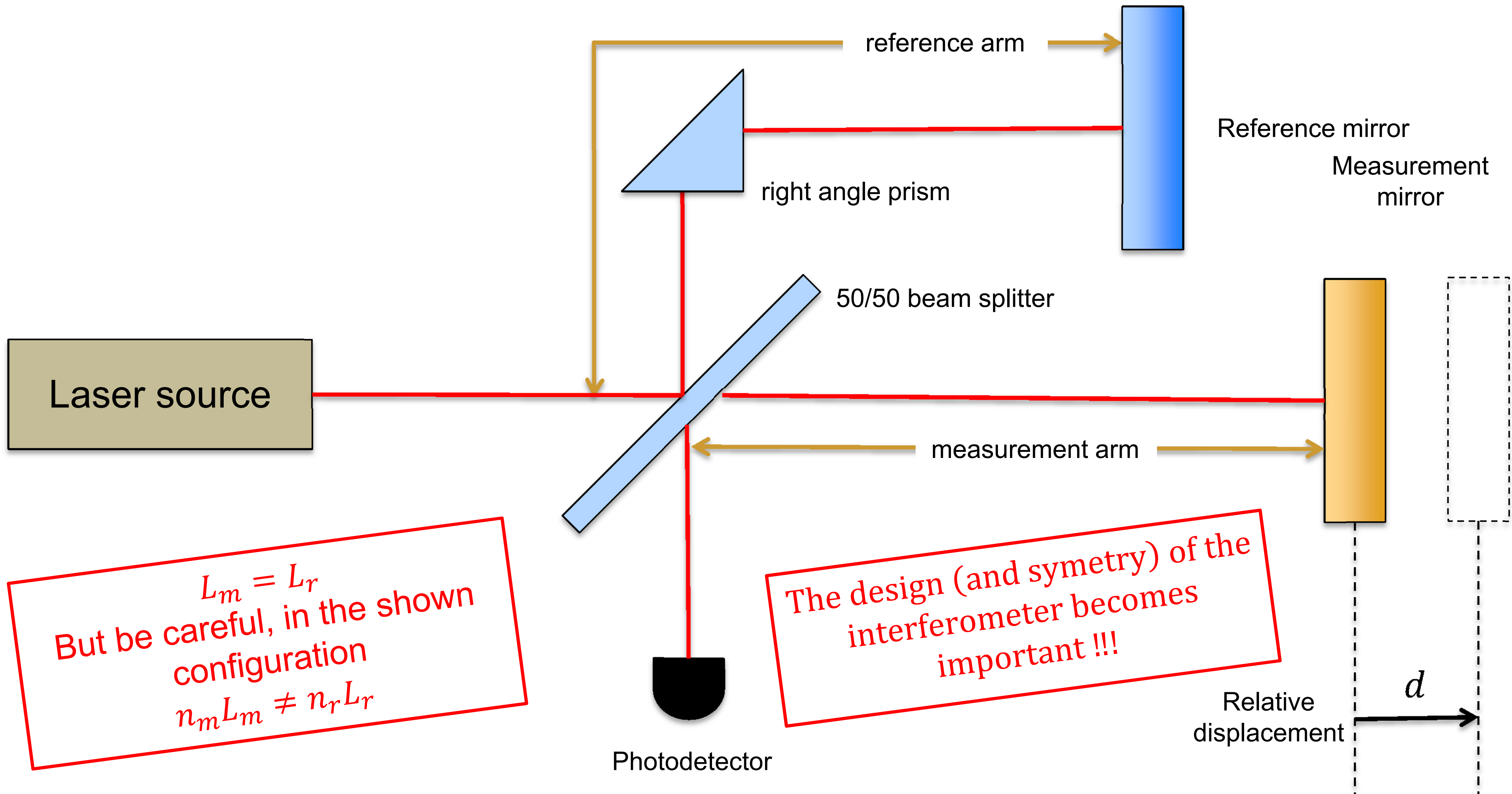
If $L_m = L_r$

then $L_{DP} = 0$

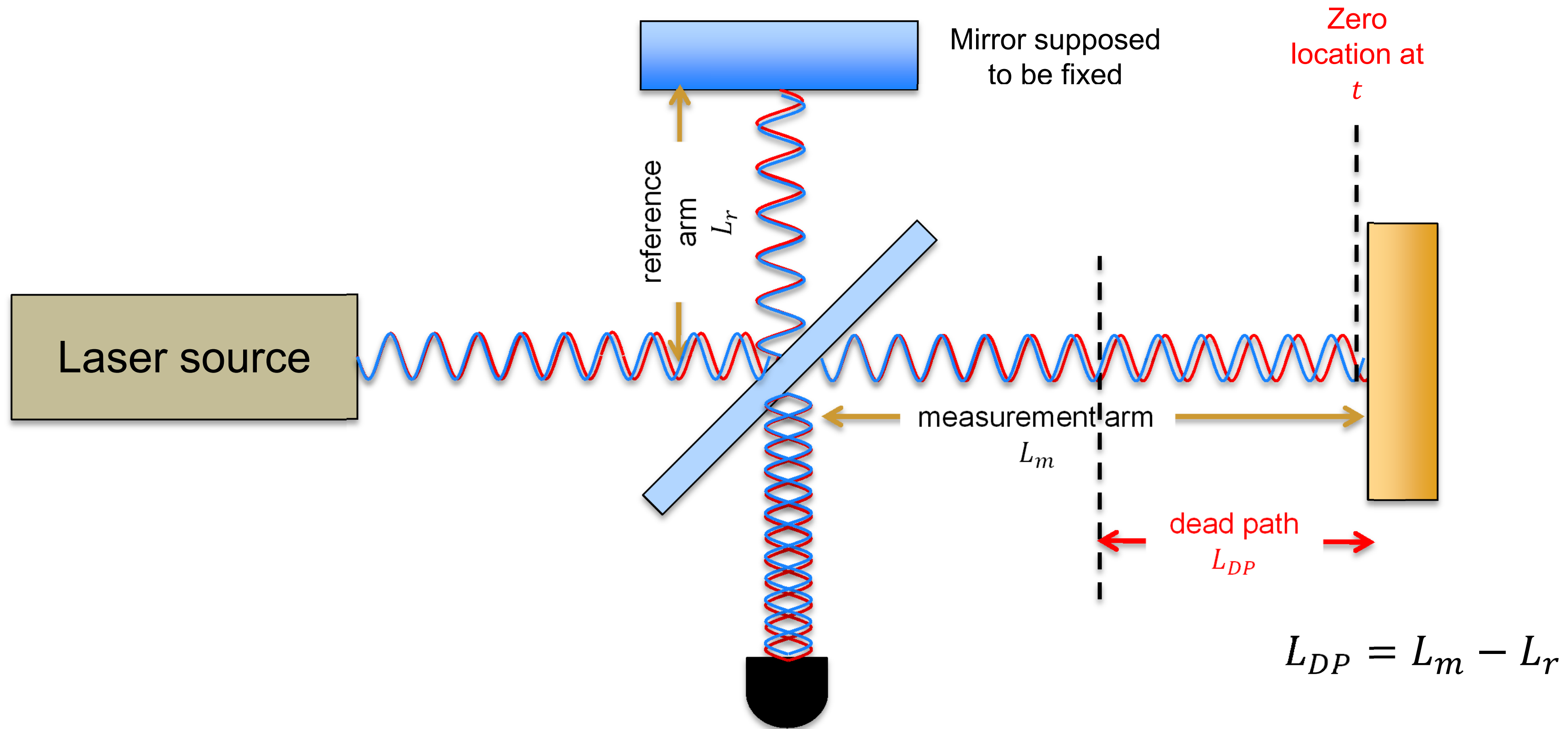
REDUCTION OF DEAD PATH ERROR

One solution consists in using a differential interferometer where the two mirrors can be placed by the user in the configuration he wants:

⇒ Possibility to balance reference and measurement path length



CORRECTION OF DEAD PATH ERROR

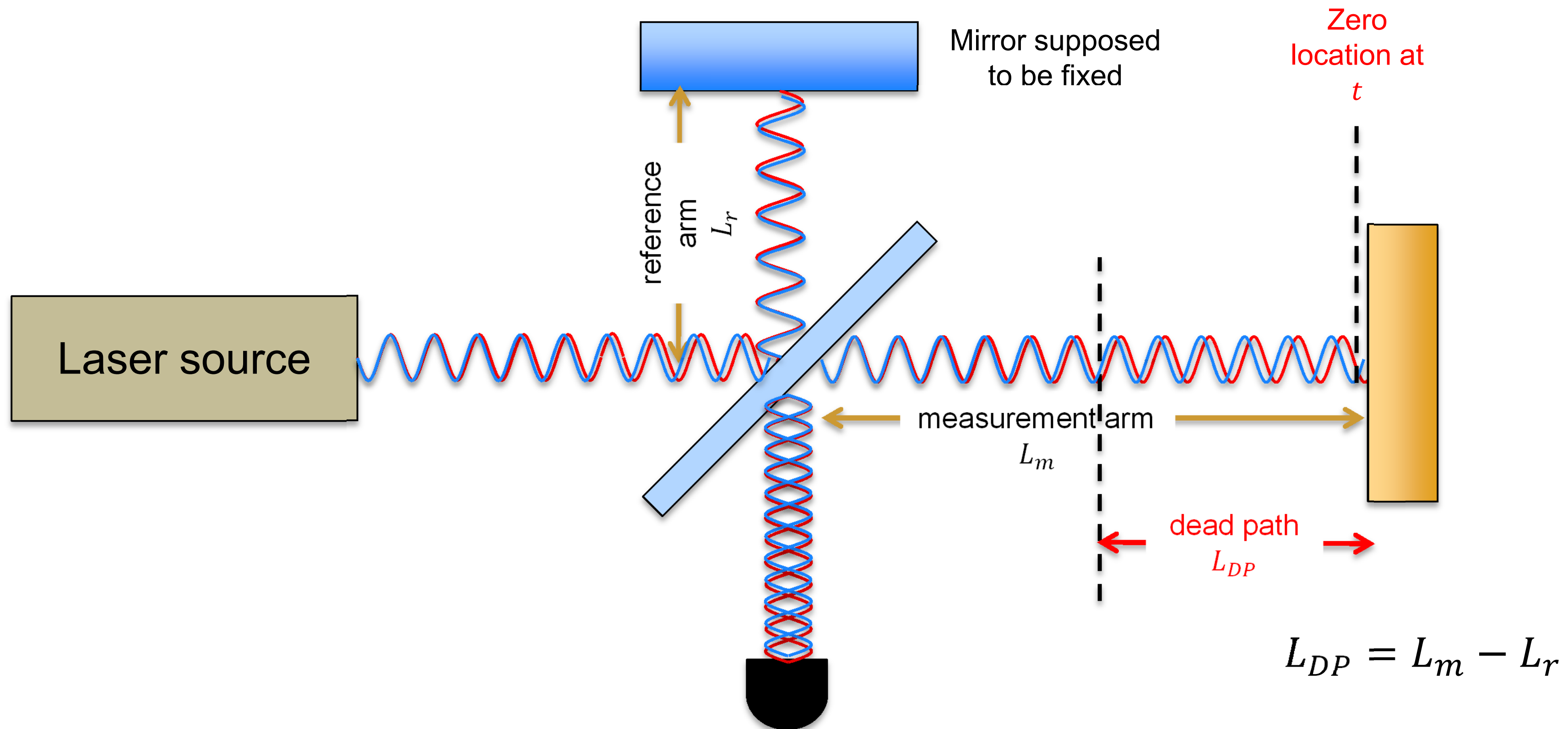


$$L_{DP} = L_m - L_r$$

If not possible due to the design of the instrument, then the solution is to compensate by applying a corrective term to the measurement:

$$d = \frac{k\lambda_v}{4n2^i} - \frac{\Delta n}{n} L_{DP}$$

CORRECTION OF DEAD PATH ERROR

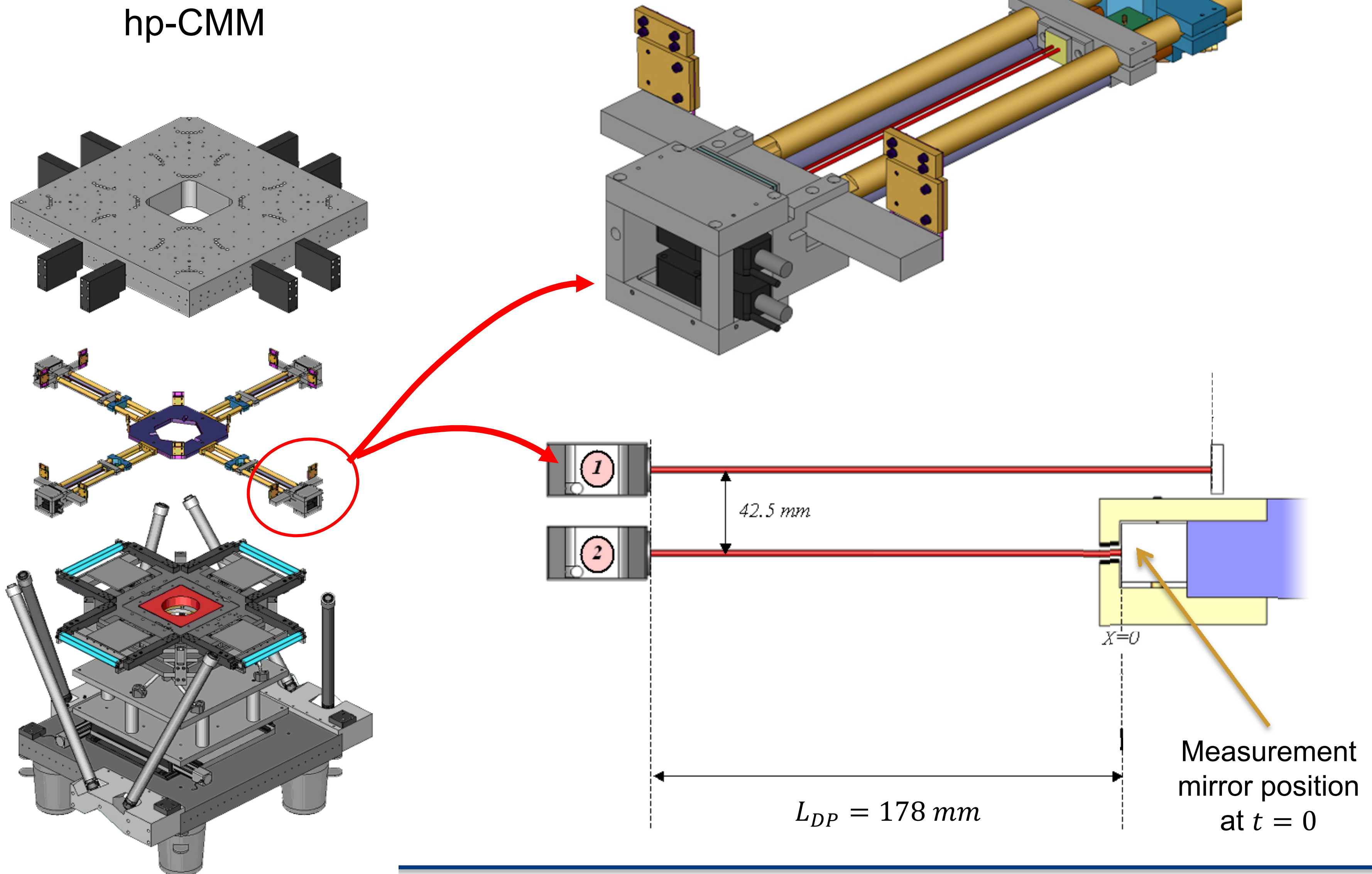


$$L_{DP} = L_m - L_r$$

Uncertainty of the correction:

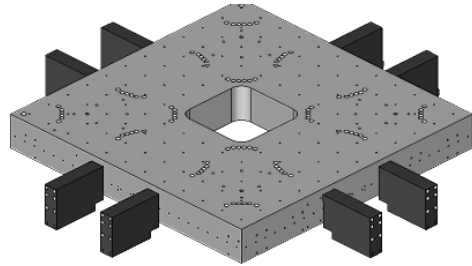
$$u_{cor_{DP}} = \sqrt{\left[u_{L_{DP}}\left(\frac{\Delta n}{n}\right)\right]^2 + \left[L_{DP}\left(\frac{u_{\Delta n}}{n}\right)\right]^2}$$

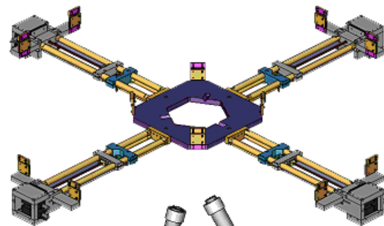
CORRECTION OF DEAD PATH ERROR

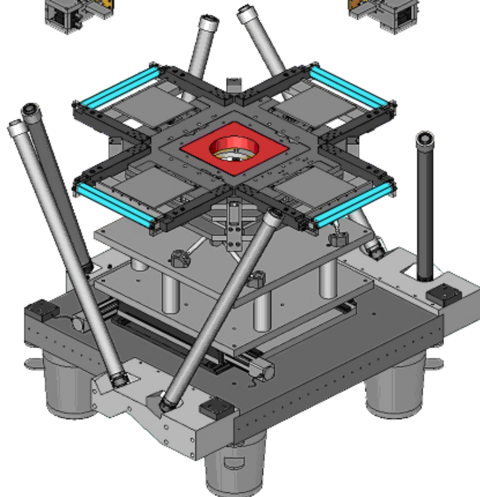


CORRECTION OF DEAD PATH ERROR

hp-CMM







$$L_{DP} = 178\ mm$$
$$u_{L_{DP}} = 5\ mm$$

$$\Delta n = 10^{-7} \text{ with } u_{\Delta n} = 5 \times 10^{-9}$$

(equivalent to
$\Delta T = 0.1\ K$
$or\ \Delta P = 0.4\ hPa$
$or\ \Delta RH = 10\%$)

