



# Applications of Caustic Methods to Longitudinal Phase Space Manipulation

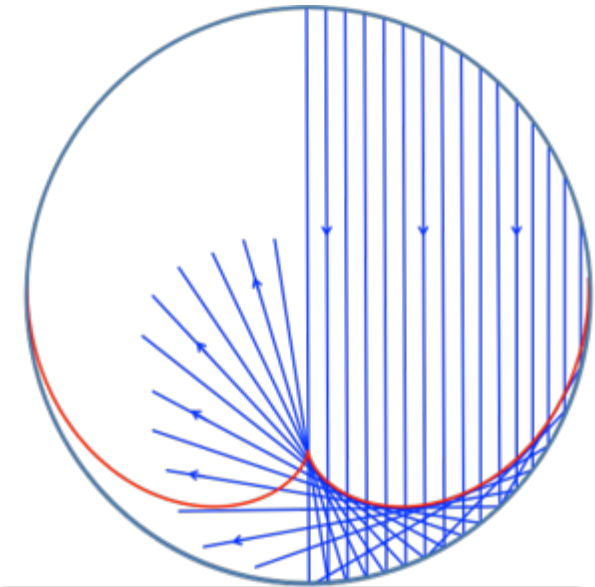
Tessa Charles<sup>1,2\*</sup>, Peter Williams<sup>3</sup>, Dave Douglas<sup>4</sup>

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2. CERN, Geneva, Switzerland
3. STFC Daresbury Laboratory, UK
4. Jefferson Lab, Newport News, Virginia, USA

\* [tessa.charles@cern.ch](mailto:tessa.charles@cern.ch)

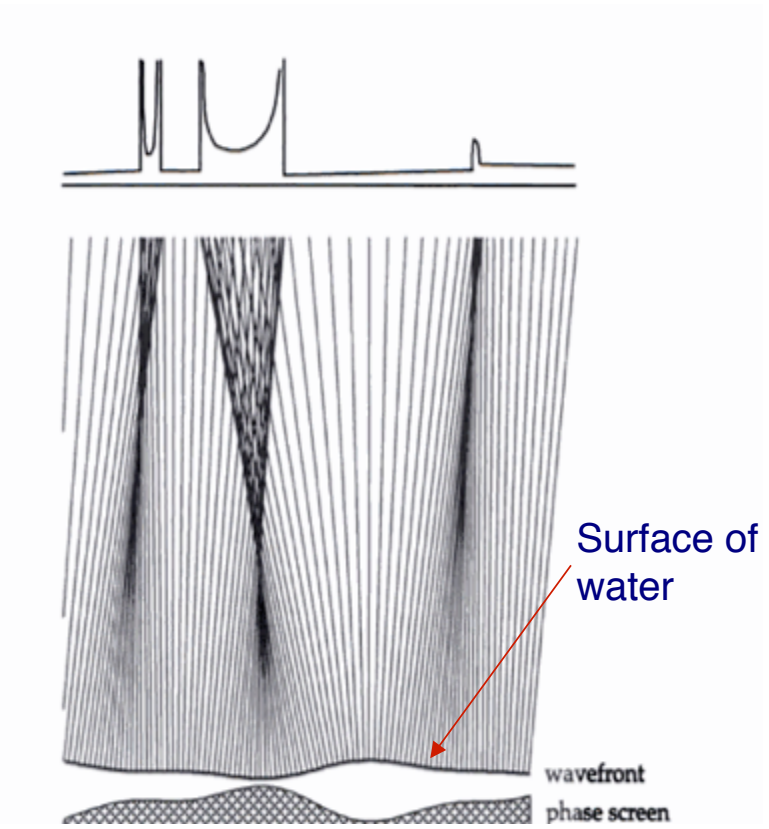


# What are caustics?



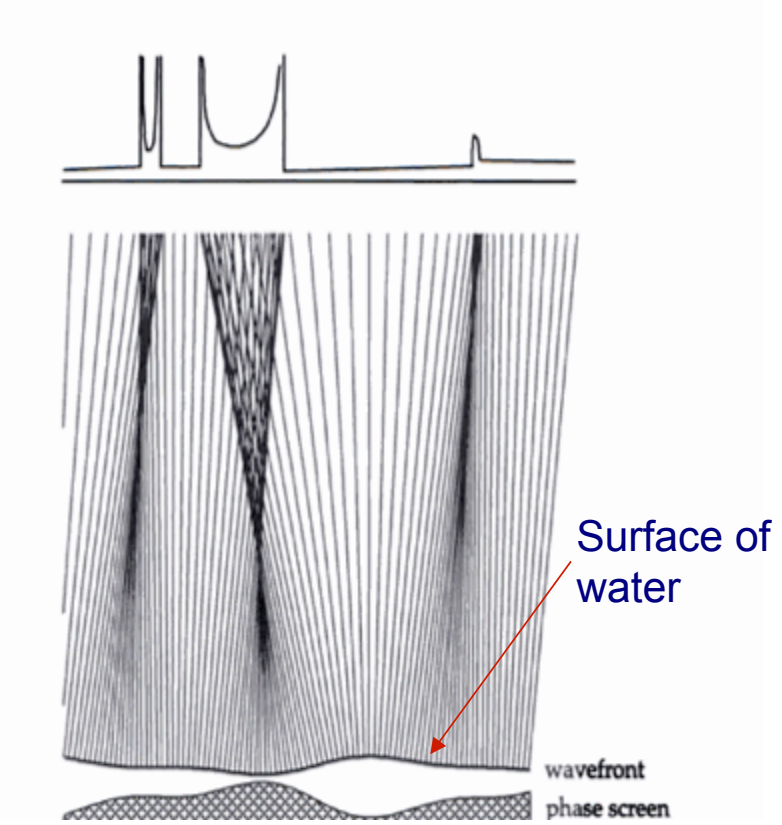
Caustics lines are an envelope of trajectories

**Caustics lines are an envelope of trajectories,  
or the projection of that envelope on another surface.**



J.F. Nye "Natural Focusing and Fine Structure of Light: Caustics and Wave Dislocations", (Taylor & Francis, Philadelphia, 1999).

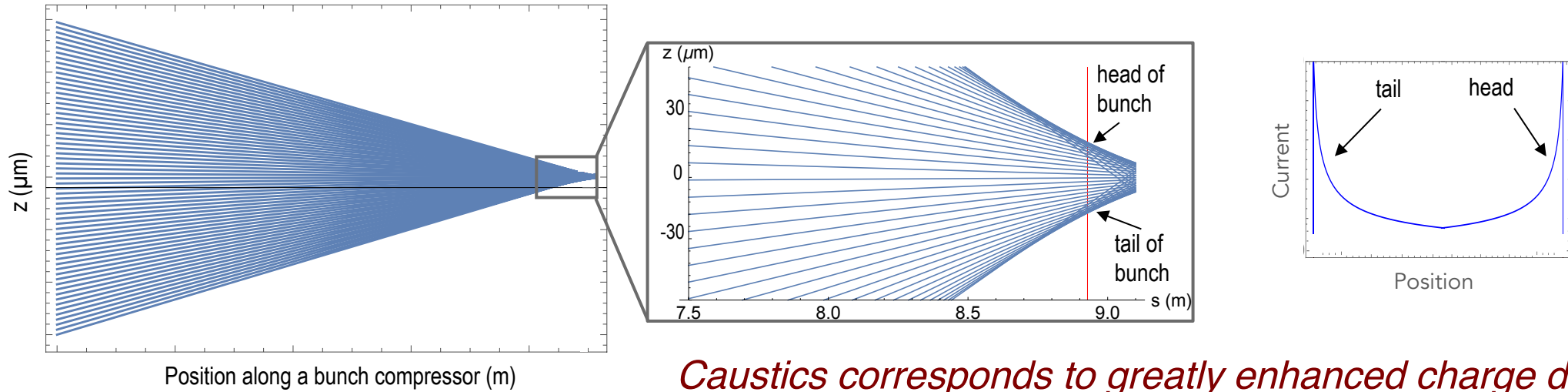
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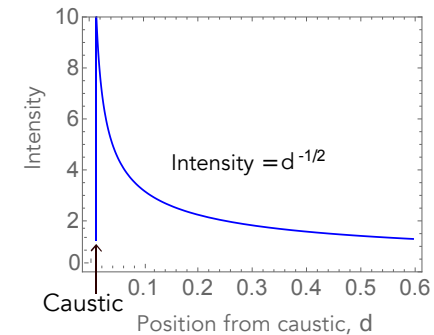
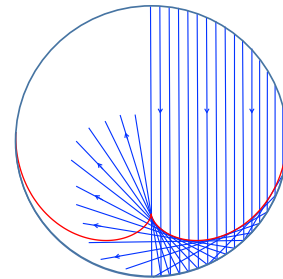
J.F. Nye "Natural Focusing and Fine Structure of Light: Caustics and Wave Dislocations", (Taylor & Francis, Philadelphia, 1999).

# Caustics in accelerator physics

Electron trajectories caustics in a bunch compressor:



Optical caustics:

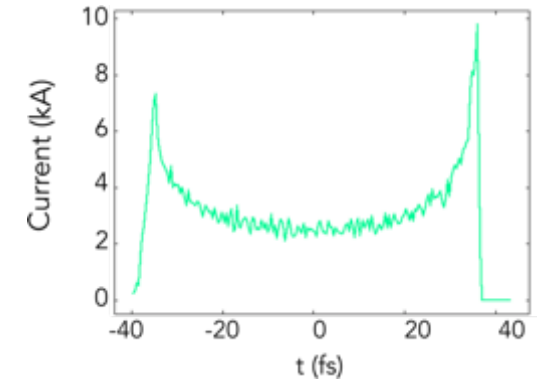
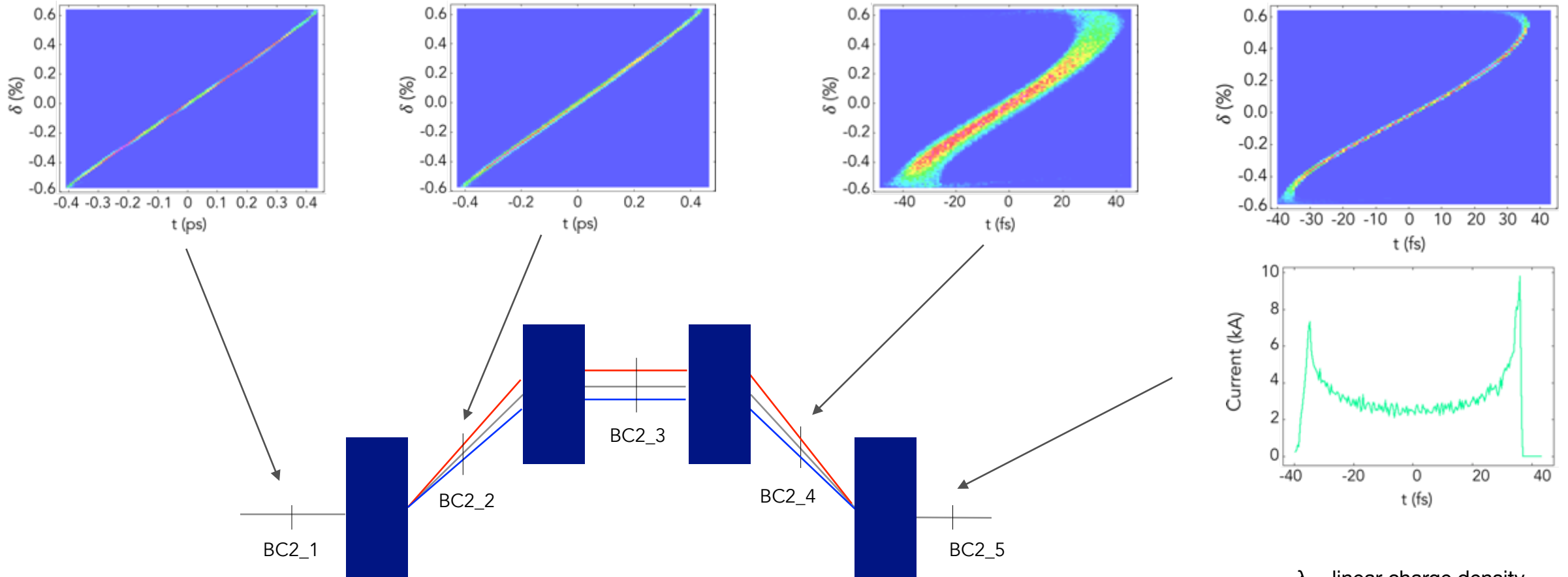


# Caustics sit in the field of Catastrophe theory



Figure: EC Zeeman (1976) Catastrophe Theory in Scientific American.

# Current horns can form during bunch compression



Current spikes are problematic, leading to greater **CSR-induced emittance growth**.

$\lambda = \text{linear charge density}$

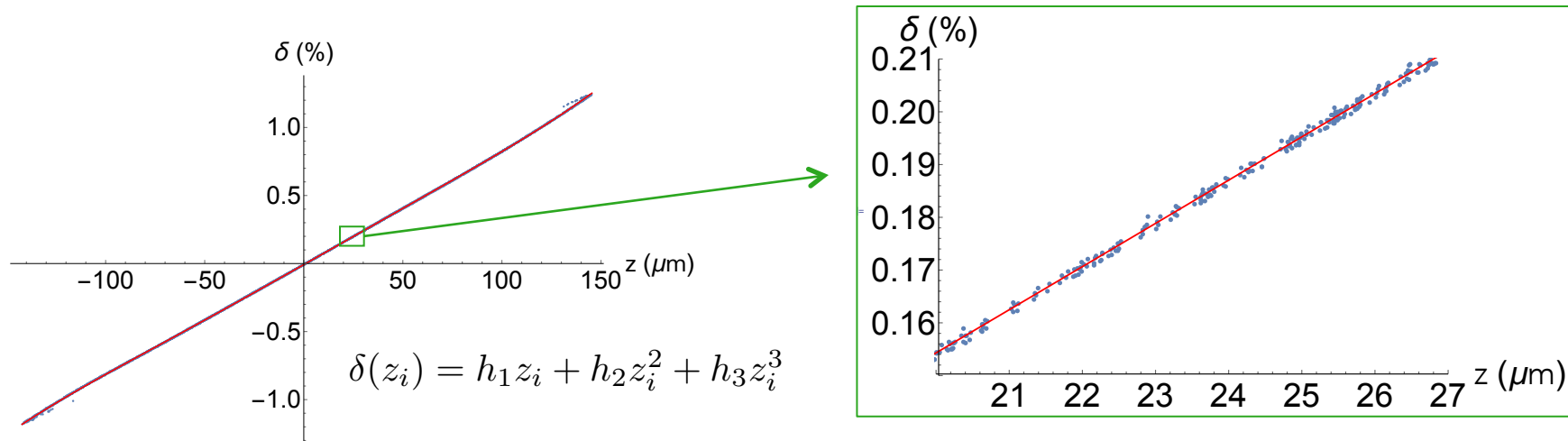
$$\frac{dE}{cdt} = \frac{-2e^2}{4\pi\epsilon_0(3R^2)^{1/3}} \int_{\tilde{z}-z_L}^{\tilde{z}} \frac{d\lambda}{dz} \left( \frac{1}{\tilde{z}-z} \right)^{1/3} dz$$

