ISBN: 978-3-95450-214-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2021-TUPAB062

EXPEDITING APS-U LONG-TERM PARTICLE TRACKING WITH ARBITRARY ORDER TAYLOR MAP*

Yipeng Sun†, ANL, Argonne, IL 60439, USA

Abstract

Truncated power series algebra was integrated within explicit symplectic integration to formulate an arbitrary order multivariate Taylor map for any given particle accelerator lattice. Tracking simulation performed with these Taylor maps shows good long term stability and physics accuracy. There is good agreement in long term particle tracking simulations between Taylor map and element by element tracking of APS-U lattice, when the particle is within 1 to 10 sigma of stored beam. It is demonstrated that most of the lower order resonance driving terms, plus chromatic and geometric aberrations are reasonably preserved by the Taylor map approach. Last but maybe most important, the computation time is reduced by a factor of 20 to 50, when compared to symplectic integration based element by element tracking.

INTRODUCTION AND MOTIVATION

There are trade offs in particle tracking simulations, which may include the following competing and conflicting items: accuracy of the physics model; symplecticity for long term tracking stability; required and available computation time. It seems that the "one-turn Taylor map" may be a good candidate [1] for particle tracking simulations, which may be much faster than element by element symplectic integration, and also much more accurate than individualized linear matrix [2]. The Taylor map approach may benefit global particle tracking simulations without much local interest, i.e., collective effects simulations. For other cases like lifetime calculations, local momentum aperture is required, and "one-turn Taylor map" may not be suitable under that situation.

In general, there are two approaches to generate a multivariate Taylor map. The approximated Taylor map may be numerically calculated from initial and final particle coordinates, as multivariate partial derivatives for arbitrary order and number of variables. Exact Taylor map with known functions form may be derived with differential algebra (DA), or truncated power series algebra (TPSA) [3], which is introduced for particle accelerator physics field in 1989 [3]. Recently algorithms have been developed by the author, employing both of these approaches.

Assume that there are m variables, the final variable vector $\mathbf{X}$ may be represented by a multivariate power series, in terms of the initial value of these m variables [1, 3, 4].

$$\mathbf{F}(\mathbf{X}) = \sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\dots\sum_{j_m=0}^{\infty}\mathbf{C}_{j_1,j_2,\dots,j_m}\prod_{h=1}^{m}x_h^{j_h}. \quad (1)$$

The coefficients vector $\mathbf{C}_{j_1,j_2,\dots,j_m}$ may be calculated through the partial derivatives of the function $\mathbf{F}(\mathbf{X})$.

$$\mathbf{C}_{j_1,j_2,\dots,j_m} = \frac{1}{j_1!j_2!\dots j_m!}\cdot\frac{\partial^{j_1+j_2+\dots+j_m}\mathbf{F}}{\partial^{j_1}x_1\partial^{j_2}x_2\dots\partial^{j_m}x_m}, \quad (2)$$

where it is assumed that the function $\mathbf{F}(\mathbf{X})$ is infinitely differentiable to $\mathbf{X}$ with known functions form.

For a truncation order of k on the Taylor map, the total number of independent monomials is given by the binomial coefficient $C(m+k,m)$, as shown below

$$C(m+k,m) = \binom{m+k}{m} = \frac{(m+k)!}{k!m!}. \quad (3)$$

For 6D particle tracking simulations with six coordinates, the count of total monomials as a function of TPSA truncation order k is shown in Fig. 1.

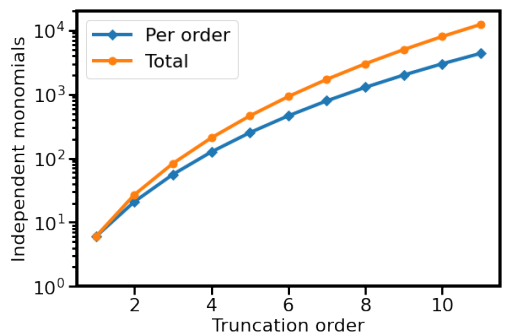

Figure 1: Count of total monomials as a function of TPSA truncation order. At TPSA truncation order 10, there are 8007 monomials for each of coordinates in (x, x', y, y', z).

PYTHON CODE FOR LATTICE AND TRACKING SIMULATIONS

To calculate and properly index all the Taylor map coefficients $\mathbf{C}_{j_1,j_2,\dots,j_m}$ for an accelerator lattice, it is necessary that TPSA algorithms need to be integrated with symplectic integration process. A Python accelerator lattice calculation and particle simulation code (with symplectic integration) was previously developed by the author. Python framework was selected as it provides object-oriented feature and many useful libraries. Symplectic integration may be performed

* Work supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-06CH11357.
† yisun@anl.gov

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up to 8th order. APS-U lattices [5] are converted from ELEGANT [2] format, to this Python code.