Without correction, dead path error = 17,8 nm

$$u_{cor_{DP}} = \sqrt{\left[u_{L_{DP}}\left(\frac{\Delta n}{n}\right)\right]^2 + \left[L_{DP}\left(\frac{u_{\Delta n}}{n}\right)\right]^2}$$

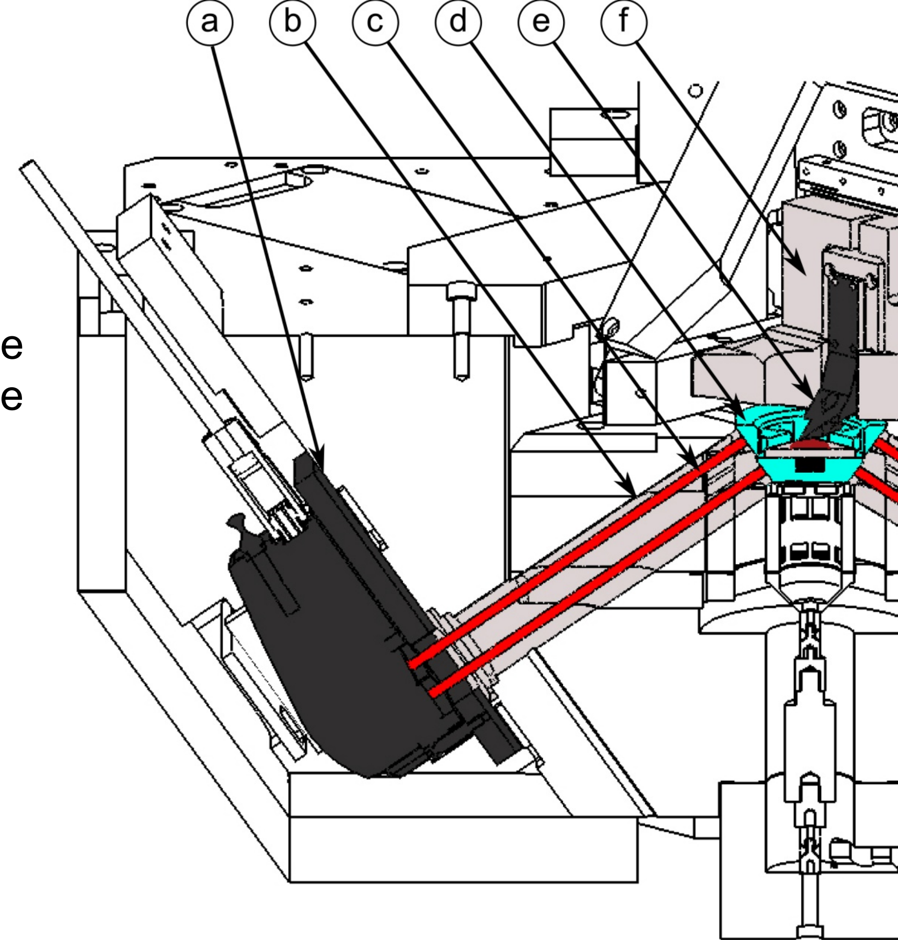
$$u_{cor_{DP}} = 1\ nm$$

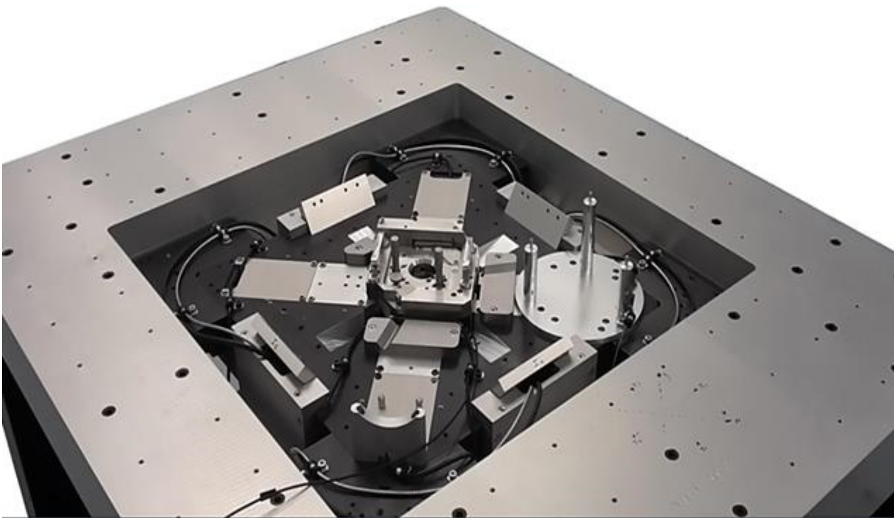
CORRECTION OF DEAD PATH ERROR

mAFM

Use of differential interferometers.

Reference and measurement mirrors are in the same plane => no dead path generated outside the interferometer head.





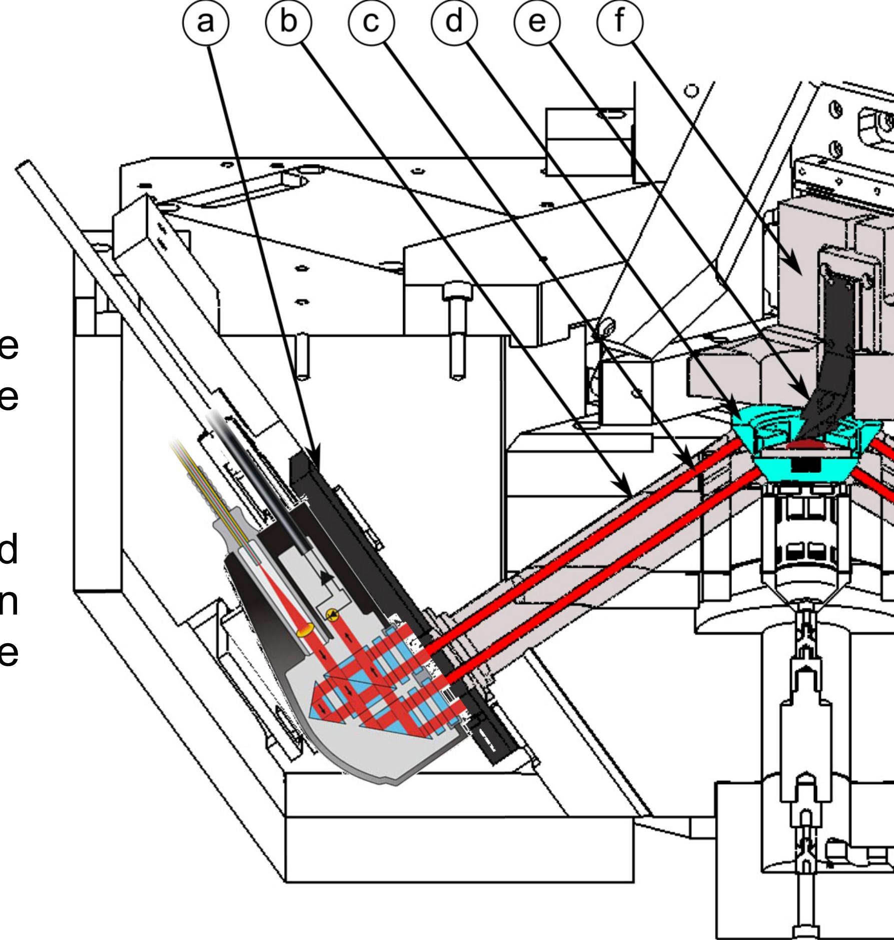
CORRECTION OF DEAD PATH ERROR

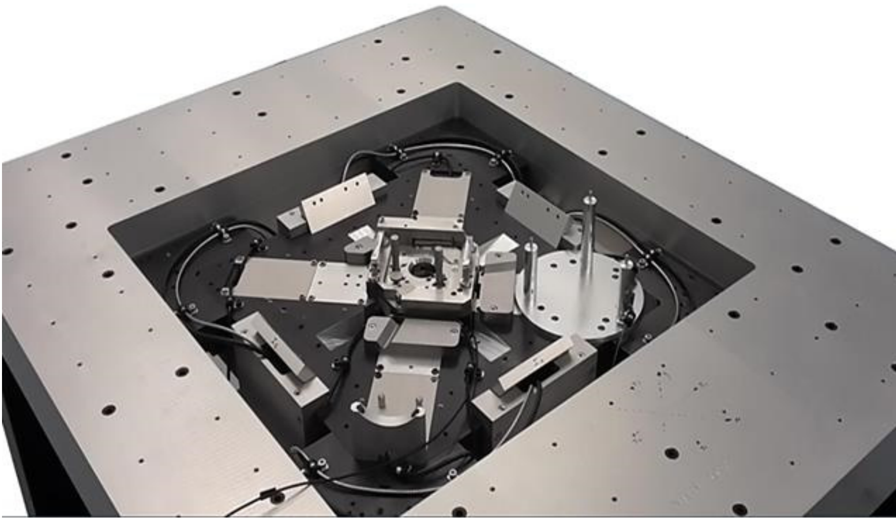
mAFM

Use of differential interferometers.

Reference and measurement mirrors are in the same plane => no dead path generated outside the interferometer head.

But unfortunately, instrument still suffers from dead path error because of the unbalance between reference and measurement paths inside the interferometer head.





CORRECTION OF DEAD PATH ERROR

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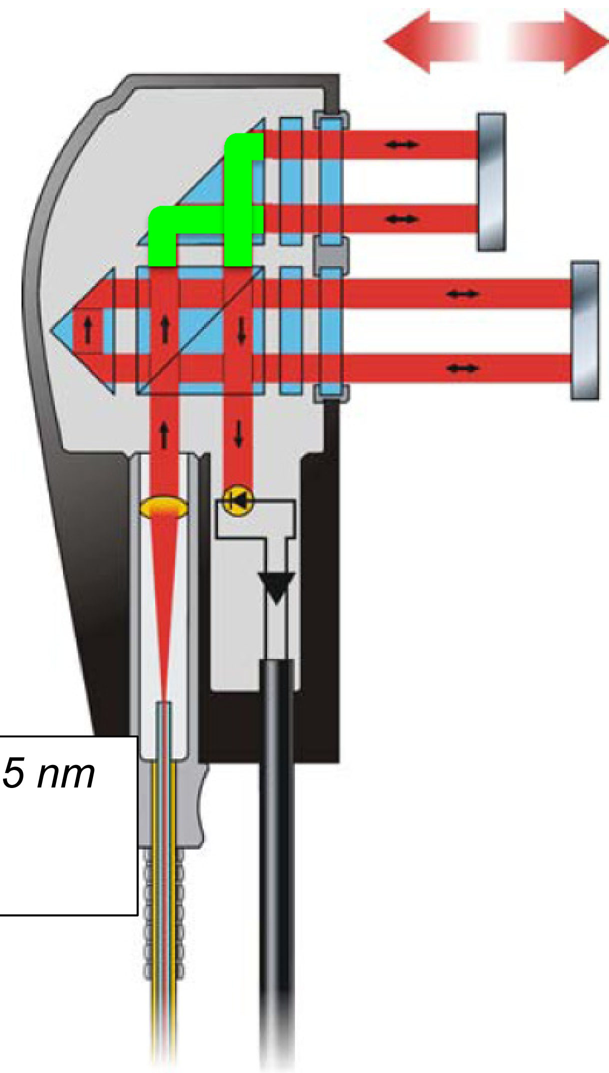
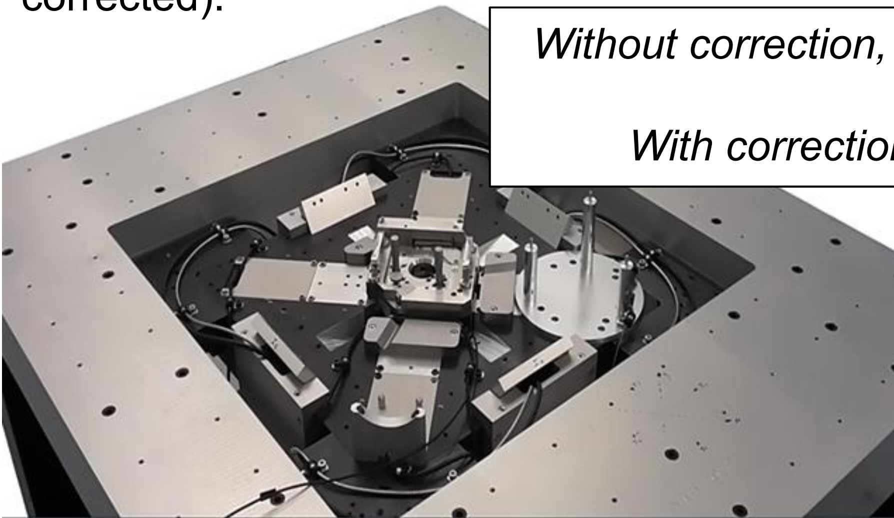
But unfortunately, instrument still suffers from dead path error because of the unbalance between reference and measurement paths inside the interferometer head (estimated to 15 ± 1 mm and corrected).



Differential interferometer detector unit

Without correction, dead path error = 1,5 nm

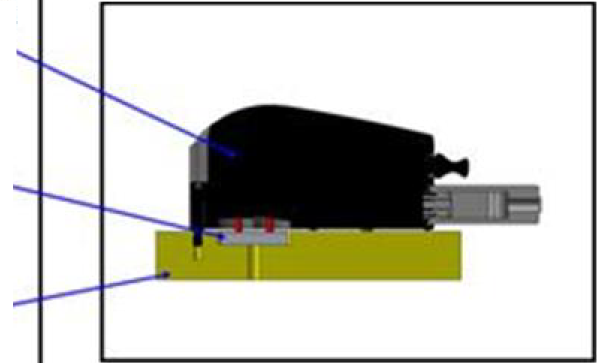
With correction $u_{cor_{DP}}$ = 0,125 nm



NON-SYMMETRY OF INTERFEROMETER

NON-SYMMETRY OF INTERFEROMETER

⇒ special attention should be given to the design of the interferometer because non symmetry of the reference and measurement paths can be a great source or errors.



Another funny experiment:

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NON-SYMMETRY OF INTERFEROMETER

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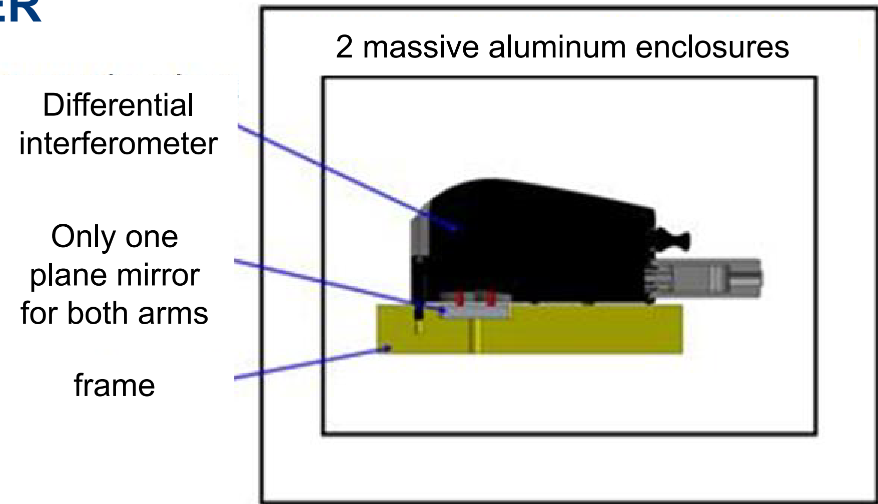

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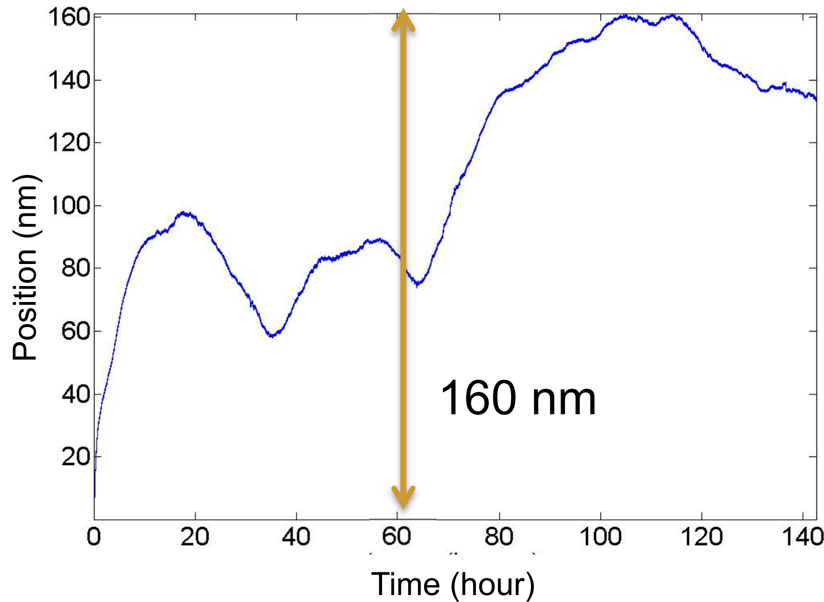

