A Liouvillian Approach
to self-consistent beam-wave modeling

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U.S. DEPARTMENT OF ENERGY
Office of Science

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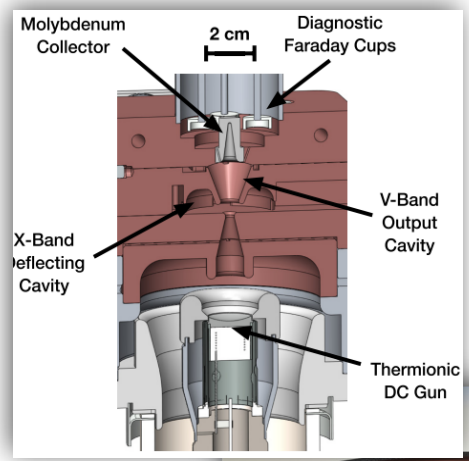
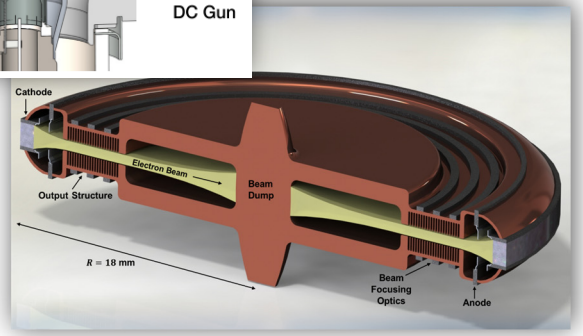
A growing *disconnect* between simulation and reality…



F. Toufexis, *Appl. Phys. Lett. 110*, 2017.



A. Vrielink, *Proc. 9th Int. Particle Accelerator Conf.* MOPMY035, 2016.

Modern manufacturing techniques
Additive manufacturing, lithography…

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Advances in materials science, optics and photonics
Metamaterials, wave-matter coupling…

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The next generation of RF power sources and accelerator structures:
Complex geometries and beams, features spanning wide spatio-temporal range

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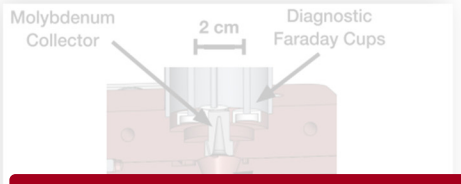

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Modern manufacturing techniques
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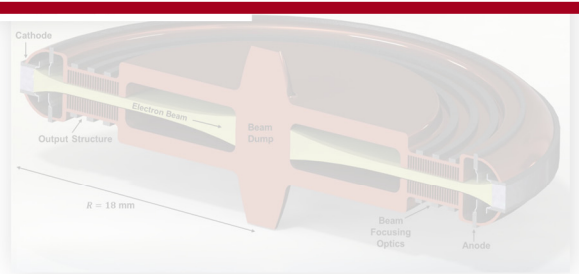
Need robust, computationally efficient multi-physics simulation tools

…



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The next generation of RF power sources and accelerator structures:
Complex geometries and beams, features spanning wide spatio-temporal range

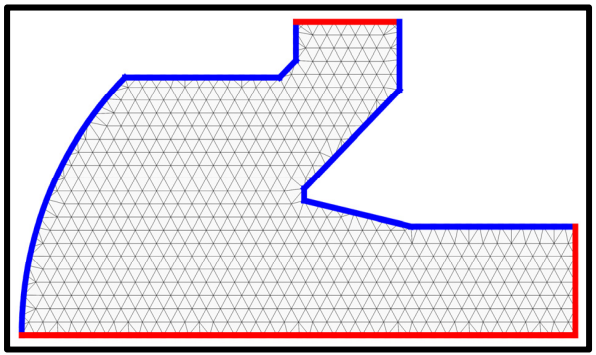
Iterative self-consistent solver

$$\mathcal{L} = \int \int \frac{\epsilon}{2} \left(|\nabla\phi + \frac{\partial \vec{A}}{\partial t}|^2 - c^2 |\nabla \times \vec{A}|^2 \right) - \rho\phi + \vec{A} \cdot \vec{J}\, dv\, dt$$

