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SPATIAL AUTORESONANT ACCELERATION OF ELECTRONS BY AN AXYSIMMETRIC TRANSVERSE ELECTRIC FIELD

O. Otero, E. A. Orozco*, Universidad Industrial de Santander, 680002 Bucaramanga, Colombia

Abstract

In this work, the autoresonance acceleration of electrons by an axisymmetric transverse electric field in the presence of a static inhomogeneous magnetic field is studied. Said acceleration scheme is known as Spatial Autoresonance Acceleration (SARA). The dynamics of electrons were determined through the numerical solution of the relativistic Newton-Lorentz equation by using a finite difference scheme. The inhomogeneous external magnetic field is generated from a three-coil system and it is calculated from the Biot-Savart law. The electrons describe a 3D motion in a cylindrical cavity excited with a TE_{011} mode, which is affected by a static magnetic field, whose axis coincides with the cavity axis. The magnetic field profile is fitted to keeps the phase-shift between the electron transverse velocity and the right-hand circular polarized electric field in the range $(\pi/2, 3\pi/2)$. It was shown that an electron injected longitudinally at the radial midpoint of the cavity can be accelerated up to an energy of 182 keV at an axial distance of 16 cm, using an electric field strength of 14 kV/cm and a frequency of 2.45 GHz.

INTRODUCTION

The resonant interaction between the transverse electromagnetic wave that propagates along a homogeneous magnetic field and a charged particle has been studied since the year 1962 by Kolomenskii and Davydovski [1, 2]. This phenomenon, known as autoresonance, consists of maintaining the equality between the frequency of the electromagnetic wave and the cyclotron frequency of electrons; and can be realized in different ways [3]. Some of these mechanisms are based on standing electromagnetic waves, as the case of the spatial auto-resonance acceleration (SARA), which uses an inhomogeneous external magnetostatic field that increases with a required rate to sustain the ECR condition along the path of the electrons [4, 5]. The design of a compact X-ray source based on the SARA mechanism was certified [6]. The SARA mechanism has been studied by using different microwave fields: the cylindrical TE_{11p} modes ($p = 1, 2, 3, \dots$) and the rectangular TE_{102} mode. In the present work, the numerical study of the spatial autoresonance acceleration of electrons accelerated by the microwave field in a TE_{011} cylindrical mode is presented, by using the single particle approximation.

SIMULATION MODEL

The autoresonance phenomenon with a standing electromagnetic wave consists of maintaining the equality between the frequency ω of said wave and the cyclotron frequency of the charged particles. For electrons in resonance conditions, the cyclotron frequency is written as

$$\Omega_c = \frac{eB}{m_e\gamma}, \tag{1}$$

where e and m_e are the electric charge and the rest mass of the electron, respectively; B is the magnetic field, $\gamma = (1-\beta^2)^{-1/2}$ is the relativistic factor with $\beta = v/c$ and c is the speed of light in vacuum. The resonance condition $\Omega_c = \omega$, can be satisfied through an increase in the magnetic field to compensate the increase in the Lorentz factor, γ. In the SARA scheme, the electrons are injected along the magnetic field axis, which is a function of the position. So the electron cyclotron frequency can be written as

$$\Omega_c(\vec{r}) = \frac{eB^c(\vec{r})}{m_e\gamma}, \tag{2}$$

where the superscript has been included to denote the external magnetic field produced by a set of coils. Choosing the z axis along the magnetic field axis, the z component along said axis can be written as

$$B_z^c(0, z) = B_0\left[\gamma_0 + b(z)\right], \tag{3}$$

where $B_0 = m_e\omega/e$ is the magnetic field for classical cyclotron resonance, γ_0 is the relativistic factor associated with the velocity of the particle at the injection point and $b(z)$ is a dimensionless function that determines the increase of the magnetic field. It is important point out that in this scheme of electrons acceleration, the diamagnetic force have an important role because it acts in the opposite direction to the growth of the magnetic field.