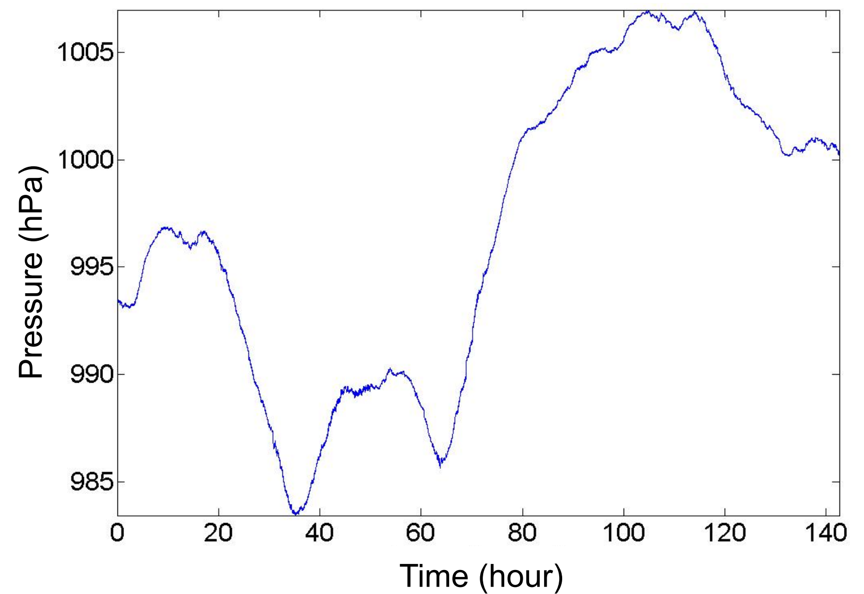

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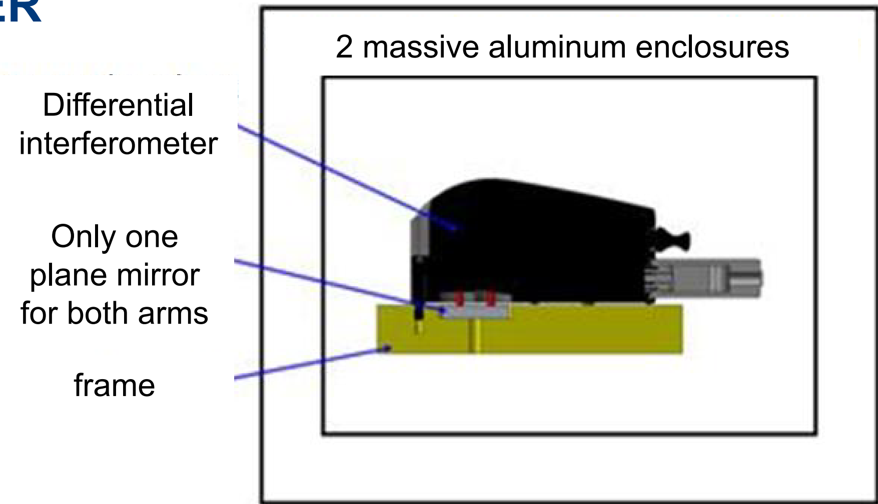

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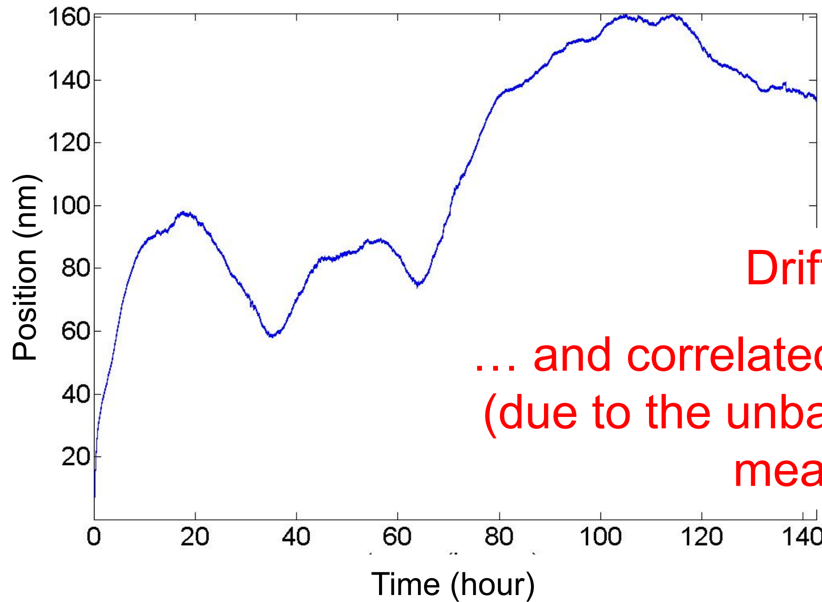

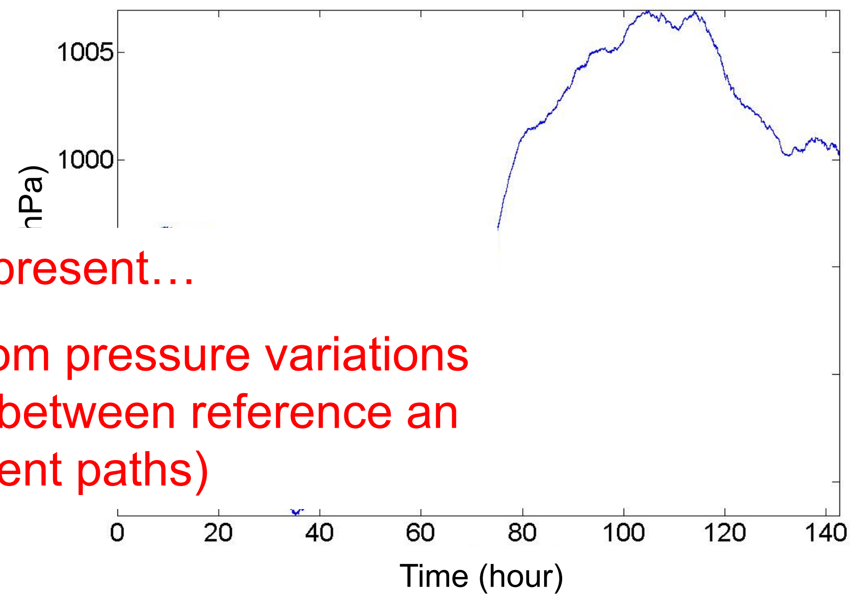

Drifts are present…

… and correlated to room pressure variations (due to the unbalance between reference an measurement paths)

NON-SYMMETRY OF INTERFEROMETER

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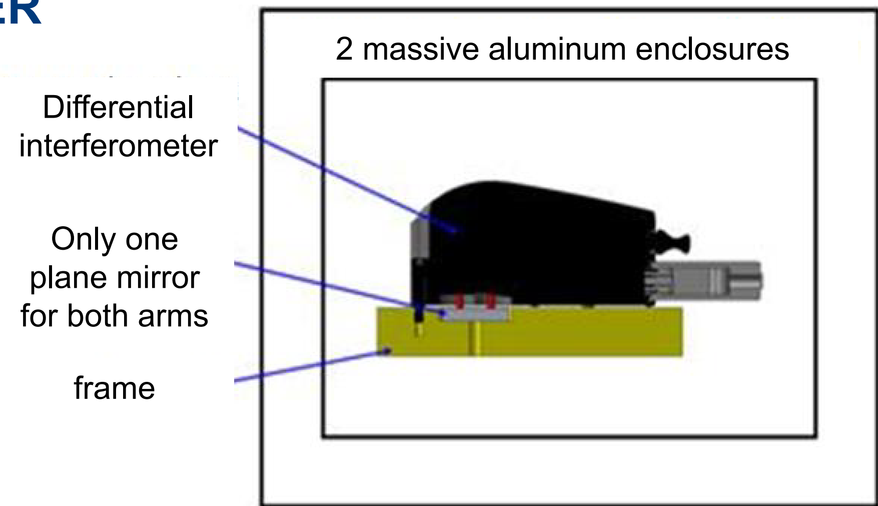
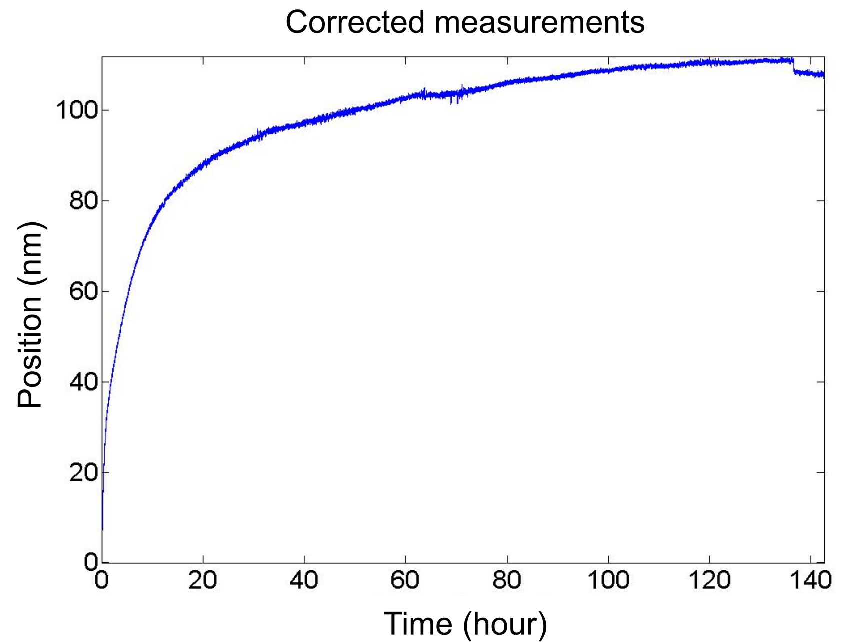


Another funny experiment:

After correction of the dead path inside the interferometer head, the measurement exhibit a thermal drift !!!

⇒ differential thermal expansion between reference and measurement paths

⇒ can be assimilated to a drift of the interferometer in *nm/hour*

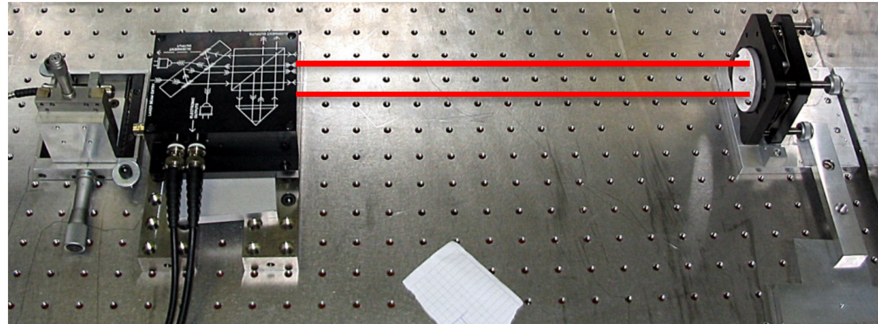


AIR TURBULENCES

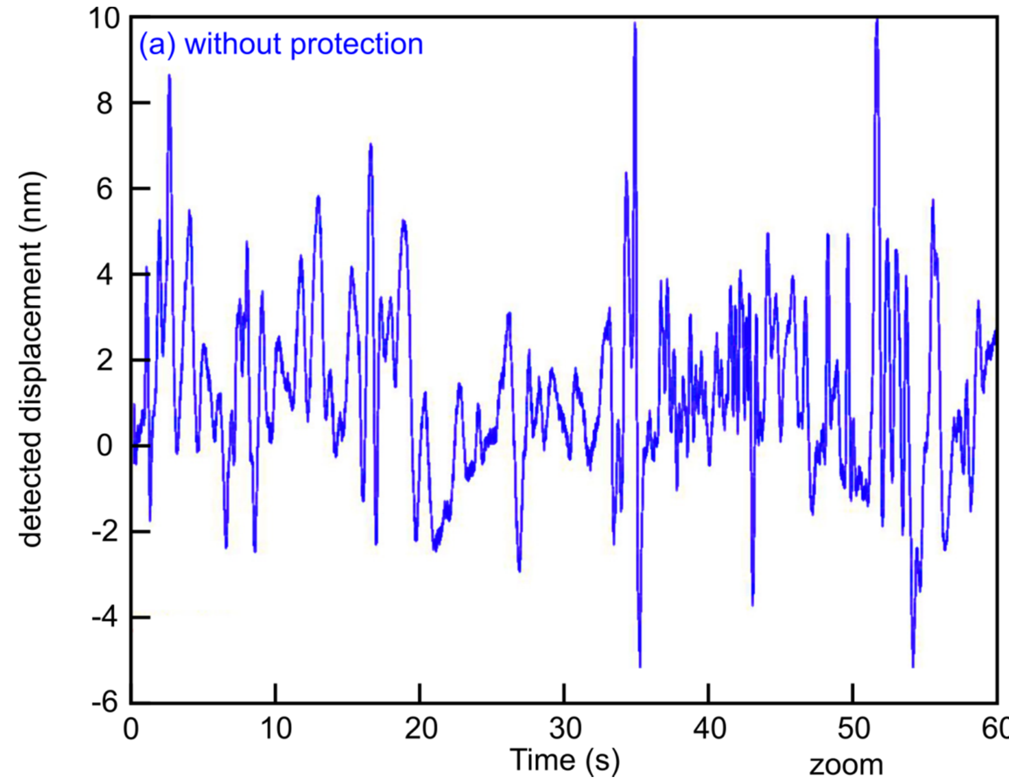
AIR TURBULENCES

Air turbulences can have a strong impact on the measurement and the stability of the positioning if no precaution are taken !

Realized with a differential interferometer and only one mirror



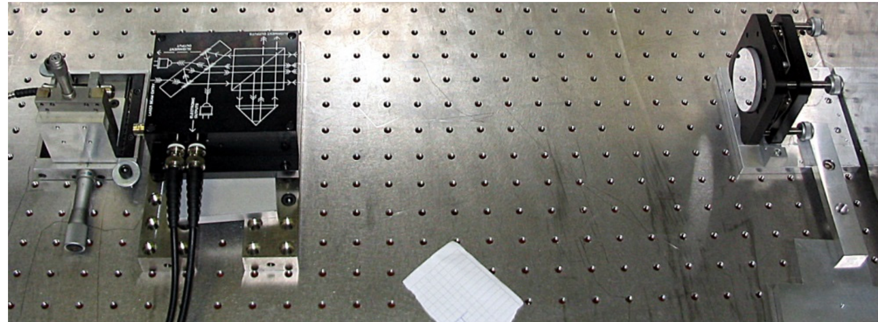
In clean room, no protection



AIR TURBULENCES

Air turbulences can have a strong impact on the measurement and the stability of the positioning if no precaution are taken !

Realized with a differential interferometer and only one mirror (only sensitive to refractive index of air variation between the two arms



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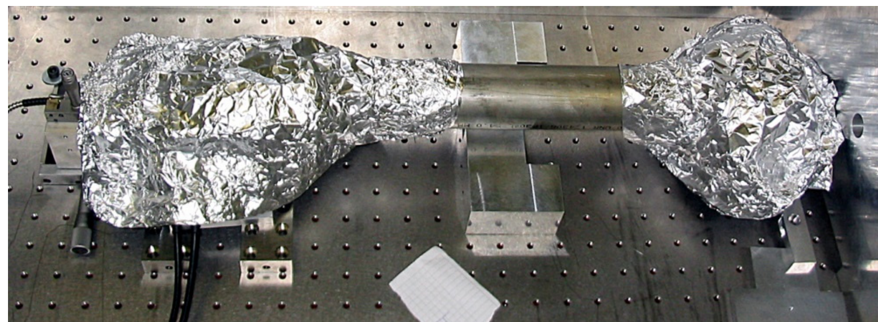
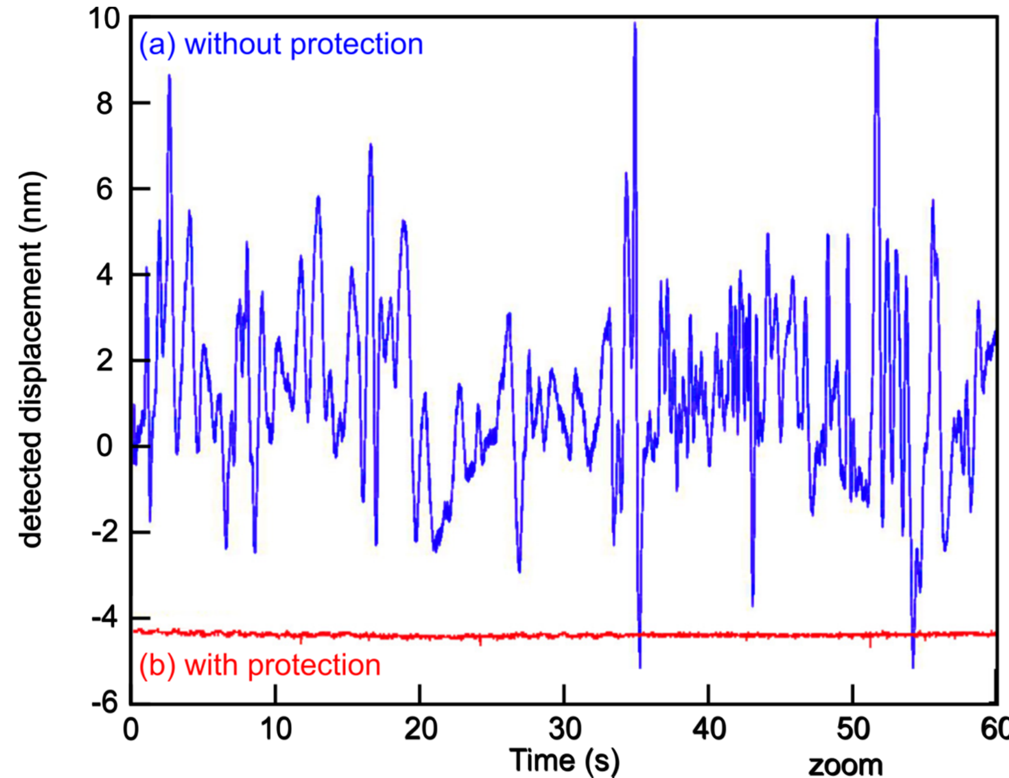