$\rho, \vec{J}$

Source Calculation

Hybrid Eulerian



Field Solver

Finite Element

$\phi, \vec{A}$

(Macro) Particle Tracking

Symplectic Integrator

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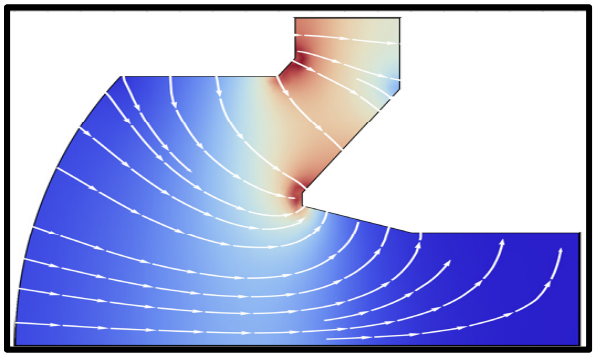

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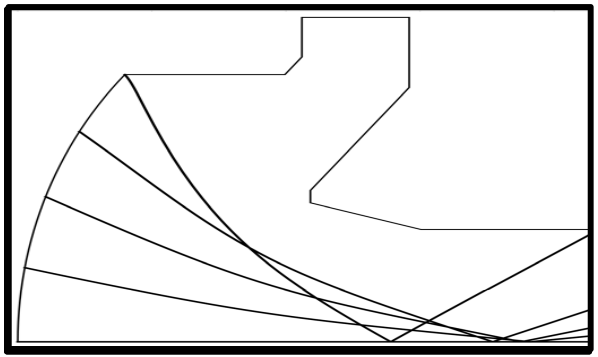

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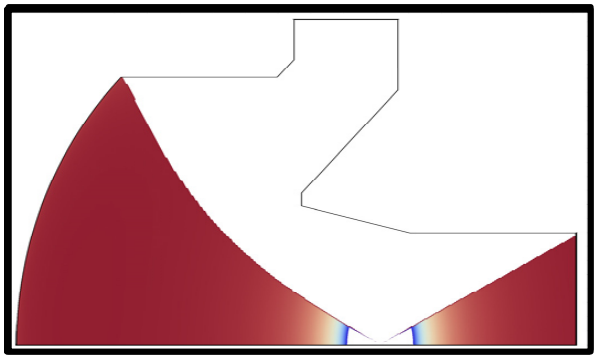

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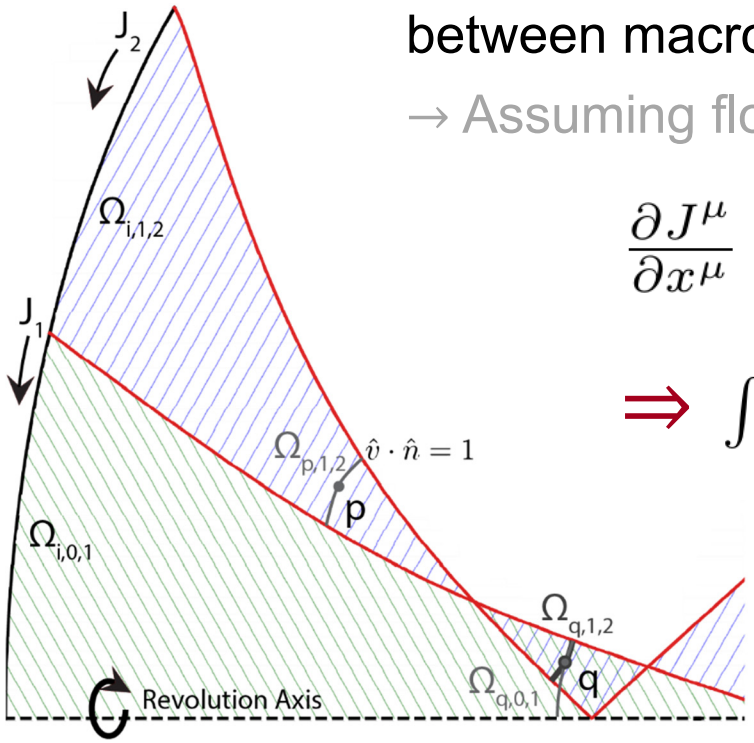
Exploiting Liouville's Theorem



