

# PHASE SPACE PAINTING OF A SELF-CONSISTENT DANILOV DISTRIBUTION IN THE SNS RING \*

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## Abstract

Self-consistent beams that linearize space charge have been a staple of accelerator physics since Kapchinskii and Vladimirskii first proposed their distribution in the 1950's [1]. Danilov *et al.* demonstrated a procedure to construct similar distributions producing a linear space charge force in two and three dimensions [2]. The case denoted {2,2} is paintable in the SNS ring. The matched solution in a ring is a uniformly filled eigenmode of the ring single-turn matrix, including the linear space charge force [3]. Because of their uniform charge density and low intrinsic emittance, such a beam could help mitigate space charge limitations in rings, minimize halo growth, or increase collider performance [4]. This work describes on-going experiments in the Spallation Neutron Source (SNS) ring to paint a so-called *Danilov Distribution* using a method we've termed *eigenpainting*. We present a procedure for tuning injection parameters with the inclusion of newly installed solenoids, and show preliminary results.

## INTRODUCTION

The {2,2}-Danilov distribution, or simply *the Danilov Distribution*, is a uniformly-filled, 2D, rotating disk of charge described in [2] with vanishing 4D emittance. In a coupled ring the matched solution is simply the eigenvector of the single turn matrix describing the lattice, including the space charge of the beam [3]. Uniform charge density linearizes space charge making space charge effects more analytically tractable, reduces terms that may drive halo formation and minimizing tune shift and spread. This offers a possible mechanism for overcoming space charge limits in rings.

A vanishing 4D emittance implies advantages beyond those due to linear space charge, particularly for colliders. A beam with vanishing emittance that can be transformed into a flat distribution using a round-to-flat transformer as described by Derbenev [5] could have multiple benefits. Burov explored this and other ideas in relation to circular modes in detail [4].

Using realistic ORBIT simulations to optimize injection, and make predictions about the attainable beam quality given at the SNS, Holmes *et al.* explored the technical challenges

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associated with painting a Danilov distribution [6]. They determined that solenoids providing roughly 0.6 T·m of integrated longitudinal field would create the coupling needed to paint a Danilov distribution with acceptable uniformity and an emittance ratio between modes of roughly 15-20.

Because painting the Danilov distribution requires painting into one eigenmode of a coupled lattice, we call the scheme developed for this work *eigenpainting*, to differentiate from 'correlated' or 'anti-correlated' schemes named for the relationship between two modes during painting. This paper will describe the configuration of the SNS ring for eigenpainting, the procedure for establishing the injection parameters, and preliminary results from our first attempt at eigenpainting in the fully-coupled SNS ring.

## THE SNS RING AND RING TO TARGET BEAM TRANSPORT LINE

The SNS ring is a 248 m, 1.0 GeV accumulation ring, with the exception of injection and extraction kickers all magnets are DC. Nominal beam intensity is 24 uC, or 1.5e14 protons, per pulse. Accumulation takes place over  $\approx 1$  ms, or 1000 turns, during which time H- ions from the linac are accumulated via charge exchange injection facilitated with a thin diamond foil.

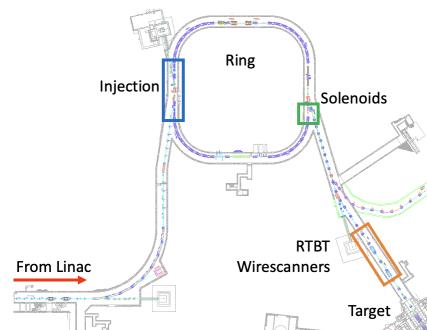


Figure 1: Relevant areas of the SNS.

Injection is fixed on the physical location of the foil, in the second magnet of a fixed chicane bump. During accumulation the position and angle of the closed orbit at the injection point (the foil) can be controlled with eight independently programmable, time-varying 'kicker' magnets [7] (4 horizontal, 4 vertical) located outside of the injection chicane. The distance of the injection point from the closed orbit is increased as the square root of time from injection start to uniformly fill the x and y phase space ellipses by correlated painting.

At full energy the kickers are not strong enough to put the closed orbit onto the foil, a requirement for painting a Danilov distribution. Thus, experiments are carried out at

a lower kinetic energy than the 1.0 GeV design. Homles *et al.* assumed an energy of 600 MeV, but the kicker power supplies have since been upgraded as part of the Proton Power Upgrade [8], and current experiments use 800 MeV. A lower energy is preferable but limitations in the timing system hardware set a threshold for complexity at around 800 MeV.

The coupling required to affect non-trivial eigenpainting of a Danilov distribution is achieved by setting the uncoupled lattice tunes equal to one another. However, the distribution is not stable in this configuration if any non-linearities are present. Holmes *et al.* showed that effect of space charge is to break the degeneracy, even at moderate intensities, but it is not sufficient for quality injection.