In clean room, with a metallic tube and aluminum foils to protect the beams. The system is enclosed in an aluminum box



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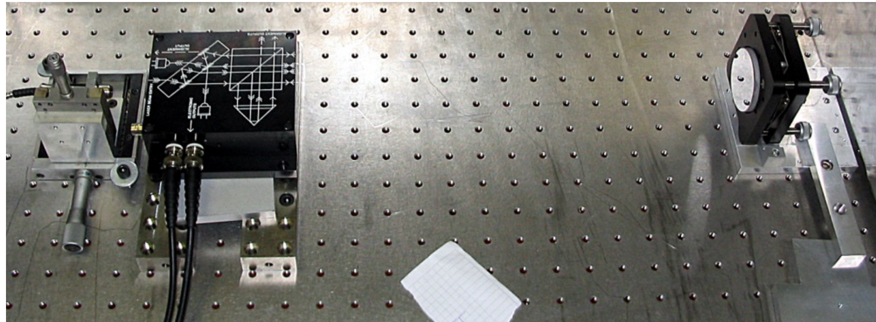
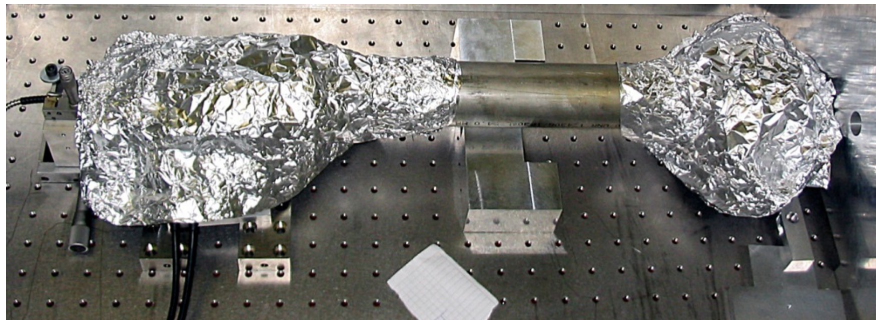
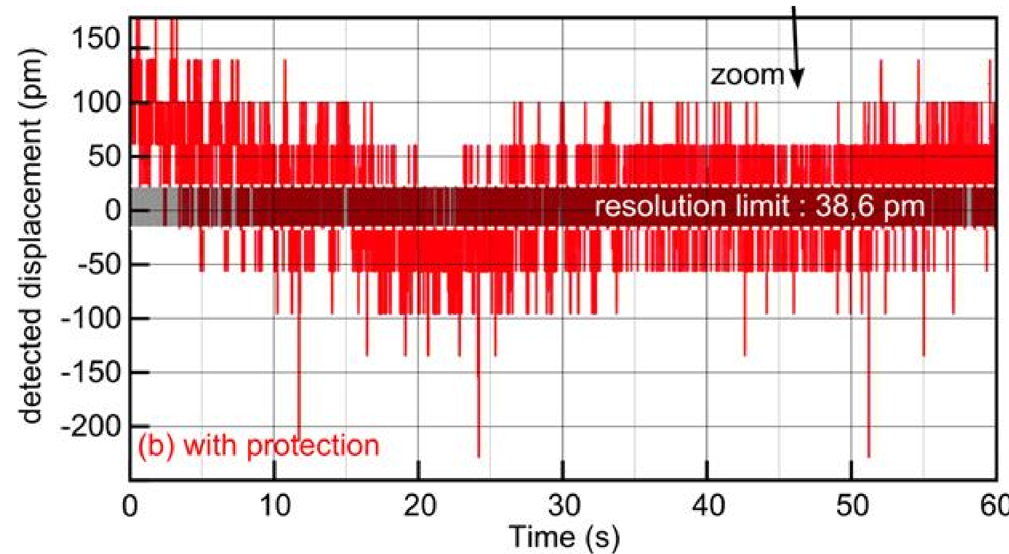


In clean room, no protection



In clean room, with a metallic tube and aluminum foils to protect the beams, The system is enclosed in an aluminum box

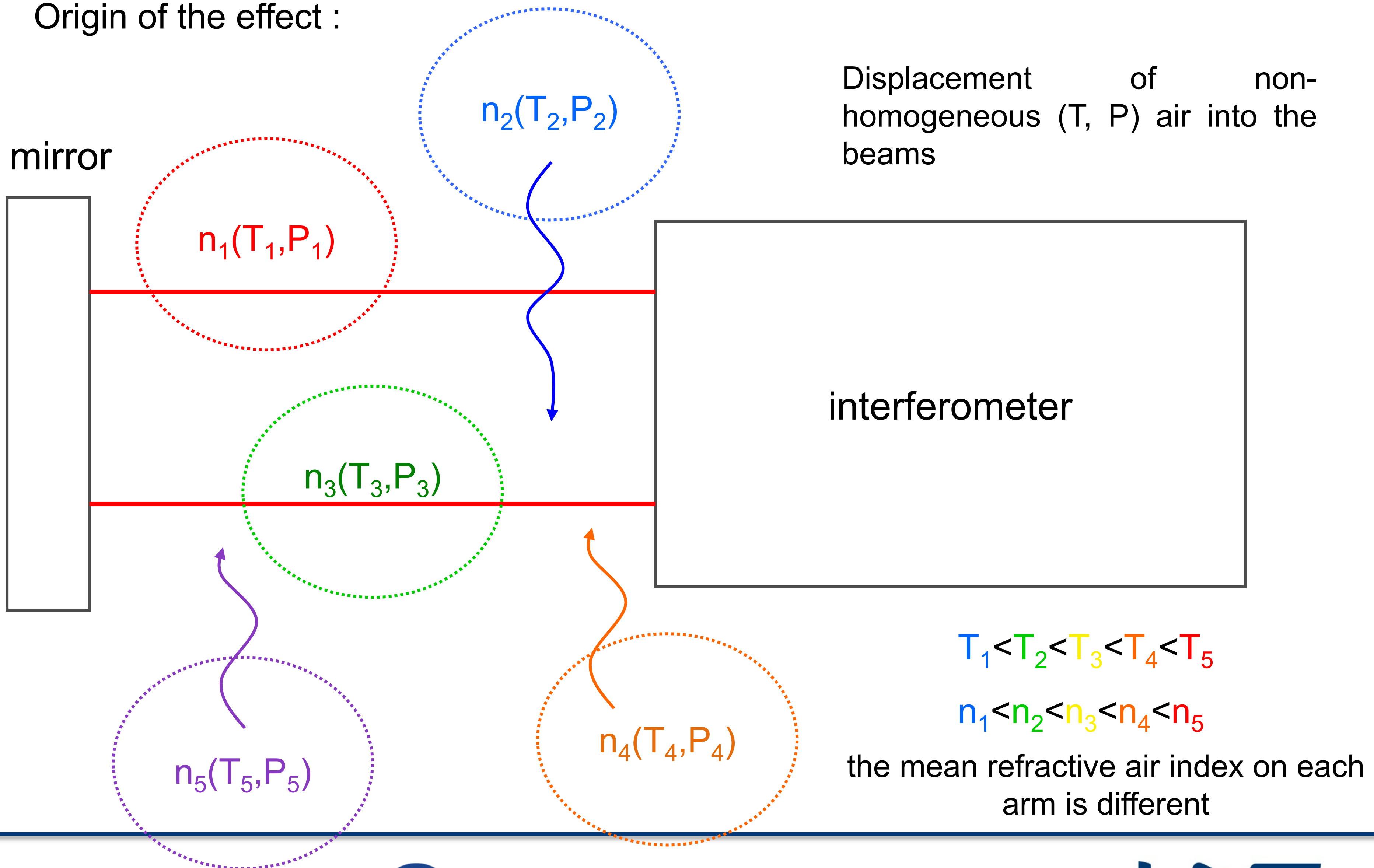
⇒ Reduction of the turbulences effect from 15 nm to only 0.1 nm



⇒ Protection of the beams can improves measurement quality

AIR TURBULENCES

Origin of the effect :



Displacement of non-homogeneous (T, P) air into the beams

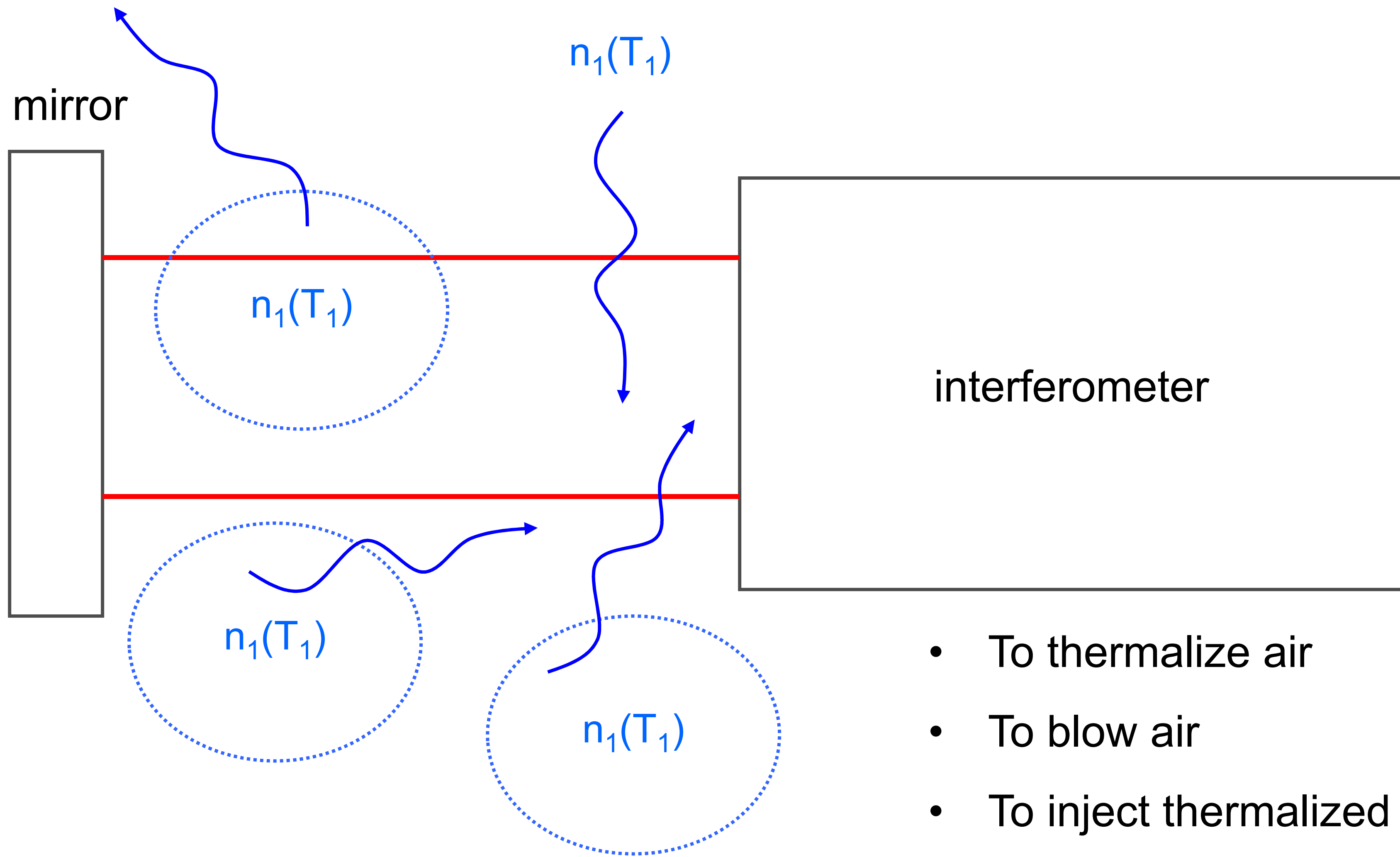
$T_1<T_2<T_3<T_4<T_5$

$n_1<n_2<n_3<n_4<n_5$

the mean refractive air index on each arm is different

AIR TURBULENCES

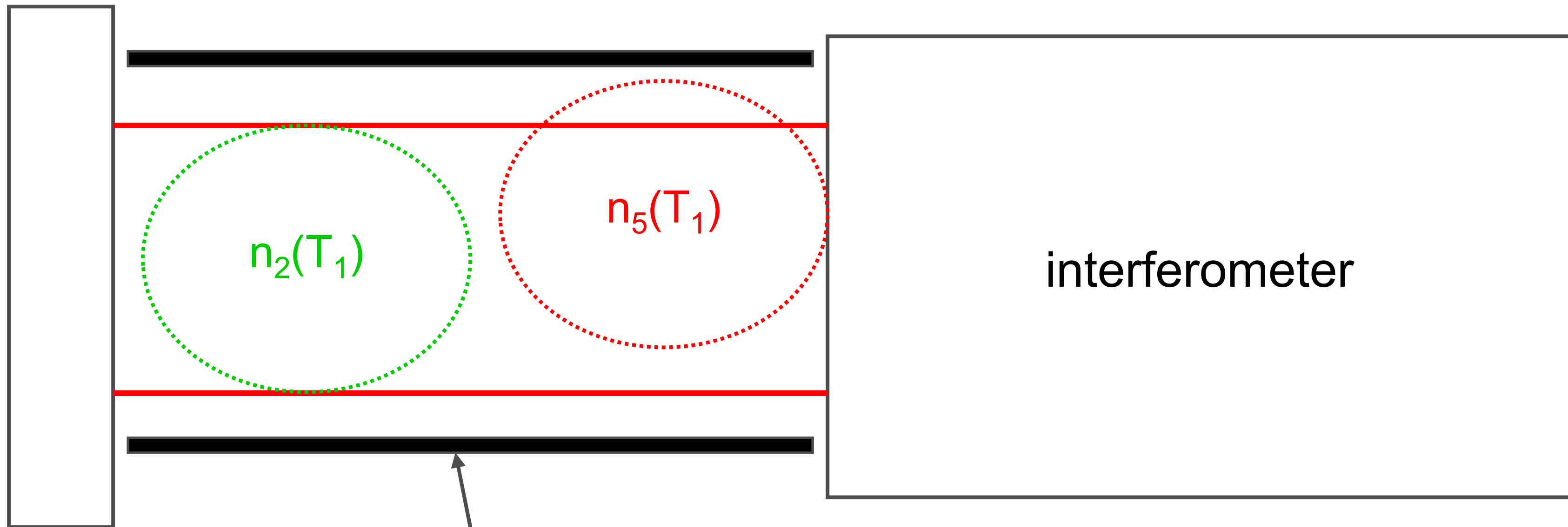
First solution :



- To thermalize air
- To blow air
- To inject thermalized air

AIR TURBULENCES

Second solution :

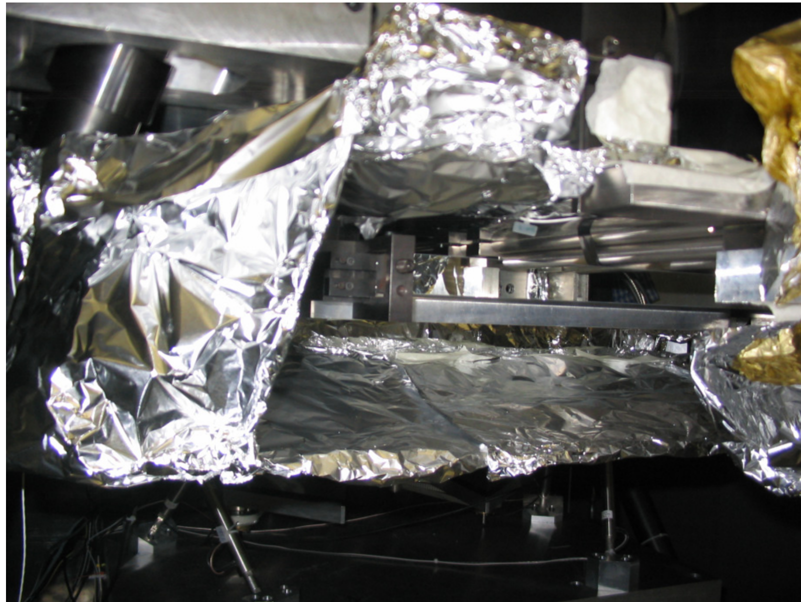


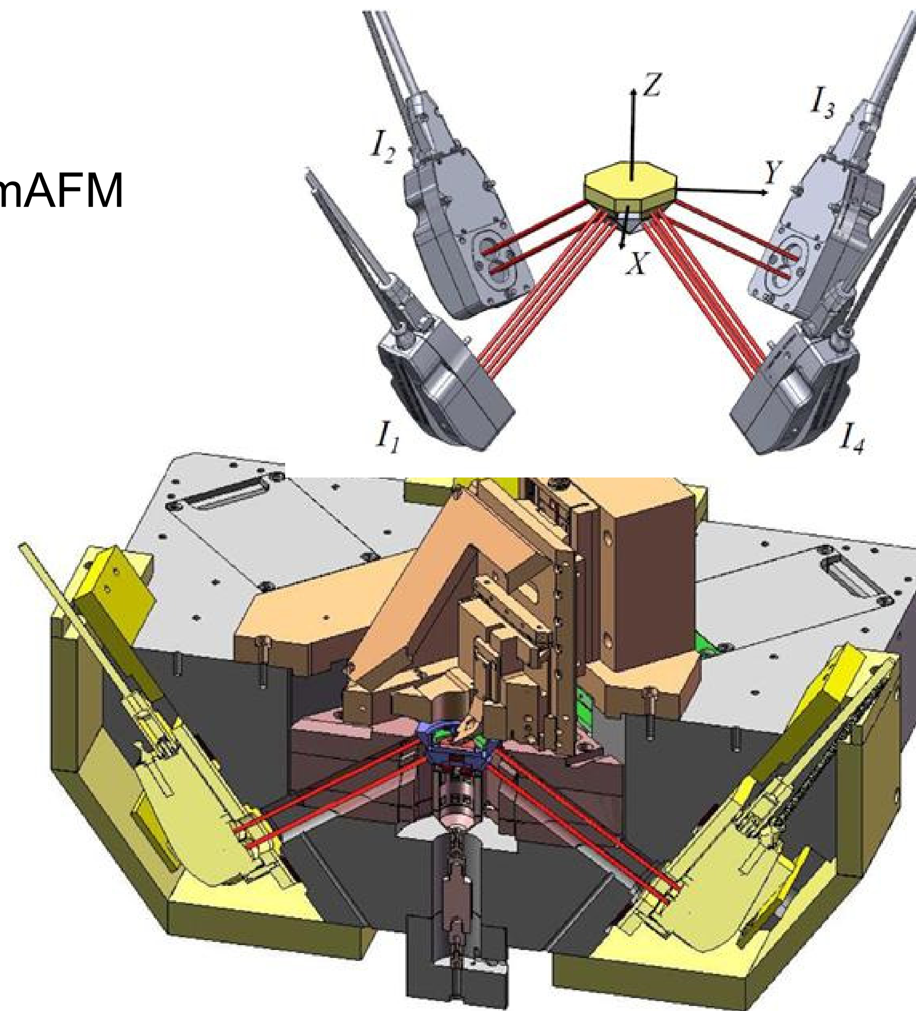
To protect the beam in order to confine and avoid air displacement

AIR TURBULENCES

hp-CMM

mAFM





Machine in clean room, enclosed in a aluminum hut, beam protected with aluminum foils.