In this work, we study the spatial autoresonance acceleration of electrons due to its interaction with an axisymmetric microwave field, the cylindrical TE_{011} mode [7]; whose electric and magnetic components are written in cylindrical coordinates as

$$E_\theta^{hf}(\vec{r}, t) = \frac{E_0}{J_1(p_{01})} J_1(k_T r)\sin(k_z z)\cos(\omega t), \tag{4}$$

$$B_r^{hf}(\vec{r}, t) = \frac{E^0}{J_1(p_{01})}\frac{k_z}{\omega} J_1(k_T r)\cos(k_z z)\sin(\omega t), \tag{5}$$

$$B_z^{hf}(\vec{r}, t) = \frac{E^0}{J_1(p_{01})}\frac{k_T}{\omega} J_0(k_T r)\sin(k_z z)\sin(\omega t), \tag{6}$$

where E_0 is the amplitude of the electric field, $p_{01} = 1.84118$, $k_T = q_{01}/R$ being $q_{01} = 3.83171$ and R the radius of the cylindrical cavity and $k_z = \pi/L$ where L is the length of the cavity. The electric field, presented in Fig. 1, can be written at each point (r, θ, z) as the superposition of two circular polarized waves: the right-hand circular polarization

* eaorozco@uis.edu.co

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(RHP) and the left-hand circular polarization (LHP) [8–10]; given by the expressions

$$\vec{E}^{RHP} = \frac{1}{2} f(r, z) \left[-\sin(\omega t + \theta)\, \hat{\imath} + \cos(\omega t + \theta)\, \hat{\jmath} \right], \quad (7)$$

and

$$\vec{E}^{LHP} = \frac{1}{2} f(r, z) \left[\sin(\omega t - \theta)\, \hat{\imath} + \cos(\omega t - \theta)\, \hat{\jmath} \right], \quad (8)$$

respectively, where

$$f(r, z) = \frac{E_0}{J_1(p_{01})} J_1(k_T r) \sin(k_z z). \quad (9)$$

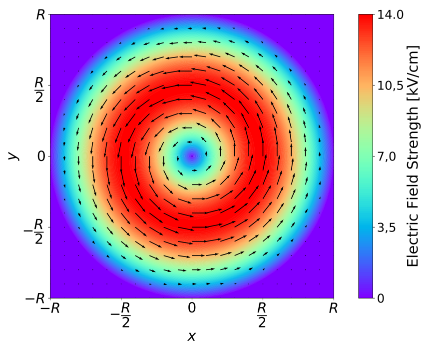

Figure 1: Electric field of TE_{011} cylindrical mode in the plane $z = L/2$.

In the present study, we only consider the interaction of the electrons with $\vec{E}^{RHP}$ because this component is the responsible for the energy transfer to the electrons [11].

The physical scheme for the spatial autoresonance acceleration is presented in Fig. 2. The microwave TE_{011} mode is excited in the cavity-1 with the frequency of 2.45 GHz and an amplitude of 14.0 kV/cm. The radius and the length of the cavity are 7.84 cm and 20.0 cm, respectively.

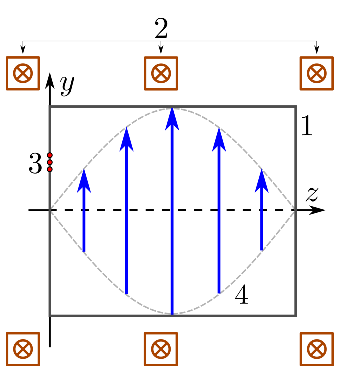

Figure 2: A physical model scheme for SARA realization: 1-cylindrical cavity, 2- set of current coils, 3-injection points, 4-longitudinal profile of the electric field.

The static inhomogeneous magnetic field is generated by a system of three currents coil-2; which are calculated from the superposition of circular loops and the Biot-Savart law. The magnetic field value in the particle position is calculated through the bilinear interpolation method. The parameters of the set coils are presented in Table 1; where R_{int}, R_{ext}, L_c and z are the inner radius, external radius, length and the position, respectively, and J is the coil current density. The two-dimensional profile of the magnetic field in units of $B_0 \approx 0.0876$ T is presented in Fig. 3.

