POSSIBLE LIMITS OF PLASMA LINEAR COLLIDERS*

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Abstract

Plasma linear colliders have been proposed as next or next-next generation energy-frontier machines for high-energy physics. I investigate possible fundamental limits on energy and luminosity of such type of colliders, considering acceleration, multiple scattering off plasma ions, intrabeam scattering, bremsstrahlung, and betatron radiation. The question of energy efficiency will also be addressed.

INTRODUCTION

Plasma acceleration [1] is one of the proposed technologies for realizing a highest energy e^+e^- collider, e.g. [2,3]. A plasma linear collider would accelerate electrons and positrons in the strong fields achievable in a plasma wake, and collide them at an interaction point (IP). The plasma wake itself can be excited in a variety of ways, e.g. by short high-power laser pulses [4], by driving electron bunches [5], or by a self-modulated proton beam [6]. The maximum energy reach, luminosity and electrical power efficiency are crucial figures of merit, which might be limited by various physical effects in such novel type of accelerator.

ACCELERATION AND FOCUSING

In the blow-out [7, 8] or bubble regime [9], the maximum accelerating electric field is [1] $E_s \approx m_e c^2 \sqrt{4\pi n_e r_e}/e$, where $m_e c^2$ denotes the electron rest mass, r_e the classical electron radius, and n_e the plasma (electron) density. In practical units we have E_s [V/m] $\approx 0.1 \left(n_e[\mathrm{m}^{-3}]\right)^{1/2}$. Typical plasma densities proposed for future plasma linear colliders are of order 10^{22} – 10^{24} m^{-3} [2], corresponding to gradients $E_s \approx 10 - 100$ GV/m. Introducing $a \equiv eE_s/(m_e c^2)$, for constant acceleration the beam energy E_b grows as $E_b(s) = (as + \gamma_0) m_e c^2$, where γ_0 is the initial value of the Lorentz factor, and

$$a \approx \sqrt{4\pi n_e r_e} \, , \tag{1}$$

the acceleration in units of 1/metre. The Lorentz factor changes as $\gamma(s) = as + \gamma_0$. An approximately linear focusing force acts on the electrons [7],

$$d^2r/ds^2 \approx -2\pi r_e n_e r/\gamma(s) \, , \tag{2}$$

where r refers to the radial distance from the axis.

Assuming the beam focusing is provided only by the plasma, the plasma-matched beta function $\beta \equiv \beta_x = \beta_y$ is energy dependent and equals

$$\beta(s) \approx \sqrt{\gamma(s)/(2\pi n_e r_e)} \, . \tag{3}$$

To avoid blow-out and to stay in the so-called "quasilinear" regime, the beam density would need to be lower than the plasma density and the beta function greater than [10]

$$\beta_{\mathrm{ql}}(s) \geq N\gamma(s)/((2\pi)^{3/2} n_e \varepsilon_N \sigma_z) \, , \tag{4}$$

where σ_z denotes the rms bunch length, N the bunch population, and ε_N the normalized emittance.

EMITTANCE EVOLUTION

Acceleration reduces the geometric emittance ε as

$$d\varepsilon/ds|_{\mathrm{ad}} \approx -a\varepsilon/\gamma \, . \tag{5}$$

Multiple small angle elastic scattering of the beam particles off the background ions (if present) leads to emittance growth, which we approximately estimate as [11]

$$\left.\frac{d\varepsilon}{ds}\right|_{\mathrm{ms}} \approx \beta(s) \left(\frac{13.6\,\mathrm{MeV}}{E_b}\right)^2 \frac{1}{X_0} \, , \tag{6}$$

where $\beta(s)$ denotes the beta function at location s, and X_0 the radiation length of the plasma ions in units of m. In the expression for the average squared scattering angle leading to (6), we have dropped a logarithmic term $(1 + 0.038 \ln(s/X_0))^2$ [11]. In the blow-out regime, a lower bound on the inverse radiation length is [12]

$$1/X_0 = 4\alpha r_e^2 n_e \left(Z^2[L_{\mathrm{rad}} - f(Z)] + (Z-1)L'_{\mathrm{rad}}\right) \, , \tag{7}$$