Influence of air turbulence reduced from 50 nm peak to peak to only 5 nm

Protection directly integrated inside the instrument frame (a 85 kg monoblock of aluminum used to thermalize the beam cavity)

Influence of air turbulence: negligible

NON LINEARITY

NONLINEARITY

- Nonlinearity is the nonlinear and periodic response of the interferometer on very short range of displacement.

- Period = a fraction of the wavelength ($\frac{\lambda}{2}$, $\frac{\lambda}{4}$, depending on the interferometer configuration, simple pass, dual pass, …)

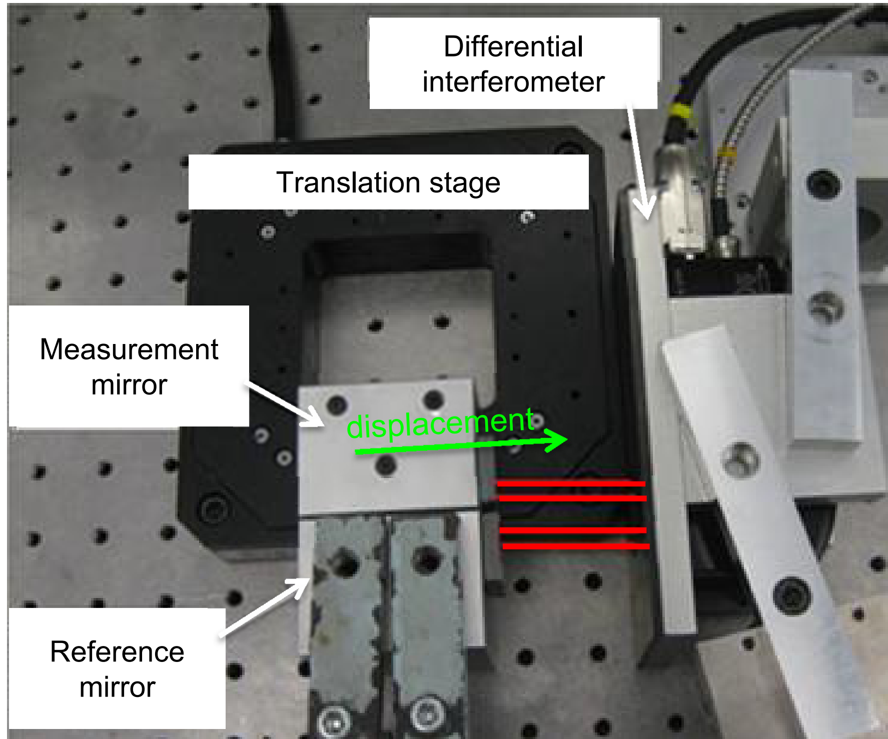
- Due to polarization mixing, non perfect alignment of the optical components, unwanted reflected or scattered beams, bad quality optical components.

- Amplitude of nonlinearity also depends on the setup:

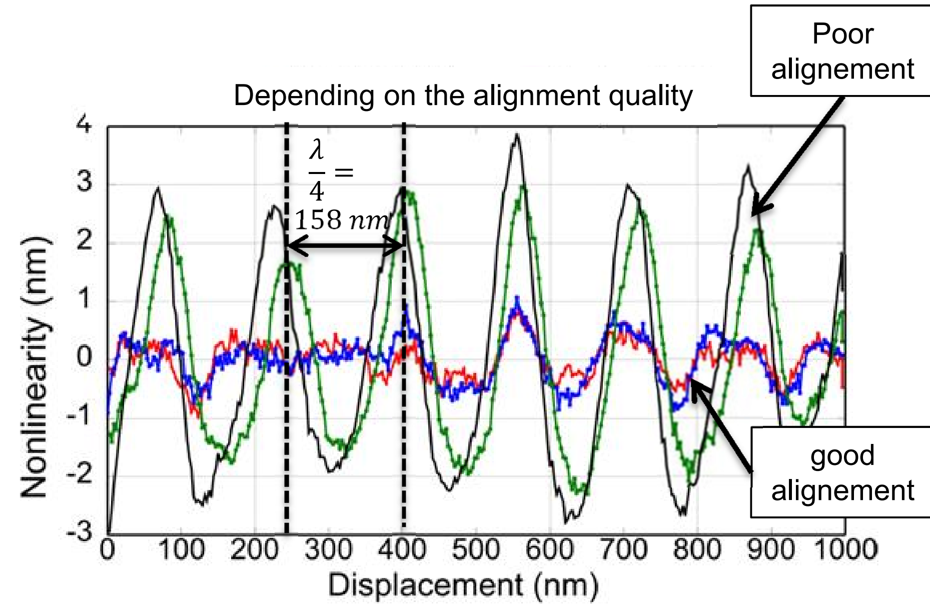
  from a fraction of nanometer < NONLINEARITIES < to few tens of nanometers

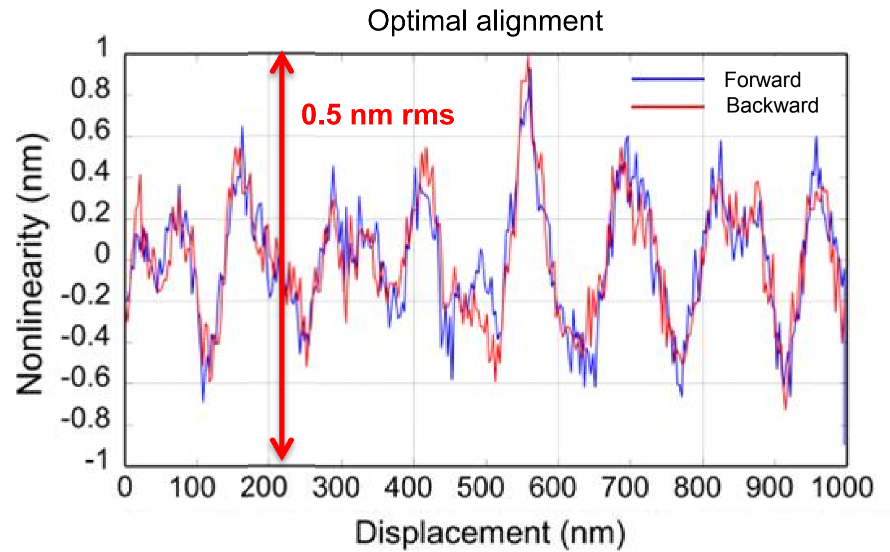
- Nonlinearity can be evaluated by using a simple translation stage to generate the displacement and a capacitive sensor to compare with interferometer.

NON LINEARITY EVALUATION



Setup used for nonlinearity evaluation





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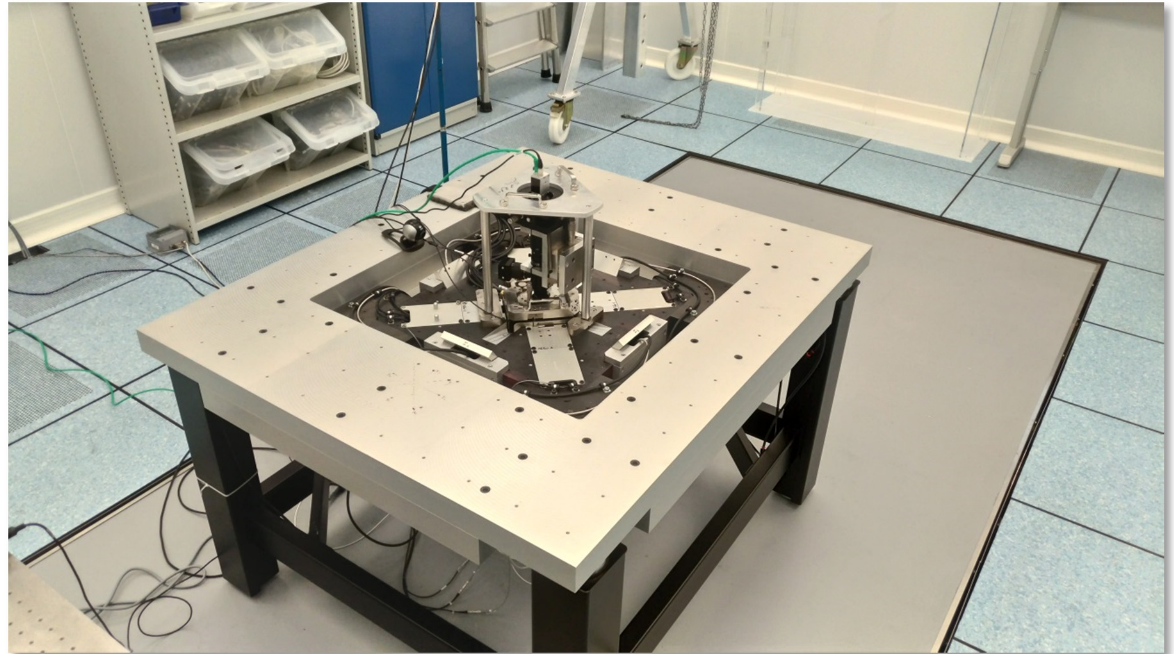
NON LINEARITY EVALUATION

hp-CMM

Nonlinearities evaluated to 1.5 nm rms with Heydemann corrections (in the range of 7 nm without corrections)

mAFM



Nonlinearities evaluated to 0.5 nm rms

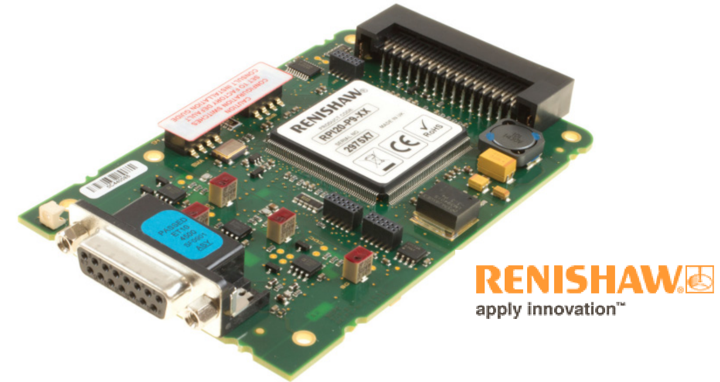
P.L.M. Heydemann. Determination and correction of quadrature fringe measurement errors in interferometers. *Applied optics*, 20(19):3382–384, October 1981.

LIMIT OF RESOLUTION DUE TO ELECTRONIC

ELECTRONIC RESOLUTION LIMIT

Mainly due to the electronic used to interpolate the signal (to subdivide the period of the sinusoidal signal).

Example :



interpolation factor = 12 (signal subdivision 4096-fold)

$$d = \frac{k\lambda_v}{4n2^i}$$

λ_v: vacuum wavelength
n: refractive index of air
d: displacement
k: counting
i: interpolation factor

With :
$\lambda_v = 632{,}8\ nm$
$n = 1$
$k = 1$
$i = 12$

Resolution $= d_{min} = 0{,}038\ nm$

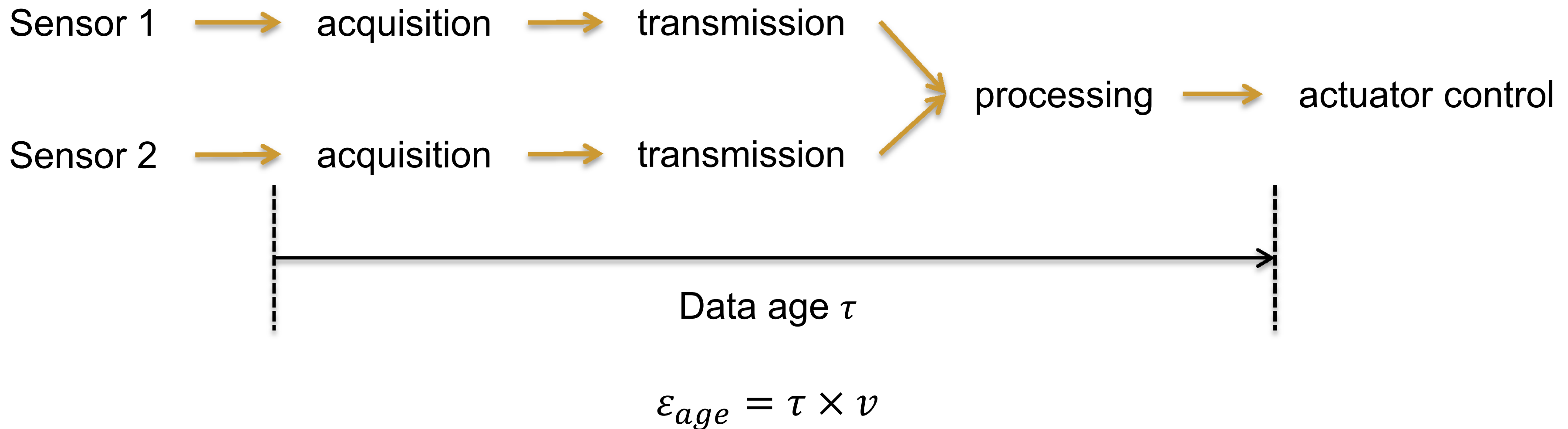
Resolution limit is not a problem

BANDWIDTH, SYNCHRONIZATION AND DATA AGING

BANDWIDTH, SYNCHRONIZATION AND DATA AGING

At high displacement speed :

- care must be taken to the age of data
- measurement board and acquisition system need to be synchronized to reduce error during processing
- controller must have high bandwidth and high processing speed with minimum delay



$$\varepsilon_{age} = \tau \times v$$

For $v = 1\ m/s\ and\ \tau = 10\ ns,\ \varepsilon_{age} = 10\ nm$

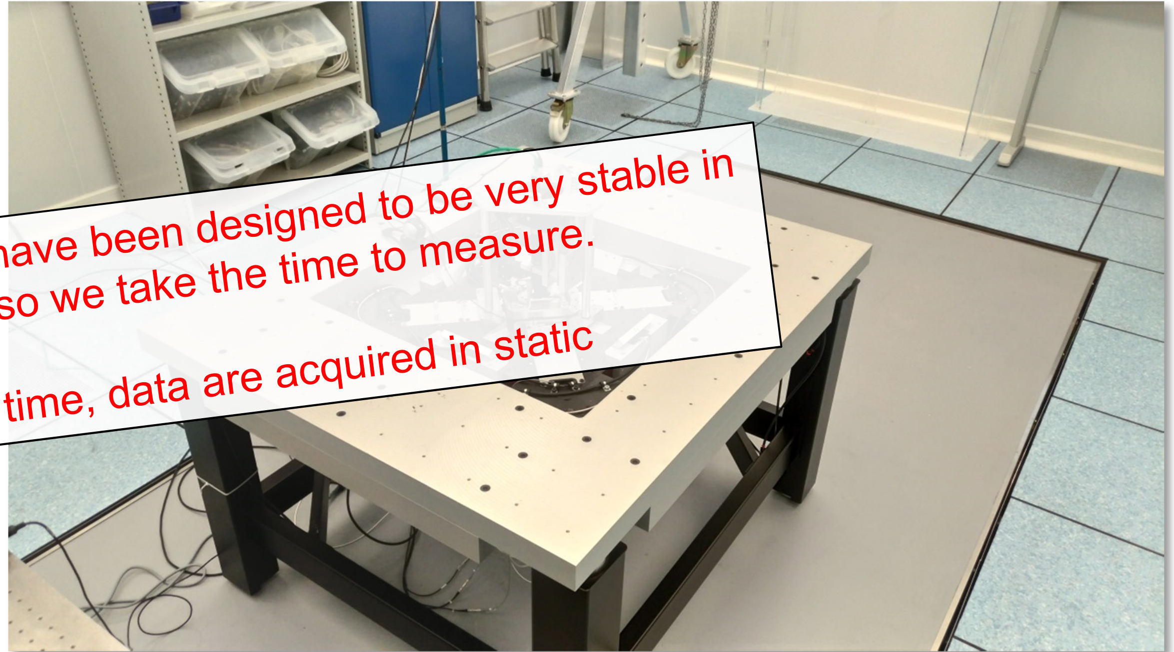
For $v = 1\ mm/s\ and\ \tau = 1\ \mu s,\ \varepsilon_{age} = 1\ nm$

$\Rightarrow$ High speed controller needed for high dynamic operation (MHz range).

