

# R&D of Mirror Bending Techniques in HEPS

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- Mirror Bending Theory
- Design of Mirror and Bender
- Measuring Results
- Zoom capability and it's optimized design
- Conclusion

# Mirror Bending Theory

- Basic Theory:
  - Pure Bent Beam curvature



$$y''(x) = \frac{M(x)}{E I(x)}$$

$y$  - deflection

$x$  - position on mirror

$M$  - Bending Moment on cross-section

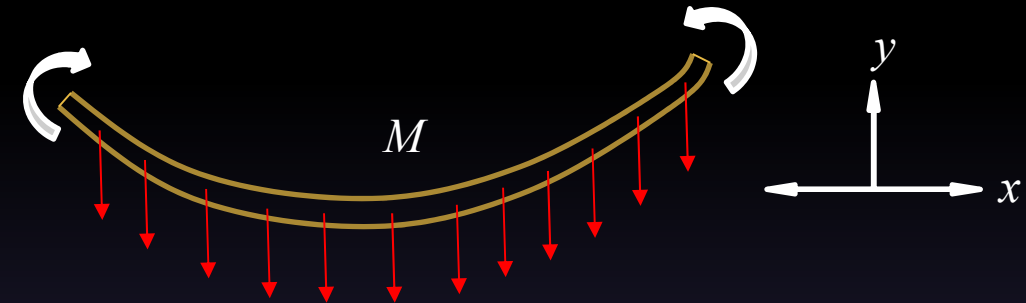
$E$  - Young's modulus

$I$  - Moment of Inertia of cross-section

# Mirror Bending Theory

- Basic Theory:
  - Pure Bent Beam

$$y''(x) = \frac{M(x)}{E I(x)}$$



When the moment is applied to both ends :

$$M_b(x) = M_0(1 + k_M x)$$

For a vertical focusing mirror, the **extra moment from gravity**:

$$M_g(x) = \frac{g\rho_m T}{4L} \left( -4L \int_x^{\frac{L}{2}} (u-x)W(u)du + (L-2x) \left( L \int_{-\frac{L}{2}}^{\frac{L}{2}} W(u)du + 2 \int_{-\frac{L}{2}}^{\frac{L}{2}} uW(u)du \right) \right)$$

$g$  – Acceleration of gravity,  $\rho_m$  - Mass density,  $T$  – Mirror Thickness,  $L$  - Mirror length,  $W$  - Mirror width

# Mirror Bending Theory

- Elastic mechanics correction 1:

- Extra transverse shear deformation:

$$y'(x) = s(x) - s(0)$$

$s$  from bending moment:

$$s_b(x) \equiv \frac{-M_0 k_M}{W(x) * TG}$$

$s$  from gravity (For a vertical focusing mirror) :

$$s_g(x) \equiv \frac{g\rho_m \left( \int_x^{\frac{L}{2}} \left( -\frac{L}{2} + u \right) W(u) du + \int_{-\frac{L}{2}}^x \left( \frac{L}{2} + u \right) W(u) du \right)}{GLW(x)}$$

$G$  - transverse shear modulus



# Mirror Bending Theory

- Elastic mechanics correction 2:
  - Extra axial shear deformation:

$$y'(x) = s(x) - s(0)$$

$$s_a(x) \equiv -\frac{M'(x)}{W(x)} \frac{6}{5G_a T}$$

$G_a$  - axial shear modulus



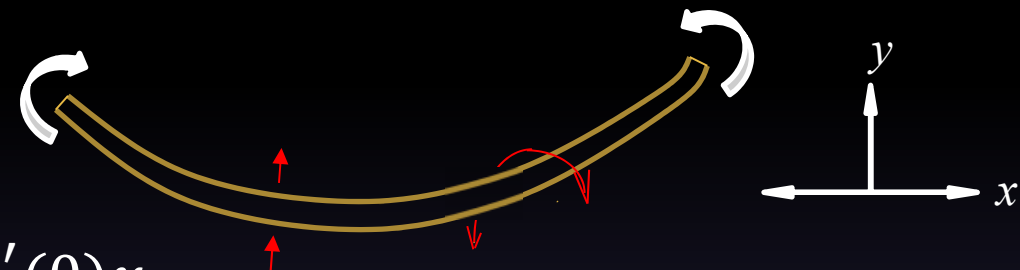
# Mirror Bending Theory

- Elastic mechanics correction 3:

- Extra transverse deformation:

$$y_c(x) = h_c(x) - h_c(0) - h_c'(0)x$$

$$h_c(x) \equiv \frac{T^2 \nu M(x)}{12 E I(x)}$$



- Curvature (including Influence of saddle deformation):

$$y''(x) = \frac{-2 \times 5^{2/3} E^{2/3} T^{8/3} + 2 \times 5^{1/3} \left( 9M(x)W(x)\nu + \sqrt{5E^2T^8 + 81M(x)^2W(x)^2\nu^2} \right)^{2/3}}{W(x)^2\nu \left( ET(9M(x)W(x)\nu + \sqrt{5E^2T^8 + 81M(x)^2W(x)^2\nu^2}) \right)^{1/3}}$$

# Bending Design

## Concept



- Key points

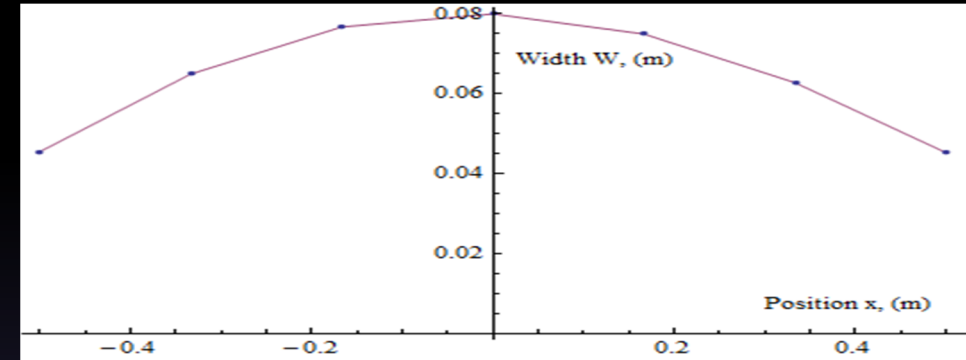
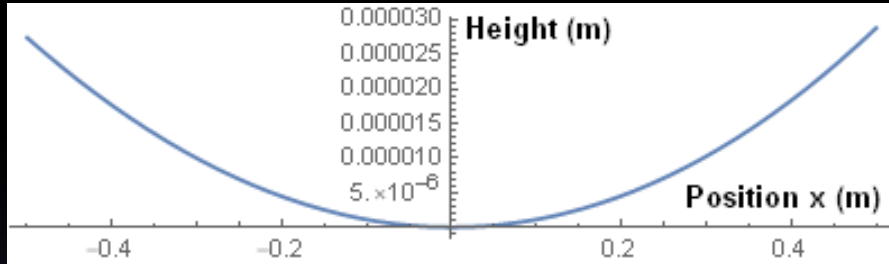
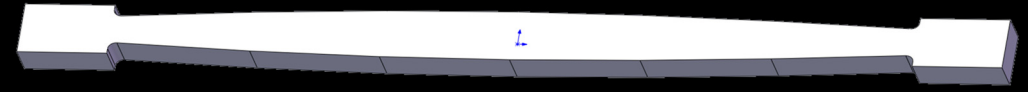
- Elliptical Bending
- Two Independent Moments
- Special Designed Torpedo Mirror
- No Gravity Compensation System, simple and stable
- Actuator spring – high resolution, high stability
- New high elastic materials, Such as: BeCu

