GYRORESONANT ACCELERATION OF ELECTRONS BY AN AXISYMMETRIC TRANSVERSE ELECTRIC FIELD

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Electron Dynamics

Classical Cyclotron Motion

$$\frac{d\vec{v}}{dt} = -\frac{e}{m_e}\,\vec{v}\times\vec{B} \quad\Rightarrow\quad \Omega_{c0} = \frac{e\,B_0}{m_e}$$

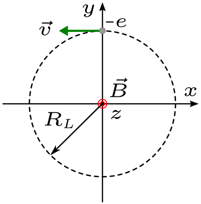
Resonant Interaction

$$\frac{d}{dt}(\gamma\,\vec{v}) = -\frac{e}{m_e}\left[\vec{E} + \vec{v}\times\vec{B}\right] \;\Rightarrow\; \Omega_c = \frac{e\,B_0}{m_e\,\gamma}$$

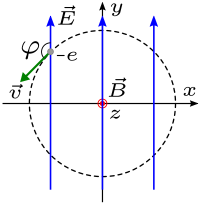
ECR Condition: $\Omega_c = \omega \;\Rightarrow$ **Acceleration Band:** $\frac{\pi}{2} \leq \varphi \leq \frac{3\,\pi}{2}$

Temporal Autoresonance

$$\omega = \Omega_c = \frac{e\,B(t)}{\gamma m_e}$$

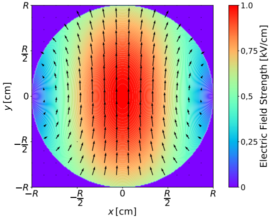


$\vec{B} = B_z\;\hat{\mathrm{k}}$

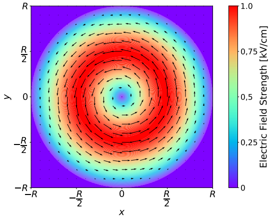


$\vec{B} = B_z\;\hat{\mathrm{k}}$ and $\vec{E} = E_0\,\cos(\omega t)\;\hat{\mathrm{j}}$

Gyroresonant Acceleration (Gyrac)



Cylindrical Mode TE_{111}



Cylindrical Mode TE_{011}

Gyrac Model

Considering:

$$\vec{E} = E_0 \left[\sin(\varphi)\, \hat{r} + \cos(\varphi)\, \hat{\theta} \right] \text{ and } \vec{B} = B_0 \left[1 + b(t) \right] \hat{k}$$

Energy and phase-shift evolution:

$$\dot{\gamma} = -\, g_0 \left(1 - \frac{1}{\gamma^2} \right)^{\frac{1}{2}} \cos(\varphi)$$

$$\dot{\varphi} = \left[b(\tau) - (\gamma - 1) \right] \frac{1}{\gamma} + g_0 \left(\gamma^2 - 1 \right)^{-\frac{1}{2}} \sin(\varphi)$$

Gyrac Regyme: $b(\tau) = \alpha\, \tau \;\Rightarrow\; \alpha \leq 1{,}19\, g_0^{\frac{4}{3}}$ where $g_0 = -\dfrac{E_0}{B_0 c}$ and $B_0 = \dfrac{\omega m_e}{e}$

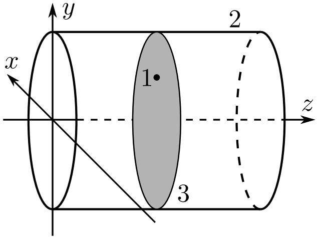
Cylindrical Mode TE_{011}

$$\vec{E}^{\,\mathrm{hf}}(\vec{r}, t) = \frac{E_0}{J_1(p_{01})} J_1\left(\frac{q_{01}}{R} r \right) \sin\left(\frac{\pi}{L} z \right) \cos(\omega t)\, \hat{\theta}$$

$$\vec{B}^{\,\mathrm{hf}}(\vec{r}, t) = \frac{E_0}{J_1(p_{01})} \left[\frac{\pi}{L\,\omega} J_1\left(\frac{q_{01}}{R} r \right) \cos\left(\frac{\pi}{L} z \right) \sin(\omega t)\, \hat{r} - \frac{q_{01}}{R\,\omega} J_0\left(\frac{q_{01}}{R} r \right) \sin\left(\frac{\pi}{L} z \right) \sin(\omega t)\, \hat{k} \right]$$

where $q_{01} = 3{,}83171$, $p_{01} = 1{,}84118$, $R = 7{,}84$ cm, $L = 20$ cm, $E_0 = 1$ kV/cm and $f = 2{,}45$ GHz.

Physical Scheme and Simulation Model



Physical scheme: (1) Electron injection point, (2) Cylindrical Cavity and (3) Cross section $z = L/2$.

Electromagnetic Field

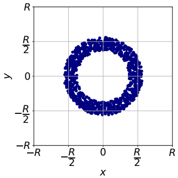
$\boxed{\text{Cylindrical Mode } TE_{011}} \Rightarrow \boxed{\vec{E} = \vec{E}^{\,hf} \text{ y } \vec{B} = \vec{B}^{\,hf} + \vec{B}^{ext}}$

Simulation Model

- Gyrac Model: Runge-Kutta Fourth Order Method.
- 2D Relativistic Newton-Lorentz equation: Boris integrator.