DATA AGING

hp-CMM

mAFM



Our instruments have been designed to be very stable in position, so we take the time to measure.

Most of time, data are acquired in static

Real time PC with a processing loop at 1 ms
Typical scanning speed: 1 cm/s

$$\varepsilon_{age} \text{ in dynamic} = 10\ \mu m$$

FPGA : acquisition at 2 MHz
1 µs < processing loop < 0.5 ms
Typical scanning speed: 2 µm/s

$$\varepsilon_{age} \text{ in dynamic} = 1\ nm$$

$$\varepsilon_{age} \text{ in static} = negligible$$

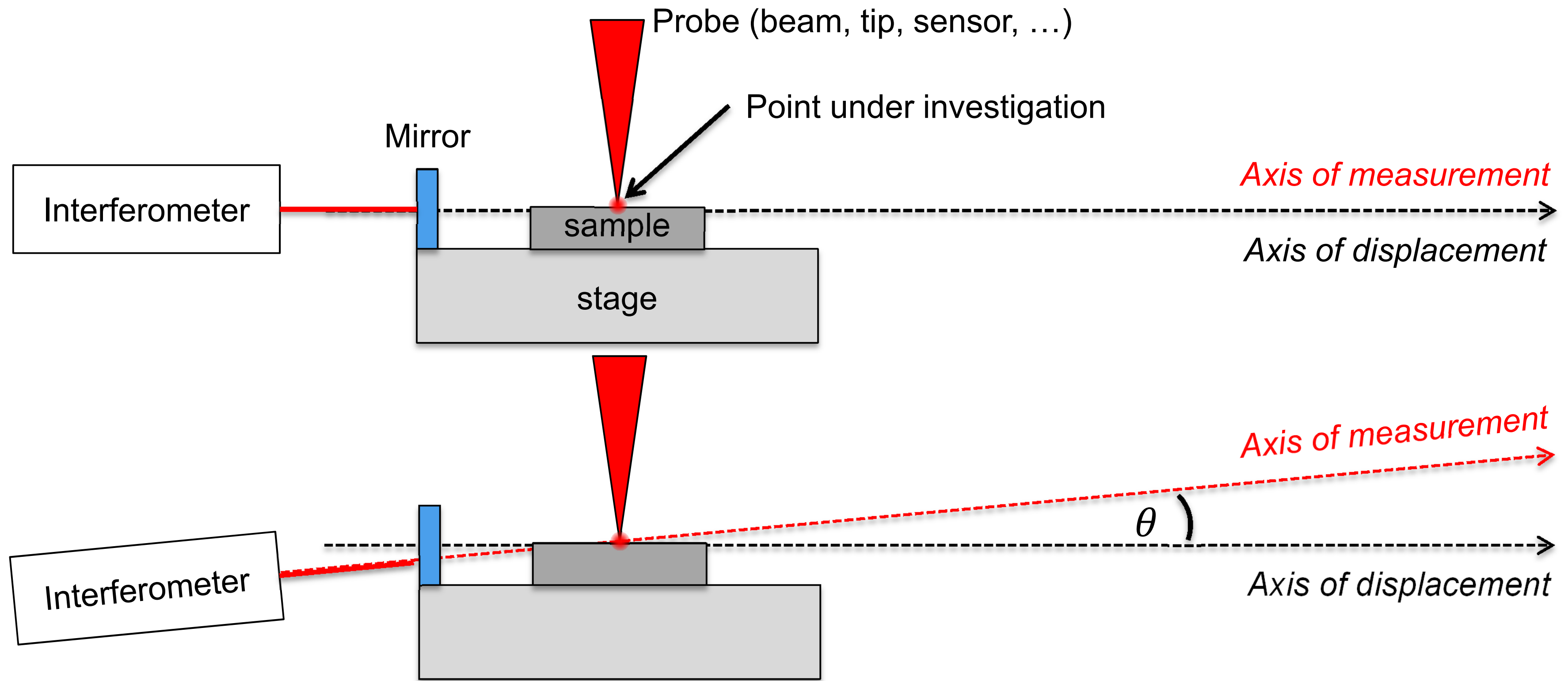
ERRORS DUE TO THE INTEGRATION OF THE INTERFEROMETER ON THE INSTRUMENT

(COMMON TO OTHER SENSORS)

COSINE ERROR

COSINE ERROR

Cosine error results from a misalignment between the interferometer beams and the axis of displacement.



$$\varepsilon_{cos} = d \times (1 - cos\theta)$$

for small angle: $\varepsilon_{cos} = d\,\frac{\theta^2}{2}$

d: displacement

θ: angular misalignement

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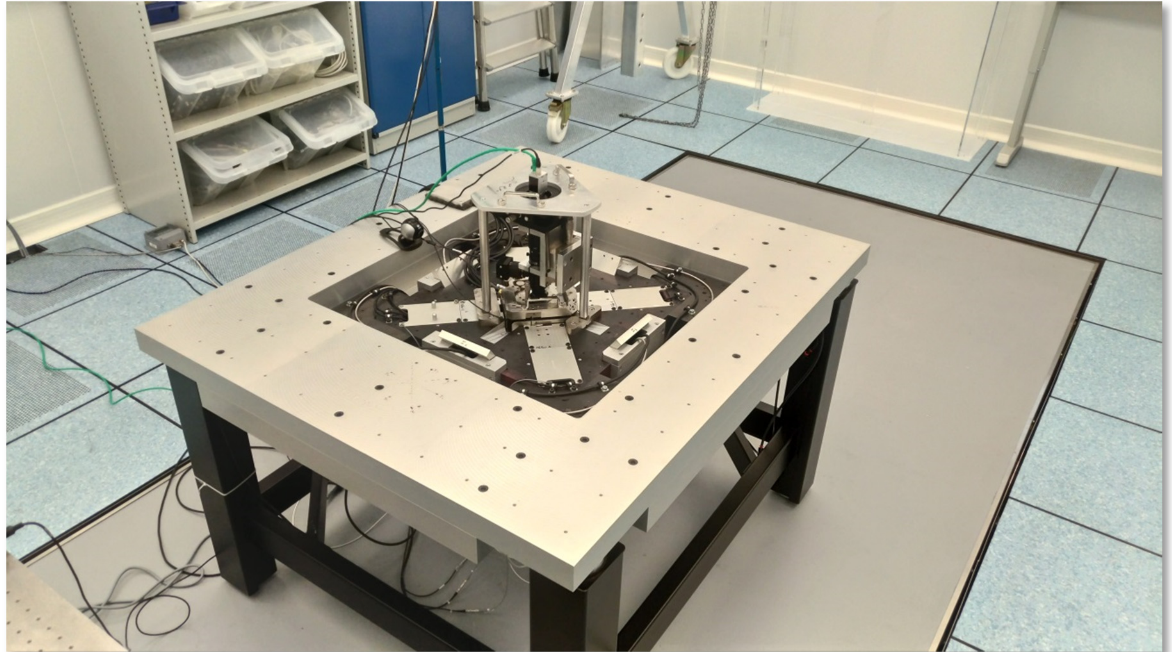
COSINE ERROR

$$\varepsilon_{cos} = d\frac{\theta^2}{2}$$

hp-CMM

mAFM



Misalignment evaluated to less than 125 µrad for a displacement of 300 mm

$$\varepsilon_{cos} = 2.4\ nm$$

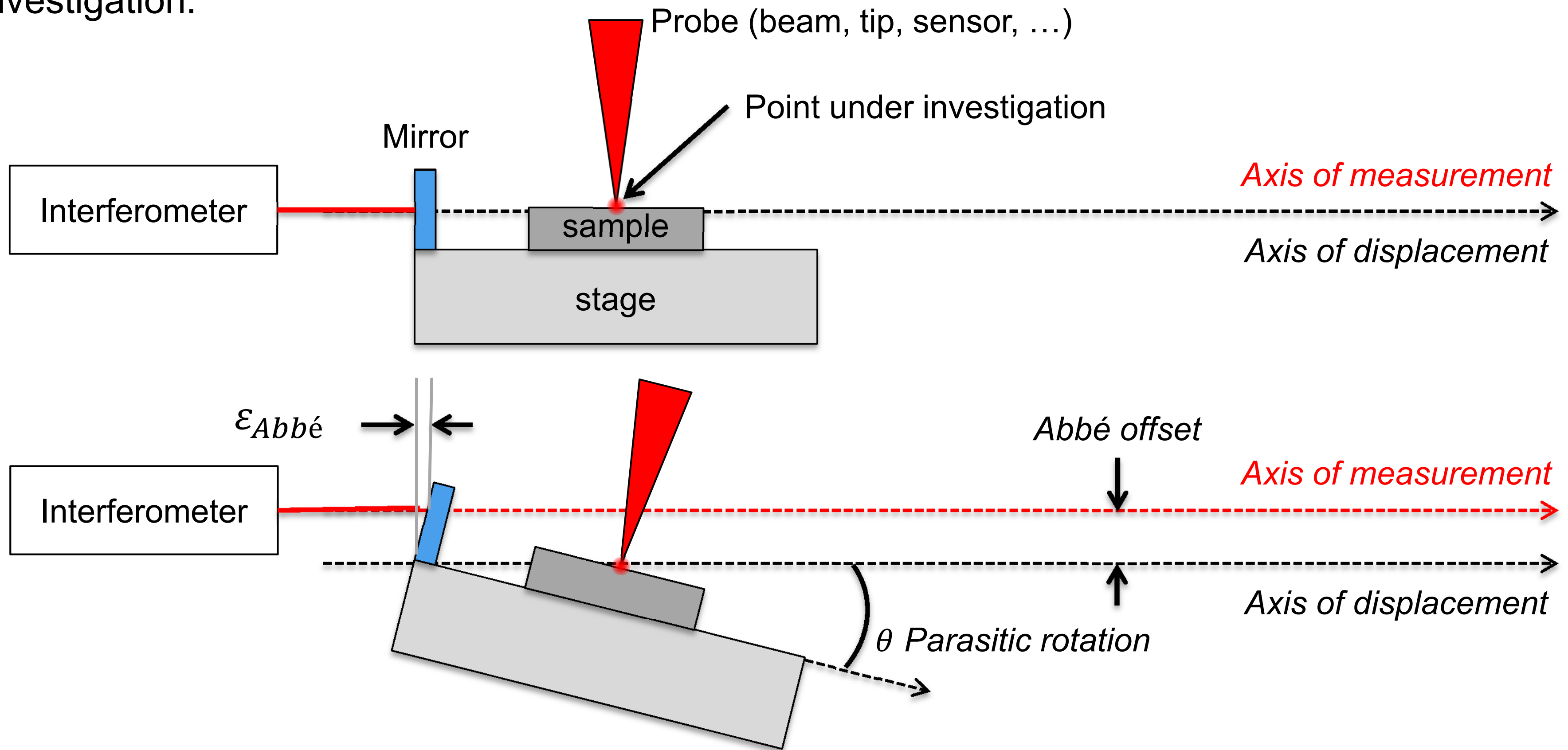
Misalignment evaluated to less than 125 µrad for a displacement of 60 µm

$$\varepsilon_{cos} = 4{,}7 \times 10^{-4}\ nm$$

ABBE ERROR

ABBE ERROR

Abbé error results from a combined action of parasitic rotations during the displacement and the presence of an offset between interferometer axis of measurement and the point under investigation.



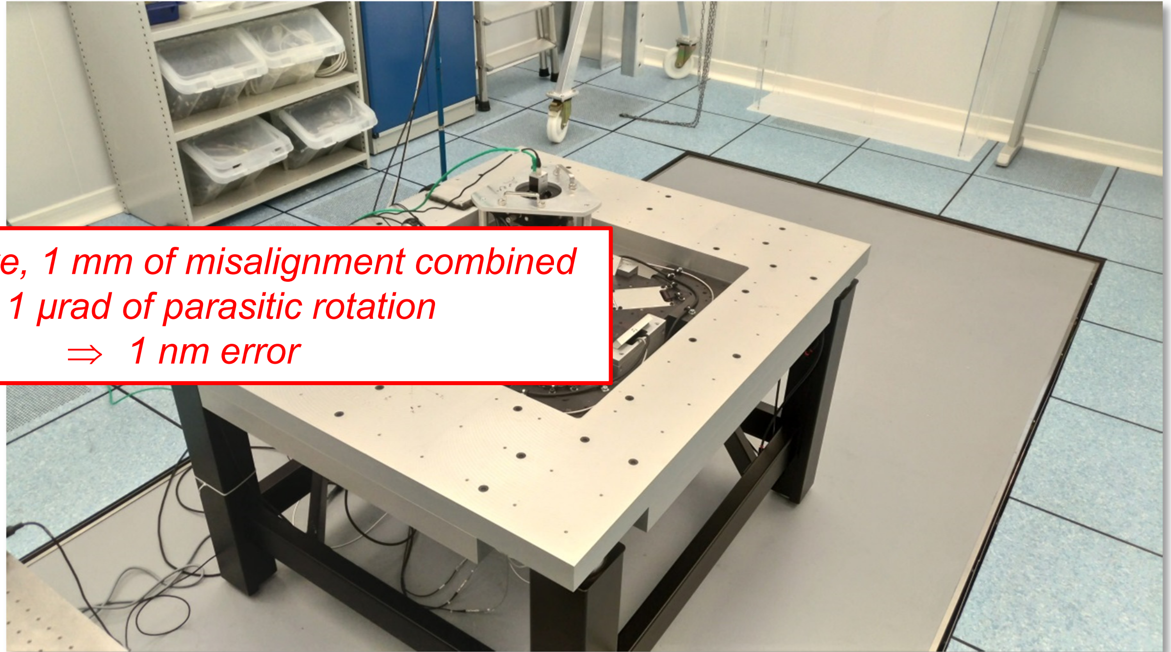
$$\varepsilon_{Abbé} = Abbé_{offset} \times tan\theta$$

ABBE ERROR

$$\varepsilon_{Abbé} = Abbé_{offset} \times tan\theta$$

hp-CMM

mAFM



Very sensitive, 1 mm of misalignment combined with 1 µrad of parasitic rotation
⇒ 1 nm error

Abbe offset = 1 mm
Parasitic rotation = 0.5 µrad

$$\varepsilon_{Abbé} = 0.5\ nm$$

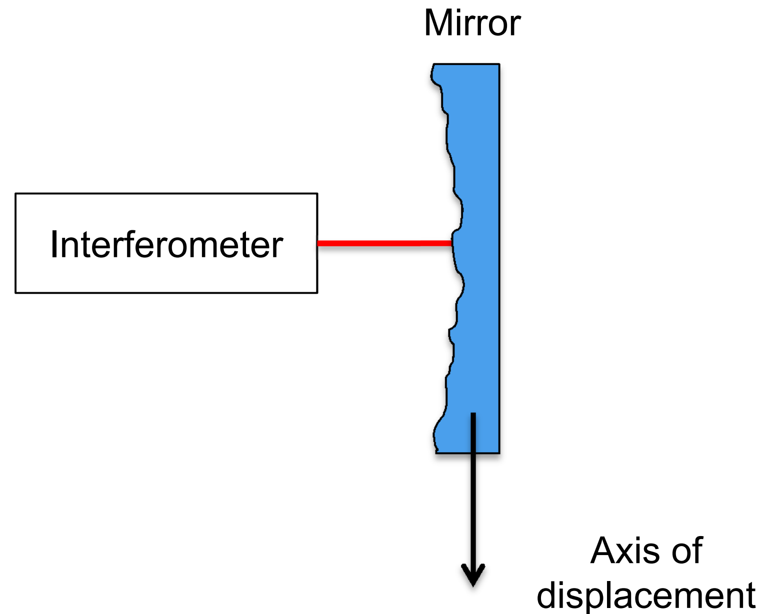
Abbe offset = 1 mm
Parasitic rotation < 10 µrad

$$\varepsilon_{Abbé} = 10\ nm$$

MIRROR CONSIDERATION

FLATNESS AND ROUGHNESS OF THE MIRROR

The flatness, roughness and tilt of the mirror can induces important measurement errors during transverse displacement.