where we have assumed singly ionized plasma ions, Z designates the atomic number, α the fine-structure constant, and the second term in the parentheses corresponds to the contribution from the remaining electrons. Up to uranium, the function $f(Z)$ can be represented as [13]

$$f(Z) \approx \frac{f^2}{1+f^2} + 0.2f^2 - 0.037f^4 + 0.008f^6 - 0.002f^8 \, ,$$

where $f \equiv \alpha Z$. The quantities L_{rad} and L'_{rad} were calculated by Tsai [12]. For $Z > 4$ they can be approximated as $L_{\mathrm{rad}} \approx \ln(184.15\ Z^{-1/3})$ and $L'_{\mathrm{rad}} \approx \ln(1194\ Z^{-2/3})$, using the Thomas-Fermi-Moliere model [12]. Table 1 shows some example values for commonly used alkali plasmas, considering a typical plasma electron density of $n_e = 10^{23}$ m^{-3}, corresponding to a peak field of about 30 GV/m.

Table 1: Scattering and bremsstrahlung parameters for alkali plasmas at a plasma electron density of $n_e = 10^{23}$ m^{-3}, assuming single ionization.

el.	Z	$f(Z)$	L_{rad}	L'_{rad}	X_0 [km]
Li	3	0.0006	4.74	5.81	1600
Rb	37	0.083	4.01	4.68	16
Cs	55	0.17	3.88	4.41	7.6

In addition to scattering off the plasma elements, the beam particles can scatter off each other. Consider a round Gaussian bunch of N electrons with transverse normalized

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rms emittances $\varepsilon_N \equiv \gamma\varepsilon_x = \gamma\varepsilon_y$, and normalized longitudinal rms emittance $\varepsilon_{z,N} \equiv \gamma\sigma_\delta\sigma_z$, where σ_δ denotes the rms relative momentum spread. Assuming that the dispersion function is zero everywhere, and that $\gamma/\sigma_\delta \gg \sqrt{\beta/\varepsilon}$ (generally fulfilled for relativistic beams), the analytical formula for the emittance growth due to intrabeam scattering (IBS) [14–16] reduces to the closed expression

$$\left.\frac{d\varepsilon}{ds}\right|_{\rm IBS} \approx -\frac{r_e^2 N \beta^{1/2} \log}{16\varepsilon_N^{3/2}\sigma_z\gamma^3}\,, \qquad (8)$$

where log $\approx 15-20$ denotes the Coulomb logarithm, and we have used $\int_0^\infty d\lambda(\lambda+b)/(\lambda^3+x\lambda^2+2bx\lambda+b^2x)^{3/2} \approx \pi/(2\sqrt{b}x^{3/2})$ for $x \gg b$. The transverse emittance shrinks (!), but more weakly the higher the energy.

Inserting the matched beta function (3), (8) becomes

$$\left.\frac{d\varepsilon}{ds}\right|_{\rm IBS} \approx -\frac{r_e^{7/4} N \log}{16(2\pi)^{1/4} n_e^{1/4}\varepsilon_N^{3/2}\sigma_z\gamma^{11/4}}\,, \qquad (9)$$

which still decreases strongly with energy.

The emittance at location s is determined by

$$\left.\frac{d\varepsilon}{ds}\right|_{\rm tot} = \left.\frac{d\varepsilon}{ds}\right|_{\rm ad} + \left.\frac{d\varepsilon}{ds}\right|_{\rm ms} + \left.\frac{d\varepsilon}{ds}\right|_{\rm IBS} \qquad (10)$$

At high energy $\gamma \gg \gamma_0$ the local "equilbrium emittance" follows from $d\varepsilon/ds|_{\rm tot} = 0$. Again assuming the beta function (3), and introducing the two auxiliary constants

$$B_{\rm ms} \equiv \frac{1}{2\pi\sqrt{2}r_e}\left(\frac{13.6\,{\rm MeV}}{m_e c^2}\right)^2 \frac{1}{X_0 n_e}\,, \qquad (11)$$

$$C_{\rm IBS} \equiv \frac{r_e^{5/4}\log}{32\,(2)^{1/4}\,\pi^{3/4}}\,, \qquad (12)$$

the equilibrium condition becomes

$$\varepsilon_{N,\rm eq} = B_{\rm ms}\gamma^{1/2} - C_{\rm IBS}\frac{N}{n_e^{3/4}\varepsilon_N^{3/2}\sigma_z\gamma^{7/4}}\,. \qquad (13)$$

In the limit of zero bunch charge ($N = 0$), the normalized equilibrium emittance is given by $\varepsilon_N = B_{\rm ms}\gamma^{1/2}$. For a Li plasma $B_{\rm ms} \approx 0.2$ μm, for Cs $B_{\rm ms} \approx 40$ μm; at 1 TeV, $B_{\rm ms}\gamma^{1/2}$ becomes 0.25 nm and 50 nm, respectively – negligible for round-beam operation.