Idea: Use charge conservation law to interpolate $\vec{j}$ between macroparticle trajectories

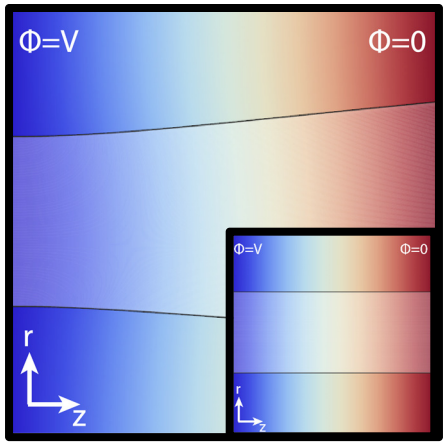
→ Assuming flow between macroparticles laminar

$$\frac{\partial J^\mu}{\partial x^\mu} = \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

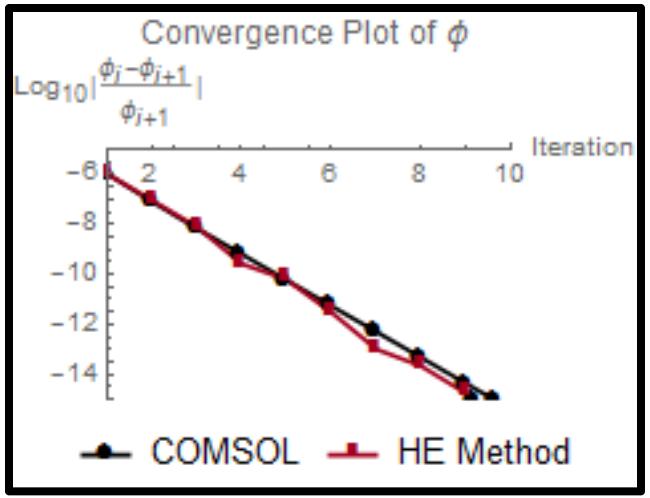
$$\Rightarrow \int_V \nabla \cdot \vec{j} dx = \int_{\partial V} \vec{j} \cdot \hat{n}\, dS = const = I$$

$$\Rightarrow \vec{j}_q = \frac{I_1}{A_{\Omega_{q,0,1}}} \hat{n}_{q,0,1} + \frac{I_2}{A_{\Omega_{q,1,2}}} \hat{n}_{q,1,2}$$

Same accuracy & convergence properties, 80x faster:



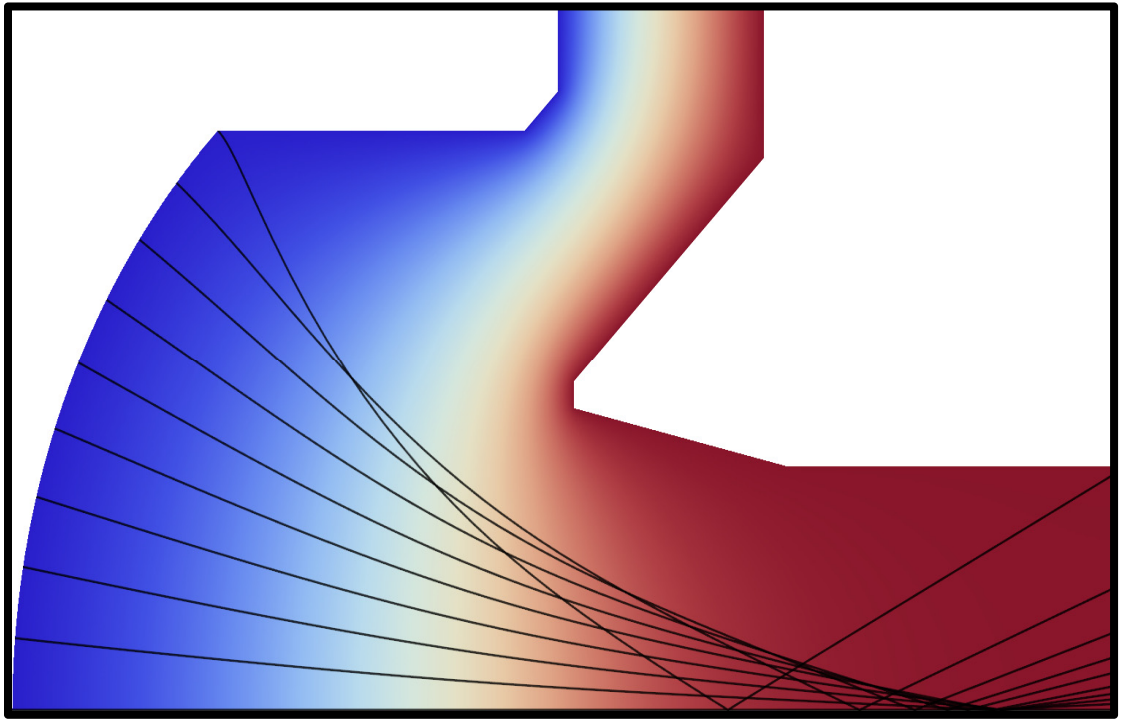
Particle trajectories plotted with electrostatic potential



