

ALGORITHM TO CALCULATE OFF-PLANE MAGNETIC FIELD FROM AN ON-PLANE FIELD MAP *

N. Tsoupas[†], J. S. Berg, F. Méot, V. Ptitsyn, D. Trbojevic
 Brookhaven National Laboratory, Upton, NY, USA
 S. Machida STFC Rutherford Appleton Laboratory England
 S. Kahn Muons, Inc., Batavia, IL, USA

Abstract

We present an algorithm to calculate the off-plane components of the magnetic field from the on-plane components of the magnetic field which are measured on a grid of the plane. The algorithm is based on the Taylor series expansion of the magnetic field components in terms of the normal to the plane location. The coefficients of the Taylor series expansion are expressed in terms of the on-plane derivatives of the field components. We describe three different ways in which these in-plane derivatives have been computed in existing codes.

INTRODUCTION

With the advent of the technology to perform more accurate magnetic field measurements the beam optics calculations very often rely on the measurements of the magnetic fields of a single magnet or many magnets over a 3D or a 2D grid. If the measurements of the magnetic fields are made on a 3D grid the field map is used to calculate the beam optics of the magnets by integrating the equation of motion in the 3D field map. However if the magnetic field measurements are made on a plane grid, Maxwell's equations have to be used to calculate the magnetic field at any point in space. In this technical note we develop an algorithm which provides the magnetic field at any given point in space from the knowledge of the magnetic field on a plane. Fig. 1 shows the grid points (intersection points of the red lines) on a plane where the magnetic field components are measured experimentally. No material exists in between the plane and the point in space where the field is calculated. The algorithm provides the values of the components of the magnetic field at a distance y from the plane. Although most of the magnets used in the applications of charged particle beam optics and in particle accelerators have median plane symmetry, some of the methods we describe are not constrained of midplane symmetric fields, and the only requirement will be the experimentally measured, or computed, field components of the magnetic field at the grid points of the plane. The algorithm is based on the Taylor series expansion of the magnetic field at the point of interest in terms of the y coordinate which is the distance of the point, the field is to be calculated, from the plane. The coefficients of the Taylor series expansion are ultimately expressed in terms of the values of the field components at grid points.

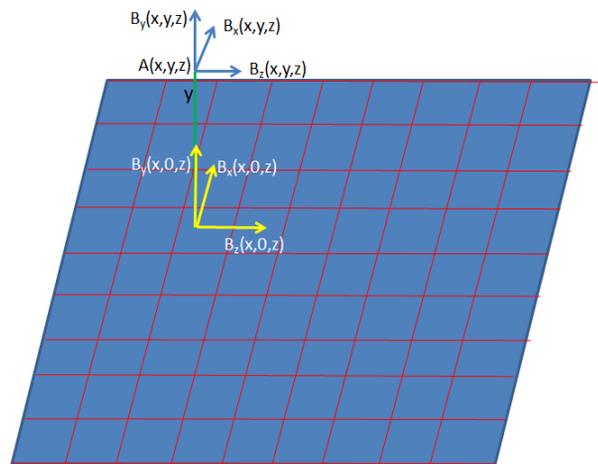


Figure 1: A schematic diagram of a grid on a plane. The magnetic field components at any grid point on the plane are measured. The algorithm calculates the magnetic field components at any given point in space at a distance y from the plane.

EXPANSION OUT OF THE MIDPLANE

We begin with the expansion of the magnetic field as a power series in y :

$$B_i(x, y, z) = \sum_{j=0}^{\infty} \frac{1}{j!} B_{ij}(x, z) y^j \quad (1)$$

where i is the component of the magnetic field. Given $B_i(x, 0, z) = B_{i0}(x, z)$, we can compute the coefficients of y^j for $j \neq 0$ using Maxwell's equations. It is useful to distinguish between the midplane-symmetric fields, where $B_x(x, 0, z) = B_z(x, 0, z) = 0$, and the midplane-asymmetric fields, where $B_y(x, 0, z) = 0$. Of course the general case can be a sum of these two cases.

Maxwell's equations can be used to find the coefficients B_{ij} for $j > 1$ using the fields in the midplane, B_{i0} . For the midplane-symmetric case, up to the fourth power of y ,

$$B_{x1} = \partial_x B_{y0} \quad (2)$$

$$B_{z1} = \partial_z B_{y0} \quad (3)$$

$$B_{y2} = -(\partial_x^2 + \partial_z^2) B_{y0} \quad (4)$$

$$B_{x3} = -(\partial_x^3 + \partial_x \partial_z^2) B_{y0} \quad (5)$$

$$B_{z3} = -(\partial_x^2 \partial_z + \partial_z^3) B_{y0} \quad (6)$$

$$B_{y4} = (\partial_x^4 + 2\partial_x^2 \partial_z^2 + \partial_z^4) B_{y0} \quad (7)$$

* Work supported by the US Department of Energy

[†] tsoupas@bnl.gov

$$c^{40} : \begin{bmatrix} & & 0 & & \\ & 0 & 0 & 0 & \\ 1 & -4 & 6 & -4 & 1 \\ & 0 & 0 & 0 & \\ & & 0 & & \end{bmatrix} \quad (22)$$

$$c^{22} : \begin{bmatrix} & & 0 & & \\ & 1 & -2 & 1 & \\ 0 & -2 & 4 & -2 & 0 \\ & 1 & -2 & 1 & \\ & & 0 & & \end{bmatrix} \quad (23)$$

The rows are from 2 (top) to -2 (bottom), columns are -2 (left) to 2 (right); note the row ordering (designed to make positive be to the top and right). The remaining partial derivatives can be obtained by exchanging x and z , and swapping rows and columns.

