

# POSSIBILITIES FOR TRANSVERSE FEEDBACK PHASE ADJUSTMENT BY MEANS OF DIGITAL FILTERS

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## Abstract

In transverse feedback systems a phase adjustment is generally required to convert a beam position signal from a pick-up into a momentum correction signal used by a transverse kicker. In this paper we outline several possibilities for phase adjustments using only single pick-ups or the vector combination of two pick-ups. Analytical expressions are given as a function of the fractional tune and the betatron phase advance between the pick-up location and the kicker. The shortest possible digital filter is formulated, including a notch for closed orbit suppression and a free parameter to adjust for betatron phase. We introduce a novel, fully parametrized digital filter with the feature to be insensitive to variations in fractional tune. Examples are given for the SPS transverse feedback system and compared with measurements.

## INTRODUCTION

In larger synchrotrons pick-ups and kickers can be generally placed at locations such that the betatron phase advance between them allows for the signal to be directly applied for feedback without further phase adjustments. These locations are however not always available for installing a dedicated monitor, or the required phase advance might change in machines with cycle-dependent optics.

The transverse feedback system (TFB) of CERN's Super Proton Synchrotron (SPS) has its pick-ups installed in close proximity to the kickers. The following analysis therefore aims on identifying potential solutions for transverse feedback phase adjustments using short finite impulse response (FIR) digital filters and one or more pick-ups.

## SIMPLE BEAM MODEL IN $z$ -DOMAIN

A first-order difference equation for the complex-valued sequence  $x[n]$ , which describes the linear portion<sup>1</sup> of the particle motion through the magnetic guidance field, is given by a recurrence formula,

$$x[n+1] = \alpha x[n], \quad n \geq 0; \alpha = r e^{j\omega_0}. \quad (1)$$

Introducing the  $z$ -Transform, defined by the  $\mathcal{Z}\{\cdot\}$ -operator (bilateral transform) [2],

$$\mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = X(z), \quad (2)$$

and applying it to the recurrence formula yields after some reformulation

$$X(z) = \frac{1}{1 - \alpha z^{-1}}. \quad (3)$$

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<sup>1</sup> We shall consider the case where the amplitude decay is dominated by active damping only, see also Ref. [1].

The rational function  $X(z)$  has a single complex-valued pole at  $z = \alpha$ , which corresponds to the fact that the sequence  $x[n]$  is complex-valued. Thereby, the convention used for Eq. (1) attributes the real part of  $x[n]$  as transverse displacement,  $y = \Re\{x\}$ , and the imaginary part as the trajectory's slope,  $y' = \Im\{x\}$ .

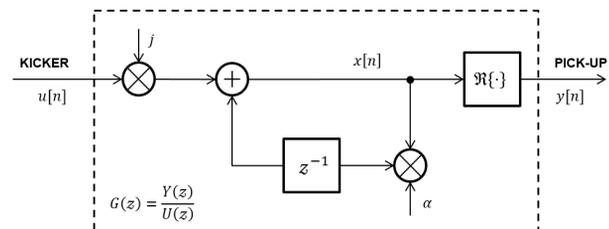


Figure 1: Simple beam model in  $z$ -domain.

The action of a kicker only changes a particle's slope. Consequently, in Fig. 1 the real-valued input sequence  $u[n]$  is first multiplied by the imaginary unit  $j = \sqrt{-1}$  and subsequently added to the complex-valued sequence  $x[n]$ . Beam position monitors detect only transverse displacement, hence  $y[n] = \Re\{x[n]\}$ , as outlined in Figure 1.

The overall beam transfer function  $G(z)$  can therefore be expressed as

$$G(z) = \frac{Y(z)}{U(z)} = j \frac{1}{2} \left( \frac{1}{1 - \alpha z^{-1}} - \frac{1}{1 - \alpha^* z^{-1}} \right). \quad (4)$$

Here,  $\alpha$  and  $\alpha^*$  are the two conjugate complex poles of  $G(z)$ . It is worth noting that the second pole seen at the output,  $Y(z) = G(z)X(z)$ , is the result of the pick-up and its ability to only detect transverse position.