# Caustic expression

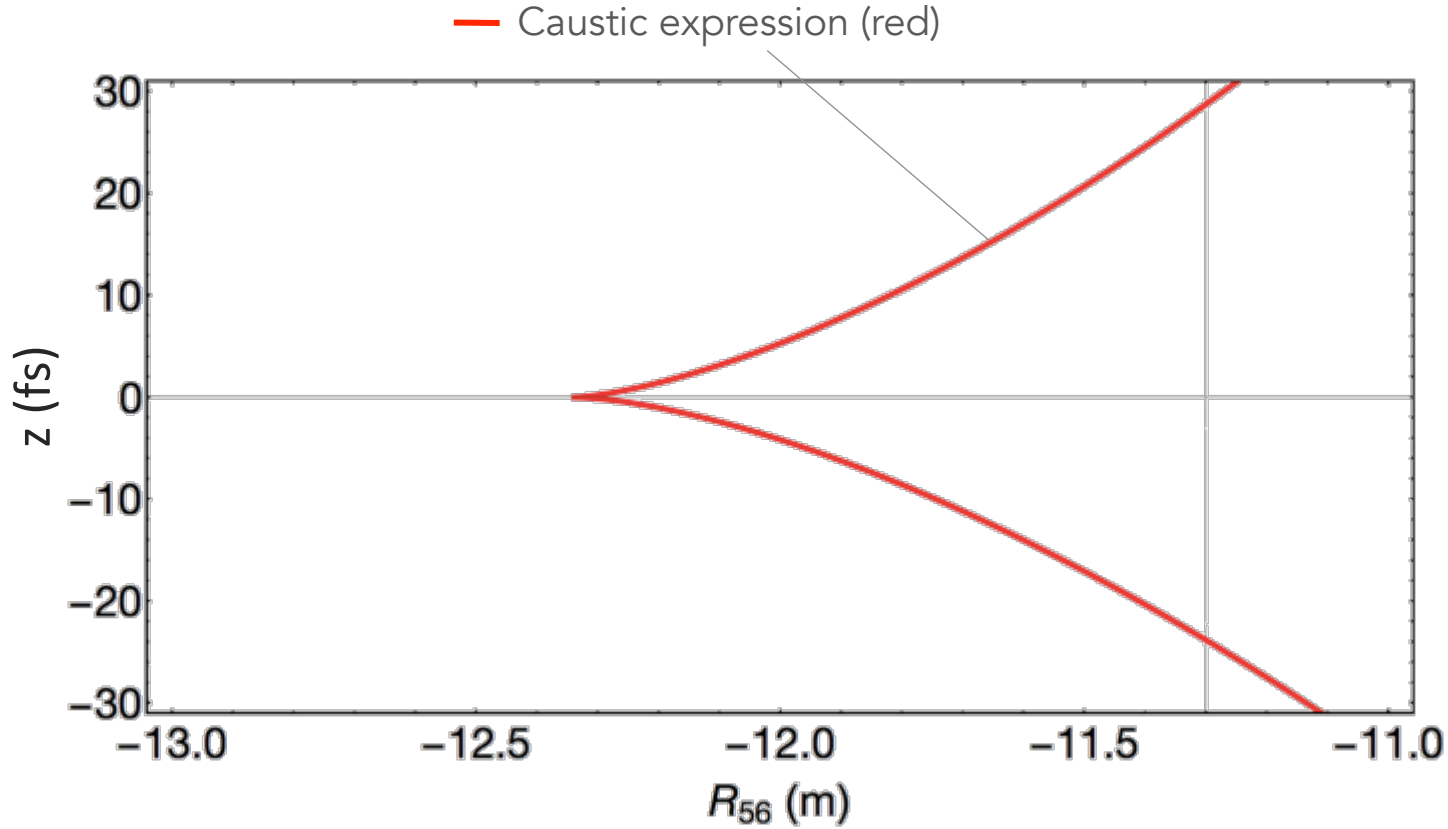
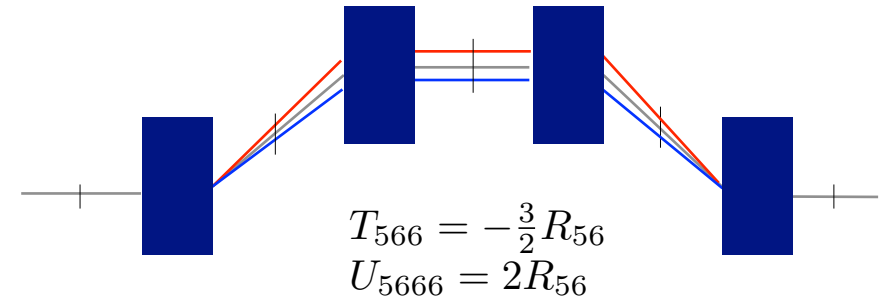
$$\tilde{z}(z_i) = z_i + \frac{\delta(z_i)(-1 + T_{566}(-2 + \delta(z_i))\delta'(z_i) + U_{5666}(-3 + \delta(z_i)^2)\delta'(z_i))}{\delta'(z_i)}$$

$$\tilde{R}_{56}(z_i) = \frac{-1 - 2T_{566}\delta'(z_i) - 3U_{5666}\delta'(z_i)}{\delta'(z_i)}$$

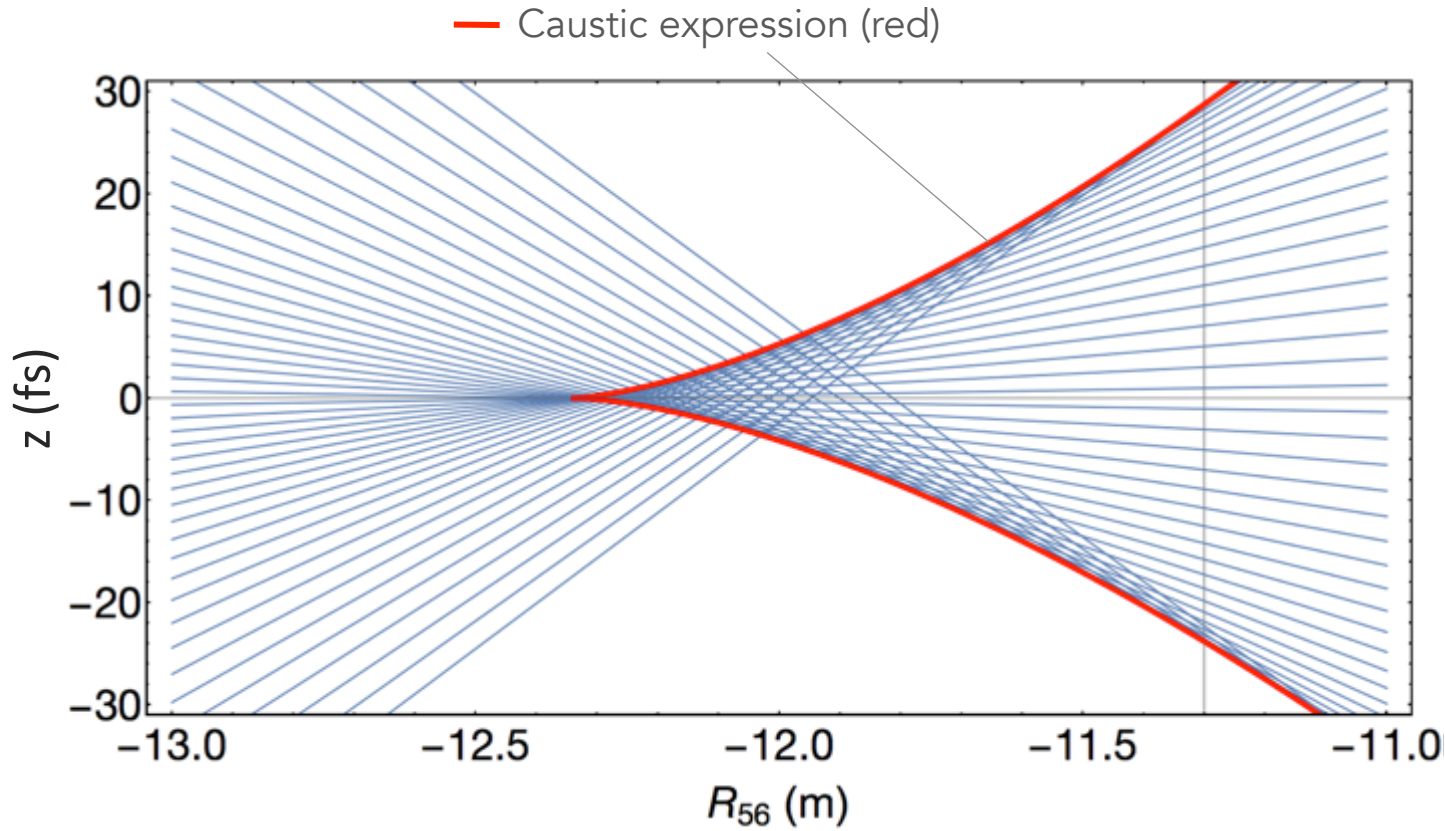
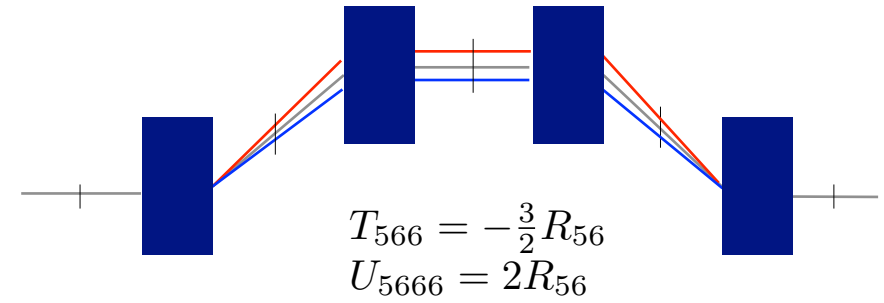
**T. K. Charles *et al.* (2016)  
Phys. Rev. AB, 19, 104402**



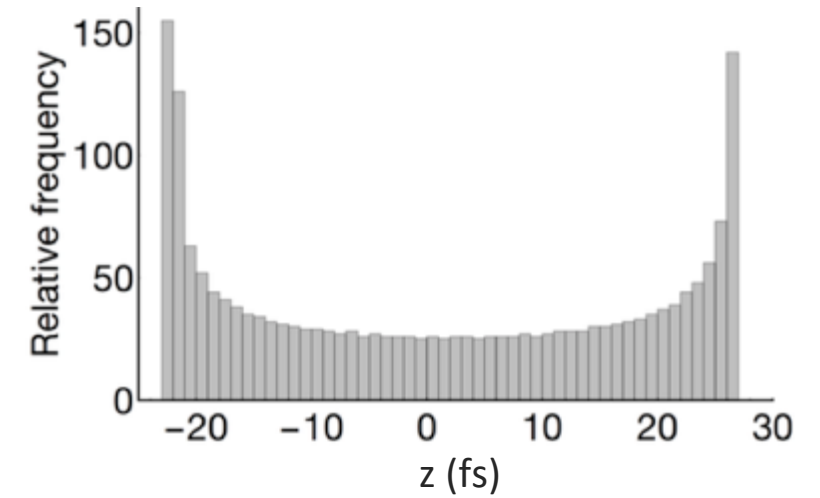
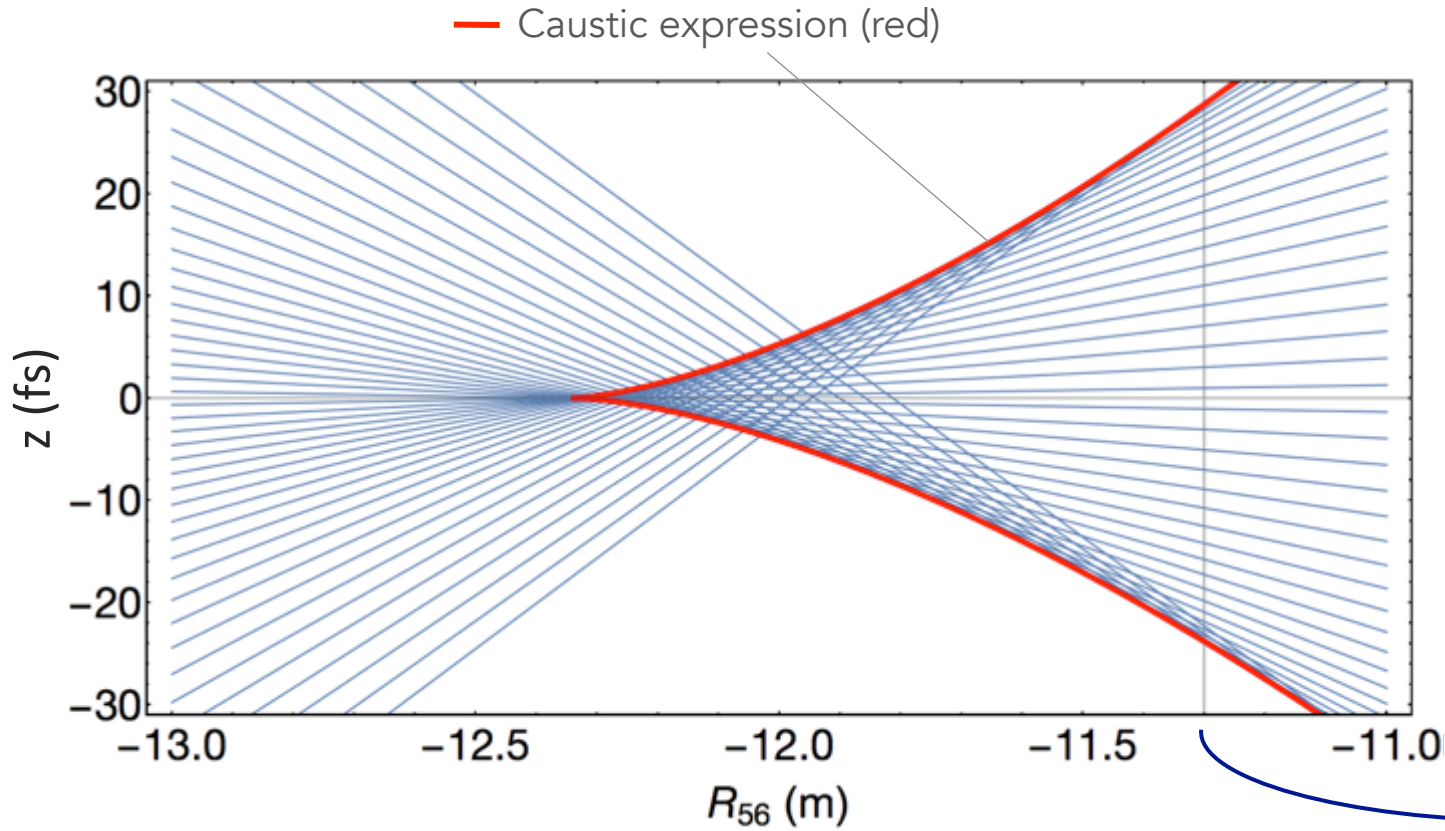
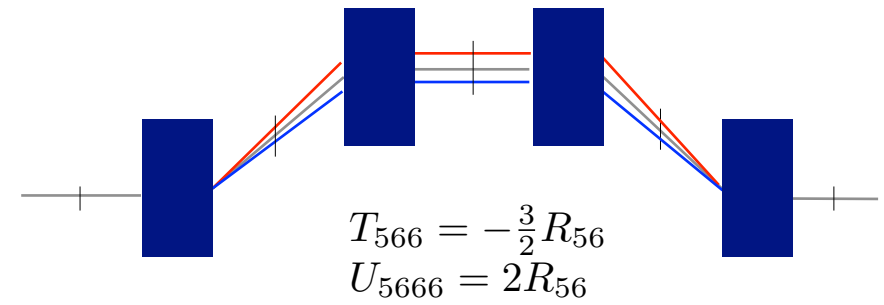
# Caustic expression for a chicane