Linear optics parameters in one nominal APS-U sector is shown in Fig. 2, as calculated using this Python code, which agree well with the ELEGANT simulation [5].

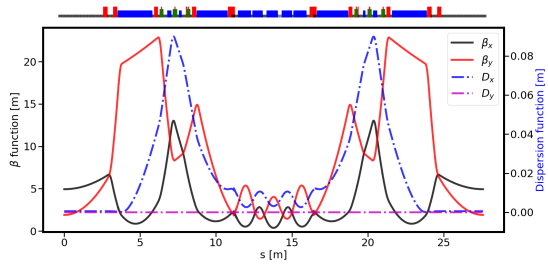

Figure 2: Linear optics parameters in one nominal APS-U sector. Blue blocks denote dipole magnets, red blocks quadrupole magnets, green blocks sextupole magnets, and gray blocks drift spaces.

TAYLOR MAPS FOR APS-U LATTICE

After integrating TPSA algorithms in symplectic integration process, it is possible to calculate arbitrary order Taylor maps for any accelerator lattice, using this Python code. As a first example, shown in Fig. 3 are the monomials index and coefficients for x with a fourth order Taylor map, as calculated for APS-U lattice. The monomials index denotes the power terms of six coordinates $(x, x', y, y', z, \delta)$.

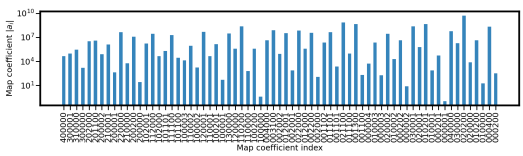

Figure 3: Monomials index and coefficients for x with the fourth order Taylor map, as calculated for APS-U lattice.

Similarly, shown in Fig. 4 are the sorted monomials index and coefficients for all five coordinates (x, x', y, y', z), with a 7th order Taylor map, as calculated for APS-U lattice.

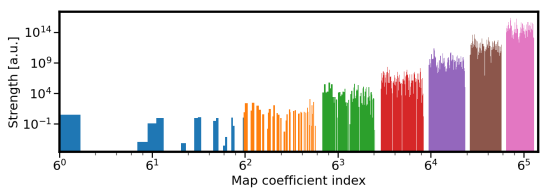

Figure 4: Monomials index and coefficients for all coordinates of (x, x', y, y', z), with the 7th order Taylor map, as calculated for APS-U lattice. Each color denotes a different order.

STABILITY AND ACCURACY OF TAYLOR MAPS FOR APS-U LATTICE

Long term stability is important for collective effects simulations of APS-U lattice. The tracking simulation is preferred to be symplectic. Although Taylor map tracking is not symplectic by design, it may work under quasi-symplectic condition as discussed below. Long term stability tracking simulations are performed with 7th order Taylor map of APS-U lattice, as shown in Fig. 5. For one particle with initial coordinate: x = 150 µm, y = 100 µm and Δp/p = 0.003, there is negligible artificial damping/excitation in 30k turns. For one particle with initial coordinate of x = 12 µm, y = 10 µm and Δp/p = 0.001, again one observes negligible artificial damping/excitation in a longer term of 300k turns.

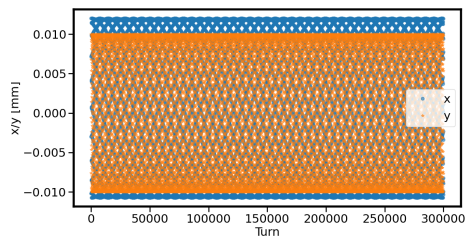

Figure 5: Long term stability tracking simulations with 7th order Taylor map of APS-U lattice. Initial coordinate: x = 12 µm, y = 10 µm and Δp/p = 0.001, with negligible artificial damping or excitation in 300k turns.

Accuracy is yet another figure of merit for particle tracking simulations. Taylor map needs to be a good approximation of the corresponding element by element tracking simulations with symplectic integration. Shown in Fig. 6 is the accuracy of 10th order Taylor map of APS-U lattice, as compared to the element by element tracking simulations with symplectic integration. It is observed that for a particle at one sigma of stored beam, the relative deviation is under 10^{-6}, when tracking is performed for 10k turns. This seems to be acceptable for long term tracking simulations of APS-U collective effects.