Equation (4) can be expanded and rewritten as

$$G(z) = \frac{r \sin \omega_0 z^{-1}}{1 - 2r \cos \omega_0 z^{-1} + r^2 z^{-2}}, \quad (5)$$

which makes it evident that due to the term  $z^{-1}$  in the numerator the output sequence is readily delayed by one sample, corresponding to the fact that a response to a transverse deflection at a particular longitudinal position is visible only after one turn.

In Eq. (5) the parameter  $r$  determines whether the oscillation at frequency  $\omega_0$  is stable ( $|r| < 1$ ), steady ( $|r| = 1$ ), or unstable ( $|r| > 1$ ).

Taking into account an arbitrary phase shift  $\phi_{PU}$  between kicker and pick-up, i.e. by extending the pick-up output such that  $y[n] = \Re\{x[n] \cdot e^{j\phi_{PU}}\}$  one obtains more generally as beam transfer function

$$G(z, \phi_{PU}) = \frac{r \sin(\omega_0 + \phi_{PU}) z^{-1} - r^2 \sin \phi_{PU} z^{-2}}{1 - 2r \cos \omega_0 z^{-1} + r^2 z^{-2}}. \quad (6)$$

As can be seen from Eq. (6) the introduction of a betatron phase advance between pick-up and kicker has no impact on the position of the poles (the denominator is unaltered). In fact the required phase shift is established solely by an additional zero in the numerator, moving on the real axis of the  $z$ -plane as the phase angle changes.

## PICK-UP PLACEMENT

The ultimate goal could be stated as follows: determine the slope of a particle's trajectory at the position of the kicker, to be able to counteract oscillations by correcting its trajectory (i.e. the transverse momentum  $p_y$ ) on a turn-by-turn basis.

Given that — at the time of writing — there is no technique known which allows to directly measure transverse momentum, it requires an indirect method to obtain the slope via position measurements.

By recalling that a pick-up measures the real part it becomes obvious that if the sequence is phase rotated, by  $-90^\circ$  or  $-j$ , then the corresponding slope is returned as position,  $y_p$ , seen by the pick-up:

$$y_p = \Re\{-jx\} = \Re\{-jy + y'\} = y' . \quad (7)$$

This procedure allows for two possible interpretations to realize the phase rotation: (a) Spatial phase shift, and (b) Temporal phase shift.

### Spatial Phase Shift

Technically speaking, Eq. (7) means nothing else than to physically place a pick-up at betatron phase advance  $-90^\circ$  with respect to the kicker.

However, if the installation of a pick-up at this phase advance is not realizable, then a phase rotation  $\varphi$  can be obtained by combining (mixing) the signals of two pick-ups accordingly, that is,

$$y_k = p_1 \cdot \cos \varphi + p_2 \cdot \sin \varphi . \quad (8)$$

Here,  $p_1 = \Re\{x \cdot e^{j\psi}\}$  and  $p_2 = \Re\{x \cdot e^{j(\psi+\pi/2)}\}$  are two pick-ups at an arbitrary longitudinal position,  $\psi$ , but with  $90^\circ$  phase advance between them. The outputs are then mixed together to obtain the sequence  $y_k$  for the kicker action.

For the combination of two pick-up signals that have a phase advance different than  $90^\circ$  a more general solution has been already formulated in Ref. [3], as *Pick-up Vector Sum*. As detailed therein, the two pick-up mixing coefficients follow from

$$b_{1,2} = -\frac{1}{2} \left( \frac{\cos(\Delta\phi_{Qkm})}{\cos(\Delta\phi/2)} \mp \frac{\sin(\Delta\phi_{Qkm})}{\sin(\Delta\phi/2)} \right) , \quad (9)$$

where  $\Delta\phi = \phi_2 - \phi_1$  describes the phase advance between the two pick-ups, and with

$$\Delta\phi_{Qkm} = -3\pi Q_f + \phi_k - \frac{\phi_2 + \phi_1}{2} . \quad (10)$$

The fractional tune is denoted as  $Q_f$ , and  $\phi_1$ ,  $\phi_2$ , and  $\phi_k$  are the betatron phase advances at pick-up and kicker

locations with respect to some fixed reference. Equation (10) is readily taking into account a one-turn delay and the phase shift of the notch filter.