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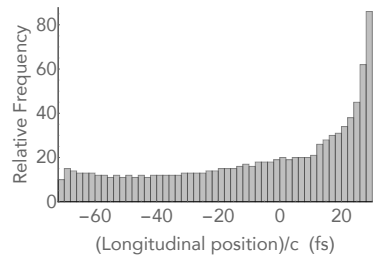
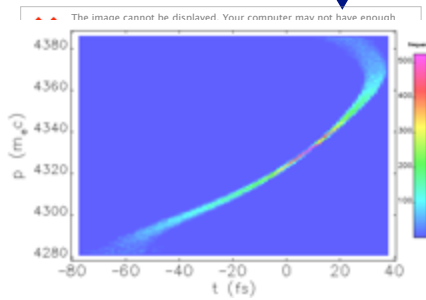
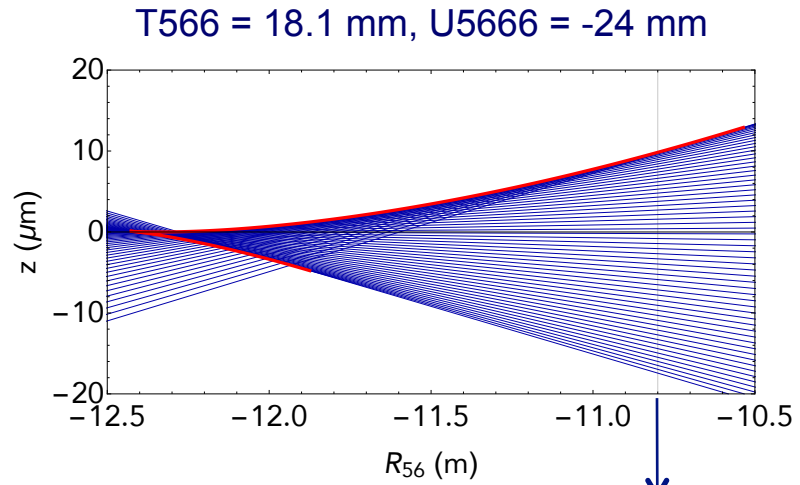


# Caustic expression for a chicane

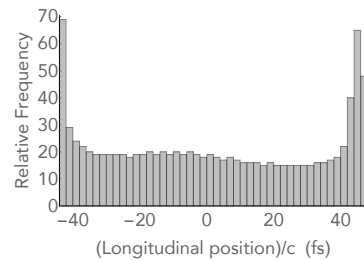
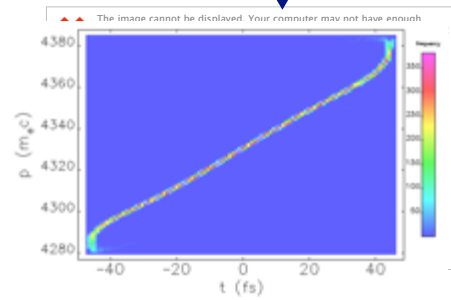
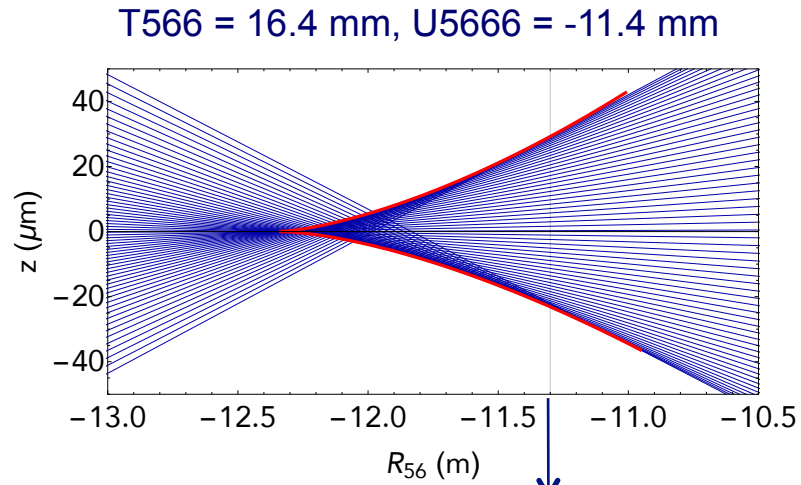


$R_{56} = -11.3$  mm

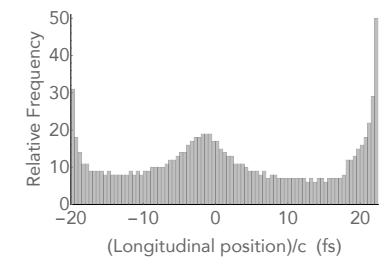
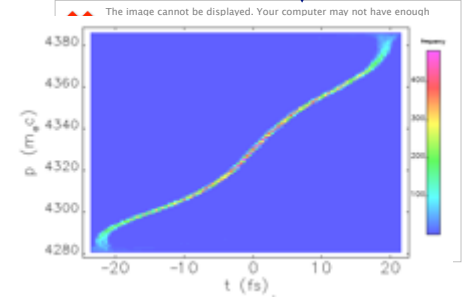
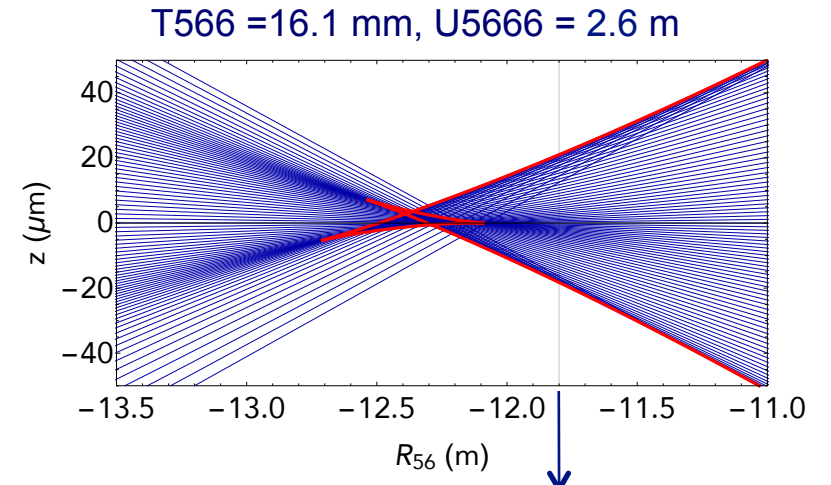
# Comparison with simulation



E.g. single spike bunch compression



E.g. LCLS [Y. Ding *et al.* Phys. Rev. AB 19, 100703]



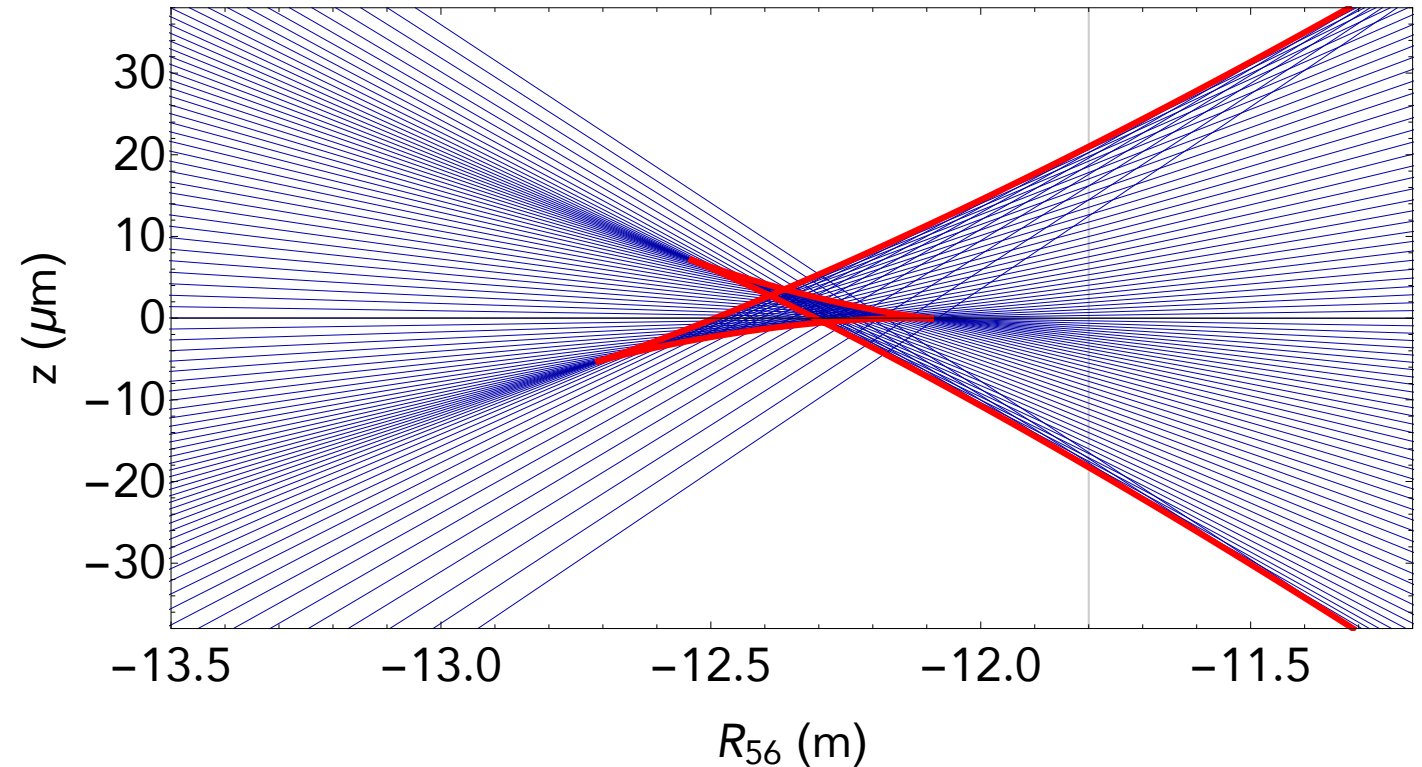
E.g. MAX IV [S. Thorin *et al.* FEL2010, WEPB34]

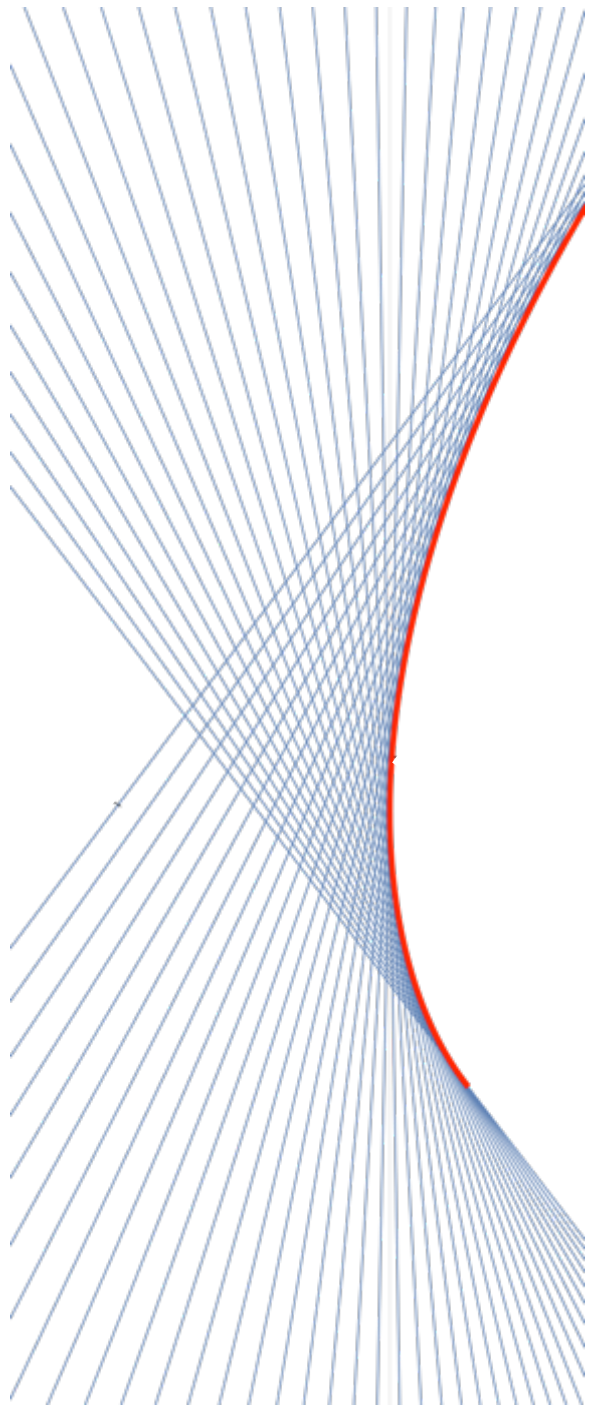
# A closer look...

$$\tilde{z}(z_i) = z_i + \frac{\delta(z_i)(-1 + T_{566}(-2 + \delta(z_i))\delta'(z_i) + U_{5666}(-3 + \delta(z_i)^2)\delta'(z_i))}{\delta'(z_i)}$$

$$\tilde{R}_{56}(z_i) = \frac{-1 - 2T_{566}\delta'(z_i) - 3U_{5666}\delta'(z_i)}{\delta'(z_i)}$$

Three types of catastrophes  
but all described by the  
same parametric  
expression.





Can we avoid caustic  
current horns?