To address this, two solenoids designed and built by Stan-genes Industries Inc. [9] were installed in the SNS ring in late 2022, in the extraction straight downstream of the extraction Lambertson for these experiments, see fig. 1. Powered in series by two 156 A Amatek SGX series power supplies, these solenoids provide a combined 0.6 Tm at 275 A.



Figure 2: Solenoids installed in the SNS ring.

The ring is equipped with 44 BPMs that are the primary diagnostic for injection tuning. No profile information is currently available in the ring, though upgrades to a non-destructive electron scanner should be complete soon. Wirescanners in the Ring to Target Beam Transport (RTBT) line provide profile information to evaluate distributions. Beam can be extracted at any point during accumulation to get a sense of how the distribution evolves.

## EXPERIMENTS AT THE SNS

Experiments at the SNS have been conducted in several stages. Prior to the installation of the solenoids, we focused on injecting beams into the ring with equal tunes to understand the tuning challenges. These experiments are described in detail in [12]. Here we report on progress since the installation of solenoids in the ring.

### Injection Tuning With Solenoids

We start with ring tunes  $\nu_x = \nu_y = 6.177$  near the SNS nominal operating values of  $\nu_x = 6.21$ ,  $\nu_y = 6.16$ . Painting

requires determining the voltage for each of eight kickers for four closed orbit bumps defining,  $t_0$  and  $t_{paint}$  in each plane, injecting beam onto the closed orbit and the envelope of the desired Danilov distribution respectively. For the equal tune + solenoid case, the model shows eigenvectors near  $x=y'$  and  $x=-y'$  at the injection, with small modifications to the other two coordinates. We use the approximate  $r = (a * x, \delta_x, \delta_y, y')$  for discussion, where  $a$  is a scaling factor, and the  $\delta$  are small corrections near 0, and  $v$  should be normalized. The eigenpainting path is then just  $A(t)r$ , where  $A(t) = A_{max} \sqrt{t/t_{paint}}$ . As a first approximation we assume  $\delta_x = \delta_y = 0$

Because the online model used for SNS tune-up does not currently support a fully-coupled parameterization we establish the injection parameters with the solenoids off, and then turn them on to verify that the kicker settings put beam into one mode. Using a single pulse from the linac stored for in the ring, TBT data is fit to a simplified Gaussian damped model used by Pelaia [10]:

$$x(n) = ae^{-\gamma n^2} \sin(2\pi\nu n + \phi) + c \quad (1)$$

Where  $n$  is the turn number,  $a$  amplitude,  $\gamma$  a damping term related to the tune spread,  $\nu$  the tune,  $\phi$  the initial phase, and  $c$  the offset.

We verify that the closed orbit does not change as the solenoids field is increased, flattening the orbit as needed, then reduce the field to 0 for tuning. Fitting 1 to TBT data at each BPM we use a calibrated linear model to extract the phase and amplitude of the beam on the first turn, then transport back to the foil and average to get the injection parameters. The online model is then used to calculate updated kicker settings for the desired coordinates. To establish the  $t_0$  bumps, injection onto the closed orbit, we minimize TBT oscillation. The flat ( $x'=0$ )  $t_{paint}$  horizontal bump is straightforward, as this is the standard bump for SNS operations. The amplitude can be tuned by scaling the horizontal magnets using appropriate coefficients. The  $t_{paint}$  vertical bump requires  $y' \neq 0$  at the foil, for which vertical kicker 2 provides the largest deflection, limiting the size of the beam we can paint. Maximizing kicker 2 output, the remaining three kickers are tuned to close the bump.

Once the  $t_{paint}$  vertical settings that maximize  $y'$  have been established the amplitude of the horizontal bump is varied to minimize beating between modes 1 and 2 - i.e. we assume  $\delta_x = \delta_y = 0$ . Here we did this optimization by hand, but we have developed software to automate the process using:

$$x(n) = e^{-\gamma_1 n^2} (a \cos(2\pi\nu_1 n) + b \sin(2\pi\nu_1 n)) + e^{-\gamma_2 n^2} (c \cos(2\pi\nu_2 n) + d \sin(2\pi\nu_2 n)) + f \quad (2)$$

Where  $n$  is turn number,  $\nu_j$ ,  $\gamma_j$ , are mode tunes and damping rates respectively, the amplitudes  $a, b, c, d$  determine the phase and amplitude of the oscillation. Because the optics are fully coupled, both planes share tunes and damping rates.