# Mirror Design

Bending Ellipse:

$p=40m$   $q=8m$   $\theta=3mrad$

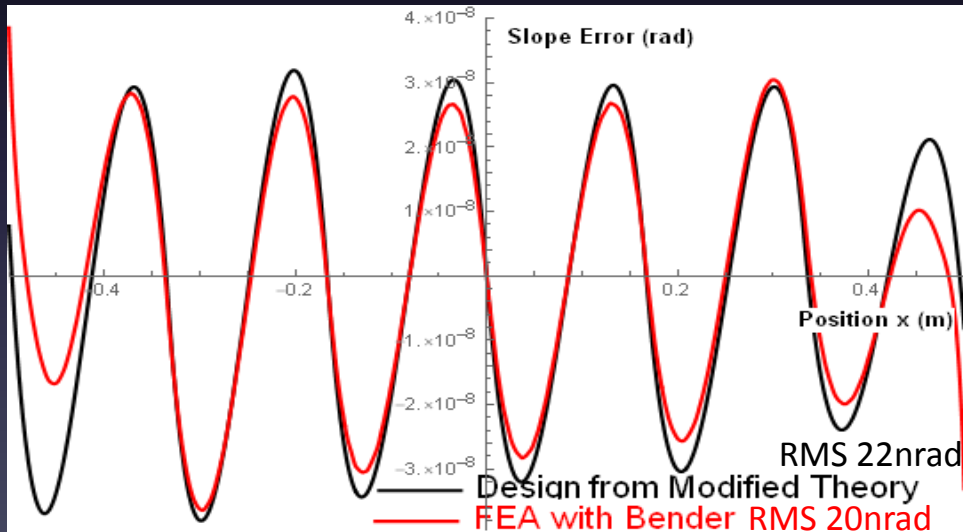
1260mm (1000mm)  $\times$  80mm(max)  $\times$  50mm



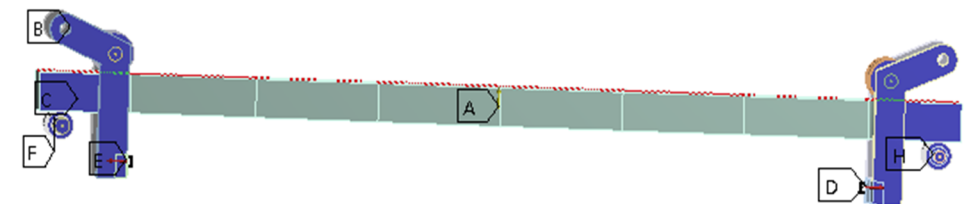
Anisotropy:

stiffness matrix: Length  $\langle 1,0,0 \rangle$ , Width  $\langle 0,1,1 \rangle$ , Height  $\langle 0,-1,1 \rangle$

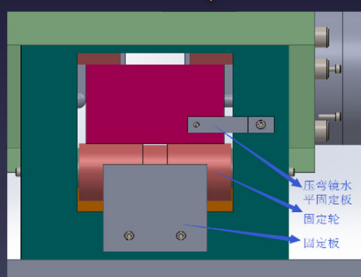
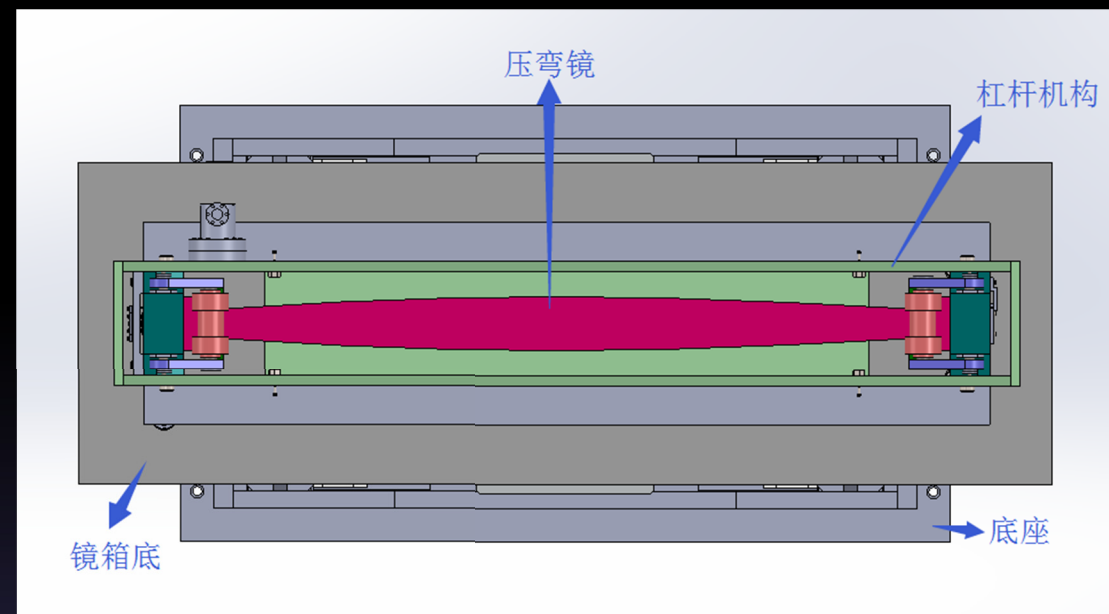
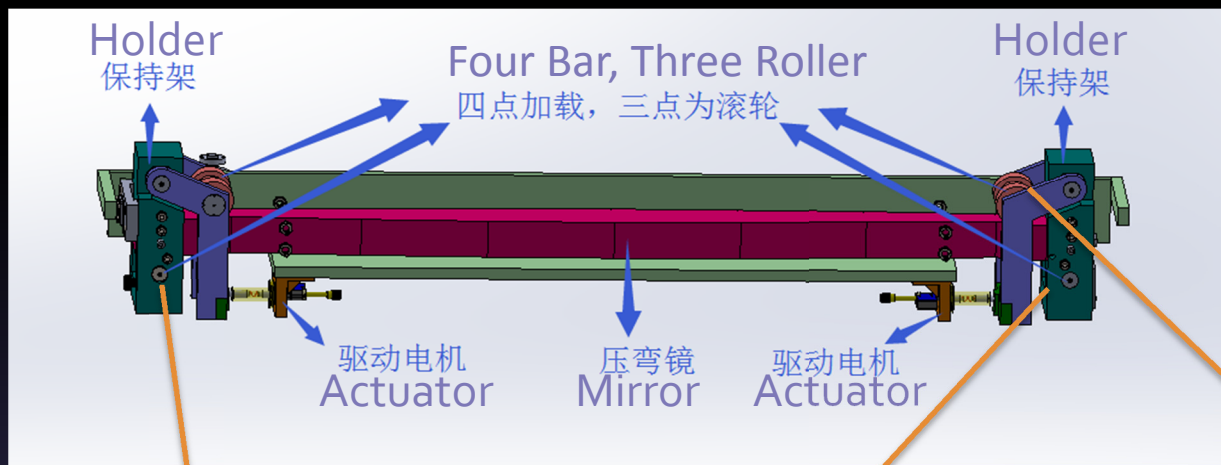
165.7	63.9	63.9	0	0	0	GPa
63.9	194.4	35.2	0	0	0	
63.9	35.2	194.4	0	0	0	
0	0	0	50.9	0	0	
0	0	0	0	79.6	0	
0	0	0	0	0	79.6	