# Caustic condition & exclusion plots

Analytical expression of boundaries:

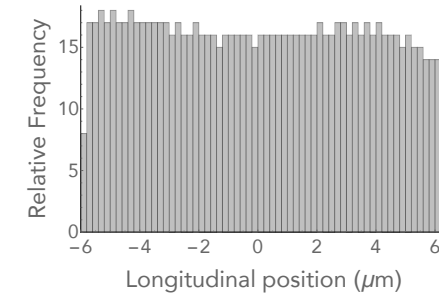
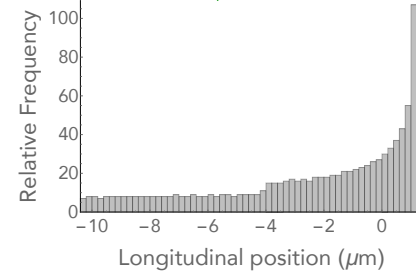
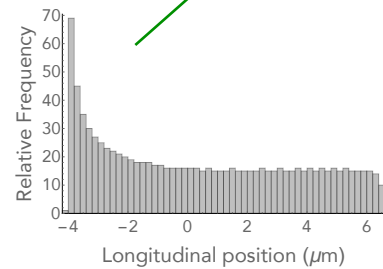
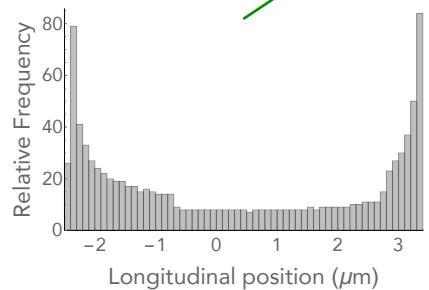
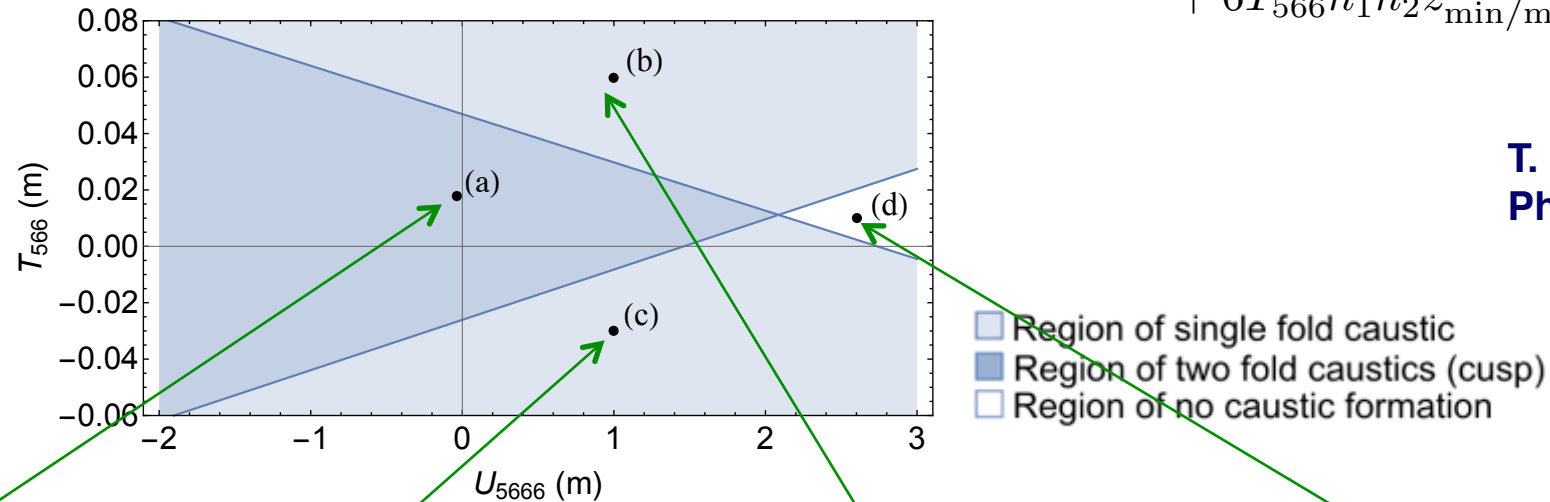
$$f(R_{56}, T_{566}, U_{5666}, h_1, h_2, h_3; z_{\min/\max}) =$$

$$1 + h_1 R_{56} + 2h_2 R_{56} z_{\min/\max}$$

$$+ 3h_3 R_{56} z_{\min/\max}^2 + 2T_{566} h_1^2 z_{\min/\max}$$

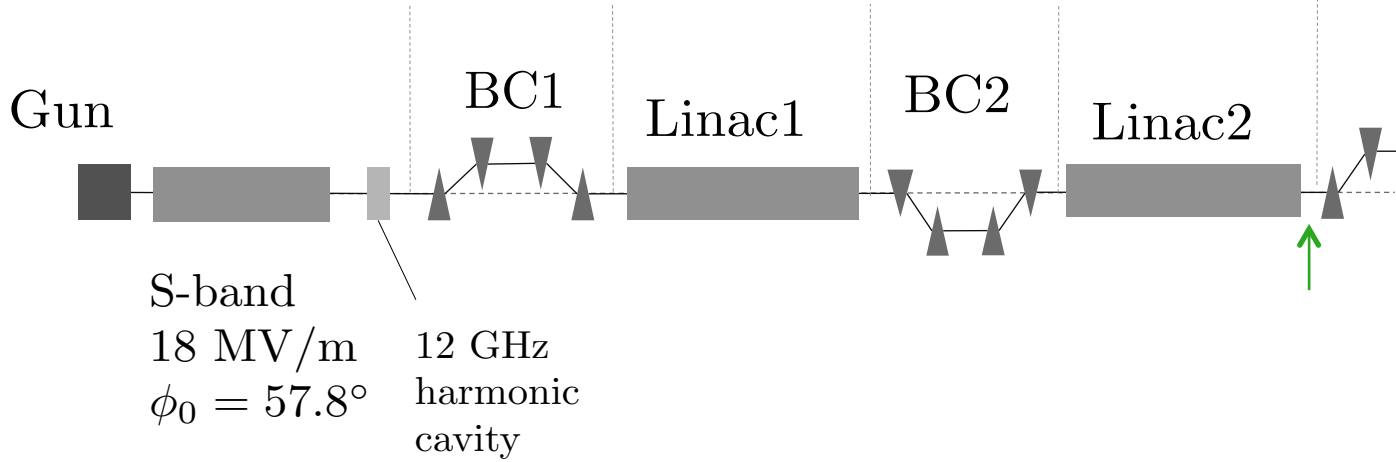
$$+ 6T_{566} h_1 h_2 z_{\min/\max}^2 + 3h_1^3 U_{5666} z_{\min/\max}^2$$

**T. K. Charles *et al.* (2017)**  
**Phys. Rev. AB, 30, 030705**

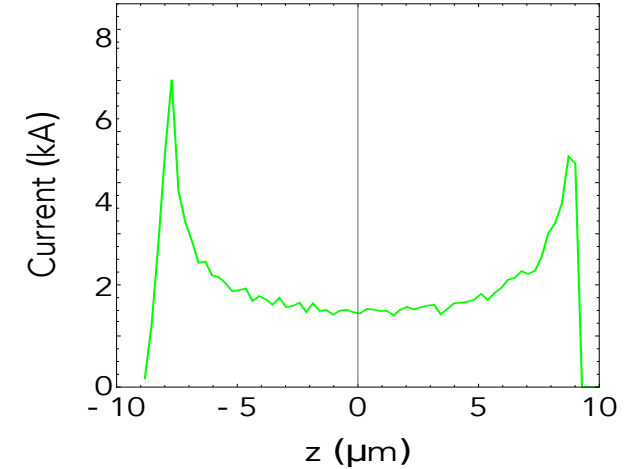
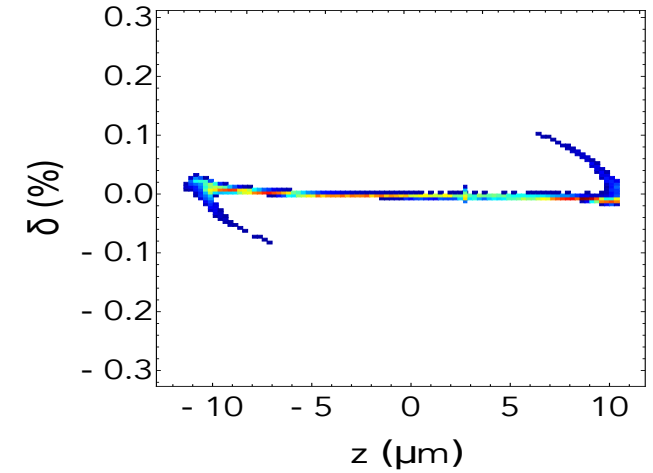


# Consider S-band FEL linac

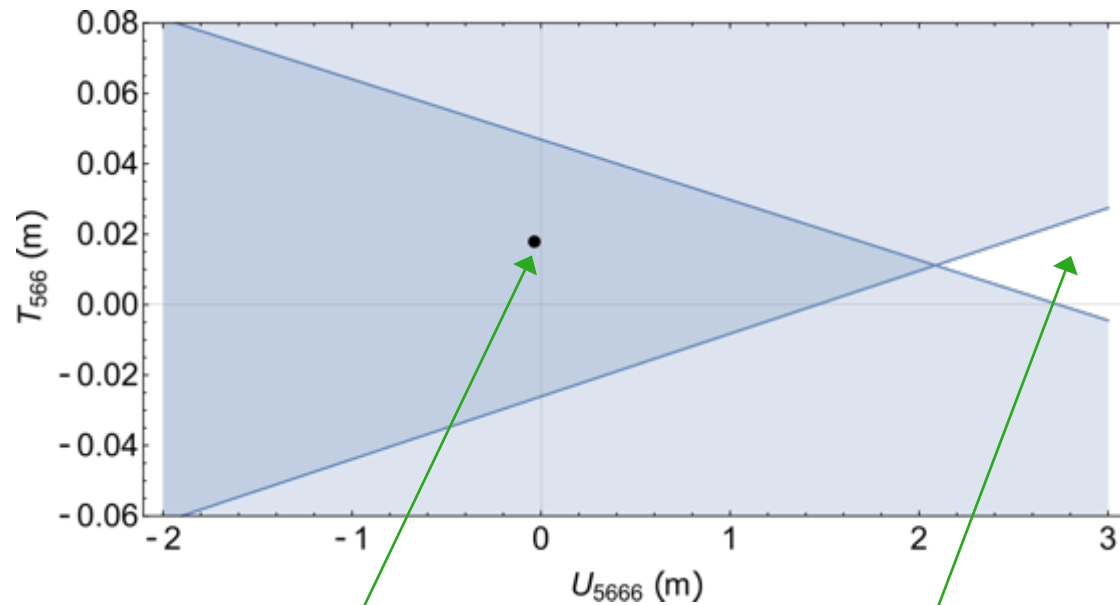
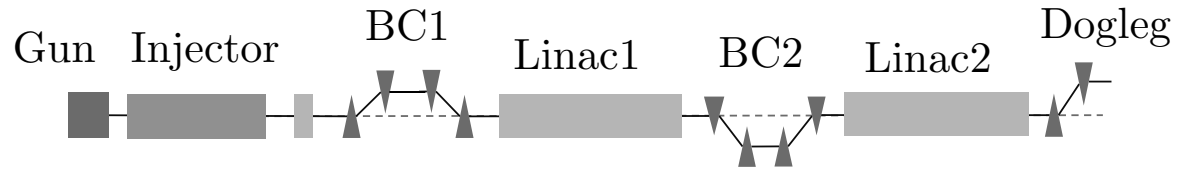
$I_{peak}$	= 0.03 kA	0.3 kA	0.3 kA	3.6 kA	3.6 kA
$\sigma_z$	= 806 $\mu\text{m}$	72.6 $\mu\text{m}$	72.3 $\mu\text{m}$	7.7 $\mu\text{m}$	7.7 $\mu\text{m}$
E	= 292 MeV	292 MeV	2.2 GeV	2.2 GeV	6.2 GeV



## End of Linac 2:



# Consider S-band FEL linac

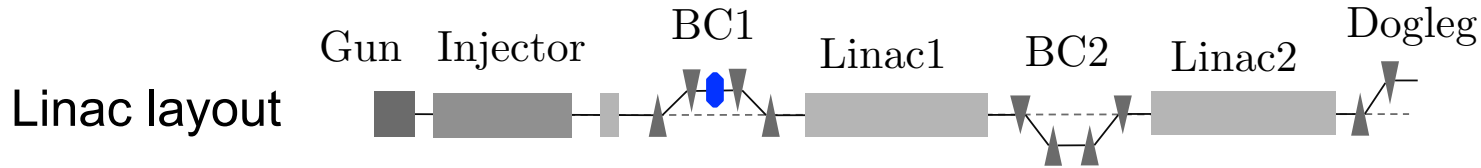


- Region of two fold caustics (cusp)
- Region of single fold caustic
- Region of no caustic formation

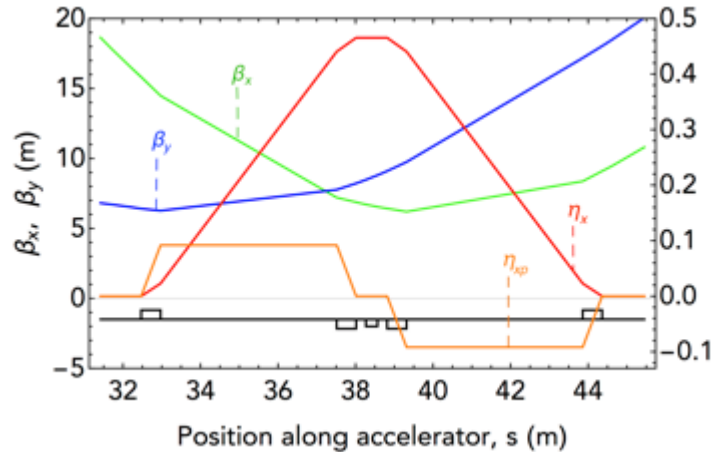
Working point of the 4-dipole chicane (BC2)

Where we'd prefer to be (non-caustic region).

# S-band linac FEL demonstration of current horn suppression



Bunch Compressor 1



BC1:

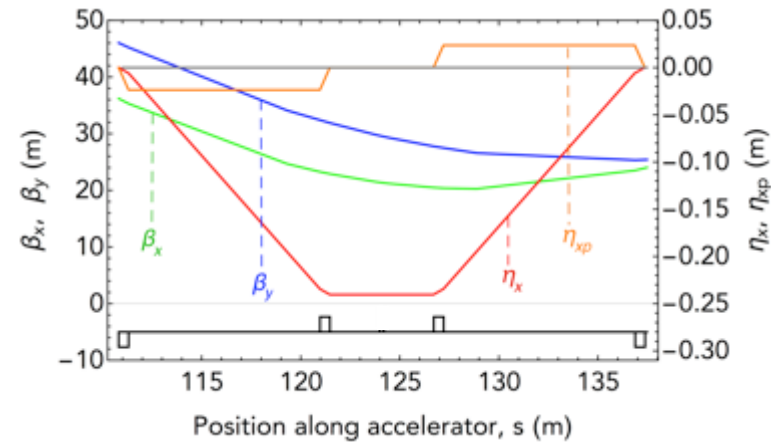
$$R56 = -82.36 \text{ mm}$$

$$T566 = 102.40 \text{ mm}$$

$$U5666 = -5.23 \text{ m}$$

Dipole bending angle,  $\theta = 5.25^\circ$   
 Octupole strength,  $K3 = -1007 \text{ m}^{-4}$

Bunch Compressor 2



BC2:

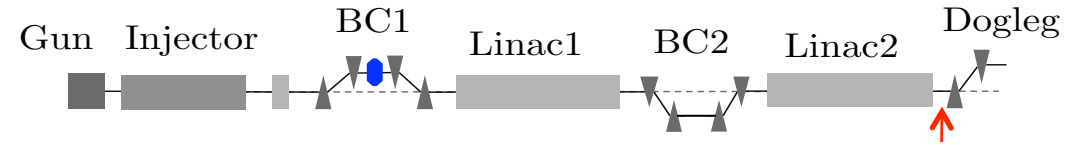
$$R56 = -11.18 \text{ mm}$$

$$T566 = 32.10 \text{ mm}$$

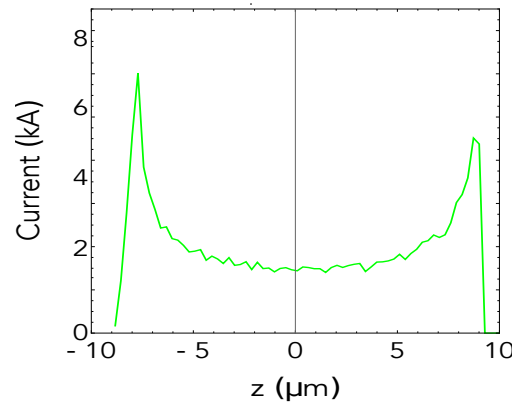
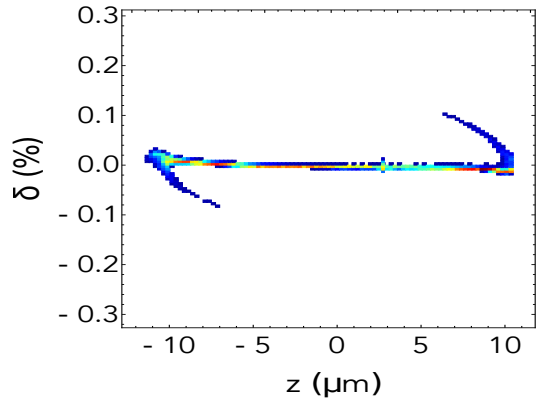
$$U5666 = -72.19 \text{ mm}$$

