OPTIMIZATION OF MULTICELL MICROWAVE CAVITIES USING YACS*

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Abstract

YACS is a 2.5D finite element method solver capable of solving for the full 3D eigenfrequency spectra of resonant axisymmetric structures while reducing the computational problem to a 2D rotation plane. The most recent revision of the code introduced arbitrary order basis functions and curved meshes, for both triangular and quadrilateral unstructured meshes. This led to significant increases in convergence rates. However, due to the utilization of curved meshes and the complex coordinate transformations that are involved, spurious modes were introduced when solving the axisymmetric problem. Although workarounds do exist that circumvent these issues by lowering the likelihood and frequency of spurious modes, linear triangular meshes with higher order basis functions were chosen due to their simplicity and spurious free solutions. In order to further support the usage of spline cavities as an alternative parameterization scheme to the well known elliptical cavities, extensive parameter space scans were carried out for non-reentrant spline shaped microwave cavities. In addition a new optimization strategy is presented that exploits the arbitrary polynomial order of Bézier curves by utilizing the degree elevation technique.

INTRODUCTION

The design and optimization of modern superconducting multicell microwave cavities heavily relies on numerical studies. Due to the complex curved shapes and the broad frequency spectra of those cavities, numerical studies typically require $\geq 10^5$ degrees of freedom to obtain accurate discretization, of both the geometry and the field. To reduce the required degrees of freedom the 2.5D finite element solver YACS has been implemented [1]. YACS is capable of solving for the full 3D eigenfrequency spectra of axisymmetric cavities while reducing the problem geometry to a 2D rotation plane. In addition, the code supports arbitrary polynomial order hierarchical function bases [2, 3] for the field discretization. Studies presented earlier showcased the usage of curved elements on 2D problems, to further reduce the required degrees of freedom [4] while introducing spurious modes when solving the axisymmetric problem. To circumvent this problem, linear triangular elements have been used in this work to explore the parameter space of cubic spline cavities and perform optimization studies on higher order spline cavities.

NUMERICAL STUDIES

All following numerical studies that involve spline cavities were tuned via the equator radius to maintain a design frequency of $\nu_{\text{design}} = 1.3\,\text{GHz}$. The cavity length was set to $L = \beta\lambda/2$ with $\beta = 1$. Discretization of the problem domain was performed with GMSH [5] and the OpenCASCADE [6] kernel, that supports Bézier curves of degree $n \leq 20$. Refer to [7] for a thorough description of the figures of merits used throughout this work.

Convergence Studies

Convergence studies involving the eigenfrequency spectra of a pillbox cavity have been carried out, in order to demonstrate the reduction of degrees of freedom required for a given discretization accuracy when using YACS. The convergence results for the first mono- and dipole mode, with varying polynomial order p of the field function basis, are displayed in Fig. 1 and 2.

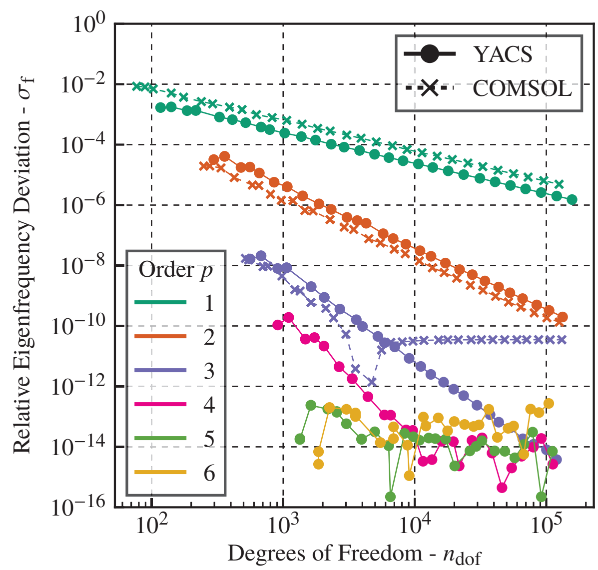

Figure 1: Relative eigenfrequency deviation σ_f of the first monopol mode of a pillbox resonator, obtained from YACS and COMSOL as a function of the number of degrees of freedom n_{dof} for different polynomial orders p of the function basis. The frequency deviations were calculated with respect to the analytical solution.