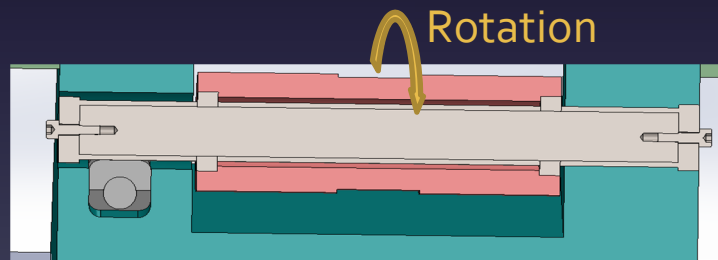
- A** Acceleration: 9.80122 m/s<sup>2</sup>
- B** Cylindrical Support: 0. m
- C** Frictionless Support
- D** Force: 73.00533027 N
- E** Force 2: 87.75414051 N
- F** Fixed Support
- H** Fixed Support 2



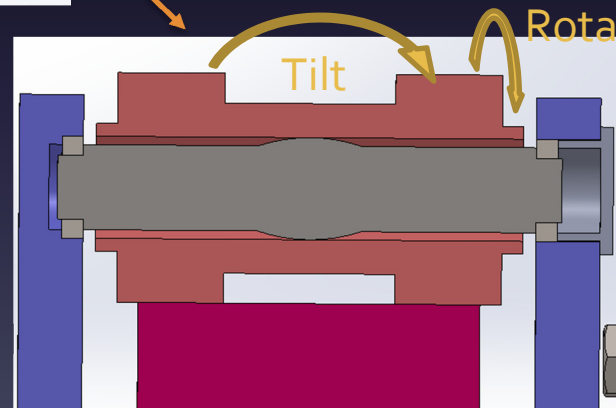
# Bender design



lower Fixed Bar



lower Roller



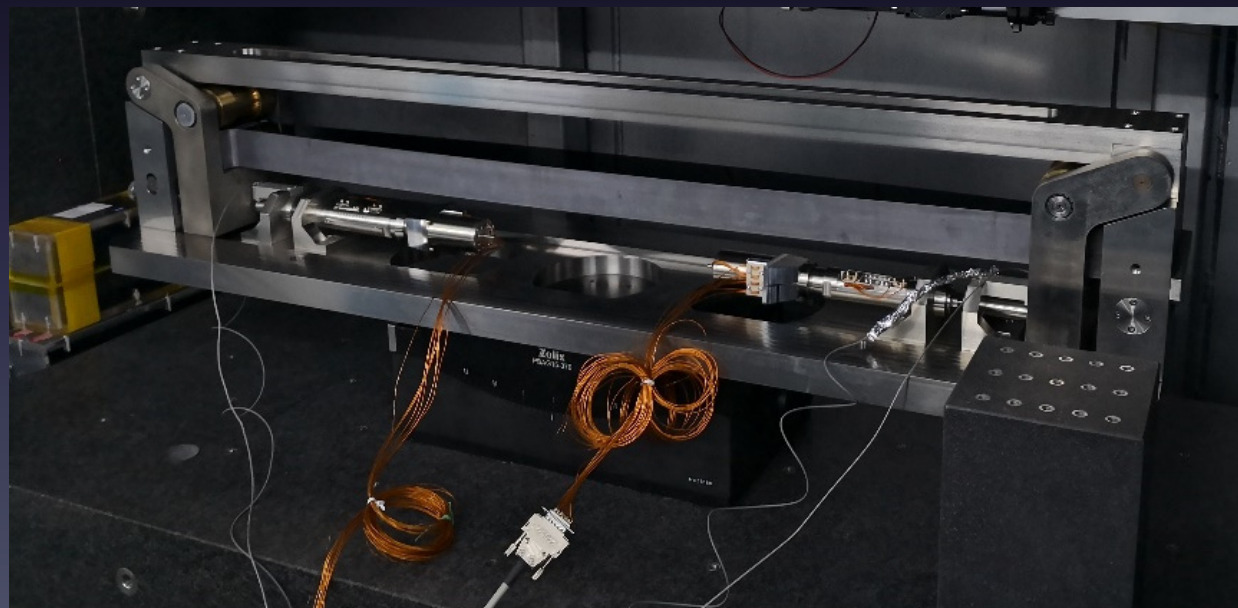
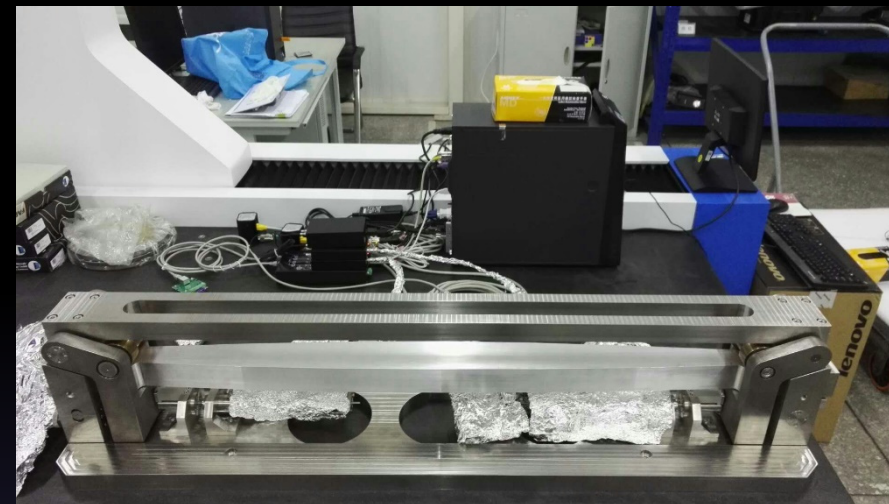
Upper Roller

Material of roller:

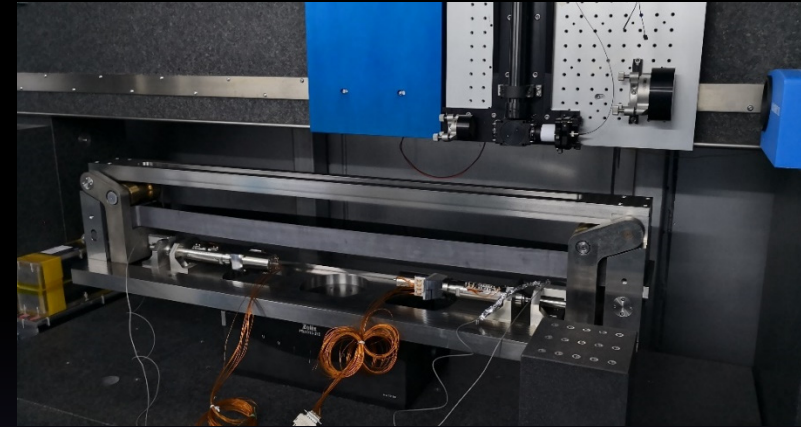
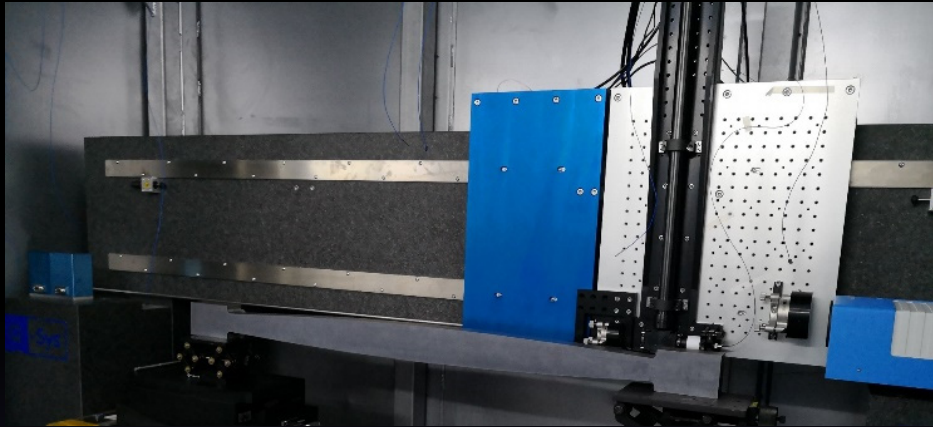
beryllium bronze

- Lower E -> Low Stress Concentration
- Higher  $\sigma_e$  -> Low Yield Deformation

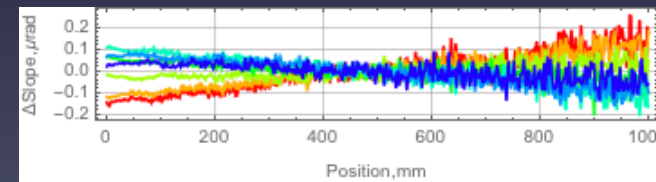
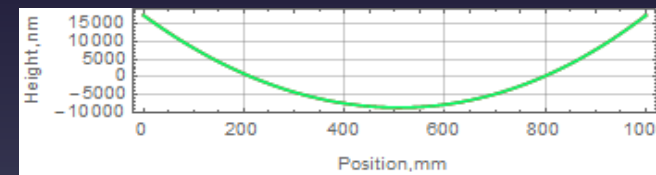
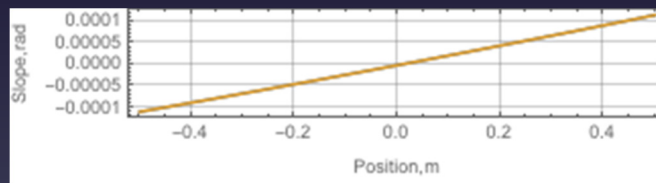
# Physical processing



# Measurement on FSP (Flag-type Surface Profiler)



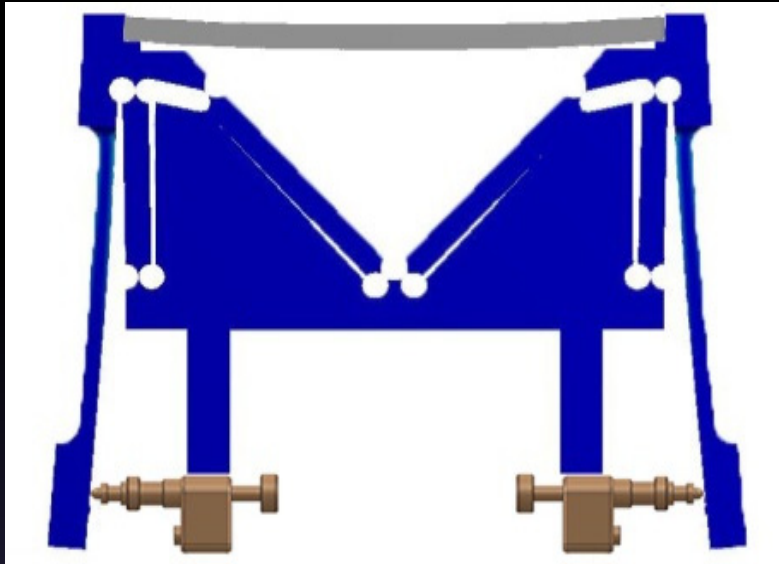
	Measuring Results
effective length	1000mm
Bending Shape	Designed Ellipse
Bending shape accuracy (Bent mirror shape – Bare mirror shape – Designed ellipse)	0.17 $\mu$ rad (Vertical focusing)



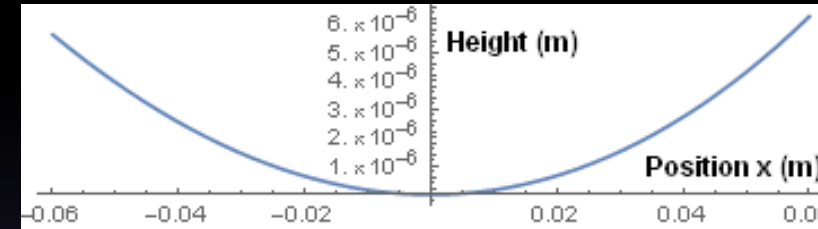
Bending shape accuracy RMS 0.17 $\mu$ rad

