New Method of Calculation of the Wake due to Radiation and Space Charge Forces in Relativistic Beams

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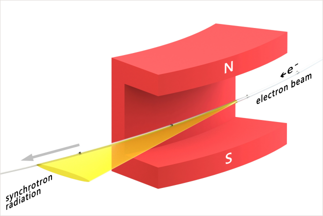
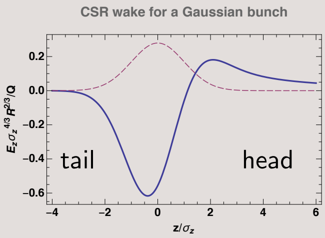
Introduction

• When the trajectory of a relativistic beam is bent by magnetic field, the beam radiates electromagnetic field and experiences a radiation reaction force (the CSR wake). This force can play an important role in beam dynamics. Studies of the CSR wake have a long history.

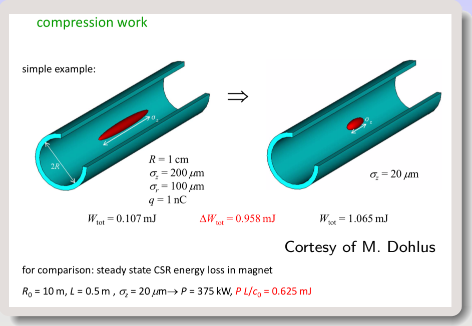


• There is a popular 1D model of the CSR wake valid for circular motion, as well as bends of finite length. The wake is especially important in bunch compressors where the peak beam current can exceed 1 kA, and the bunch length may be in the range of tens of microns or less.



Motivation

• In 2002, M. Dohlus from DESY made an observation that in a bunch compressor the energy radiated by a bunch can be smaller that the change in the electromagnetic energy of the bunch due to the compression. What kind of wakefield is responsible for the *compression work*?



• The compression also works in the transverse direction (e.g., due to the variation of the beta function). It exists in the limit $v = c$.

• Existing 3D CSR codes solve Maxwell's equations and, in principle, include all effects, but they are slow, and not easy to use.

We developed a novel computational technique that includes both the radiation wake field and the wake appearing when the bunch changes its shape.[1]

[1] Part of this work has bean developed earlier by this author in collaboration with D. Ratner.

Representation of the beam

The beam is represented by its charge density $\rho(\boldsymbol{r}, t)$ and velocity $\boldsymbol{v}(\boldsymbol{r}, t)$ with the beam current density $\boldsymbol{j}$ given by the product $\boldsymbol{j} = \rho\boldsymbol{v}$. [Note that we neglect the spread in velocity at each point due the the angular and energy spread in the beam.] For given functions $\rho(\boldsymbol{r}, t)$ and $\boldsymbol{j}(\boldsymbol{r}, t)$ we then use the *retarded potentials* to calculate the electric field in the beam, $\boldsymbol{E}(\boldsymbol{r}, t)$. The instantaneous energy change per unit time and *per unit charge* is

$$\mathcal{P}(\boldsymbol{r}, t) = \boldsymbol{v}(\boldsymbol{r}, t) \cdot \boldsymbol{E}(\boldsymbol{r}, t)$$

The result is expressed as an integral over the volume around the beam trajectory at preceding times $t' < t$. We loosely call $\mathcal{P}$ the *longitudinal wake*.

The longitudinal wake

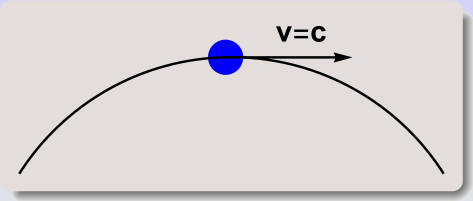
The wake $\mathcal{P}$:

$$\frac{1}{c}\mathcal{P}(\boldsymbol{r},t) =$$
$$-\int \frac{d^3r'}{|\boldsymbol{r}'-\boldsymbol{r}|}\left[\boldsymbol{\beta}(\boldsymbol{r},t)-(\boldsymbol{\beta}(\boldsymbol{r},t)\cdot\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret}))\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret})\right]\cdot\partial_{\boldsymbol{r}'}\rho(\boldsymbol{r}',t_{\rm ret})$$
$$+\int \frac{d^3r'}{|\boldsymbol{r}'-\boldsymbol{r}|}(\boldsymbol{\beta}(\boldsymbol{r},t)\cdot\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret}))\rho(\boldsymbol{r}',t_{\rm ret})\partial_{\boldsymbol{r}'}\cdot\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret})$$
$$-\int \frac{d^3r'}{|\boldsymbol{r}'-\boldsymbol{r}|}\rho(\boldsymbol{r}',t_{\rm ret})\boldsymbol{\beta}(\boldsymbol{r},t)\cdot\frac{1}{c}\partial_{t_{\rm ret}}\boldsymbol{\beta}(\boldsymbol{r}',t_{\rm ret}),$$