As a reference the studies include the convergence rates obtained from COMSOL's [8] axisymmetric and 3D solver [1]. It is immediately visible that YACS drastically lowers the degrees of freedom required to yield discretization accuracies

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[1] The version of COMSOL used in this study yields spurious modes when using an azimuthal mode number $m \neq 0$ within the axisymmetric solver module.

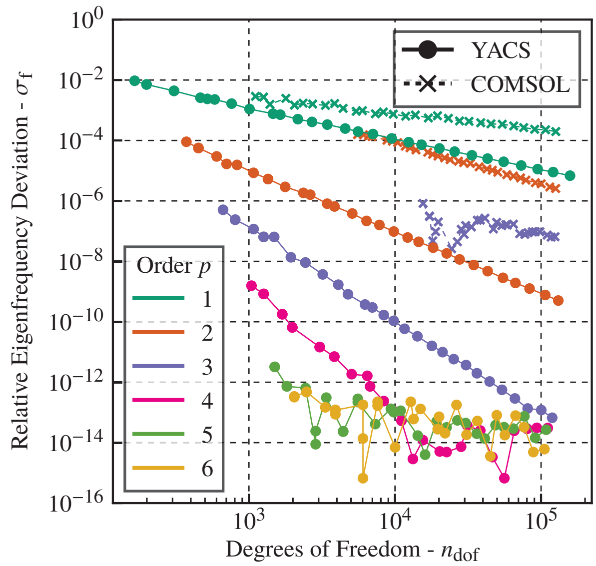

Figure 2: Relative eigenfrequency deviation σ_f of the first dipole mode of a pillbox resonator, obtained from YACS and COMSOL (3D) as a function of the number of degrees of freedom n_{dof} for different polynomial orders p of the function basis. The frequency deviations were calculated with respect to the analytical solution.

that are similar to commercial 3D and even 2.5D codes. The convergence rates show expected behavior with increasing polynomial degree of the field function basis, except for the last two orders which yield discretization accuracies close to machine precision, considering the complex sparse matrix algorithms used [9, 10]. In addition it was observed that spurious free solutions for multipole modes could only be obtained with the out of plane function basis order increased by one with respect to the in plane function basis.

Spline Cavities

The curved boundary of superconducting multicell microwave cavities is usually parameterized by ellipses and extensive studies for these shapes have been performed e.g. [11]. However there are alternative parameterizations available. This work explores the subset of cavities that utilize Bézier curves to represent the curved boundaries, hereafter referred to as spline cavities [12]. A Bézier curve is a parametric curve with the control points $\underline{P}_i$ defined as

$$\underline{B}(t) = \sum_{i=0}^{n} b_{i,n}(t)\underline{P}_i, \quad 0 \leq t \leq 1$$

and the Bernstein basis polynomials

$$b_{i,n}(t) = \binom{n}{i} t^i (1-t)^{n-i}, \quad i = 0, \ldots, n.$$

This parameterization is inherently flexible since it can represent curves of arbitrary polynomial degree and thus is very well suited for optimization problems.

Parameter Space Scan

Previous studies [12] already explored parts of the parameter space of cubic spline cavities using SUPERFISH [13]. However, those studies did not include the whole subset of cubic spline cavities in the non-reentrant regime. To support the results obtained with SUPERFISH, and to further explore the parameter space, parameter scans for the complete non-reentrant regime were carried out. The results of the parameter scan are displayed in Fig. 3. Since the $E_{surf,max}/E_{acc}$ ratio is very sensitive to the discretization of the curved boundary, only values below ≤ 3.0 have been considered in the figure as values above 3.0 usually are not appropriate for high power superconducting microwave cavities and will often be discarded by cavity designers.