Dipole bending angle,  $\theta = 1.35^\circ$

# Current horn suppression

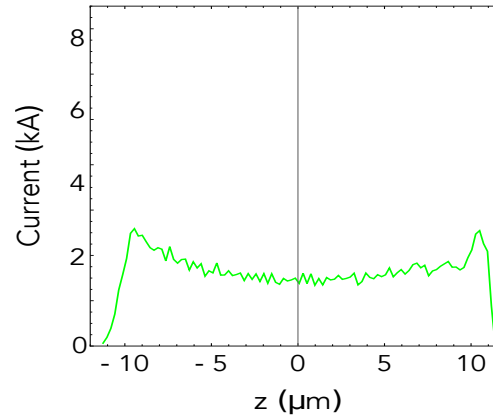
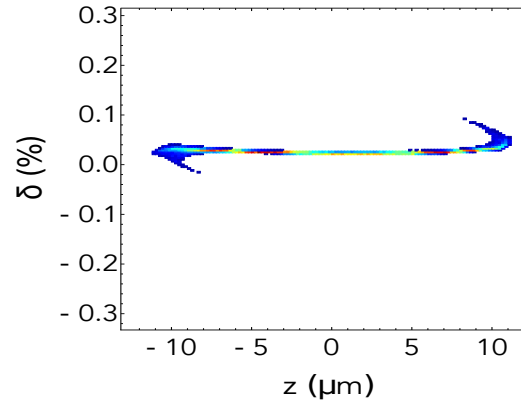


Without octupole:



$$\epsilon_{nx} = 1.394 \text{ mm mrad}$$

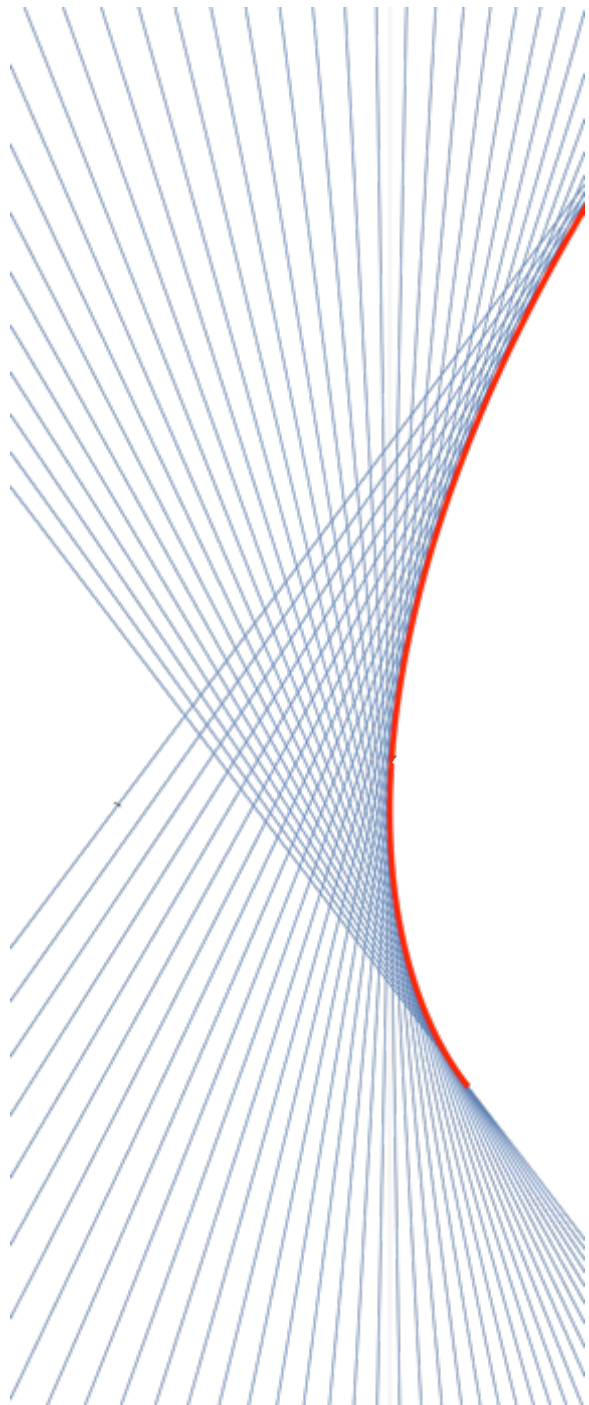
With octupole:



$$\epsilon_{nx} = 0.842 \text{ mm mrad}$$

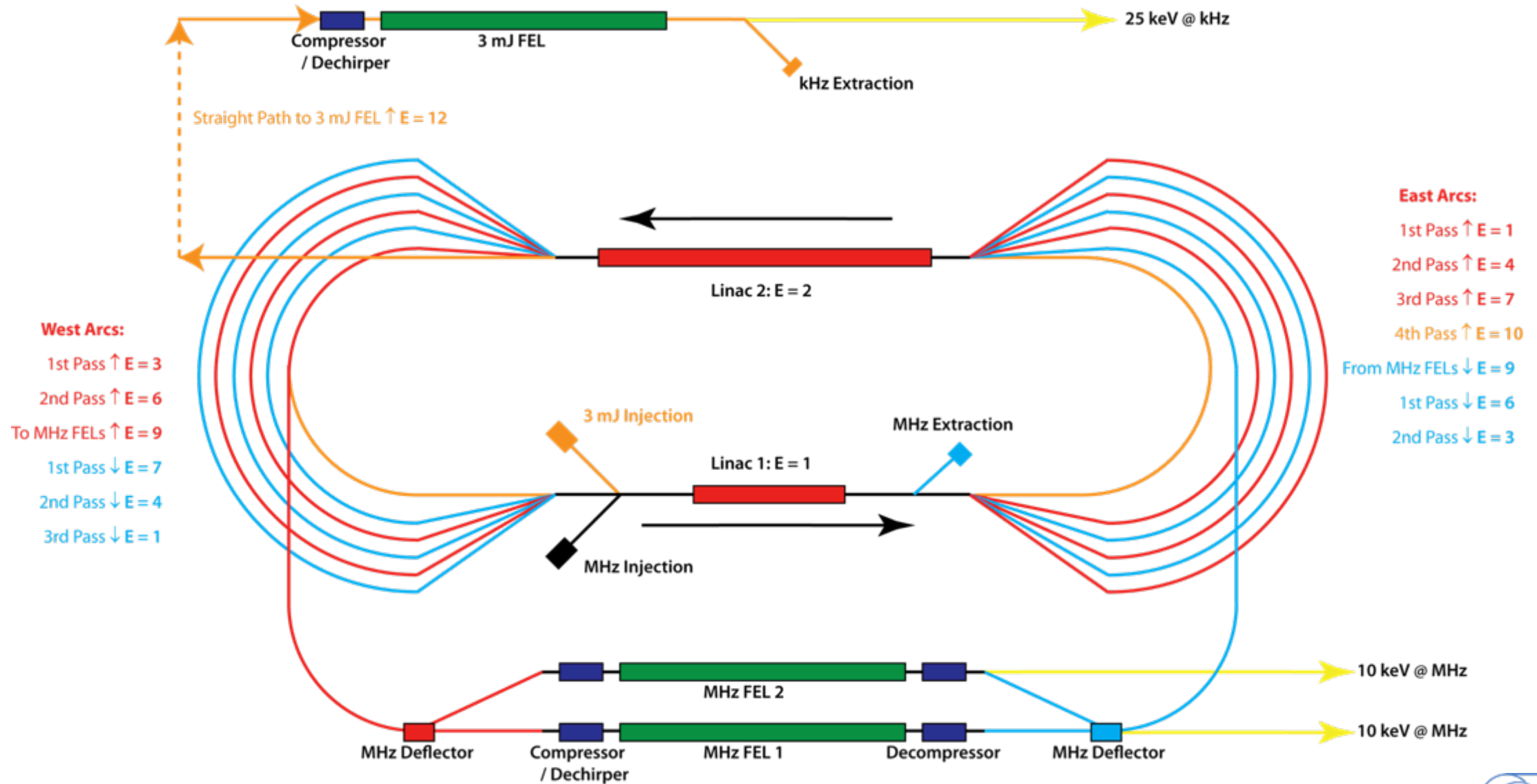
*49% reduction in the CSR-induced emittance growth.*

**T. K. Charles et al. (2017)**  
**Phys. Rev. AB, 30, 030705**



We can also apply this caustic approach  
to recirculating energy recovery  
machines ...

# A UK recirculating FEL concept



# Caustic exclusion plots for each arc of a recirculating machine during acceleration

Arc no.

1

2

3

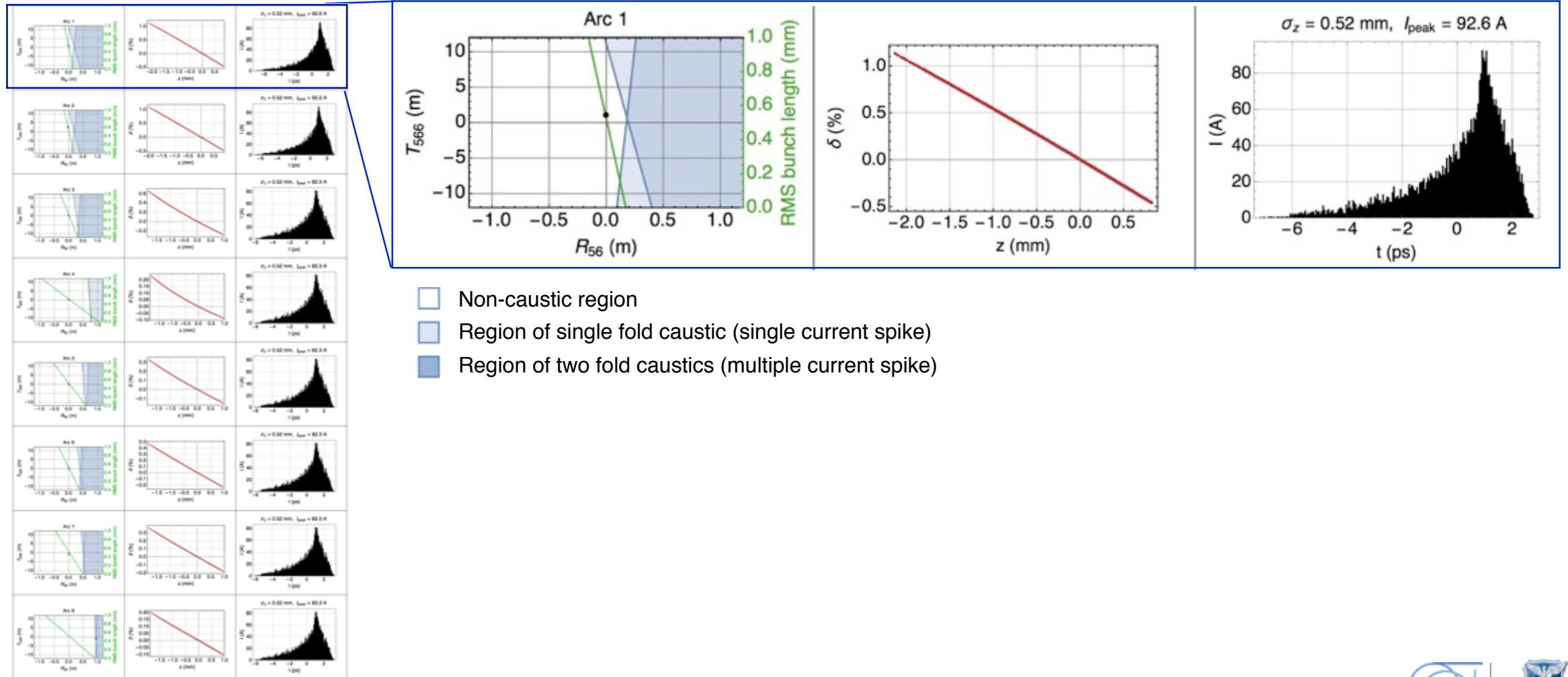
4

5

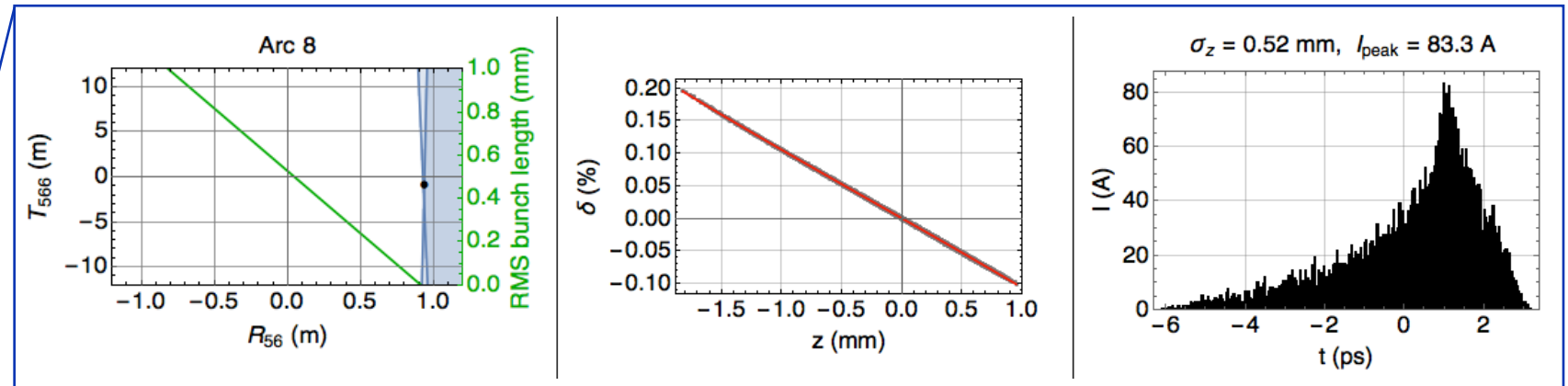
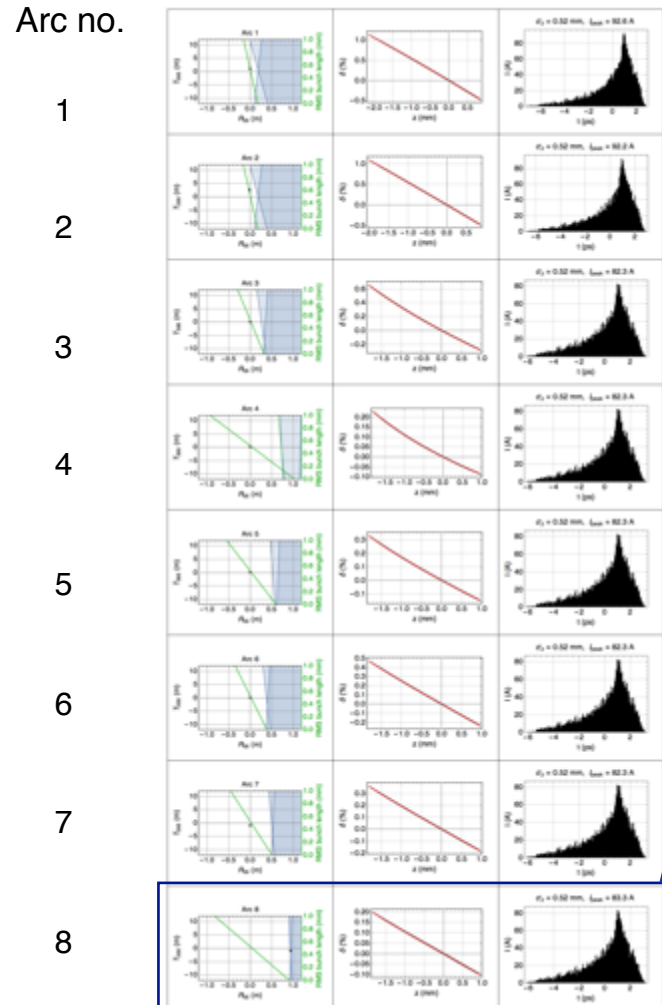
6