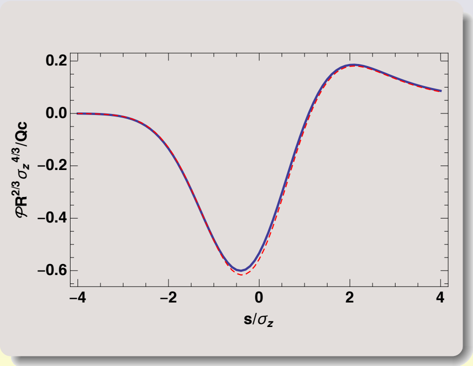
where $\boldsymbol{\beta}=\boldsymbol{v}/c$ and $t_{\rm ret}(\boldsymbol{r},\boldsymbol{r}',t)=t-|\boldsymbol{r}'-\boldsymbol{r}|/c$. The formula is valid for finite γ. If the variation of the velocity $\boldsymbol{v}$ within the bunch can be neglected, the last two integrals can be dropped.
For the known functions $\rho(\boldsymbol{r},t)$ and $\boldsymbol{v}(\boldsymbol{r},t)$ the integration can be carried out in 3D or in 2D (say, horizontal, and longitudinal).

CSR in circular motion - I

- 2D (disk-like) bunch is moving on a circle with $v = c$: a Gaussian charge distribution with $\sigma_x = \sigma_z = 1$ mm, circle radius $R = 1$ m. Only the first integral in $\mathcal{P}$ is used.

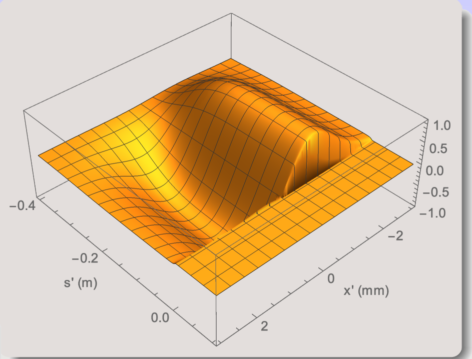


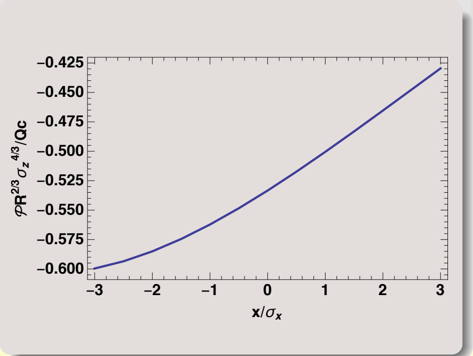
- The result (black solid curve) agrees very well with 1D CSR (red dashed line).



CSR in circular motion - II

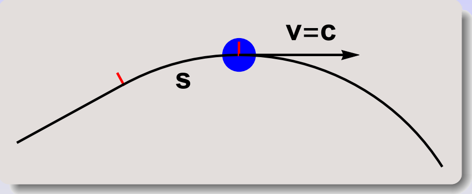
- Plot of the integrand as a function of s' (arc length) and x' (coordinate in the horizontal plane) for the observation point $x = s = 0$. Note different scales along s' and x' (the spread of the integrand along s is the *formation length* of the wake).



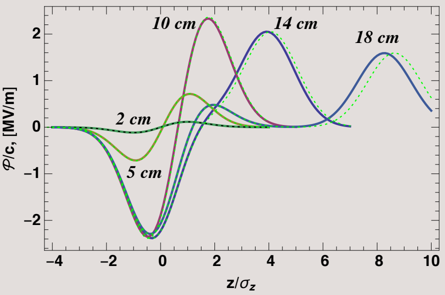


- Transverse dependence of the wake versus x at $s = 0$ ($x = s = 0$ is the center of the bunch) [this is important for the slice energy spread induced by CSR].

Entrance to a bend[2]



- 2D beam enters a bending magnet from a straight orbit: $R = 1.5$ m, $\sigma_x = \sigma_z = 50\,\mu$m, $Q = 1$ nC.

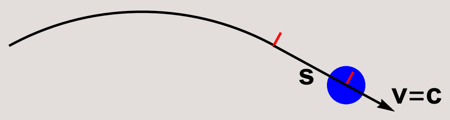


- The longitudinal wake on the axis of the beam at different distances from the entrance. The dashed lines are the result of 1D calculations.

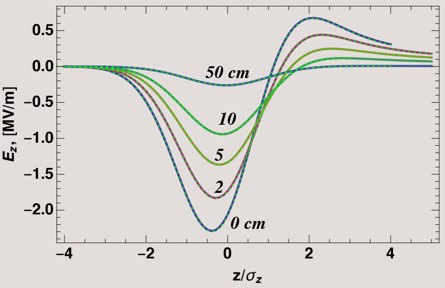
[2] A 1D model for this case was developed in: Saldin, Schneidmiller, Yurkov, NIMA, **A398**, 373, (1997); G. Stupakov and P. Emma, EPAC, p. 1479, (2002).

Exit from a bend

- 2D beam exits a bending magnet to a straight orbit: $R = 1.5$ m, $\sigma_x = \sigma_z = 50\,\mu$m, $Q = 1$ nC.