Stability: 72h, deformation 66nrad RMS

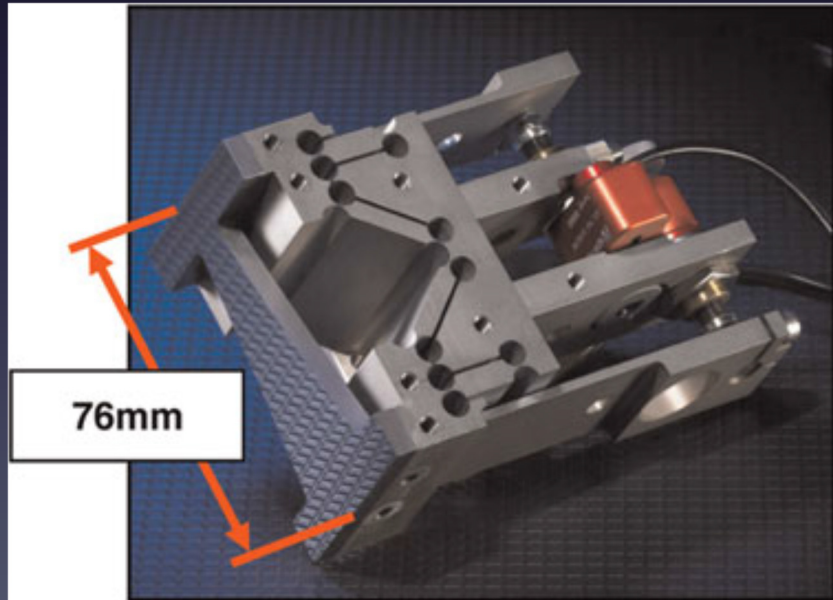
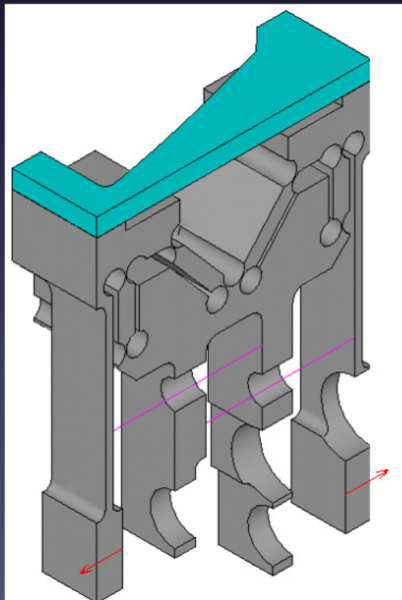
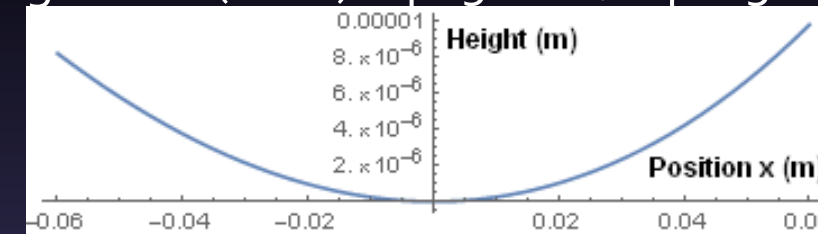
# K-B Mirror Design



**Bending Ellipse:** Mirror Length: 146mm (120mm)  
**Vertical Focusing Mirror (VFM) :**  $p=5\text{m}$ ,  $q=0.5\text{m}$ ,  $\theta=3\text{mrad}$



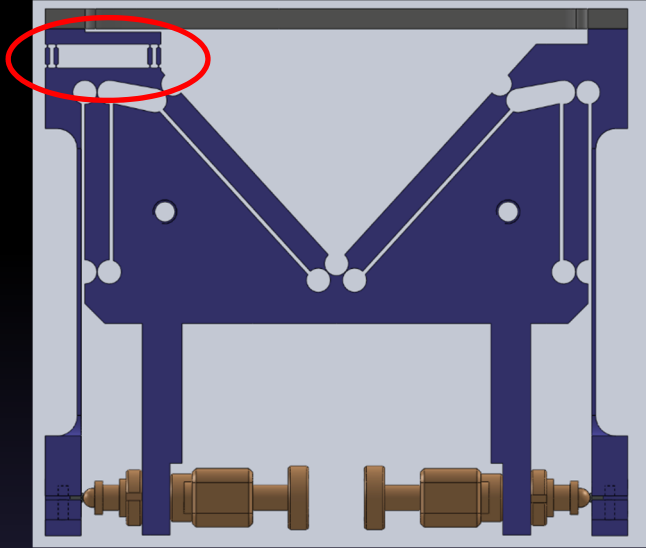
**Horizontal Focusing Mirror (HFM) :**  $p=5.18\text{m}$ ,  $q=0.32\text{m}$ ,  $\theta=3\text{mrad}$



Original Design Idea From:  
 Lin Zhang et al. (2010) ESRF  
 R. Baker et al. (2010) ESRF

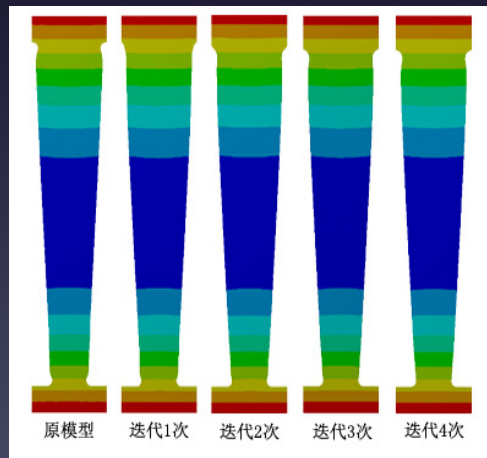
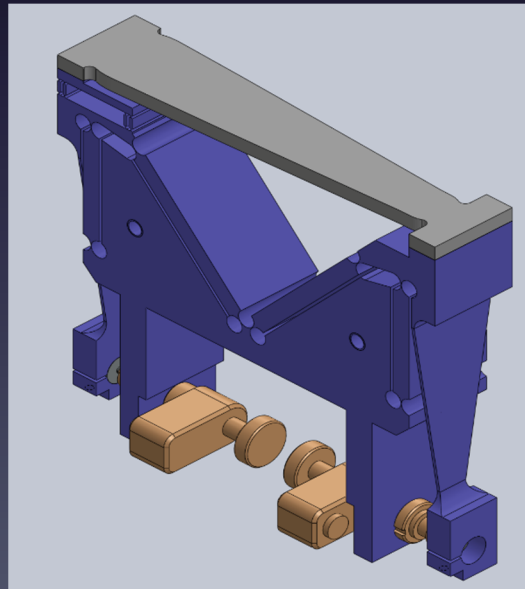
# K-B Mirror Design

## New Design: add Translation Flexure Hinge (TFH)



Machining Error of Flexure Bender  
Bonding Error between Bender and Mirror } → Bending Shape Error