	COMSOL	HE Method
Number of Mesh Cells	6282	6109
Number of Particles	200	2
Time / Iter. (s)	**7.80**	**0.10**

Highly non-laminar flow: 120x faster

	COMSOL	HE Method
Number of Mesh Cells	9318	9549
Number of Particles	500	10
Time / Iter. (s)	**33.8**	**0.29**



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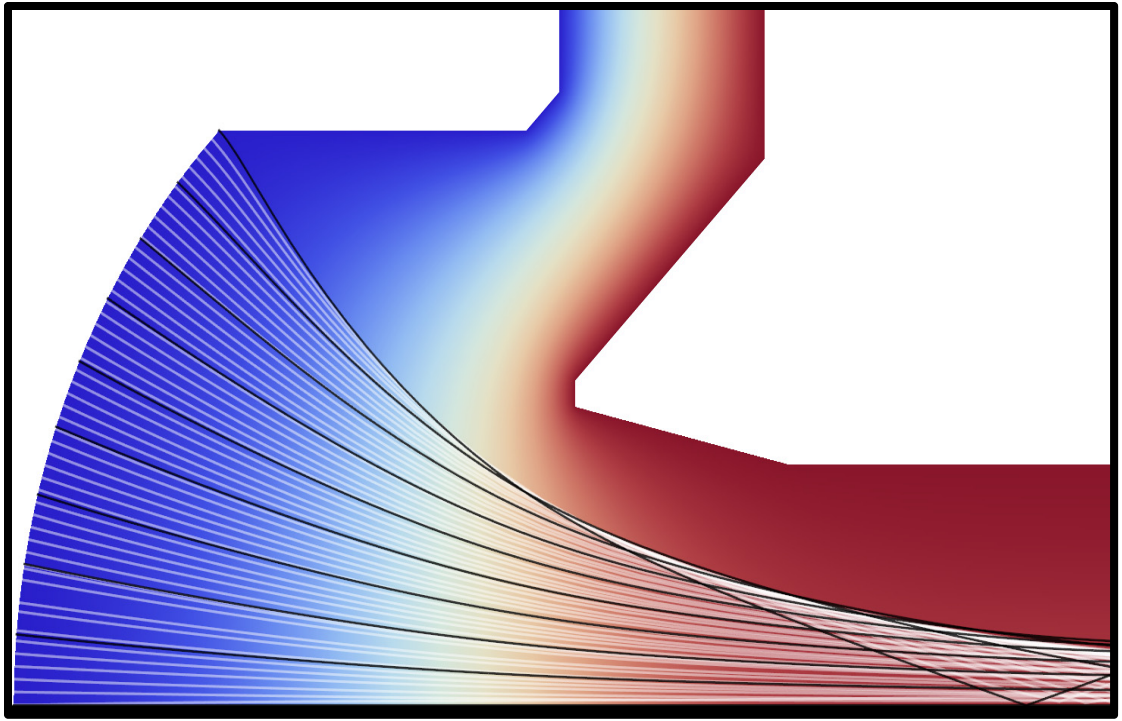

Acknowledgements

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T. Abel, O. Hahn and R. Kaehler, "Tracing the dark matter sheet in phase space," Mon. Notices Royal Astron. Soc. vol. 427, pp. 61-76. July, 2012.

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