Table 1: Magnetic Coil System Parameters

Coils	R_{int} cm	R_{ext} cm	L_c cm	J A/mm^2	z cm
1	9.84	15.84	6.0	2.48	−4.25
2	9.84	15.84	6.0	2.43	9.85
3	9.84	15.84	6.0	3.11	22.5

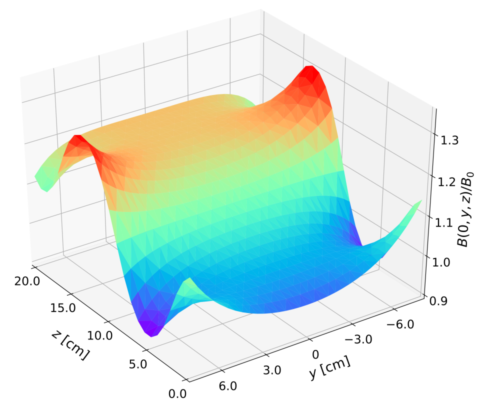

Figure 3: The profile of the magnetostatic field at the plane $x = 0$.

The motion of the electron is described by the relativistic Newton-Lorentz equation [12], which is expressed in a dimensionless form as

$$\frac{d\vec{u}}{d\tau} = \vec{g}_0 + \frac{\vec{u}}{\gamma} \times \vec{b}, \quad (10)$$

where $\vec{u} = \vec{p}/mc$ is the momentum of the electron, $\vec{g}_0 = -\vec{E}/B_0 c$ is the microwave electric field, $\vec{b} = -\vec{B}/B_0$ is the total magnetic field ($\vec{B} = \vec{B}^{hf} + \vec{B}^{c}$), $\tau = \omega t$ the time and $\gamma = \sqrt{1 + u^2}$. The Eq. (10) in a finite difference form is solved as described in [13].

RESULTS AND DISCUSSIONS

In the realized numerical experiments, the electrons are injected into the cavity in a direction parallel to the z-axis, through three points, P_1, P_2 and P_3, located at the radial distances $R/2$, $3R/8$ and $9R/16$, and injected with the energies of 3, 4 and 5 keV, respectively, see Fig. 2.

The Fig. 4 shows the trajectory for an electron injected at the point P_1 with the energy of 4 keV. The projection of the trajectory onto transverse planes are like-concentric-rings around the injection point. Additionally, it is possible to appreciate that in the region $5 < z < 15$ cm the Larmor radius grows, which is associated with the gain of the electron energy.

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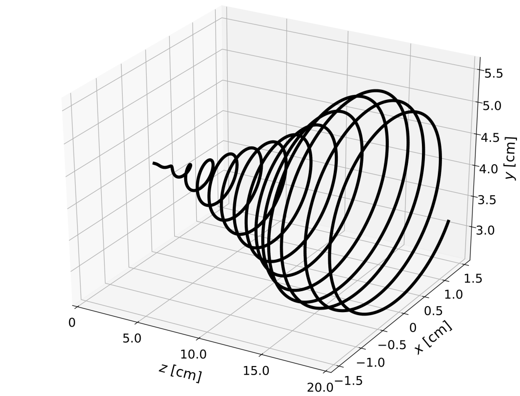

Figure 4: The helical trajectory of an electron accelerated through SARA mechanism.

The phase-shift between the RHP component of the electric field and the transverse electron velocity as a function of the z coordinate, for electrons injected with the energy of 4 keV is presented in the Fig. 5. The corresponding energies for said cases are presented in Fig. 6. We can note that in the region 0< z <5 cm, φ is in the acceleration band ($\pi/2 < \varphi < 3\pi/2$), see Fig. 5, however the electrons do not gain energy considerably, see Fig. 6, because on that region the strength of the electric field is still small, see Fig. 2.