7

8



# Caustic exclusion plots for each arc of a recirculating machine during acceleration

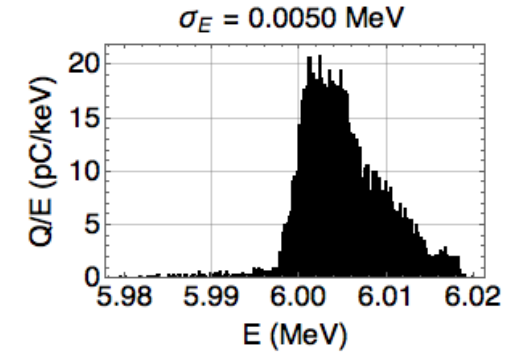
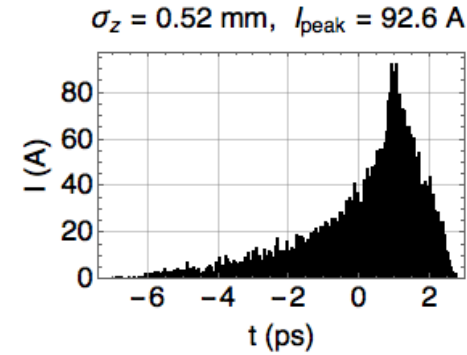
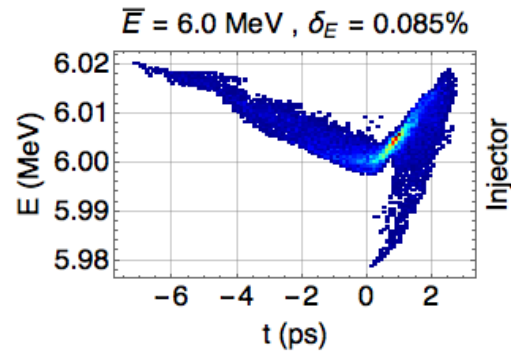


- Non-caustic region
- Region of single fold caustic (single current spike)
- Region of two fold caustics (multiple current spike)

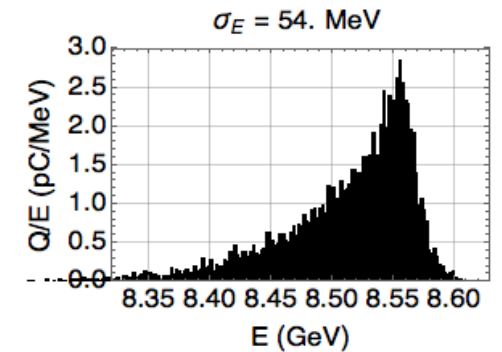
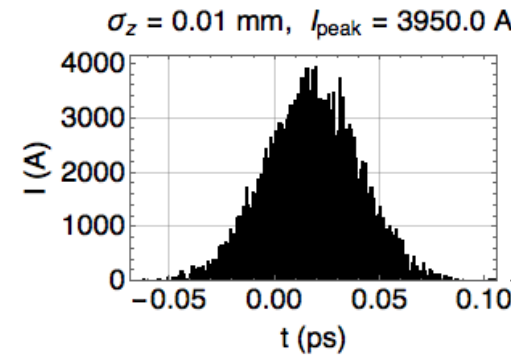
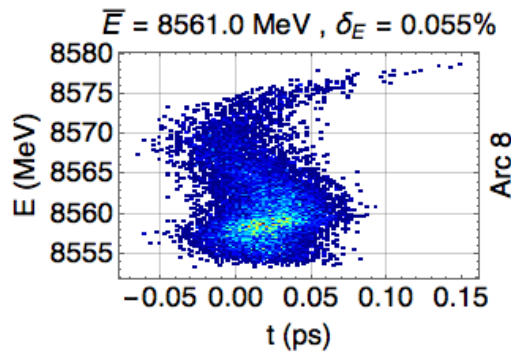
# Bunch distributions at various positions around the accelerator

## Bunch Distribution from gun:

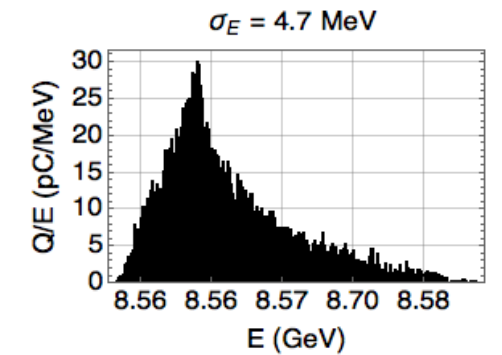
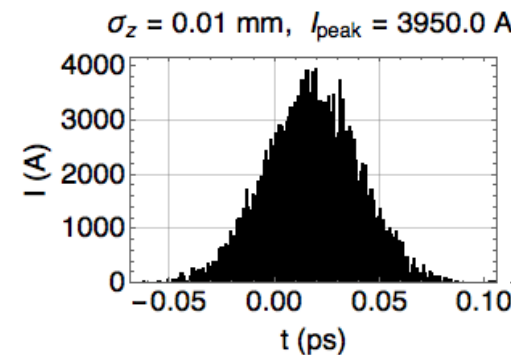
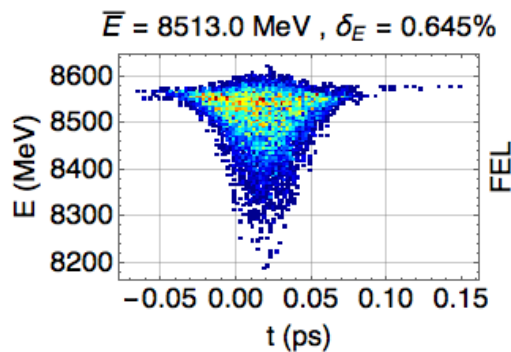
With thanks to **Julian McKenzie** (Daresbury, UK) for this distribution.



## Distribution at end of arc 8:



## Distribution after lasing:



# Caustic exclusion plots for each arc of a recirculating machine during deceleration

Arc no.

9

10

11

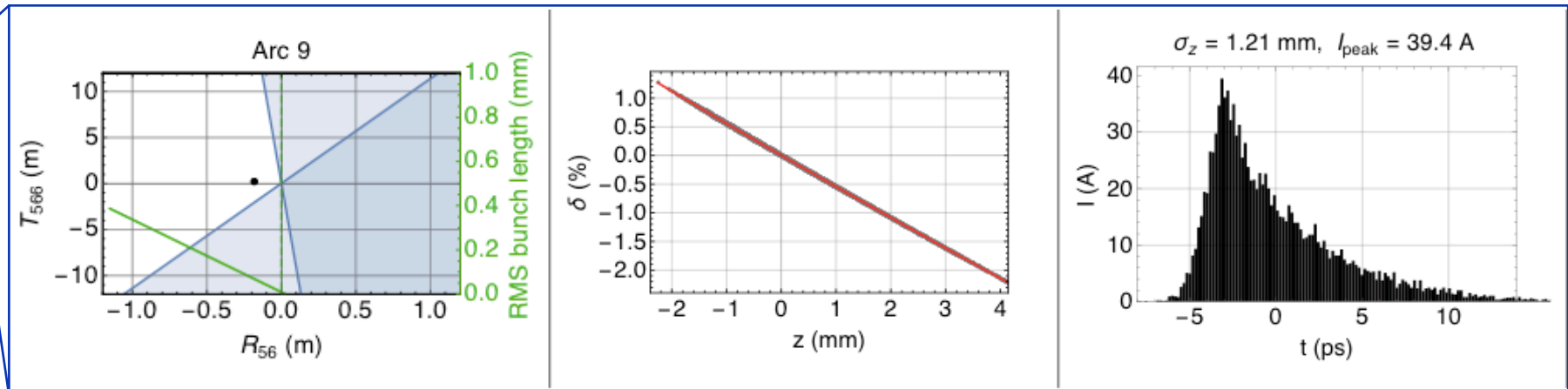
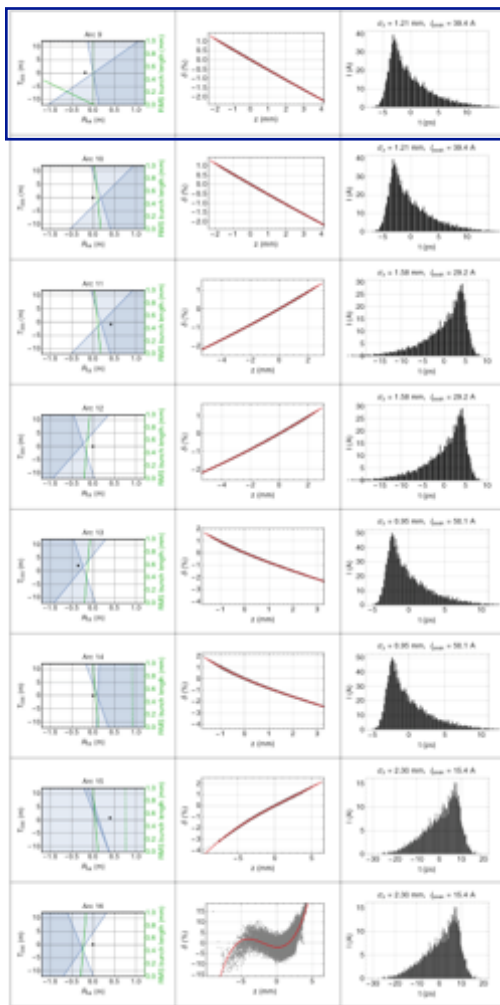
12

13

14

15

16



- Non-caustic region
- Region of single fold caustic (single current spike)
- Region of two fold caustics (multiple current spike)

# Caustic exclusion plots for each arc of a recirculating machine during deceleration

Arc no.

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11

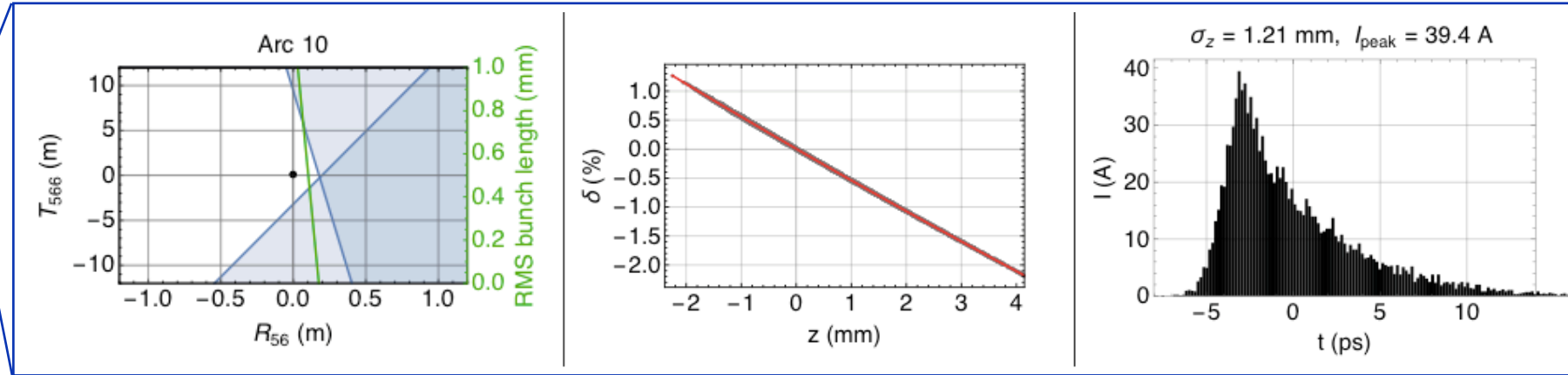
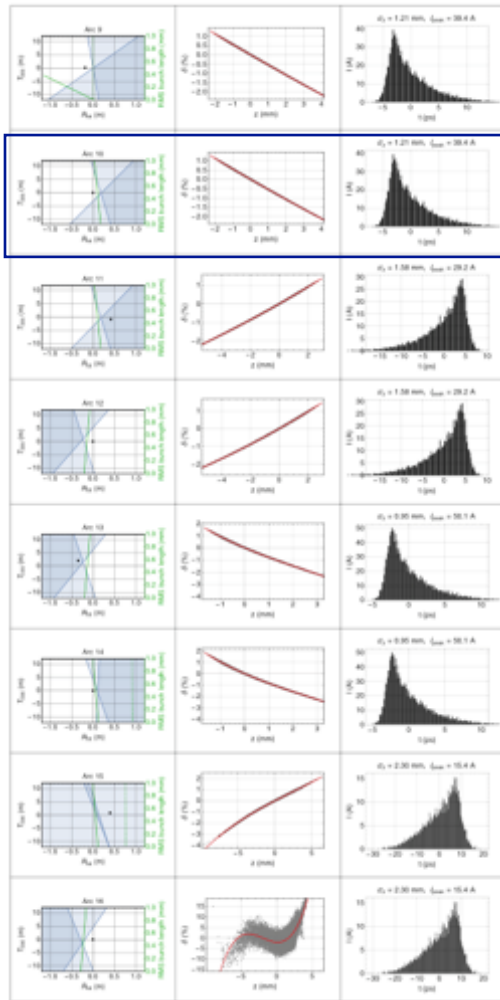
12

13

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# Caustic exclusion plots for each arc of a recirculating machine during deceleration