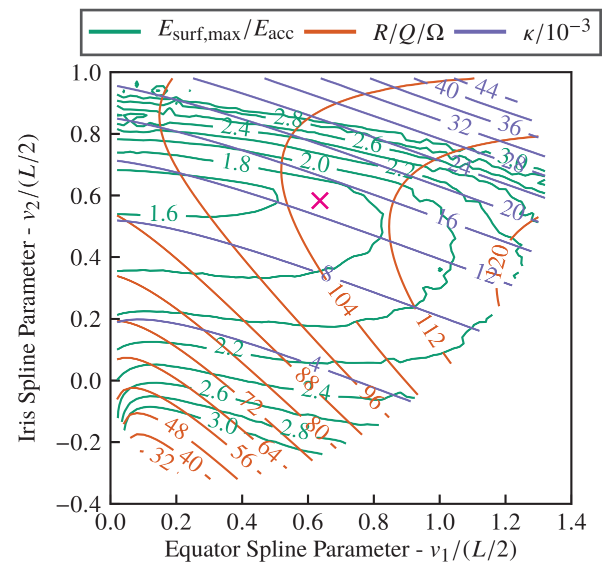

Figure 3: Parameter space scan for the subset of spline parameters within the non-reentrant regime of cubic spline cavities. Displayed are the figures of merit $E_{surf,max}/E_{acc}$, R/Q and the coupling constant κ for the $TM_{010,\pi}$-Mode. The red cross represents the final cubic spline parameters for the optimization of the cost function (see Fig. 4).

Higher Order Spline Cavity Optimization

In contrast to the well known elliptical parameterization of superconducting multicell cavities, spline cavities can represent arbitrary curves due to their polynomial definition of arbitrary degree. This property was exploited in order to implement an efficient optimization algorithm based on a technique called degree elevation. Using a simple algorithm the polynomial degree of a Bézier curve can be increased without altering the shape. If a lower degree spline cavity can not be optimized to reach certain design criteria, its shape can be used as the starting point for a higher degree spline cavity for further optimizations. Since lower degree spline cavities have less degrees of freedom in the form of control points, those optimizations can usually be performed far more efficient with less iterations required. The control

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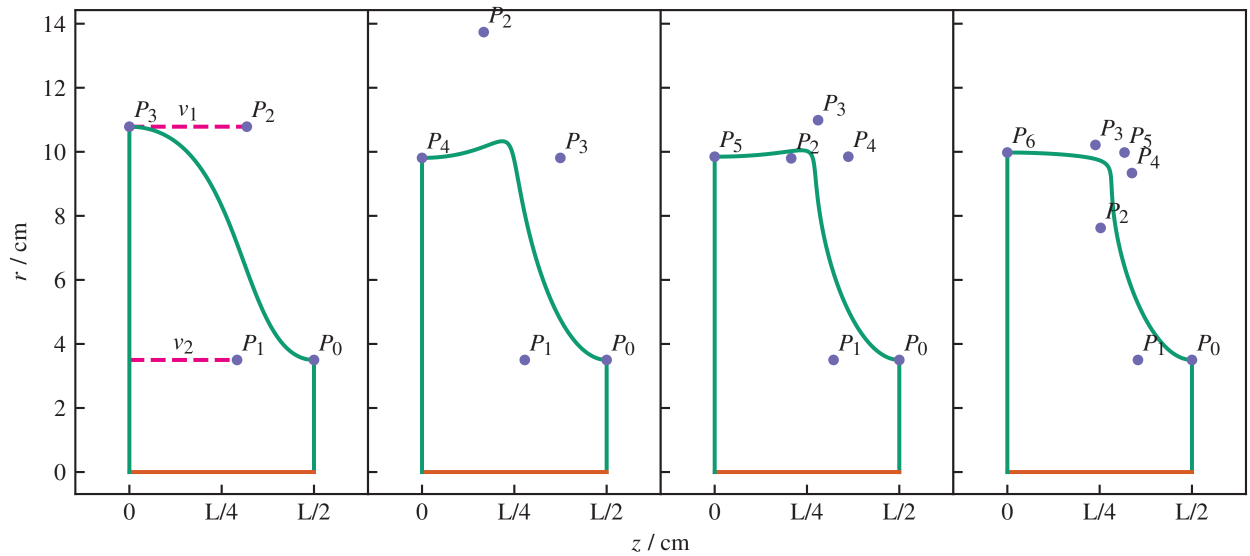