### Temporal Phase Shift

During one complete revolution a particle exhibits a precise number of oscillations, thus if the fractional tune  $Q_f$  is 0.25 then a phase rotation of  $90^\circ$  is achieved between consecutive turns. By reconsidering Eq. (7) we can exploit this as

$$y_p[n] = \Re\{jx[n-1]\} = -y'[n-1] , \quad (11)$$

which states that the position  $y_p$  at turn  $n$  represents the negative slope,  $-y'$ , of the previous turn,  $n-1$ .

In reality a fractional tune close to the quarter integer resonance is usually not very practical. Therefore the same Pick-up Vector Sum algorithm as for the two pick-up case can be applied for a single pick-up at subsequent turns, as outlined in Fig. 2. In this case we define  $\phi_1 = 0$  and  $\phi_2 = 2\pi Q_f$ , to be used with Eq. (9) respectively Eq. (10).

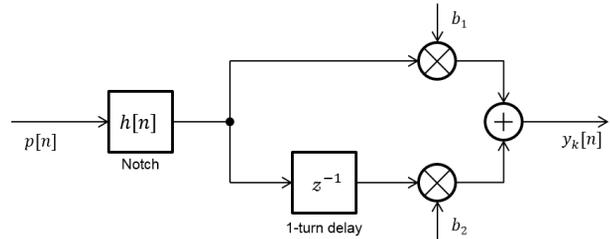


Figure 2: Block diagram of temporal phase shift.

Figure 2 suggests that by considering a notch filter using  $h[n] = [1, -1]$  the phase shift between  $y_k[n]$  and  $p[n]$  can be carried out by a short FIR, with the filter coefficients resulting from

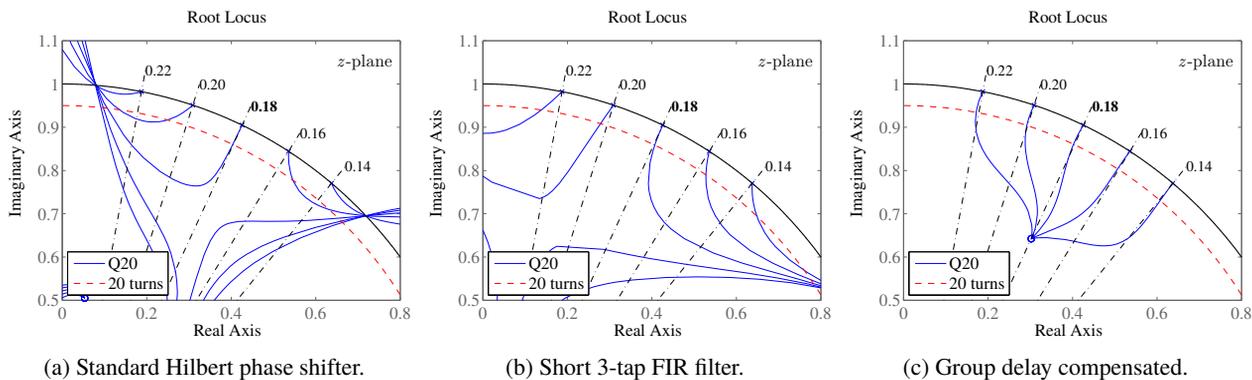
$$y_k[n] = b_1 p[n] + (b_2 - b_1) p[n-1] - b_2 p[n-2] . \quad (12)$$

With only three taps and including a notch for DC-orbit suppression Eq. (12) describes the shortest possible digital filter. The design has two parameters, the fractional tune  $Q_f$  and a free parameter  $\phi_k$  which allows for direct phase adjustment.

Optionally, improved noise suppression can be achieved with zeros added at  $z = \pm 1$ , which can be absorbed into the notch filter, including an extra phase term,  $\Delta\theta = -\pi Q_f$  into Eq. (10) for every additional tap the notch filter is extended.