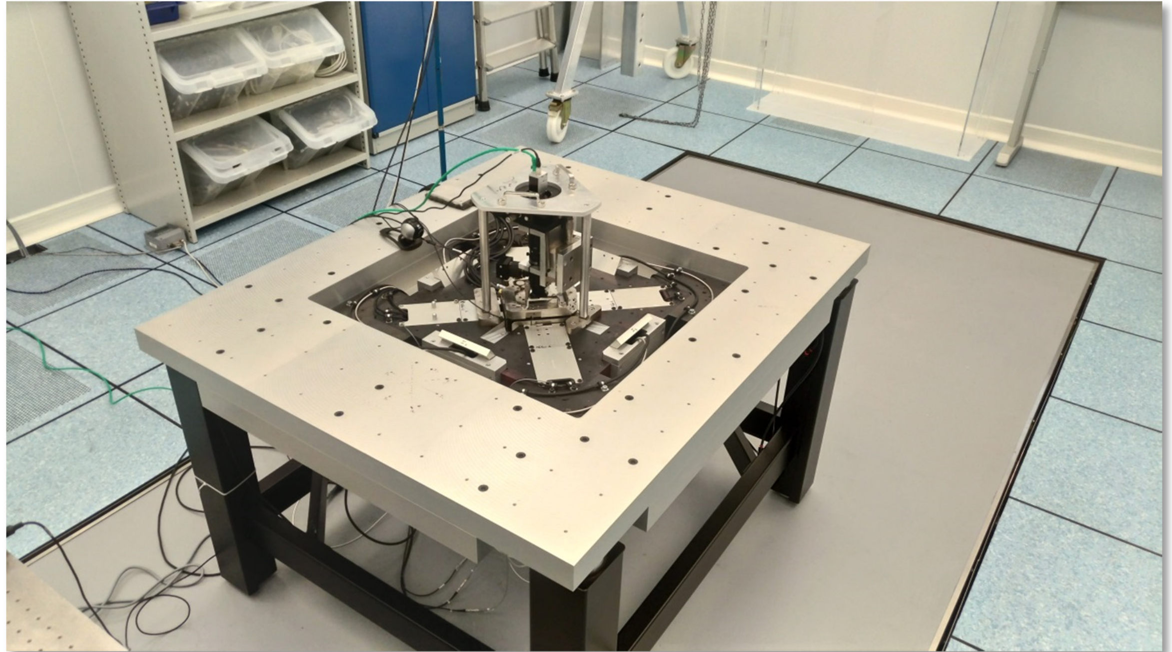
If mirror flatness is given to $\frac{\lambda}{10}$, contribution can be up to 63 nm.

Correction necessary !!

FLATNESS OF THE MIRRORS

hp-CMM

mAFM



400 mm mirror flatness = 5 µm
- Need to be corrected
- Measured using permutation and reversal method

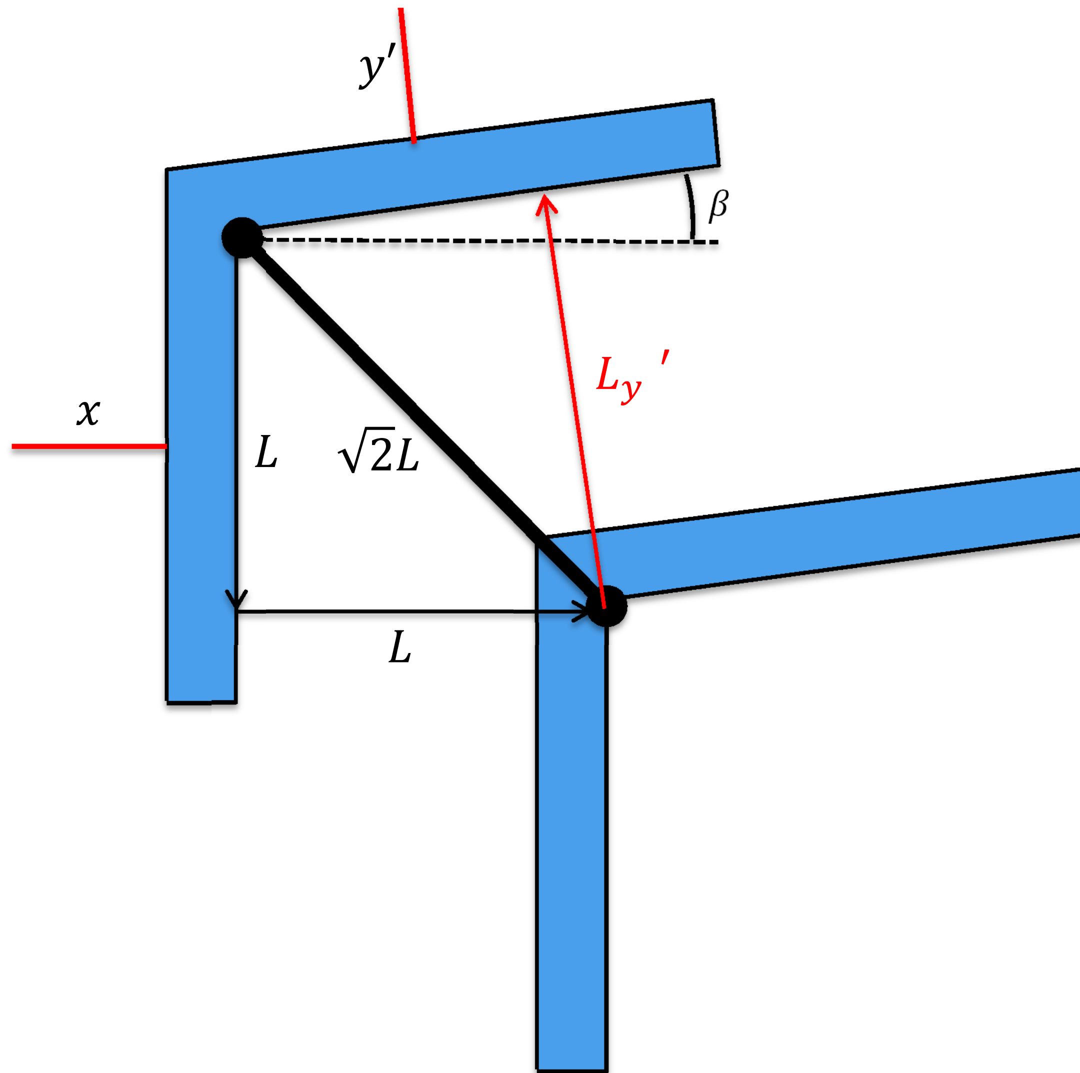
After correction

$$\varepsilon_{flatness} = 25\ nm$$

14 mm x 7 mm mirror flatness = 30 nm pp
Displacement = only 60 µm

$$\varepsilon_{flatness} = 0.36\ nm$$

OUT-OF-SQUARENESS



Due to out of squareness

$$\sqrt{2}L \neq \sqrt{L_x{}^2 + L_y{}^2}$$

Need to be corrected

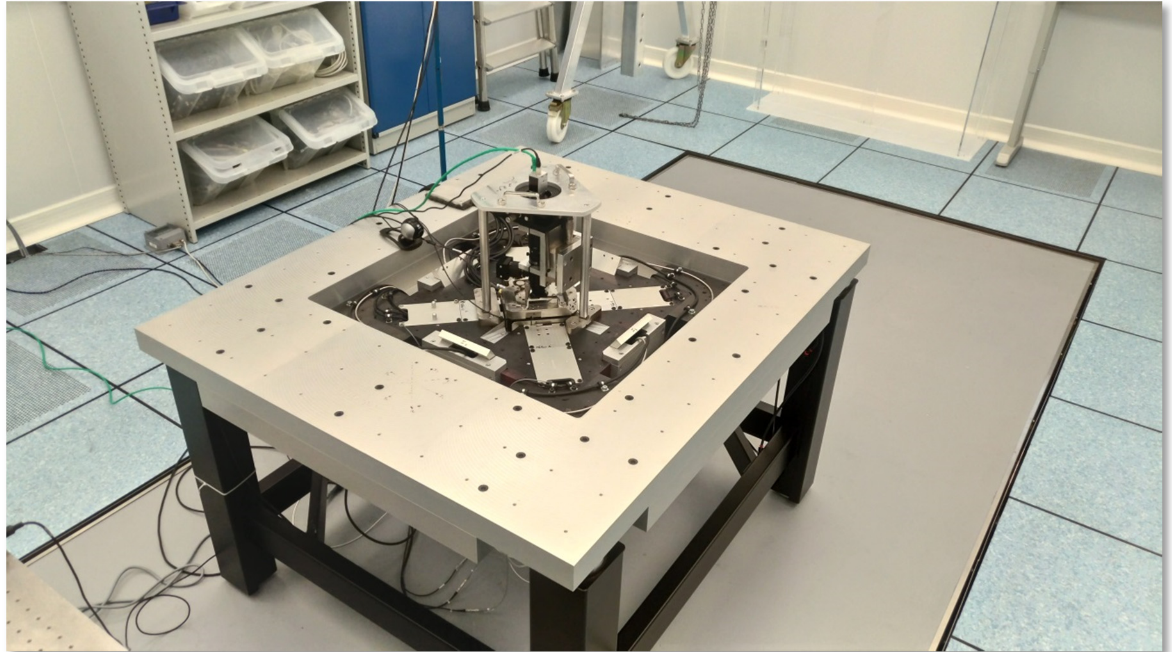
$$L_y\,' = L\cos\beta \ + L\sin\beta$$

OUT-OF-SQUARENESS

hp-CMM

β estimated to 0.2 mrad
Need to be corrected

mAFM



β estimated to less than 20 µrad
Not corrected

After correction

$$\varepsilon_{squareness} = 30\ nm$$

Without correction

$$\varepsilon_{squareness} = 42\ \mu m$$

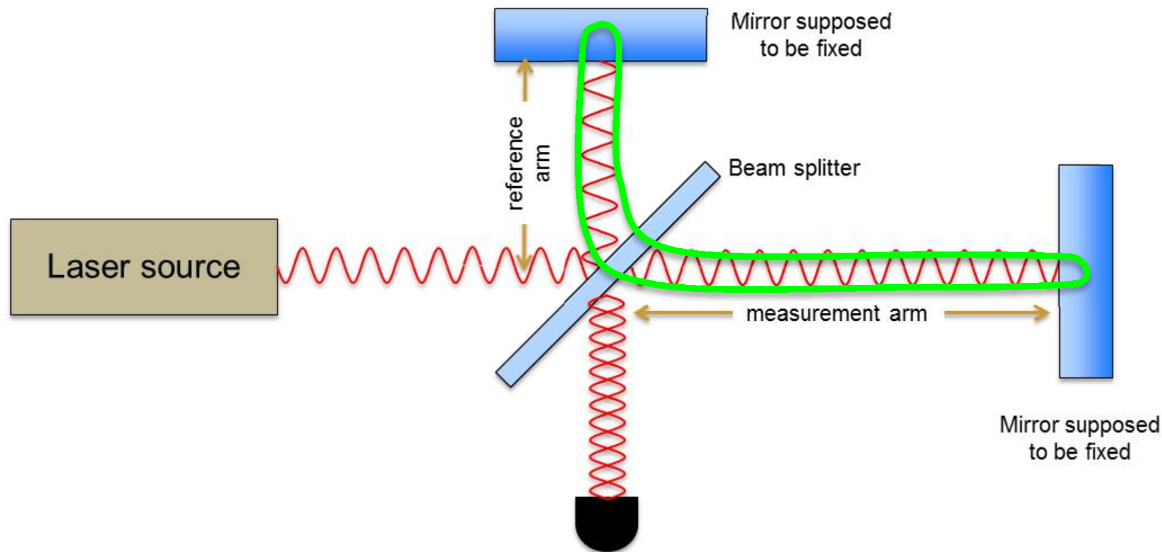
$$\varepsilon_{squareness} < 0.8\ nm$$

METROLOGY LOOP

METROLOGY LOOP IDENTIFICATION

The metrology loop is the virtual closed path containing all the components through which the information of position is passing and that can influence the measurement result.

At the interferometer scale:

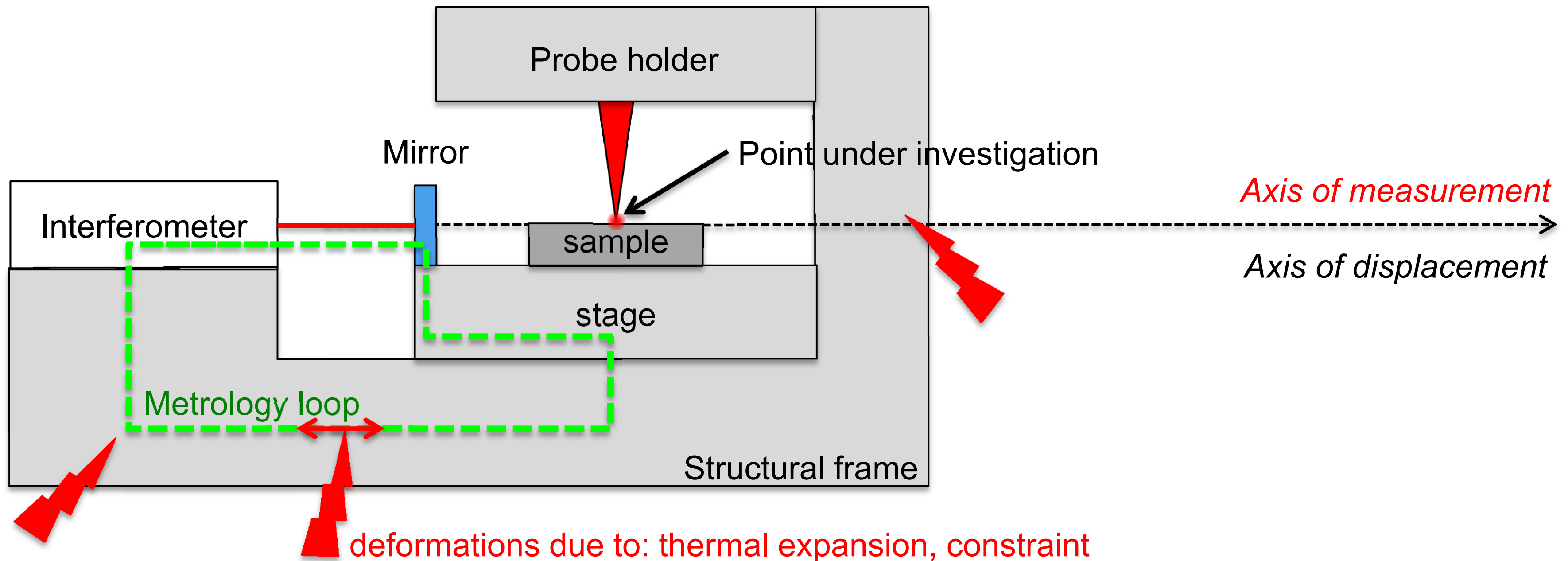


But the interferometer needs to be linked to the instrument !

METROLOGY LOOP IDENTIFICATION

Metrology loop is the virtual closed path containing all the components through which the information of position is passing an that can influence the measurement result.

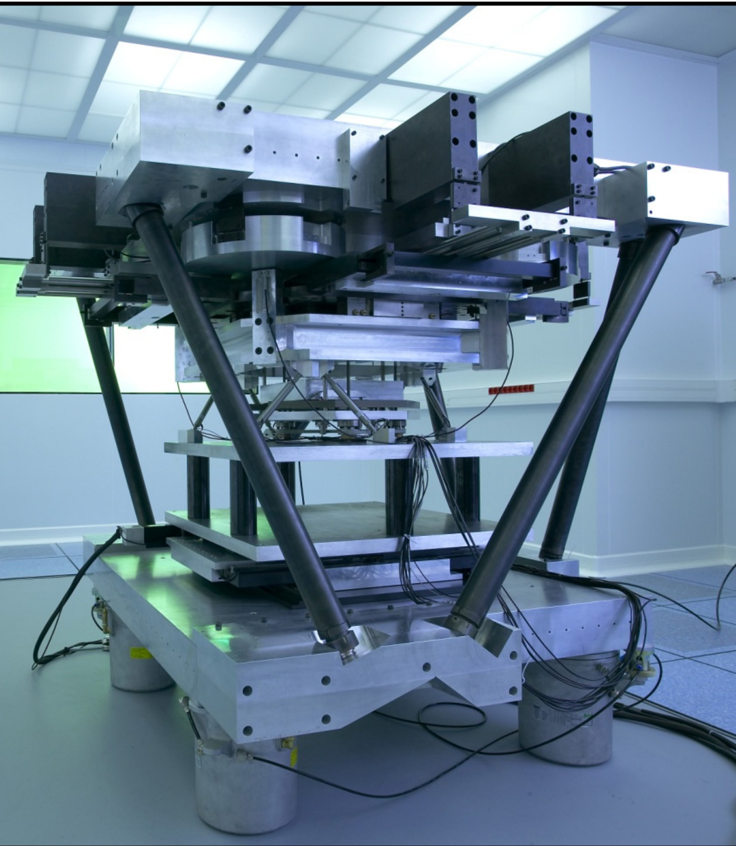
Advance



The metrology loop deformations will introduce errors in the measurements !!!