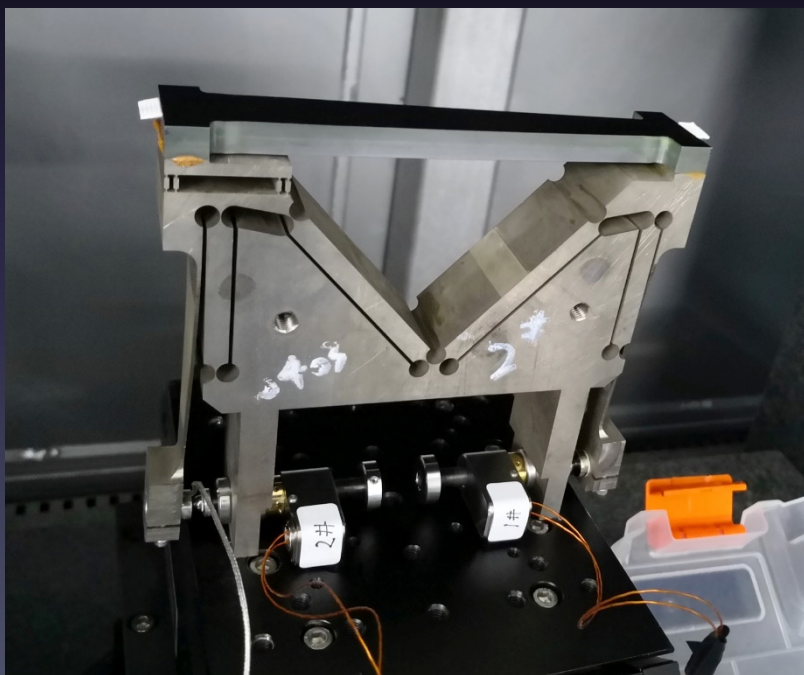
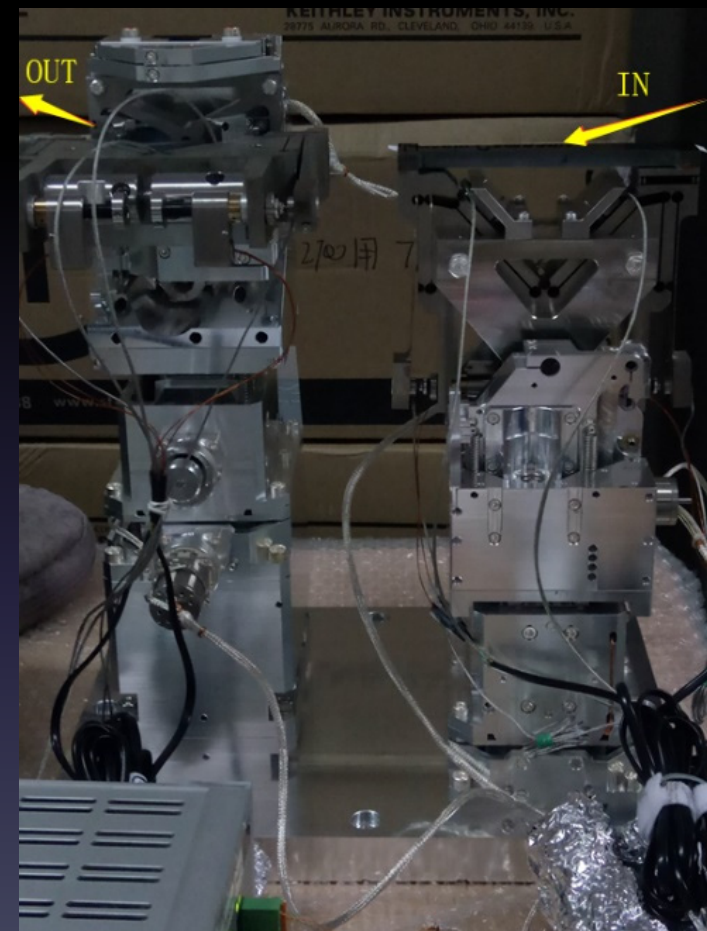
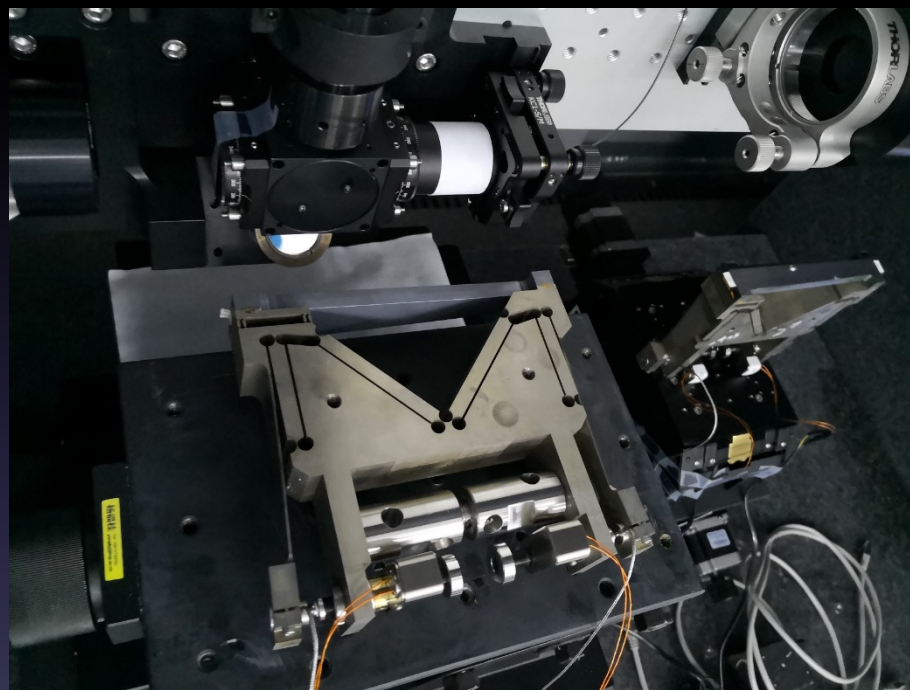
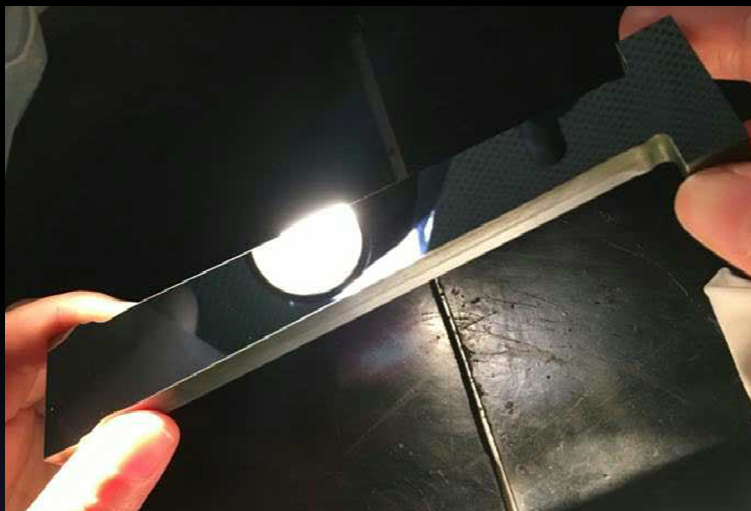
**Translation Flexure Hinge**  
- Reduce the Sensitivity of Bending Shape to Bender Errors.