Numerical experiments

1. An electron is released from rest at point 1 using a set of α parameters.
   $\alpha = \{1.0 \times 10^{-4}, 1.5 \times 10^{-4}, 2.0 \times 10^{-4}, 2.5 \times 10^{-4}, 2.75 \times 10^{-4}, 3.0 \times 10^{-4}\}$
2. Particle System: Ring-like electron injection from rest using said set of α parameters.



Results

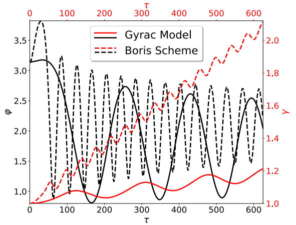


Fig 1: Evolution of γ and φ.

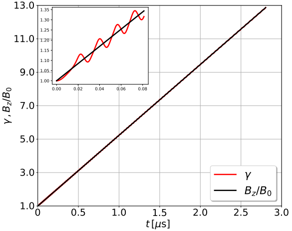


Fig 2: Evolution of γ and $B_z\,/\,B_0$.

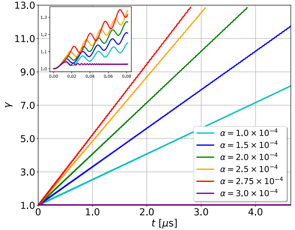


Fig 3: Evolution of γ for different α parameters.

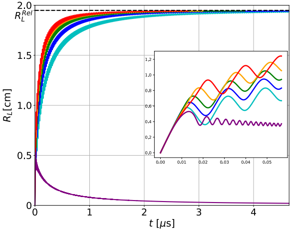


Fig 4: Evolution of R_L for different α parameters.

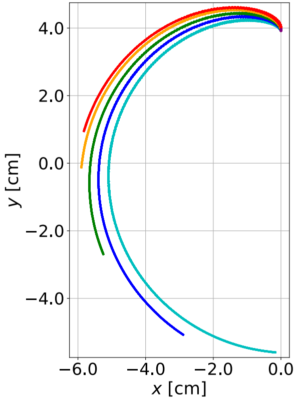


Fig 5: Guide center trayectory.

Results

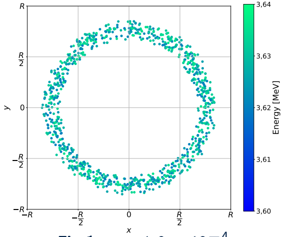


Fig 1: $\alpha = 1{,}0 \times 10^{-4}$

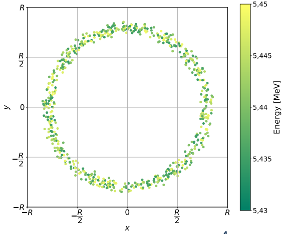


Fig 2: $\alpha = 1{,}5 \times 10^{-4}$

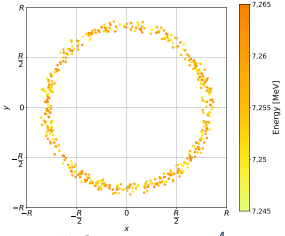


Fig 3: $\alpha = 2{,}0 \times 10^{-4}$.

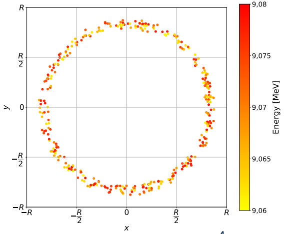


Fig 4: $\alpha = 2{,}5 \times 10^{-4}$.

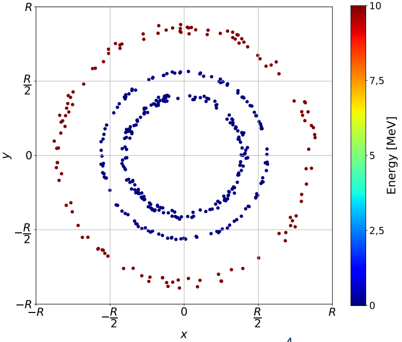


Fig 5: $\alpha = 2{,}75 \times 10^{-4}$

Evolution of the particle systems after 4.65 μs for different α parameters.

Conclusions

- It was showed by numerical experiments that it is possible to accelerate electrons under electron cyclotron resonance conditions in time-varying magnetic fields using the TE_{011} cylindrical mode.
- A set of values for α parameter that allow to maintain the resonance condition over time was determined.
- It was found that there is a region ring-like $(3R/8 < r < 9R/16)$ where the electrons are captured in the autoresonance regyme.

Future Works

We will study the 3D dynamic of an electrons cloud in magnetic fields varying in time using the cylindrical mode TE_{011}.

References

V P Milant'ev.
Cyclotron autoresonance—50 years since its discovery.
Physics-Uspekhi, 56(8):823, 2013.

K. S. Golovanivsky.
The gyrac: A proposed gyro-resonant accelerator of electrons.
IEEE Transactions on Plasma Science, 10(2):120–129, June 1982.

J.M.M. Pantoja.
Ingeniería de microondas: técnicas experimentales.
Prentice práctica. Pearson Educación, 2002.

C.K. Birdsall and A.B. Langdon.
Plasma Physics via Computer Simulation.
Series in Plasma Physics and Fluid Dynamics. Taylor & Francis, 2004.