By anticipating that the group delay — a measure for linearity of the phase — introduced by a filter lowers the stable phase margin of a closed loop system we made an attempt to compensate this effect by introducing an additional pair of conjugate complex zeros,  $c_{1,2} = \zeta e^{\pm j\omega_d}$ , at the desired tune frequency,  $\omega_d = 2\pi Q_f$ , which adds negative group delay as a function of the magnitude,  $\zeta$ . The group delay as specified in Ref. [2] follows for a direct-form FIR transfer function as

$$\tau(\omega) = -\sum_{k=1}^M \frac{|c_k|^2 - \Re\{c_k e^{-j\omega}\}}{1 + |c_k|^2 - 2\Re\{c_k e^{-j\omega}\}} . \quad (13)$$

Figure 3: Root locus plots, fractional tune  $Q_f = 0.18 \pm 0.04$ .

By taking into account the secondary phase term resulting from these zeros the overall group delay was compensated to  $\tau(\omega) = 0$  at the design tune,  $\omega = \omega_d$ . Note that a system having zero group delay is able to transmit the signal's envelop without delay.

## RESULTS

The introduced beam model of transverse oscillations (Eq. (6)) allows numerical evaluation of the analytic expressions for the described phase adjustment possibilities. We use root locus plots to study the evolution of the beam's closed-loop poles at selected frequencies,  $\omega_0 = 2\pi Q_f$ , and as a function of the feedback gain.

The three plots in Fig. 3 detail a portion of the upper right quarter of the complex  $z$ -plane. As an example closed-loop pole trajectories for the SPS vertical plane are shown, assuming a fractional tune of  $Q_f = 0.18$  and variations of up to  $\pm 0.04$  (blue, solid). Their origin is at the unit circle (black, bold solid) for zero loop gain (corresponding to the undamped open loop response). By increasing the feedback gain the damping time reduces gradually until the trajectories cross the circle at  $|z| = 0.95$  (red, dashed) which is the design value of the SPS TFB corresponding to 20 turns. For as long as a trajectory points towards the origin of the  $z$ -plane (black, dash-dotted lines) it will follow the desired closed loop negative feedback of  $-180^\circ$ .

Figure 3(a) shows the beam response for a standard FIR Hilbert phase filter, using seven taps for the case of the SPS TFB. As can be seen this filter is working perfectly fine at the target tune of 0.18, however, the feedback phase appears to be sensitive to tune variations, with the active feedback adversely pushing the tune further away the more its value deviates from the desired value. This effect is attributed to the constant group delay of the FIR filter causing the feedback phase to be optimum only for a single frequency and to roll off quickly for long filters.

As already anticipated, shortening the FIR filter length to only 3 taps, as provided by Eq. (12), has a positive effect on tune variations, as can be seen by inspecting Fig. 3(b), with the closed loop response remaining stable over a larger range of tune values.

Figure 3(c) shows that the loop stability is ultimately improved by carefully compensating the group delay of the digital filter. With its flat phase response around the design tune this filter is robust against changes of the fractional tune in the order of  $\pm 22\%$ . This filter was tested in the SPS TFB by changing the machine's fractional tune and performing beam transfer function measurements. The results listed in Table 1 confirm the theory of negative feedback over the same range of fractional tunes.

Table 1: Measured Loop Response vs. Tune

Tune	0.14	0.16	<b>0.18</b>	0.20	0.22
Phase	$-167^\circ$	$-179^\circ$	$-182^\circ$	$-182^\circ$	$-187^\circ$

## SUMMARY AND CONCLUSION

New FIR phase shift filters have been designed and analysed for the SPS TFB based on a simple beam model in  $z$ -domain and with root locus plots, to assess their closed loop performance with beam and in presence of active feedback. The obtained results were found to be in good agreement with measurements carried out in the SPS. Limitations on system gain [4], the performance in the presence of noise [5, 6], and the ability to reject disturbances are subject of further studies.

The shortest digital filter described has only 3-taps, including DC suppression by a notch filter (for other realisations of short phase shift filters see for example [7–12]). Moreover, the tune sensitivity was further compensated with an additional pair of conjugate complex zeros, effectively lowering the filter's group delay to zero at the fractional tune. Compensation of tune-dependent phase variations were reported also in Ref. [13].

The described digital filters are reasonable candidates for replacing the Hilbert phase shifter currently in use in the transverse feedback system of the SPS, mostly due to their favourable response over a larger range of tune values.

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