The emittance growth due to multiple scattering has been examined previously, e.g., in [17] (for crystals) and [2, 10, 18, 19] (for plasmas), in each case invoking somewhat different assumptions and approximations. Our estimate with emittance values around 1 nm is of the same order of magnitude as those in [2, 18]. Reference [10] obtained a larger value of 10 μm, by considering the beta function (4) for the quasilinear regime, in place of the plasma-matched beta function (3).

Concerning the IBS contribution, $C_{\rm IBS} \approx 1.3 \times 10^{-19}$ m$^{5/4}$ (with log $\approx$ 17.5). Then, for example, using $n_e \approx 10^{23}$ m^{-3}, $\varepsilon_N \approx 1$ μm, $N \approx 10^9$, $\sigma_z \approx 1$ μm, $\sigma_\delta \approx 10^{-3}$ at a beam energy of 1 TeV, we obtain $C_{\rm IBS}N/(n_e^{3/4}\varepsilon_N^{3/2}\sigma_z\gamma^{7/4}) \approx 10^{-23}$ m, an insignificant effect.

IBS also changes the beam energy spread, namely as

$$\left.\frac{d\sigma_\delta^2}{ds}\right|_{\rm IBS} \approx 2\frac{r_e^2 N \log}{16\beta^{1/2}\varepsilon_N^{3/2}\sigma_z\gamma}\,. \qquad (14)$$

Comparing with (8), we note that

$$\frac{d}{ds}\left[\frac{\sigma_\delta^2}{\gamma^2} + 2\frac{\varepsilon}{\beta}\right]_{\rm IBS} \equiv \frac{d}{ds}\left[\frac{\sigma_\delta^2}{\gamma^2} + \frac{\varepsilon_x}{\beta_x} + \frac{\varepsilon_y}{\beta_y}\right]_{\rm IBS} = 0\,, \qquad (15)$$

a relation already derived by Piwinski [20].

Using (8) and (3), we find for the plasma-focused case

$$\left.\frac{d\sigma_\delta}{ds}\right|_{\rm IBS} = -\frac{\gamma^2}{\sigma_\delta\beta}\left.\frac{d\varepsilon}{ds}\right|_{\rm IBS} \approx \frac{r_e^{9/4}(2\pi)^{1/4}\,\log N n_e^{1/4}}{16\varepsilon_N^{3/2}\varepsilon_{z,N}\gamma^{1/4}}\,. \qquad (16)$$

or, for large γ, $\Delta\sigma_\delta \to 4\,d\sigma_\delta/ds|_{\rm IBS}\,\gamma/(3a)$. Assuming the same parameters as above, this amounts to $d\sigma_\delta/ds|_{\rm IBS} \approx 2\times10^{-8}$/m, and $\Delta\sigma_\delta \approx 10^{-6}$ — negligible.

ENERGY LOSS

At very high energies and except at the high-energy tip of the bremsstrahlung spectrum, in the "complete screening case" (though in our case atoms are partially ionized and nuclei not fully shielded) the cross section can be approximated as [12]

$$\left.\frac{d\sigma}{dk}\right|_{\rm bs} \approx \frac{1}{k}4\alpha r_e^2\left\{\left(\frac{4}{3}-\frac{4}{3}y+y^2\right)\left(Z^2[L_{\rm rad}-f(Z)] +(Z-1)L'_{\rm rad}\right) - \frac{1}{9}(1-y)(Z^2+Z-1)\right\}\,, \qquad (17)$$

where k is the photon energy, and $y = k/E_b$ the fraction of the electron energy transferred to the radiated photon. If the second term (of order of 2% of the total) is ignored the above can be expressed through the radiation length as

$$\left.\frac{d\sigma}{dk}\right|_{\rm bs} \approx \frac{1}{X_0 n_e k}\left(\frac{4}{3}-\frac{4}{3}y+y^2\right)\,. \qquad (18)$$