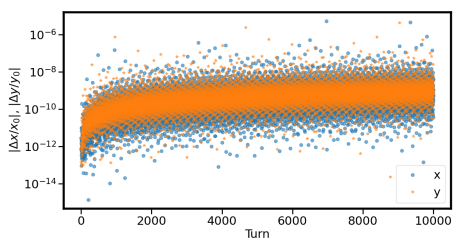

Figure 6: Accuracy of 10th order Taylor map of APS-U lattice, as compared to the element by element tracking simulations with symplectic integration. Initial coordinate: x = 10 µm, y = 10 µm and Δp/p = 0.001, tracking of 10k turns.

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BENCHMARK ON TUNE SHIFT AND FREQUENCY MAP ANALYSIS

Benchmark study is performed on tracking simulations with Taylor map of APS-U lattice, and element by element tracking of symplectic integration. As shown in Fig. 7, there is good agreement on tune shift with horizontal amplitude x, and also for chromatic tune shifts.

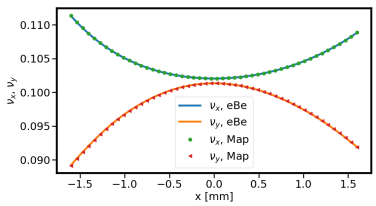

Figure 7: Benchmark on tune shift between tracking simulations with Taylor map of APS-U lattice, and element by element tracking of symplectic integration.

As shown in Figs. 8, 9 and 10, there is good agreement on frequency map analysis (FMA) between tracking simulations with Taylor map of APS-U lattice, and element by element tracking of symplectic integration. This is an illustration in features of Taylor map approach. It demonstrates that most of the lower order resonance driving terms, plus chromatic and geometric aberrations may be correctly preserved by the Taylor map approach. This preservation is valid for transverse amplitudes up to 1 mm and energy offset up to 0.005, at least in a short term of hundreds of turns.

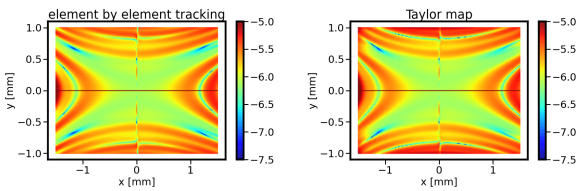

Figure 8: $\Delta p/p = 0.0$, on momentum frequency map analysis (FMA) comparison for APS-U lattice. Left: element by element tracking of symplectic integration; Right: tracking simulations with 7th order Taylor map.

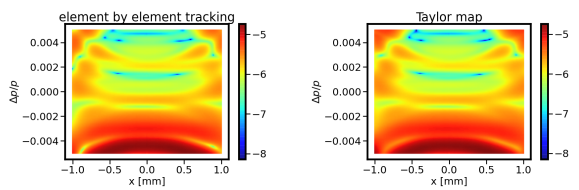

Figure 9: $\Delta p/p$-x frequency map analysis (FMA) comparison for APS-U lattice. Left: element by element tracking of symplectic integration; Right: tracking simulations with 7th order Taylor map.

Benchmark study is performed on computation time between tracking simulations with Taylor map of APS-U lattice, and element by element tracking with symplectic integration.

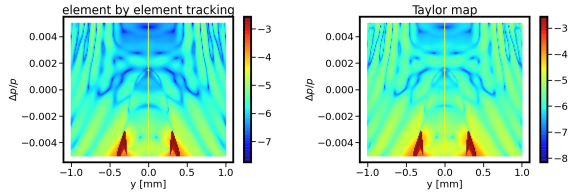

Figure 10: $\Delta p/p$-y frequency map analysis (FMA) comparison for APS-U lattice. Left: element by element tracking of symplectic integration; Right: tracking simulations with 10th order Taylor map.

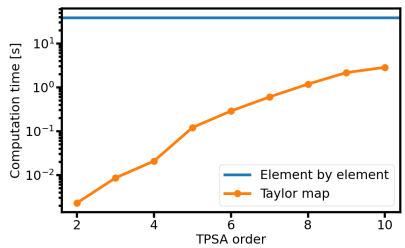

Figure 11: Benchmark on computation time between tracking simulations with Taylor map of APS-U lattice, and element by element tracking with symplectic integration.

Tracking is performed for 20 turns. As shown in Fig. 11, using a 7th order Taylor map of APS-U lattice, the computation time is reduced by a factor of 50, when compared to element by element tracking. For 10th order Taylor map of APS-U lattice, the computation time is reduced by a factor of 20.

ACKNOWLEDGMENTS

Several *Python* libraries are employed for development of the accelerator lattice and simulation code. The figures are generated using the *matplotlib* libraries in the *Python* code. The work is supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-06CH11357.

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