ZGOUBI [5,6]

ZGOUBI [5,6] assumes an underlying grid of points in a 2-D plane. First, the grid point nearest the evaluation point in question is determined. Then a 5×5 grid of points centered at that point is used to solve Eq. (15) in the least squares sense. The resulting coefficients are used in Eq. (14), which is then differentiated to obtain the expansion out of the midplane. The rows of c to obtain the coefficients using this method are:

$$c^{00} : \frac{1}{1225} \begin{bmatrix} 51 & -99 & 96 & -99 & 51 \\ -99 & -24 & 246 & -24 & -99 \\ 96 & 246 & 541 & 246 & 96 \\ -99 & -24 & 246 & -24 & -99 \\ 51 & -99 & 96 & -99 & 51 \end{bmatrix} \quad (24)$$

$$c^{10} : \frac{1}{420} \begin{bmatrix} 31 & -44 & 0 & 44 & -31 \\ -5 & -62 & 0 & 62 & 5 \\ -17 & -68 & 0 & 68 & 17 \\ -5 & -62 & 0 & 62 & 5 \\ 31 & -44 & 0 & 44 & -31 \end{bmatrix} \quad (25)$$

$$c^{20} : \frac{1}{2940} \begin{bmatrix} -289 & 904 & -1230 & 904 & -289 \\ 71 & 724 & -1590 & 724 & 71 \\ 191 & 664 & -1710 & 664 & 191 \\ 71 & 724 & -1590 & 724 & 71 \\ -289 & 904 & -1230 & 904 & -289 \end{bmatrix} \quad (26)$$

$$c^{11} : \frac{1}{600} \begin{bmatrix} 44 & -63 & 0 & 63 & -44 \\ -63 & -74 & 0 & 74 & 63 \\ 0 & 0 & 0 & 0 & 0 \\ 63 & 74 & 0 & -74 & -63 \\ -44 & 63 & 0 & -63 & 44 \end{bmatrix} \quad (27)$$

$$c^{30} : \frac{1}{10} \begin{bmatrix} -1 & 2 & 0 & -2 & 1 \\ -1 & 2 & 0 & -2 & 1 \\ -1 & 2 & 0 & -2 & 1 \\ -1 & 2 & 0 & -2 & 1 \\ -1 & 2 & 0 & -2 & 1 \end{bmatrix} \quad (28)$$

$$c^{21} : \frac{1}{70} \begin{bmatrix} 4 & -2 & -4 & -2 & 4 \\ 2 & -1 & -2 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ -2 & 1 & 2 & 1 & -2 \\ -4 & 2 & 4 & 2 & -4 \end{bmatrix} \quad (29)$$

$$c^{40} : \frac{1}{5} \begin{bmatrix} 1 & -4 & 6 & -4 & 1 \\ 1 & -4 & 6 & -4 & 1 \\ 1 & -4 & 6 & -4 & 1 \\ 1 & -4 & 6 & -4 & 1 \\ 1 & -4 & 6 & -4 & 1 \end{bmatrix} \quad (30)$$

$$c^{31} : \frac{1}{20} \begin{bmatrix} -2 & 4 & 0 & -4 & 2 \\ -1 & 2 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 0 & 2 & -1 \\ 2 & -4 & 0 & 4 & -2 \end{bmatrix} \quad (31)$$

$$c^{22} : \frac{1}{49} \begin{bmatrix} 4 & -2 & -4 & -2 & 4 \\ -2 & 1 & 2 & 1 & -2 \\ -4 & 2 & 4 & 2 & -4 \\ -2 & 1 & 2 & 1 & -2 \\ 4 & -2 & -4 & -2 & 4 \end{bmatrix} \quad (32)$$

This method works for fields with or without midplane symmetry, since it allows us to compute all needed derivatives. It was used extensively for the simulation of the partial siberian snakes in the AGS at BNL, 3 meter long helical dipoles, using a 2D diametral field map [7]; it proved to allow the simulation of a complete polarized proton bunch acceleration cycle (150000 turns of the 807 m circumference AGS), with very satisfactory accuracy.

Least Squares in Conjunction with RAYTRACE

For a study of the AGS [4], the method described above for RAYTRACE was used, but it expects a function that can be obtained at an arbitrary point. The magnets in question had fields measured in the midplane on a grid. To obtain the function needed by RAYTRACE, a rectangular grid of 7 points horizontally and 5 points vertically was fit in a least squares sense to a polynomial (14) to 5th order in x and 3rd order in z . c matrices could be determined similarly to the above. The fits were not made on every possible 7×5 grid, but around every 3rd grid point horizontally and 2nd point longitudinally. To determine which function to evaluate in RAYTRACE, they determined which of these grids had its center nearest the point to be evaluated, then used the corresponding polynomial.

REFERENCES

- [1] S.B. Kowalski and H.A. Enge “The Ion-Optical Program Ray-trace” NIM A258 (1987) 407.
- [2] N. Tsoupas et. al. “Effects of Dipole Magnet Inhomogeneity on the Beam Ellipsoid” NIM A258 (1987) 421-425.
- [3] H.A. Enge, Deflecting Magnets, in “Focusing of Charged. Particles”, edited by A. Septier, Academic Press, 1967.
- [4] N. Tsoupas et. al. “Closed Orbit Calculations at AGS and Extraction Beam Parameters at H13” AD/RHIC/RD-75.
- [5] F. Méot, Zgoubi Users’ Guide, <https://sourceforge.net/projects/zgoubi/>.
- [6] Pavel Akishin, JINR, Dubna. Installed in Zgoubi [5] (1992).
- [7] F. Méot *et al.*, Spin tracking simulations in AGS based on ray-tracing methods, Tech. Note C-A/AP-452, BNL C-AD, 2009.