Figure 4: Optimized shapes and their respective control points for the different spline orders in ascending order. Each shape optimization for $n > 4$ used the optimization result of the previous order as the initial guess, utilizing the degree elevation algorithm. In the case of cubic spline cavities the parameters v_1 and v_2 represent the only degrees of freedom of the curve, since the r-coordinates are fixed due to the C^1 continuity requirements [12].

points $\underline{P}'_i$ of a Bézier curve with degree $n + 1$ that has the same shape as the underlying Bézier curve of degree n with the control points $\underline{P}_i$ can be calculated with

$$P'_i = \frac{i}{n+1} P_{i-1} + \frac{n+1-i}{n+1} P_i, \quad i = 0, \ldots, n+1.$$

Bézier curves have another interesting property for cavity designers, since their derivative $\underline{B}'(t)$ at the endpoints of the curve depend only on the first resp. last two control points, C^1 continuity can always be guaranteed for arbitrary degrees, by simply constraining the mentioned control points. To demonstrate the capabilities of this optimization algorithm a cost function based on the observations of the parameter scan of cubic spline cavities (see Fig. 3) was used.

$$f(\Lambda) = \left(\frac{E_{\mathrm{surf,max}}}{E_{\mathrm{acc}}}(\Lambda)/1.6 - 1\right)^2 + \left(\frac{R}{Q}(\Lambda)/114\,\Omega - 1\right)^2$$

where Λ represents the problem domain. This cost function was chosen on purpose since it fails to be minimized to zero for cubic spline cavities. The obtained optimization results for the different iteration stages with varying degree of the spline cavity are listed in Table 1. The corresponding cavity shapes and the respective control points are displayed in Fig. 4. The spline parameters of the cubic spline cavity are also display in Fig. 3. Using spline cavities up to a degree of 6 the cost function could almost be halved. It should be obvious that the obtained shapes could not possibly be obtained with elliptical parameterizations. Thus it can be concluded that spline cavities are far more flexible, can be optimized more efficiently and due to their polynomial nature can be constrained far more easily.

Table 1: Optimization results for the different spline cavities with polynomial order p of the Bézier curve

p	$\frac{E_{\mathrm{surf,max}}}{E_{\mathrm{acc}}}$	$\frac{R}{Q}/\Lambda$	$f(\Lambda)/10^{-3}$
3	1.668	106.779	5.835
4	1.641	108.538	2.961
5	1.650	109.210	2.757
6	1.647	109.414	2.498

CONCLUSION

The latest iteration of YACS supports arbitrary order hierarchical function bases within linear triangular elements. It could be shown that this combination results in a vastly reduced number of degrees of freedom required to obtain comparable discretization accuracies compared to 3D codes. The parameter space of cubic spline cavities has been further explored and now covers the whole non-reentrant regime. To further support the usage of spline cavities as an alternative parameterization, a new optimization strategy has been introduced. It was demonstrated that this strategy can help reaching design criteria that would otherwise not be reached with simple cubic or even elliptical cavities. Thus rendering spline cavities to be far more flexible than common microwave cavity parameterizations. In addition it was observed that spurious free solutions could only be obtained for out of plane function bases that have a higher polynomial degree than their corresponding in plane function bases, when solving for multipole modes. This observation could be beneficial for the ongoing work to support curved elements for axisymmetric problems.

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