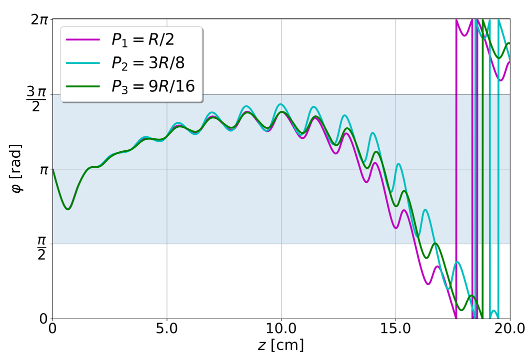

Figure 5: Phase-shift between the electric field and the transverse velocity of the electron for electrons injected at the points P_1, P_2 and P_3 as a function of z coordinate.

The electrons gain energy significantly on the region 5< z <15 cm for all injection points, reaching a maximum value about of 182 keV for the case of the electron injected from the point P_1, see Fig. 6. In the region 15< z <20 cm, the energy decrease due to the loos of the resonant condition which lead to the phase-shift outside of the acceleration band, see Fig. 5. We can note that the maximum difference in energy for the considered cases is about of 4%. The oscillations observed in Fig. 5 and 6 are generated because as an electron describes a cyclotron motion, the phase of the electric field changes not only in time but also with position (r, θ), see Eq. (8). This effect is negligible in the region z <5 cm because the Larmor radius is still small, but it is an significant effect as the Larmor radius increases.

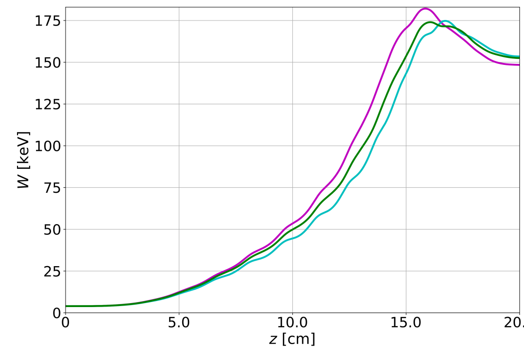

Figure 6: Energy of the electrons injected at the points P_1, P_2 and P_3 as a function of z coordinate.

The Fig. 7 shows the longitudinal velocity as a function of the z coordinate, for an electron injected with the energy of 4 keV from different injection points. The longitudinal velocity depends mainly on the diamagnetic force, which acts in the opposite direction to the growth of the magnetic field. For example, if we neglect the oscillations, we can see that in the range 5< z <15 cm v_z decreases, because in that region the magnetic field increases; while in the region 15< z <18 cm v_z increases because the magnetic field decreases. The oscillations are attributed to the radial magnetic component of the microwave field, whose direction at the electron position depends not only on time but also on the (r, θ) position as was described previously.

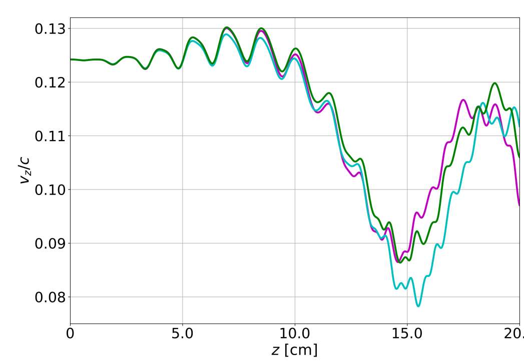

Figure 7: The longitudinal component of the electron for electrons injected at the points P_1, P_2 and P_3 as a function of z coordinate.

CONCLUSION

It was shown by numerical experiments that it is possible to accelerate electrons under electron cyclotron resonance conditions by using a microwave field TE_{011} cylindrical mode, in an inhomogeneous magnetostatic field. It was found an inhomogeneous magnetostatic field which maintains the electron acceleration regime close to the exact resonance condition along almost its entire trajectory. The used parameters of the physical scheme let a gain of energy about 45.5 times the injection energy.

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