The average energy loss per unit length becomes

$$\left.\frac{dE_b}{ds}\right|_{\rm bs} = n_e\int_{E_{\rm min}}^{E_b} dk\left.\frac{d\sigma}{dk}\right|_{\rm bs} k \approx \frac{E_b}{X_0}\,. \qquad (19)$$

This can be a noticeable effect for plasma atoms of higher atomic number (e.g., Rb, Cs) — see Table 1.

The energy loss due to scattering off thermal photons [21, 22] might need to be taken into acount if the plasma temperature is high. The additional contribution from scattering off atomic electrons [22] is negligible.

More important is the following effect. A nonzero betatron amplitude together with the linear focusing force of the plasma leads to the emission of a special type of synchrotron radiation called betatron radiation (br) [23]. The resulting energy loss is given by

$$\left.\frac{dE_b}{ds}\right|_{\rm br} = -\frac{2}{3}r_e m_e c^2\gamma^2(2\pi r_e n_e)^2 r^2\,. \qquad (20)$$

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Averaging over a Gaussian bunch we have $< r^2 >= 2\beta(s)\varepsilon()$, and with the beta function of (3)

$$\left\langle \frac{dE_b}{ds}\bigg|_{\text{br}} \right\rangle = -\frac{4}{3}\left(r_e^{5/2}(2\pi)^{3/2}n_e^{3/2}m_e c^2\right)\gamma(s)^{5/2}\varepsilon(s)\ .$$

The acceleration comes to a standstill is

$$a m_e c^2 + \frac{dE_b}{ds}\bigg|_{\text{bs}} + \frac{dE_b}{ds}\bigg|_{\text{br}} = 0\ . \quad (21)$$

For simplicity, dropping the bremsstrahlung and retaining only the betatron-radiation loss, this condition becomes

$$1/(\gamma_{\text{max,br}}(s)\varepsilon(s)) = 4\sqrt{2}\pi r_e^2 n_e \gamma_{\text{max,br}}(s)^{3/2}/3\ . \quad (22)$$

The right side of the equation increases strongly with γ. A lower emittance boosts the energy reach:

$$\gamma_{\text{max,br}} = (4\sqrt{2}\pi r_e^2 \varepsilon_N n_e/3)^{-2/3}\ . \quad (23)$$

Let us consider an example: With $\varepsilon_N \equiv \gamma\varepsilon = 10\ \mu$m, $n_e = 10^{23}\ \text{m}^{-3}$, the inverse of (23) yields $\gamma_{\text{max}} \approx 7.7\times 10^6$, or a maximum energy of 4 TeV, where betatron radiation loss outweighs the acceleration.

Combining the relations (1) and (23) allows us to estimate the minimum length of a plasma accelerator. Assuming a constant (maximum possible) plasma density determined by the final energy, this minimum length becomes

$$l_{\text{min}} = \gamma/a_{\text{max}} = 2^{1/4} r_e^{1/2}/3^{1/2}\gamma^{7/4}\sqrt{\varepsilon_N}\ . \quad (24)$$

This length increases roughly quadratically with energy, also taking into account (13). The relation (24), for fixed ε_N, is illustrated in Fig. 1. Weaker focusing (quasi-linar regime) yields a more favorable conclusion on the accelerator length [2], e.g., by considering the beta function of (4).

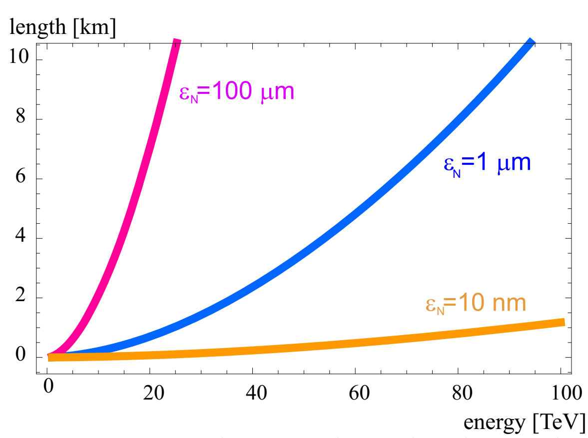

Figure 1: Minimum plasma accelerator length versus beam energy, according to (24); the various curves correspond to different values of the transverse emittance as indicated.