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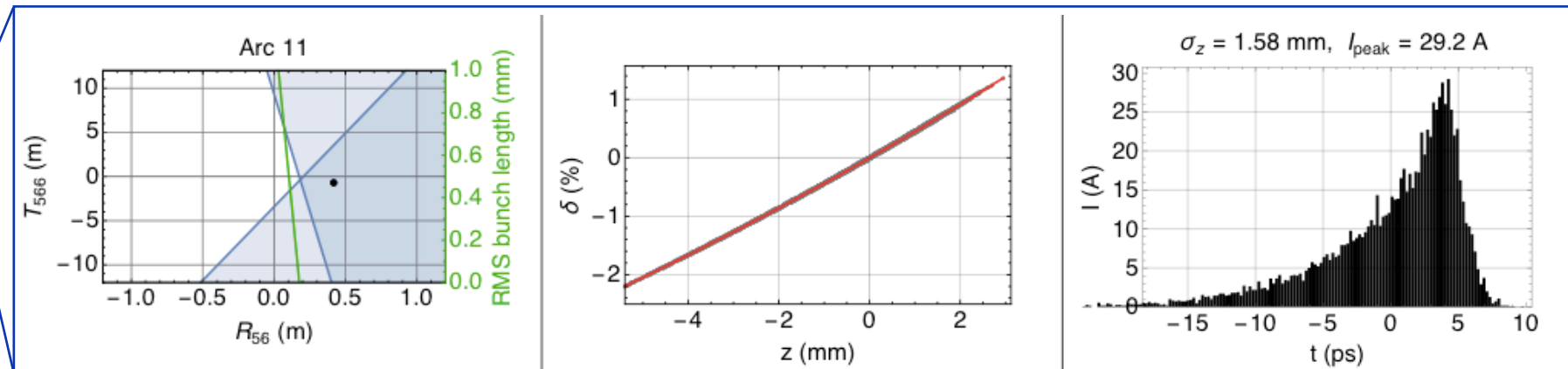
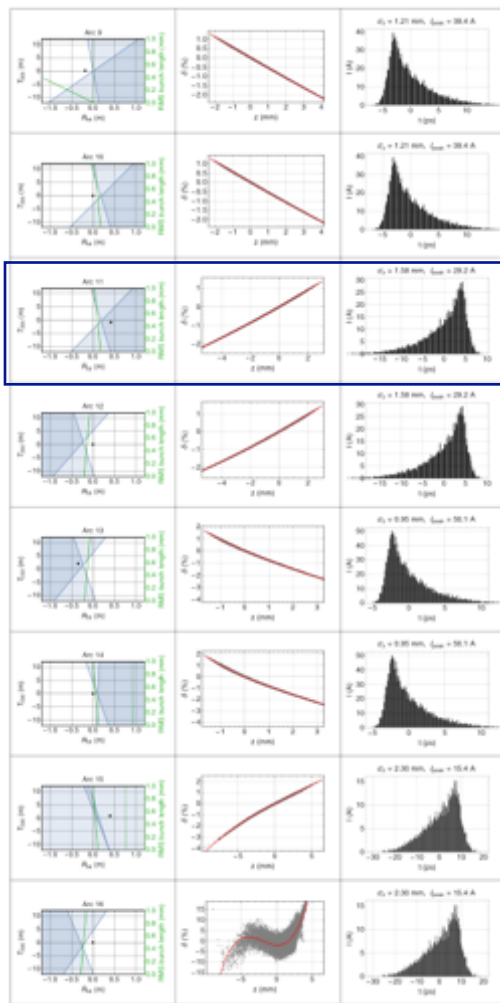
12

13

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# Caustic exclusion plots for each arc of a recirculating machine during deceleration

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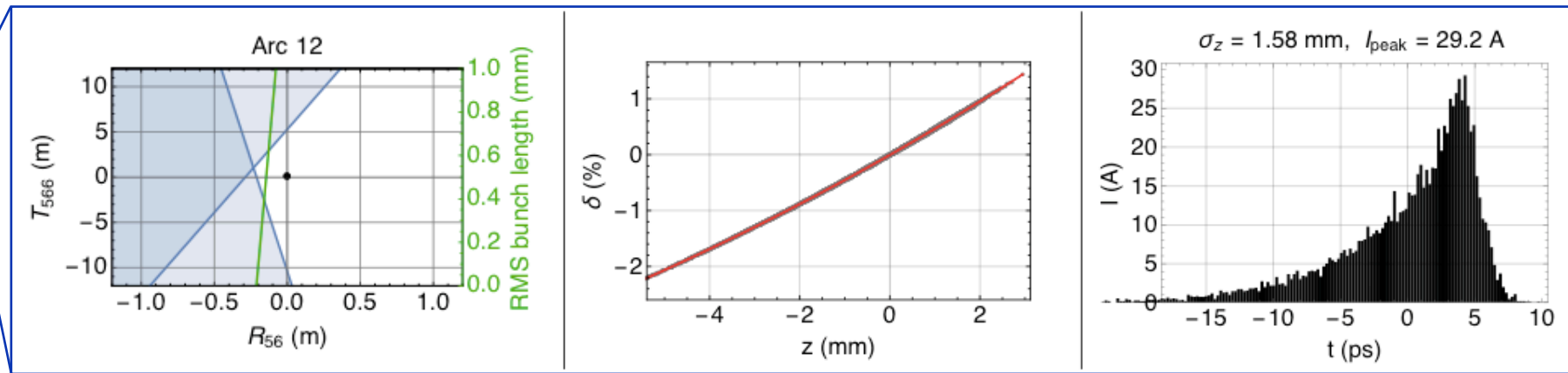
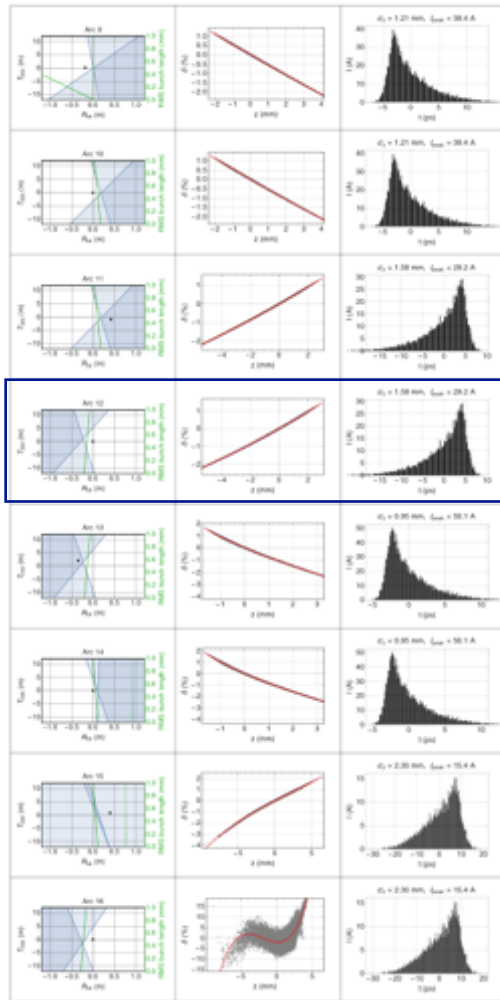
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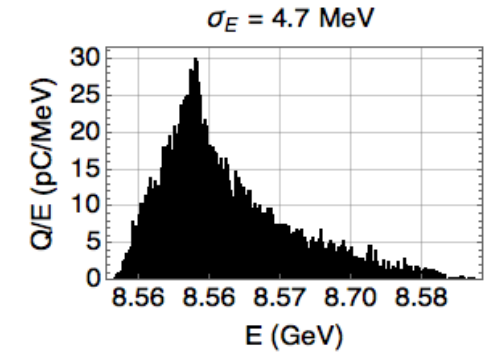
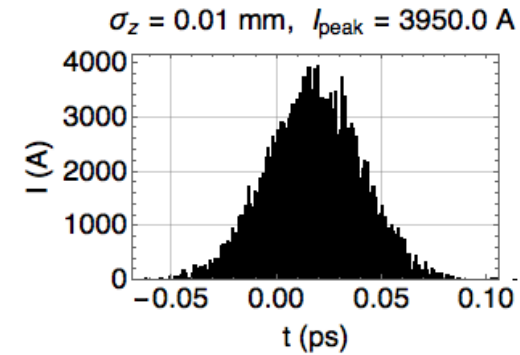
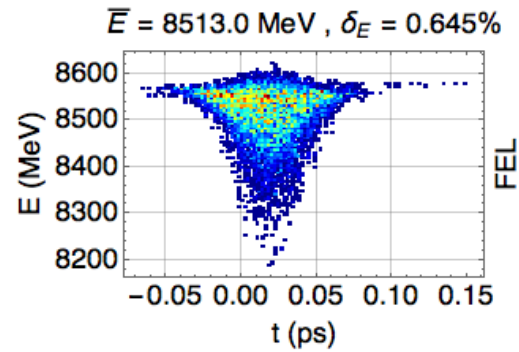
16



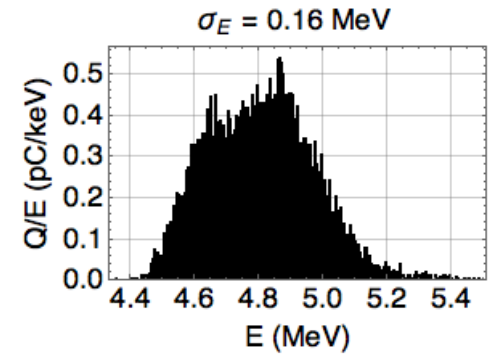
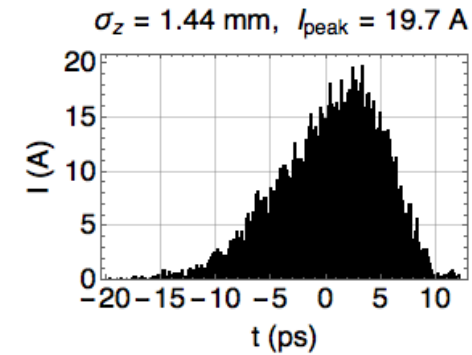
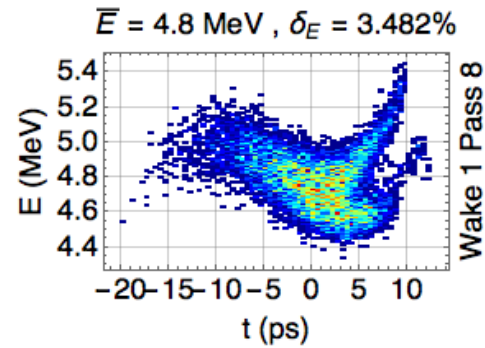
- Non-caustic region
- Region of single fold caustic (single current spike)
- Region of two fold caustics (multiple current spike)

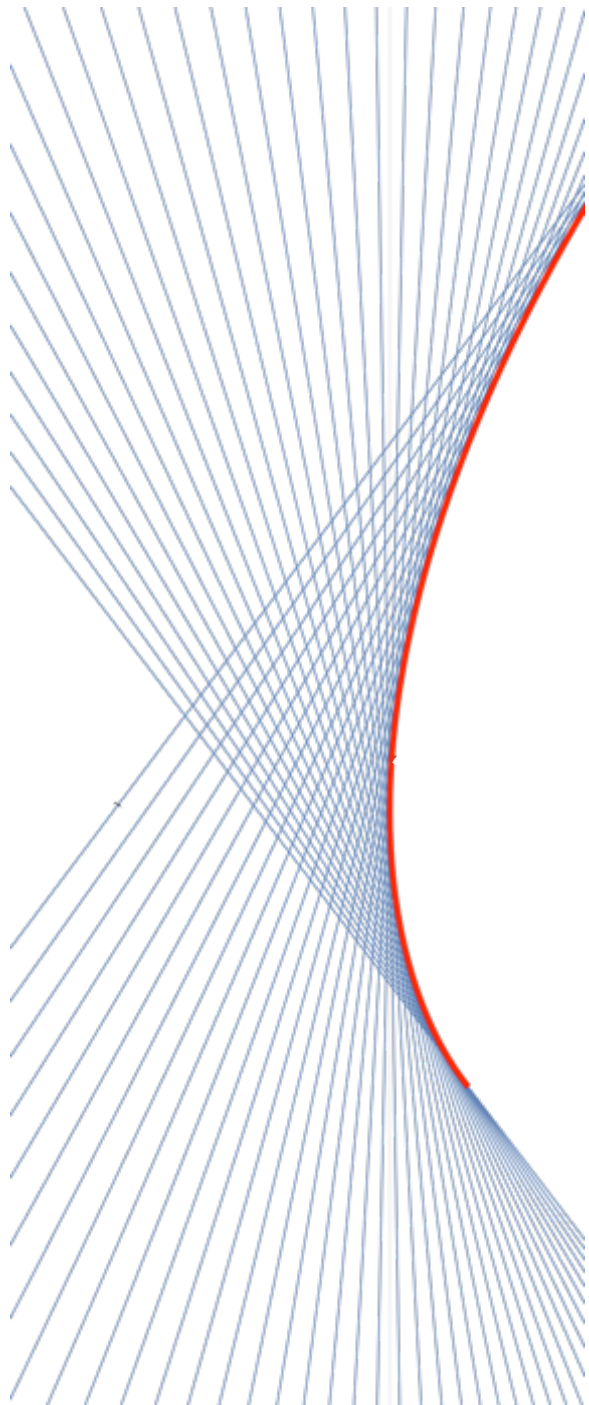
# Bunch distributions at various positions around the accelerator

Distribution after lasing:



Distribution at dump:

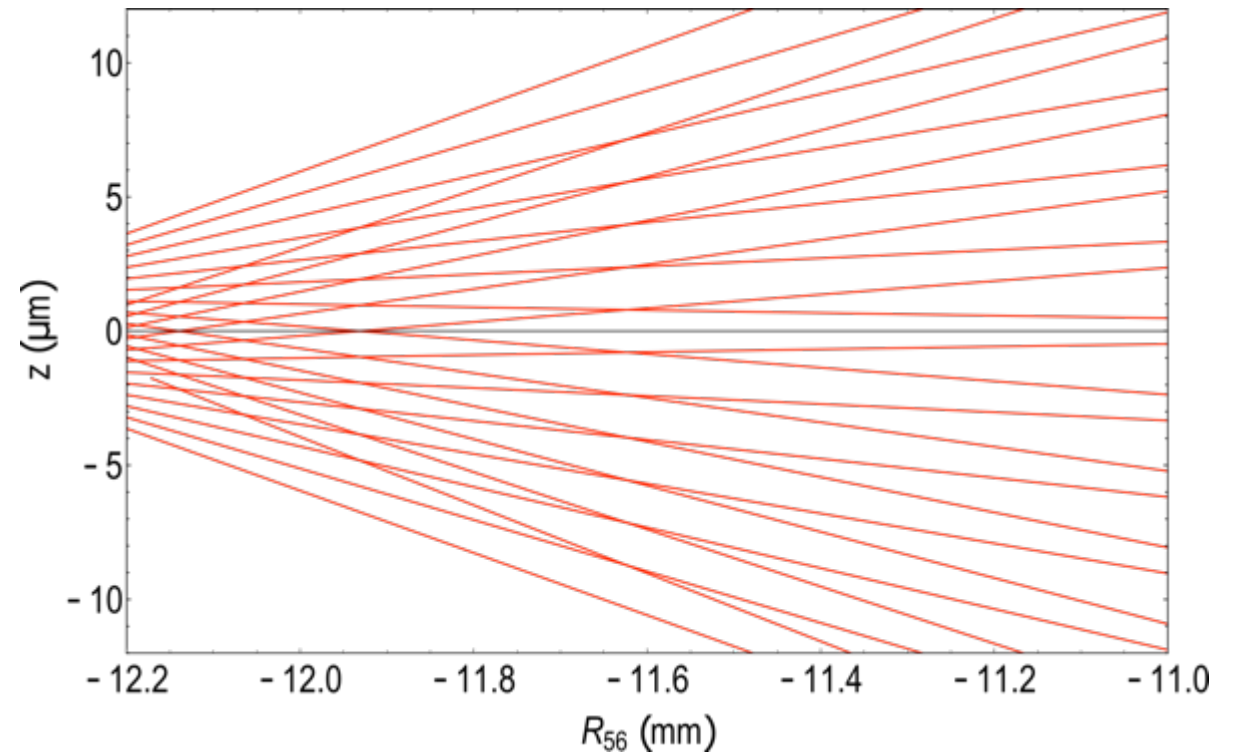
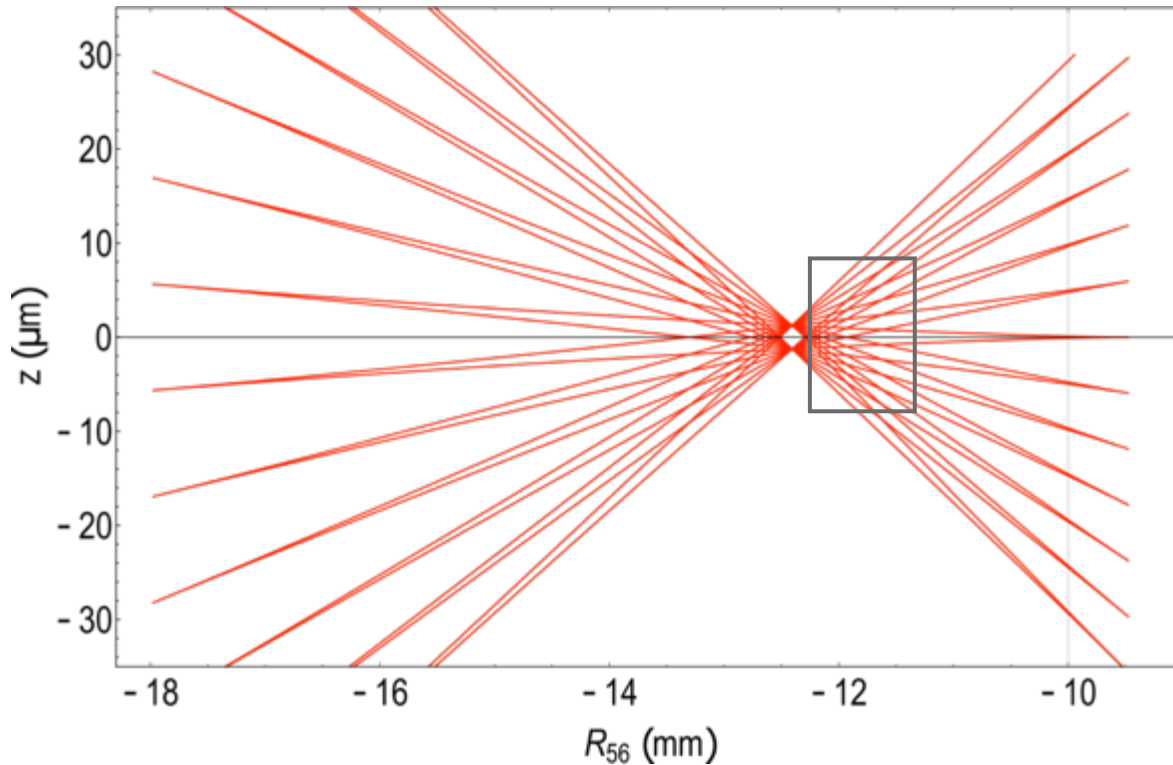




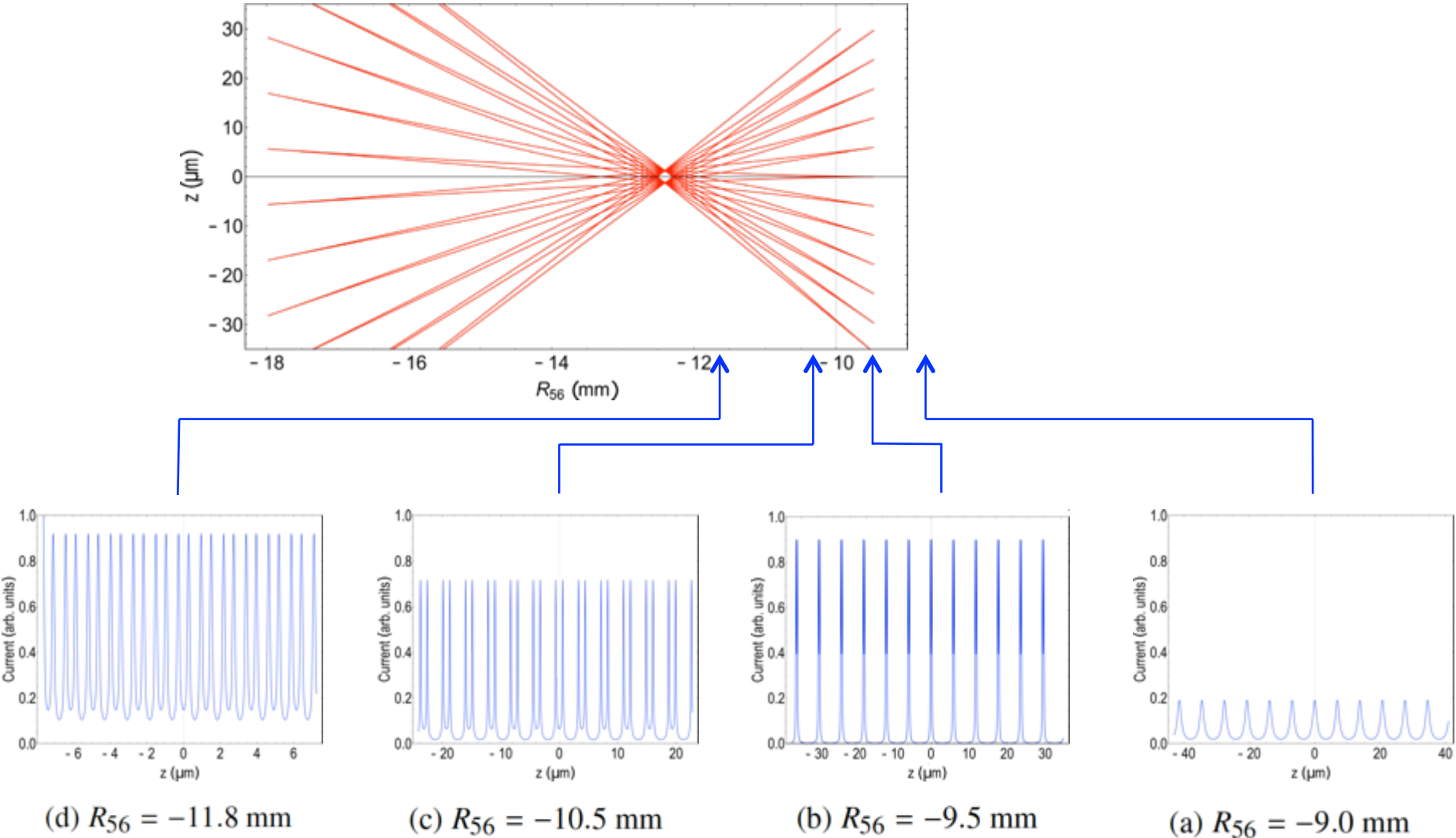
Is micro-bunching a caustic phenomenon?

# Micro-bunching caustics

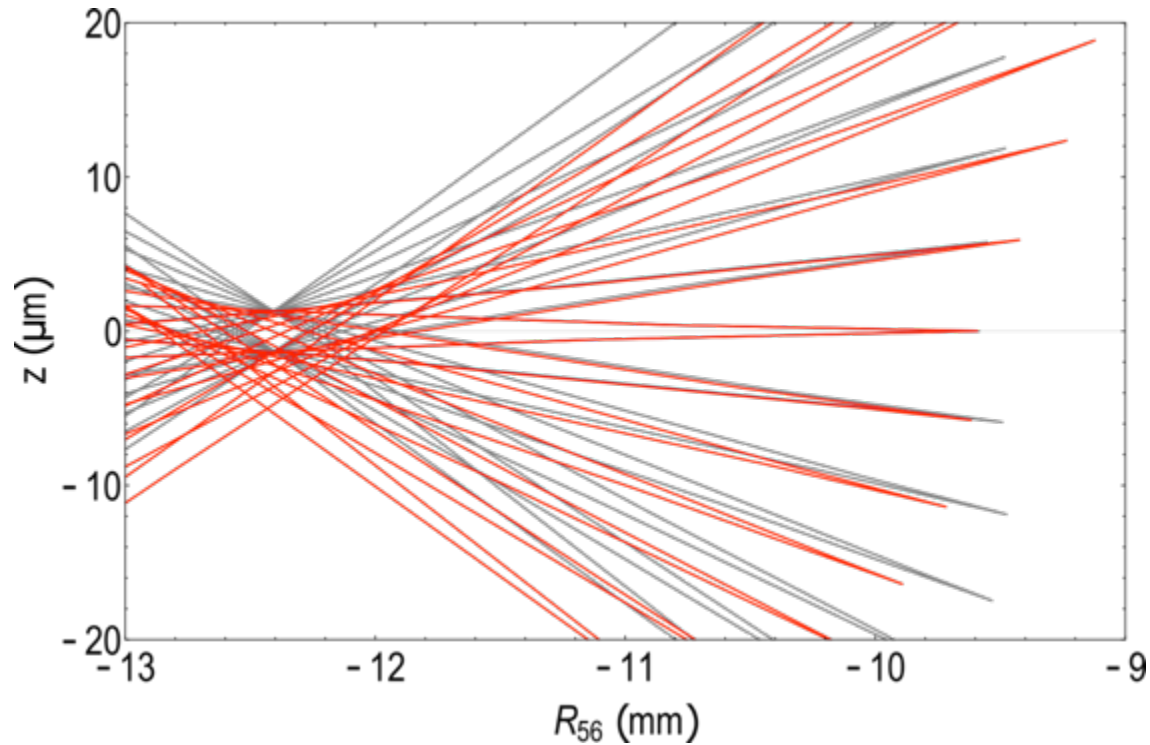
A caustic expression can also be derived for micro-bunching. Each line shows where a current spike will be seen.



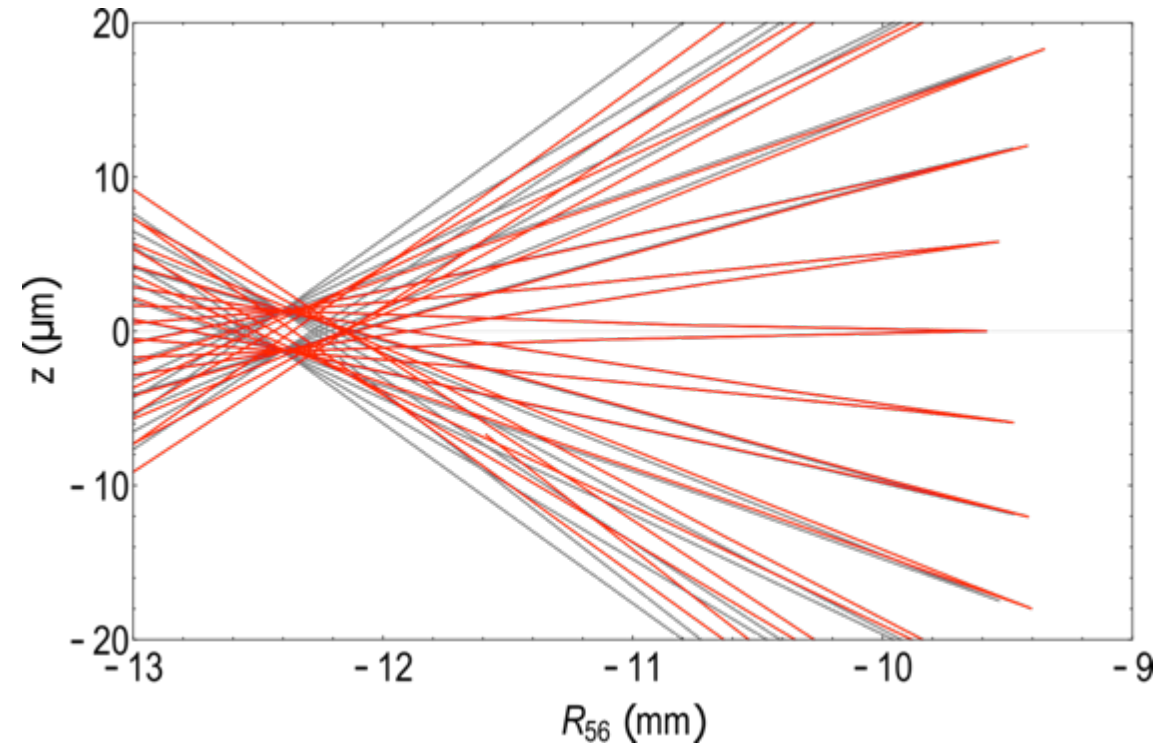
# Caustic nature leads to bifurcated current peaks



# Micro-bunching with higher-order effects



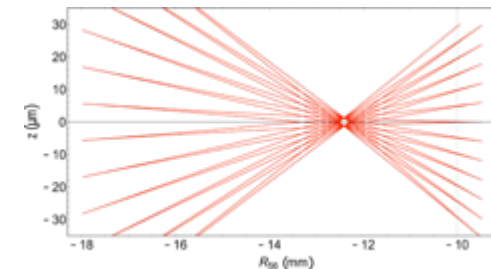
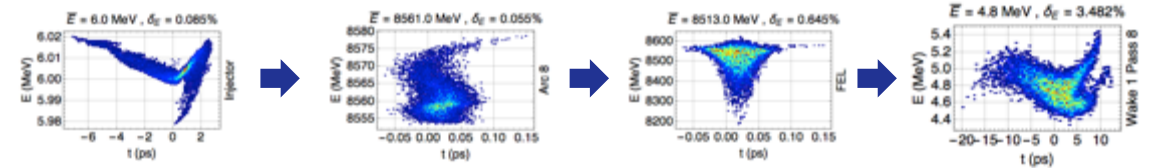
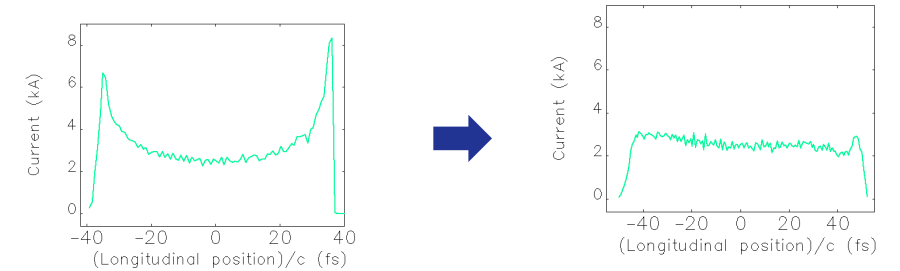
$T_{566} = 0, U_{5666} = 0$  mm (gray)  
 $T_{566} = -30$  mm,  $U_{5666} = 0$  mm (red)



$T_{566} = 0, U_{5666} = 0$  mm (gray)  
 $T_{566} = -2$  m,  $U_{5666} = 0$  mm (red)

# Conclusions

- Caustics can form as singularities in particle trajectory densities which are associated with current spikes.
- Current horns from strong bunch compression can be avoided with non-linear magnetic elements.
- The caustic approach can be applied to longitudinal phase space matching challenges in recirculating (and energy recovery)
- Micro-bunching can also be described as caustics



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