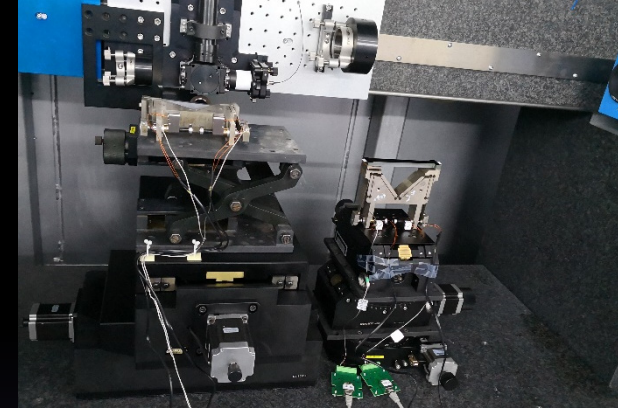
Mirror Profile Iteration by FEA

	VFM Without TFH	VFM With TFH	HFM Without TFH	HFM With TFH
Total Axial Stress F (N)	2.1	0.19	5.1	0.43
$\Delta F$ caused by 30 $\mu$ m Thickness Error of Glue Layer (N)	0.23	0.023	1.16	0.13
$\Delta F$ caused by 30 $\mu$ m Machining Error of Flexure Bender (N)	0.32	0.032	1.6	0.09

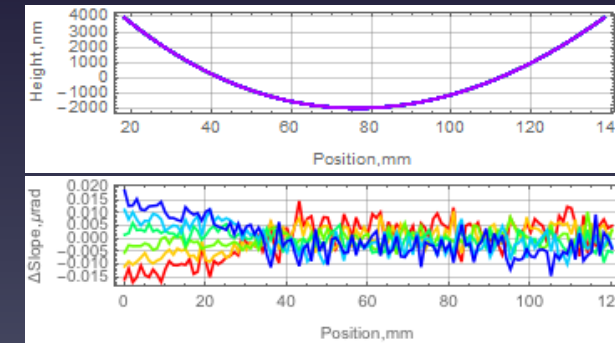
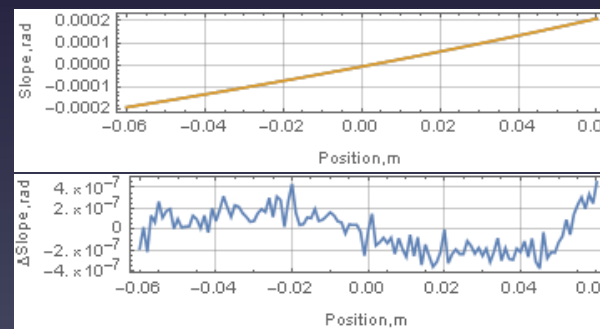
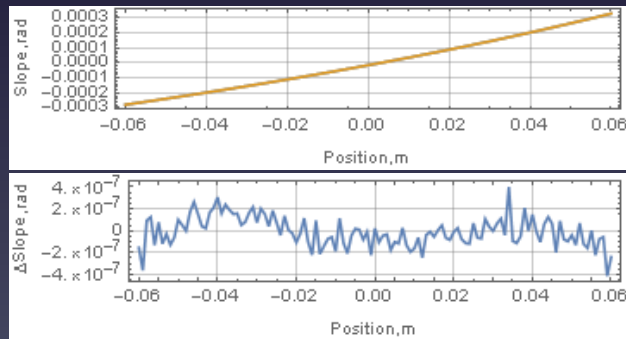
# Physical processing



# Measurement on FSP (Flag-type Surface Profiler)



Measuring Results	
effective length	<b>120mm (146mm total)</b>
Bending Shape	<b>Designed Ellipse(40m, 120mm, 3mrad)</b>
Bending shape accuracy (Bent mirror shape – Bare mirror shape – Designed ellipse)	<b>0.13<math>\mu</math>rad(HFM) 0.19<math>\mu</math>rad (VFM)</b>



Bending shape accuracy RMS **0.13 $\mu$ rad & 0.19 $\mu$ rad**

Stability : 72h, transformation **6nrad RMS**

# Next Bending Techniques in HEPS

— Zoom capability and optimized design

## Zoom Variation Capability is new important requirement for Beam-lines

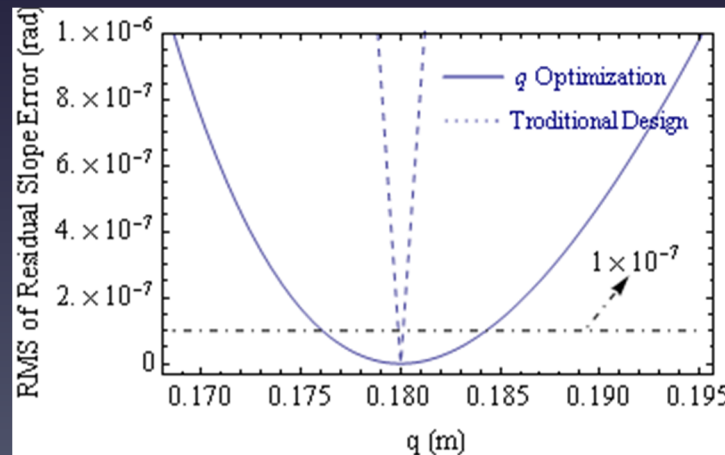
Whether Pure Bent Beam Theory or Elastic Mechanics,

For any  $M(x)$ , we can find matched  $I(x)$  to get Perfect Designed Elliptical Bending

$$y''(x) = \frac{M(x)}{E I(x)}$$

But for different  $M(x)$  design, typically different  $M_1$  and  $M_2$  -> different  $k_M$  design  
the **Zoom Variation Capability of bent mirror** is different.