EFFICIENCY

In the classical beamstrahlung regime (i.e. critical photon energy E_c smaller than E_b, or $\Upsilon \equiv (2/3)E_c/E_b \ll 1$), the luminosity of a linear e^+e^- collider may be written as [24]

$$L \approx \left(\frac{1}{16\pi\alpha r_e}\right)\frac{P_{\text{wall}}}{E_b}N_\gamma\frac{\eta_{\text{tot}}}{\sigma_y^*}\ , \quad (25)$$

where P_{wall} is the wall plug power, and η the conversion efficiency of wall-plug (w.p.) power into total average beam power, $P_{\text{beam}} = 2f_{\text{rep}}NE_b$, and f_{rep} the (average) bunch repetition rate. N_γ denotes the average number of beamstrahlung photons per colliding particle, and should not much exceed 1, in order to preserve a well-defined luminosity spectrum. Since cross sections decrease as $1/\gamma^2$ the luminosity should increase with energy as γ^2. The vertical beam size at the collision point, σ_y^*, is limited by various effects. Values of σ_y^* below 1 nm have rarely been proposed.

Equation (25) reveals the general feature that the luminosity is proportional to the overall conversion efficiency η_{tot}. For linear colliders like CLIC and ILC the value of η_{tot} varies between 4 and 6%. For a plasma collider the efficiency is the product of the efficiency of generating the driver, and the transfer efficiencies from the driver to the plasma wake, and from the wake to the main beam, i.e.:

$$\eta_{\text{tot}} = \eta_{\text{w.p.}\to\text{driver}}\ \eta_{\text{driver}\to\text{wake}}\ \eta_{\text{wake}\to\text{beam}}\ . \quad (26)$$

The driver can be a proton beam, an electron beam, or a laser pulse. From [3, 25–27] we estimate that

$$\eta_{\text{w.p.}\to\text{driver}} \approx \begin{cases} <0.5\% & \text{protons} \quad (5\times \text{faster LHC}) \\ \approx 40\% & e^- \quad (\text{cw SC linac}) \\ \approx 50\% & \text{laser} \quad (\text{future fibre lasers}) \end{cases}$$

According to simulations with Gaussian bunches a realistic transfer efficiency from driver to wake is $\eta_{\text{driver}\to\text{wake}} \approx 77\%$ [3]. For $\eta_{\text{wake}\to\text{beam}}$ a peak value of about 50% was demonstrated at FACET [5], and 65% or above appears attainable [3, 28]. However, transverse stability constraints may yet reduce these optimistic efficiency figures [29].

CONCLUSIONS

Equations (13), (16) and (23) impose constraints on high-energy plasma-based accelerators. The scattering off plasma nuclei and plasma electrons is almost negligible in the blow-out regime, but may become significant in the opposite, quasilinear regime. The inverse dependence is true for betatron radiation. Both effects might be mitigated by accelerating in a hollow plasma channel [30, 31], at the expense of a lower accelerating gradient and, perhaps, additional complication [31]. The betatron radiation is then determined by the electromagnetic fields in this channel [32].

It appears that (only) if driven by fibre lasers plasma accelerators might achieve overall efficiencies approaching, or exceeding, those of conventional accelerators: $\eta_{\text{tot}} \approx 50\%\times 77\%\times 65\% \approx 25\%$. Considering only the efficiency constraint, from (25) a 3 TeV e^+e^- collider with this optimistic value of $\eta_{\text{tot}} \approx 25\%$, along with $E_b = 1.5$ TeV, $\sigma_y^* \approx 1$ nm, $N_\gamma \approx 1$ and a wall-plug power of 500 MW could reach a luminosity of $5\times 10^{34}\ \text{cm}^{-2}\text{s}^{-1}$ (or possibly 2× more if operating at $\Upsilon \gg 1$ [3]), which is 6× (or 3×) lower than the luminosity of a 100 TeV circular pp collider, FCC-hh [33], at comparable electric power.

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