=> The metrology loop needs to be dimensionally stable !!

hp-CMM METROLOGY LOOP



Probe

1st metrology frame made of invar (1.8 m long)

Interferometers

Metrology loop dissociated from the rest of the instrument

Sample holder

2nd metrology frame made of steel (1.3 m long)

400 mm Zerodur mirrors

Thermal expansion coefficient

- for invar: $1.2\ \mu m\ K^{-1}\ m^{-1}$
- for steel: $13\ \mu m\ K^{-1}\ m^{-1}$

Thermal stability: $20\ \pm 0.01\ °C$

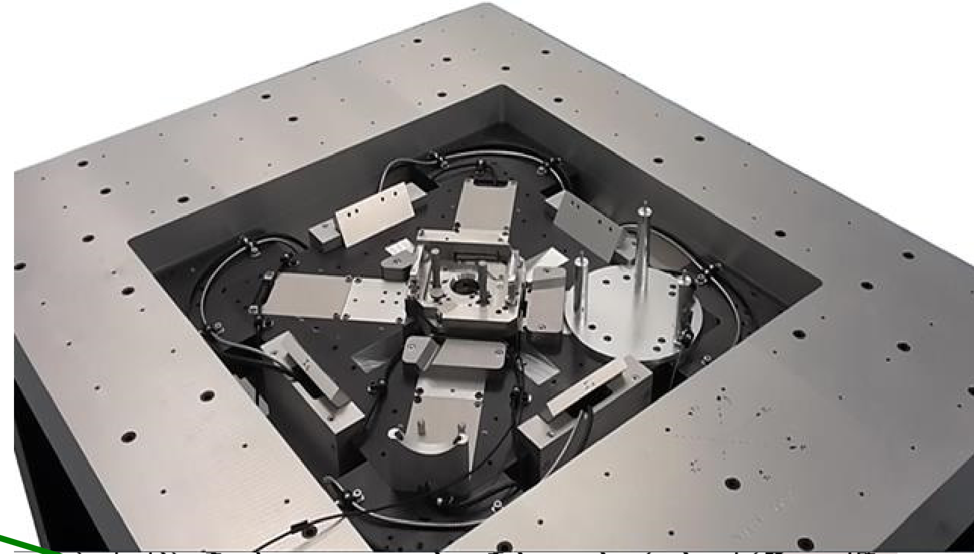
Metrology frame differential expansion: $147\ nm$

mAFM METROLOGY LOOP

Differential interferometers

500 kg of aluminum to increase thermal inertia

Thermal stability: $< 20 \ \pm 0.005\ °C$



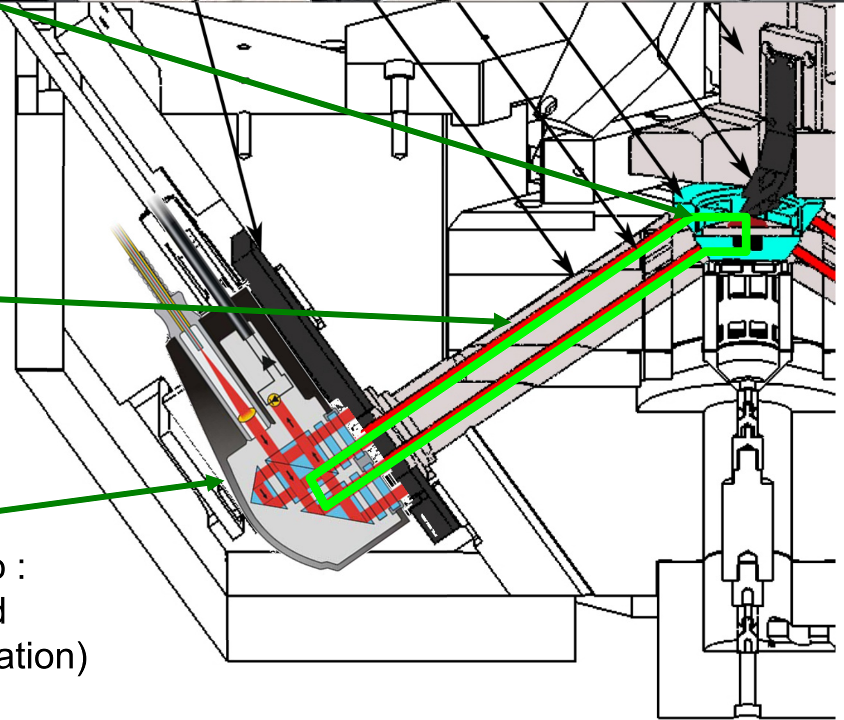
Part of the metrology loop in the Zerodur prisms

- prism typical dimension 5-7 cm
- for Zerodur: $0.05\ \mu m\ K^{-1}\ m^{-1}$
- thermal expansion: $0.01\ nm$
- Mirrors in the same plan => no dead path error

Part of the metrology loop in air

- air confined and very stable in temperature

$\Rightarrow$ no differential effect on interferometer measurement



Part of the metrology loop in the interferometer:

presence of an internal dead path (non-symmetry) subject to :

- air index variation (mainly pressure) : compensated
- thermal expansion: $< 50\ nm\ °C$ (Renishaw specification)

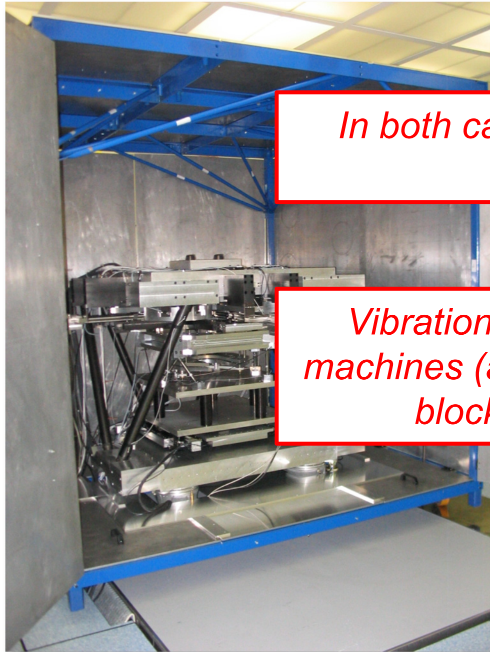
$\Rightarrow$ $0.25\ nm$

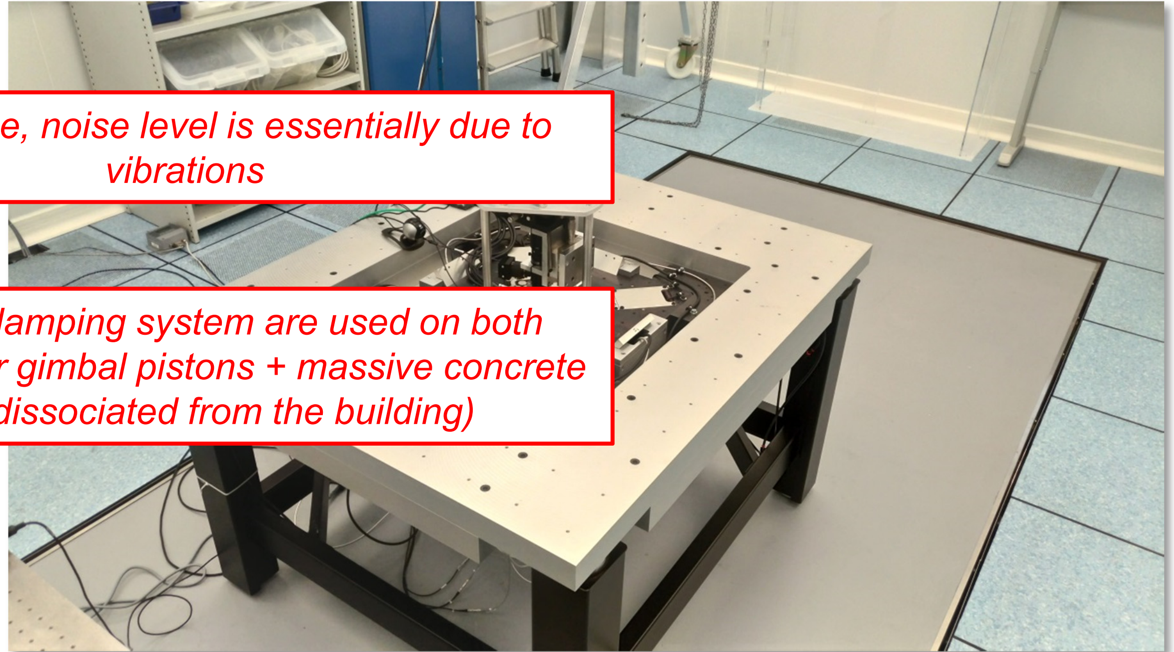
VIBRATION

VIBRATION

hp-CMM

mAFM





In both case, noise level is essentially due to vibrations

Vibration damping system are used on both machines (air gimbal pistons + massive concrete block dissociated from the building)

hp-CMM

Noise level reached (open loop at 2kHz):
X, Y: 10 - 30 nm
Z : 10 nm

Noise level reached (closed loop at 2kHz):
X, Y: 5 nm
Z : 2 nm

mAFM

Noise level reached (open loop at 2kHz):
X, Y: 3 nm
Z : 0.5 nm

Noise level reached (closed loop at 2kHz):
X, Y: 0.15 nm
Z : 0.05 nm

Stability in position:
X, Y: < 1 nm per hour
Z: ≈ 10 nm per hour

SUMMARY AND COMPARISON OF THE TWO INSTRUMENTS

SUMMARY AND COMPARISON OF THE TWO INSTRUMENTS

Components	Contribution for hp-CMM (300 mm)		Contribution for mAFM (60 µm)	
	Without	With	Without	With
Wavelength stability:	-	$3{,}8\ nm$	-	$8 \times 10^{-4}\ nm$
Edlen formulae:	-	$6\ nm$	-	$10^{-3}\ nm$
Index compensation	$\gg 400\ nm$ without correction	$30\ nm$	$\gg 2\ nm$	$8 \times 10^{-3}\ nm$
Dead path:	$7.8\ nm$ without correction	$1\ nm$	$1{,}5\ nm$	$0.125\ nm$
Turbulences:	$15\ nm$ without protection	$< 5\ nm$	$15\ nm$	*negligible*
Non-linearity:	$7\ nm$ without correction	$1.5\ nm$	-	$0.5\ nm$
Resolution limit:	-	$0.038\ nm$	-	$0.038\ nm$
Data age	$10000\ nm$ (in dynamic)	negligible in static	$1\ nm$	*negligible*
Cosine error:	-	$2.4\ nm$	-	$4.7 \times 10^{-4}\ nm$
Abbé error:	-	$0.5\ nm$	-	$10\ nm$
Mirror flatness:	$5000\ nm$ without correction	$25\ nm$	-	$0.36\ nm$
Out-of-squareness:	$42000\ nm$ without correction	$30\ nm$	-	$< 0.8\ nm$
Metrology frame thermal expansion:	-	$147\ nm$	-	$0.25\ nm$
Noise level:	$10 - 30\ nm$ in open loop	$2 - 5\ nm$	$0.5 - 3\ nm$	$0.05 - 0.14\ nm$

SUMMARY AND COMPARISON OF THE TWO INSTRUMENTS

Components	Contribution for hp-CMM (300 mm)		Contribution for mAFM (60 µm)	
	Without	With	Without	With
Wavelength stability:	-	$3{,}8\ nm$		$8 \times 10^{-4}\ nm$
Edlen formulae:				$10^{-3}\ nm$
Index compensation				$8 \times 10^{-3}\ nm$
Dead path:				$0.125\ nm$
Turbulences:				*negligible*
Non-linearity:				$0.5\ nm$
Resolution limit:				$0.038\ nm$
Data age				*negligible*
Cosine error:				$4.7 \times 10^{-4}\ nm$
Abbé error:				$10\ nm$
Mirror flatness:				$0.36\ nm$
Out-of-squareness:			-	$< 0.8\ nm$
Metrology frame thermal expansion:	-	$147\ nm$	-	$0.25\ nm$
Noise level:	$10 - 30\ nm$ in open loop	$2 - 5\ nm$	$0.5 - 3\ nm$	$0.05 - 0.14\ nm$

The uncertainty budget depends on the design of the instrument and must be evaluated:

During the design of the instrument:

- *to identify the main contributions and try to reduce them before construction,*
- *to find the interferometer that will fit with the required performances,*

When the instrument is working:

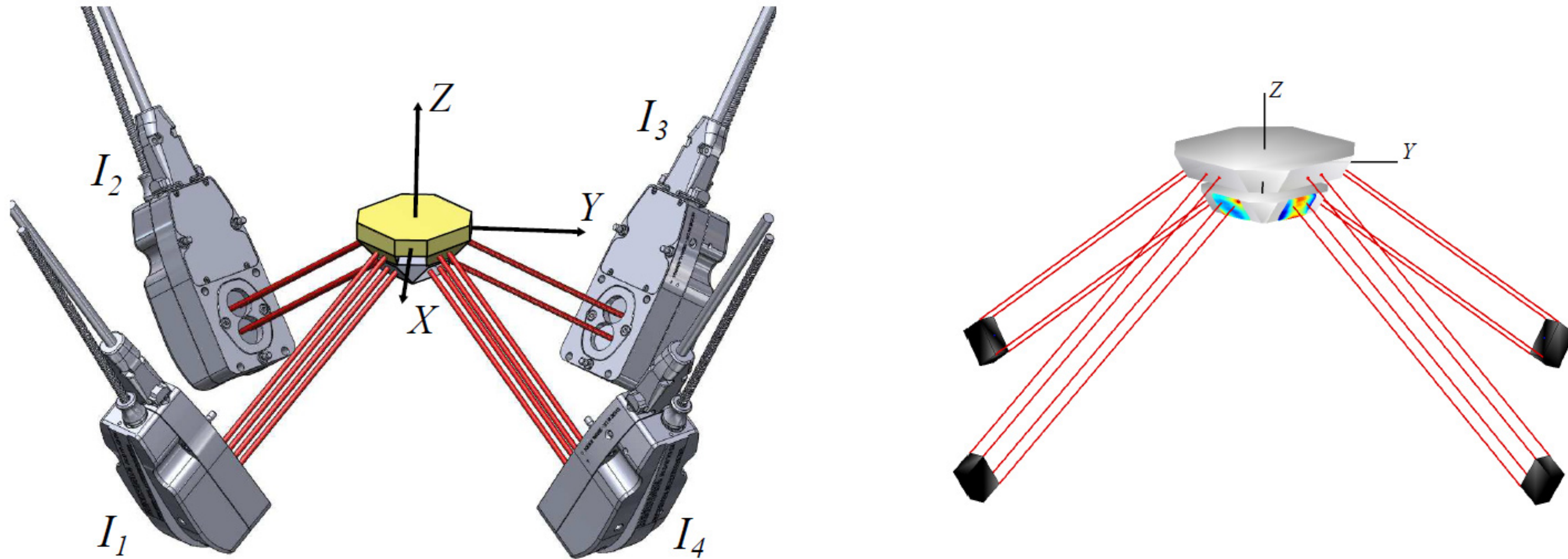
- *to evaluate its achieved performances*
- *to estimate its positioning uncertainty*
- *to improve instrument*

TO GO FURTHER: MODELLING OF THE METROLOGY LOOP

MODELLING OF THE METROLOGY LOOP

When all the above possible errors have to be simultaneously taken into account on a complex system, it becomes very difficult to have a clear view.

⇒ Then modeling can be of precious help before and after construction of the instrument



Assessment of the main sources of uncertainties for the mAFM by modelling of the metrology loop combined with a Monte Carlo approach

P. Ceria « Développement d'un AFM virtuel pour l'évaluation du bilan d'incertitude de l'AFM métrologique du LNE ». Université Toulouse III Paul Sabatier, 2010, http://thesesups.ups-tlse.fr/3652/

P. Ceria *et. al.* « Modelling of the X,Y,Z positioning errors and uncertainty evaluation for the LNE's mAFM using the Monte Carlo method » 2017 *Meas. Sci. Technol.* **28** 034007 https://doi.org/10.1088/1361-6501/28/3/034007

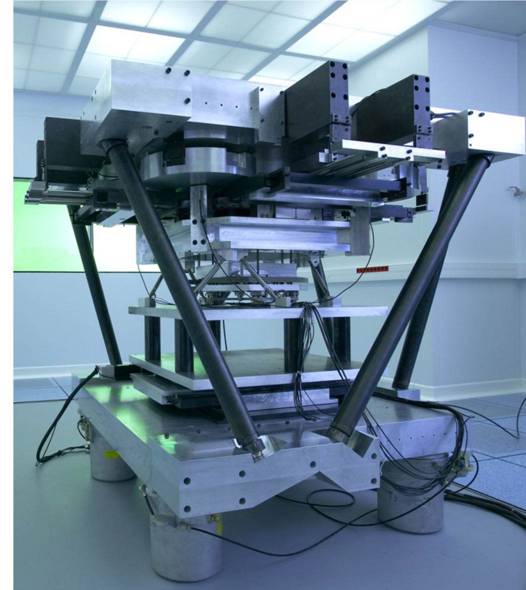
CONCLUSION AND RECOMMENDATIONS

CONCLUSION

- It was just an overview: measurements of displacement using Interferometry for high precision applications are more complex than explained during this presentation.

$$\begin{cases} X = \dfrac{1}{4(n_0 + \Delta n)}\left[\lambda_{v_2} \cdot \Delta k_2 - 4\Delta n \cdot L_2\right] - \delta L_2 \\ \Delta n = \dfrac{\lambda_{v_1} \cdot \Delta k_1}{4(L_1 + \delta L_1)} - \dfrac{n_0 \cdot \delta L_1}{(L_1 + \delta L_1)} - \delta n \end{cases}$$

X : displacement to measure
n_0 : air index for t=0 (given by a weather station)
Δn : air index variation
λ_v : wavelength in vacuum
Δk : interpolated signal
L : length of the beam in air
δL : metrology loop dilatation
δn : air index deviation between the two interferometers



hp-CMM - equation used for displacement evaluation along one axis using two interferometers (one for position, one for index tracking)

RECOMMENDATIONS

- When purchasing an interferometer, pay attention to its design.
- When interferometer is purchased, do not trust the measurement, compare the measurements with the one provided by another displacement sensor, a sensor that you know and that you have perfectly characterized. Then take time to understand the difference of measurements between the two sensors.
- Test the interferometer on very simple setup before an integration on a more complex one.
- Pay attention to the way the interferometer will be integrated in the instrument. Identify the metrology loop and optimize it. For this task, an uncertainty analyses can be a very powerful tool to identify the most important contributions during the design of the instrument.
- If possible, model the metrology loop with the interferometers. A lot can be learned from it. You can identify and minimize the main error sources and test several configurations.
- Keep in mind that lots of things are describe theoretically in literature but you have to create your own experience to better use interferometers and to find the best compromise to implement it (need hindsight or good advices).

Thank you for your attention

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[5] Aerotech Inc « Machine positioning uncertainty with laser interferometer feedback »

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Thesis:

[7] Cosijns, S.J.A.G. « Displacement laser interferometry with sub-nanometer uncertainty », https://doi.org/10.6100/IR579456

Recent

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[15] A. Slocum, « Precision machine design », Society of Manufacturing Engineers, 1992 ISBN 0872634922 9780872634923