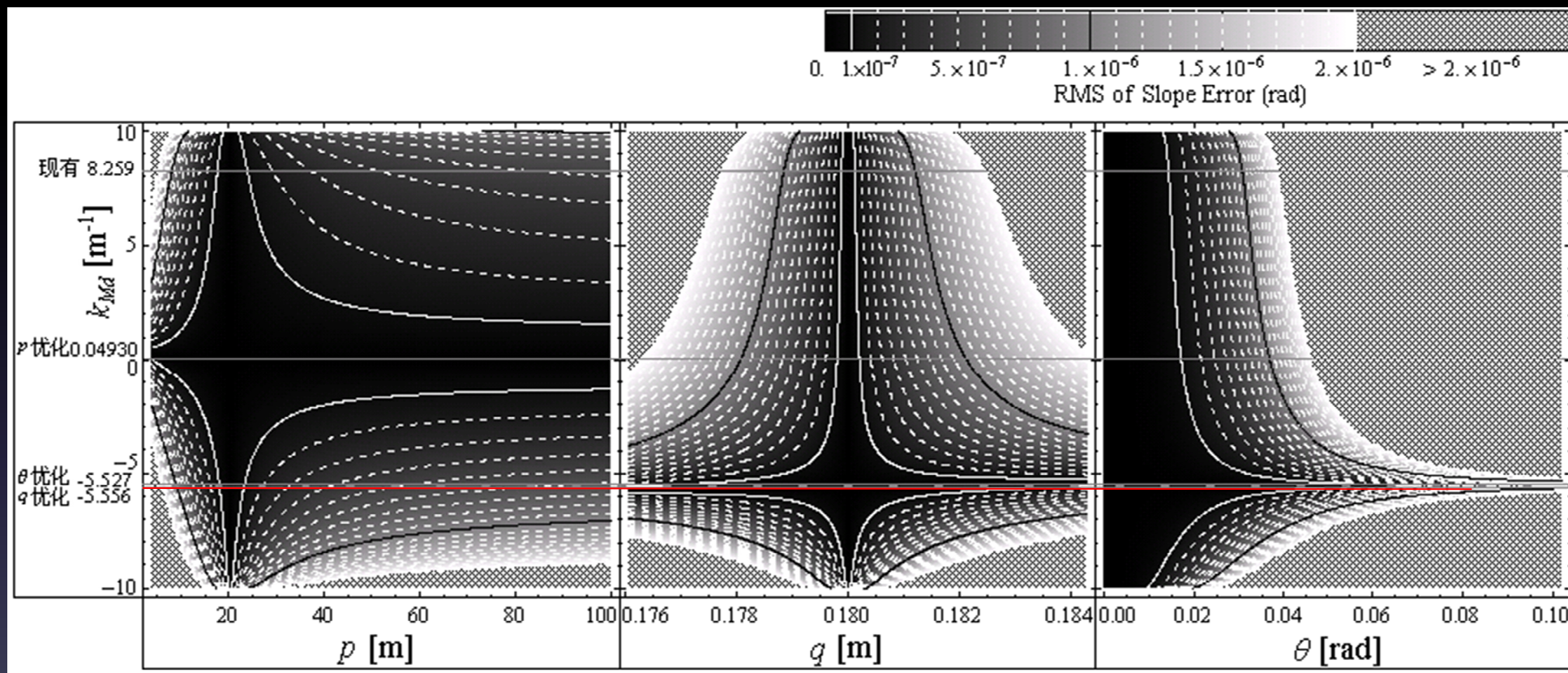
For example:  $p=20.3\text{m}$ ,  $q=0.18\text{m}$ ,  $L=0.2\text{m}$ ,  $\theta= 2.2\text{mrad}$



# Next Bending Techniques in HEPS

— Zoom capability and optimize design

For example:  $p=20.3\text{m}$ ,  $q=0.18\text{m}$ ,  $L=0.2\text{m}$ ,  $\theta=2.2\text{mrad}$  **HFM**



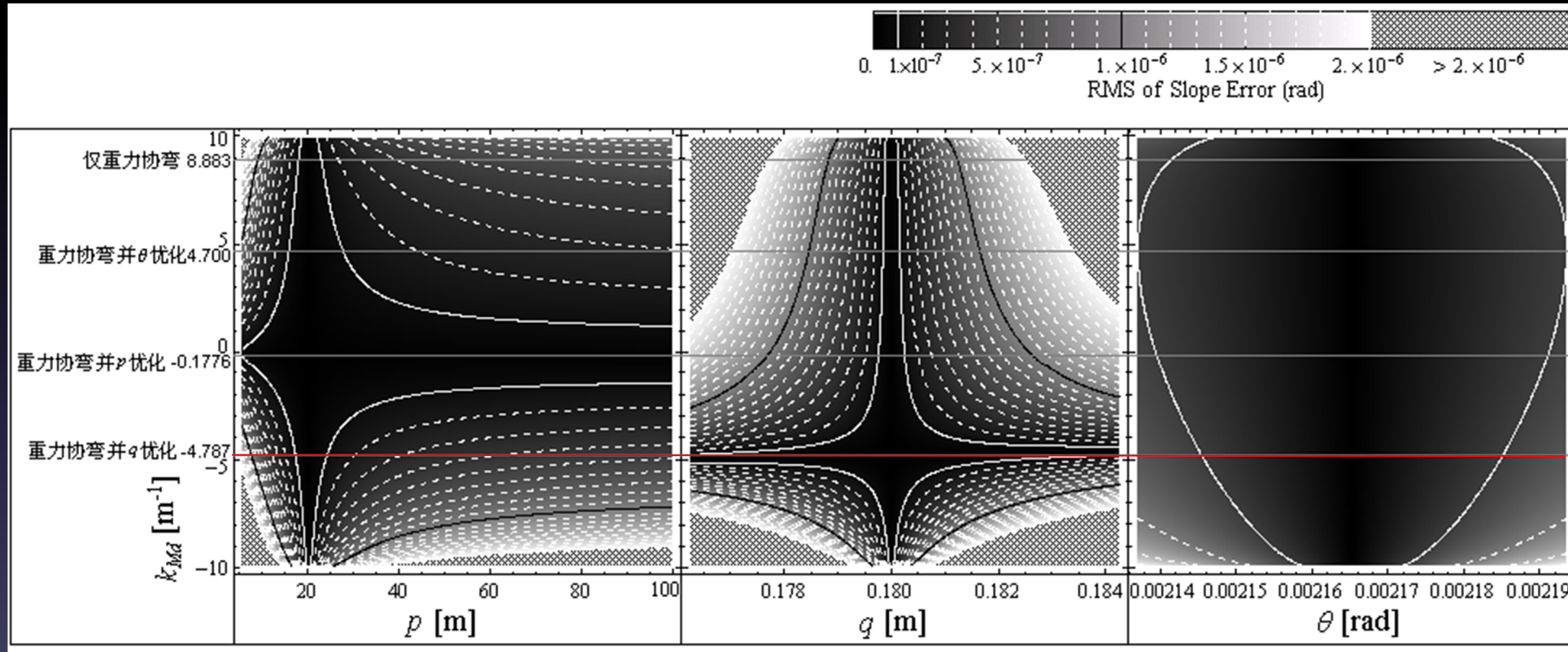
$$k_{Md} \doteq -\frac{\cos \theta_d}{q_d} - \frac{\sin \theta_d \tan \theta_d}{p_d} \approx -\frac{1}{q_d}$$

Subscript “d” means design value.

# Next Bending Techniques in HEPS

— Zoom capability and optimized design

For example:  $p=20.3\text{m}$ ,  $q=0.18\text{m}$ ,  $L=0.2\text{m}$ ,  $\theta=2.2\text{mrad}$ ,  $T=5\text{mm}$  **VFM**



Subscript “d” means design value.

# Conclusion

- Elastic mechanics corrections on bending theory
- Long Bent Mirror and K-B Bent Mirrors are R&D, sub-200nrad bending accuracy is got with high stability.
- Zoom capability optimized bending design is discussed.

# Thanks!

## Our Team:

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- Lidan Gao
- Shanzhi Tang
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- Fugui Yang
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