Figures 3 and 4 show the horizontal and vertical TBT data averaged over 10 pulses from BPM B10 after maximizing the  $y'$  component of injection, both before and after optimizing the x bump amplitude by hand. The fits were done off-line using a generalization of [10] to coupled optics with eqn. 2. Each mode is plotted independently to show contributions to the observed oscillation. After tuning the amplitude of the x bump, the beating has been visibly reduced, and the amplitude of mode 1 oscillation has decreased by roughly a factor of 3, as measured by comparing the ratio of mode 1 to mode 2 amplitudes before and after optimization for each BPM. A slight change in the offset,  $f$ , indicates the at least one of the bumps could be closed better.

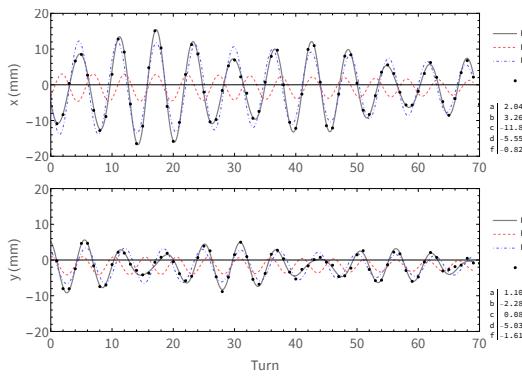


Figure 3: TBT data, solenoids on, no bump optimization.

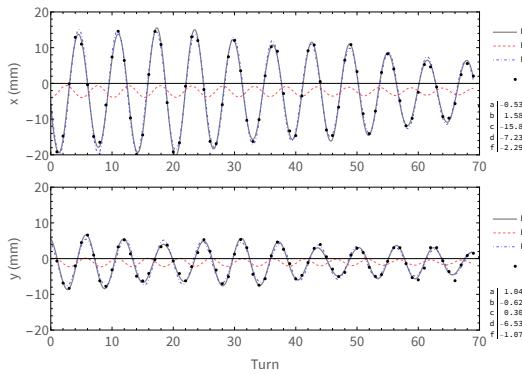


Figure 4: TBT data, solenoids on, bump optimized.

Global parameters were fit once on the un-optimized data using a non-linear optimizer. The result was used to determine the local values for both cases using linear least squares. Measured tunes and damping rates are:  $\nu_1 = 0.195$ ,  $\nu_2 = 0.158$ ,  $\gamma_1 = 0.000137$ ,  $\gamma_2 = 0.000157$ .

### Measuring the Accumulated Beam

Tuning the optics allows reconstruction of the full 4D matrix of second order moments in the RTBT [11], providing intrinsic emittances of the beam. Figure 5 shows wiresscans from each plane x, y, and  $u=45^\circ$  of beam accumulated over 600 turns (5.5E13 ppp) using the hand-tuned bump from 4. For comparison, guassian and half-elliptical traces with the same RMS width as the profile are overlaid. Several of the

profiles are a closer match to the elliptical form, indicating a uniformly filled ellipse. Depending on the phase advance of each of the eigenvectors from the ring to the measurement location, a mix of Gaussian and elliptical projections is expected from a scheme uniformly filling one mode, with the second mode occupation determined by the linac beam. Further analysis is needed to verify that observed profiles are consistent with the painting scenario carried out in this experiment.

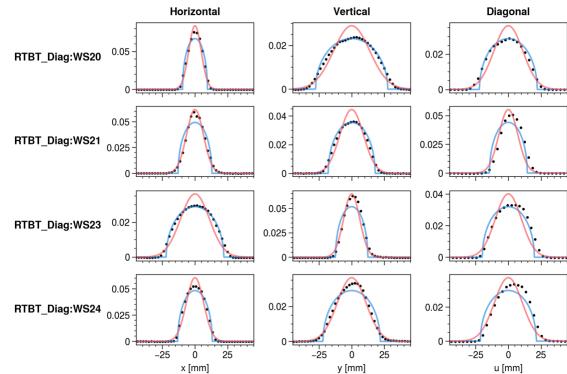


Figure 5: RTBT wiresscans of accumulated beam, optimized injection.

Compared to previous experiments [12], several profiles are more elliptical, but emittance ratios are similarly far from the best simulation. Reconstructed values in  $mm \cdot mrad$  are:  $\epsilon_1 = 11.5$ ,  $\epsilon_2 = 4.5$ ,  $\epsilon_x = 6.4$ ,  $\epsilon_y = 10.1$ . We suspect this is due to the stringent requirements on the phase space painting path required when solenoids are included and modes well-defined. A new procedure to minimize the content of one mode during  $t_{paint}$ -bump tuning that relies on a fast, robust fit of local parameters and allowing small variation of  $x'$  and  $y$  should help refine injection.

## CONCLUSION

We discussed recent upgrades to the SNS ring to support eingenpainting of a Danilov distribution. We have presented the outline of a procedure to tune eigenpainting in a fully-coupled ring lattice, and shown preliminary results demonstrating injection primarily into one mode. The results shown here are still significantly worse than predicted by ORBIT simulations, but issues were identified, and solutions to improve tuning presented.

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