- The longitudinal wake on the axis of the beam at different distances from the exit. The dashed lines are the result of 1D calculations.



Linear optics and a Gaussian bunch (in 2D)

We assume a Gaussian distribution function at the beginning of the beam line:

$$F = \frac{N_b}{2\pi\epsilon}\frac{1}{2\pi\sigma_\eta\sigma_{z0}}\exp\left(-\frac{J}{\epsilon}-\frac{(\eta-hz)^2}{2\sigma_\eta^2}-\frac{z^2}{2\sigma_{z0}^2}\right)$$

J – the betatron action variable
η – relative energy deviation
ϵ – the emittance
h – the energy chirp

The charge density and velocity at coordinate s ($z = s - ct$)

$$\rho(x,z,s) = \frac{eN_b}{2\pi\Sigma}\exp\left(-\frac{A}{2\Sigma^2}\right), \qquad \beta_x(x,z,s) = \frac{B}{\Sigma^2}, \qquad \beta_z \approx 1$$

$$A = z^2\left(\beta_\mathrm{f}\epsilon + D^2\left(h^2\sigma_{z0}^2+\sigma_\eta^2\right)\right) - 2CDxz + Ex^2,$$
$$B = z\epsilon(\alpha D + \beta D')C + DD'\sigma_\eta^2\sigma_{z0}^2 x - \alpha x\epsilon E,$$
$$C = h^2 R_{56}\sigma_{z0}^2 + h\sigma_{z0}^2 + R_{56}\sigma_\eta^2,$$
$$E = (hR_{56}+1)^2\sigma_{z0}^2 + R_{56}^2\sigma_\eta^2$$

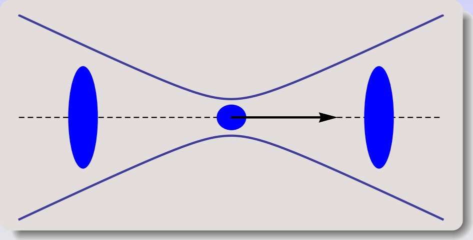
D, D' – dispersion and its derivative,
β_f – the beta function,
α – the alpha function
R_{56} – 5-6 element of the transport matrix

and

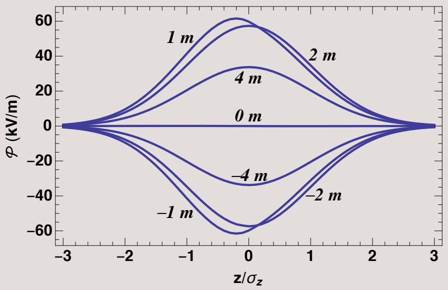
$$\Sigma^2 = D^2\sigma_\eta^2\sigma_{z0}^2 + \beta_\mathrm{f}\epsilon\left[(hR_{56}+1)^2\sigma_{z0}^2 + R_{56}^2\sigma_\eta^2\right].$$

Compression work in 3D

• A bunch moving in free space through a focal point is being compressed (before the focus) and decompressed (after the focus). No CSR in this case. We used a 3D integral to calculate $\mathcal{P}$.



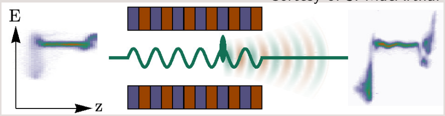
• Parameters of the orbit: $\beta_* = 1$ m, $\sigma_x^* = \sigma_y^* = \sigma_z = 50\,\mu$m, $Q = 1$ nC. The plot shows that wake on the axis of the beam at several locations along the path.



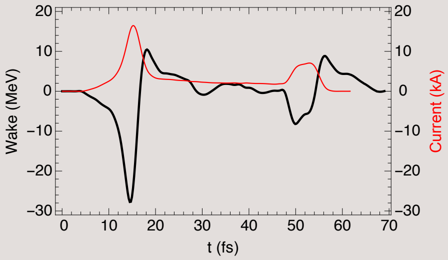
XLEAP wiggler wake[3]

- A short bunch propagates through a 6-period wiggler: $K = 51$, $Q = 200$ pC, $E = 3.9$ GeV, $\sigma_\perp \approx 72$ µm. The radiation wake of the current spike in the tail of the bunch modulates its central part. What is this wake? Is transverse size of the bunch is important?

Cortesy of J. MacArthur



- Bunch current distribution (courtesy of J. MacArthur) and the CSR wake generated in the wiggler. The wake is calculated with the 1D and 2D models. The transverse size of the beam does not play a role.



[3] See the poster by J. MacArthur et al., TUPRB098.

Summary

- In addition to the CSR wake (responsible for the energy loss of the beam that goes into radiation) there is a “space charge” force in relativistic beams even in the limit $\gamma \to \infty$. This force is responsible for the compression work.
- We worked out a formula for the longitudinal wake that includes both forces. To calculate the wake, one needs to do 3D/2D integrals along the beam orbit.
- For a bunch with a general Gaussian distribution function moving along a curvilinear path in free space, there is an analytical expression for the integrand in terms of the beam and orbit parameters.
- With the new algorithm, we have reproduced many known results from the literature. Our next step is to implement the new algorithm in a computer code.