NOISE IN RADIO/OPTICAL COMMUNICATIONS

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List of figures: Noise in radio/optical communications

Fifth Solvay International Conference on Electrons and Photons (October 1927). The leading figures Albert Einstein and Niels Bohr disagreed:

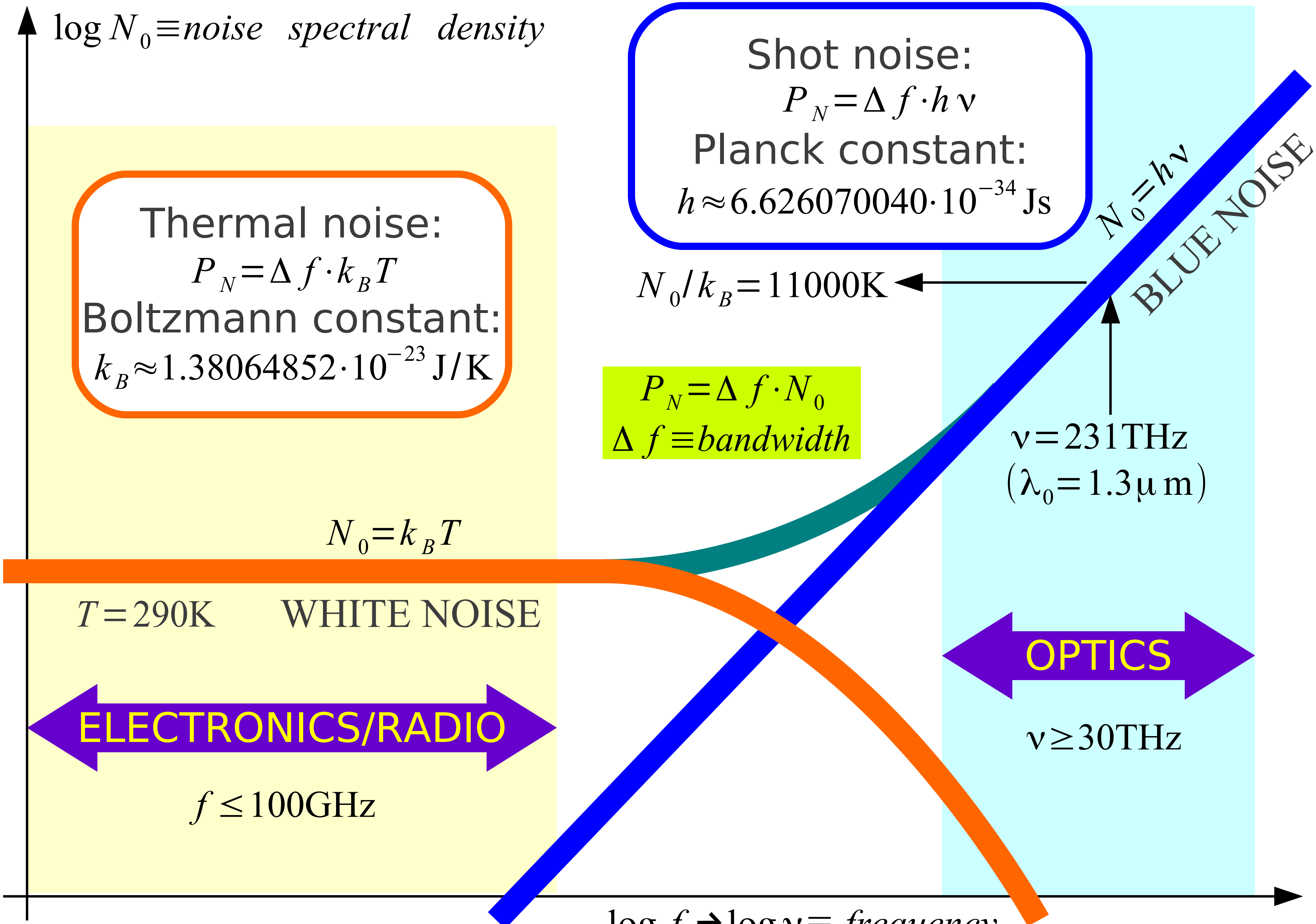
Albert Einstein: „God does not play dice!“

Niels Bohr: „Einstein, stop telling God what to do!“

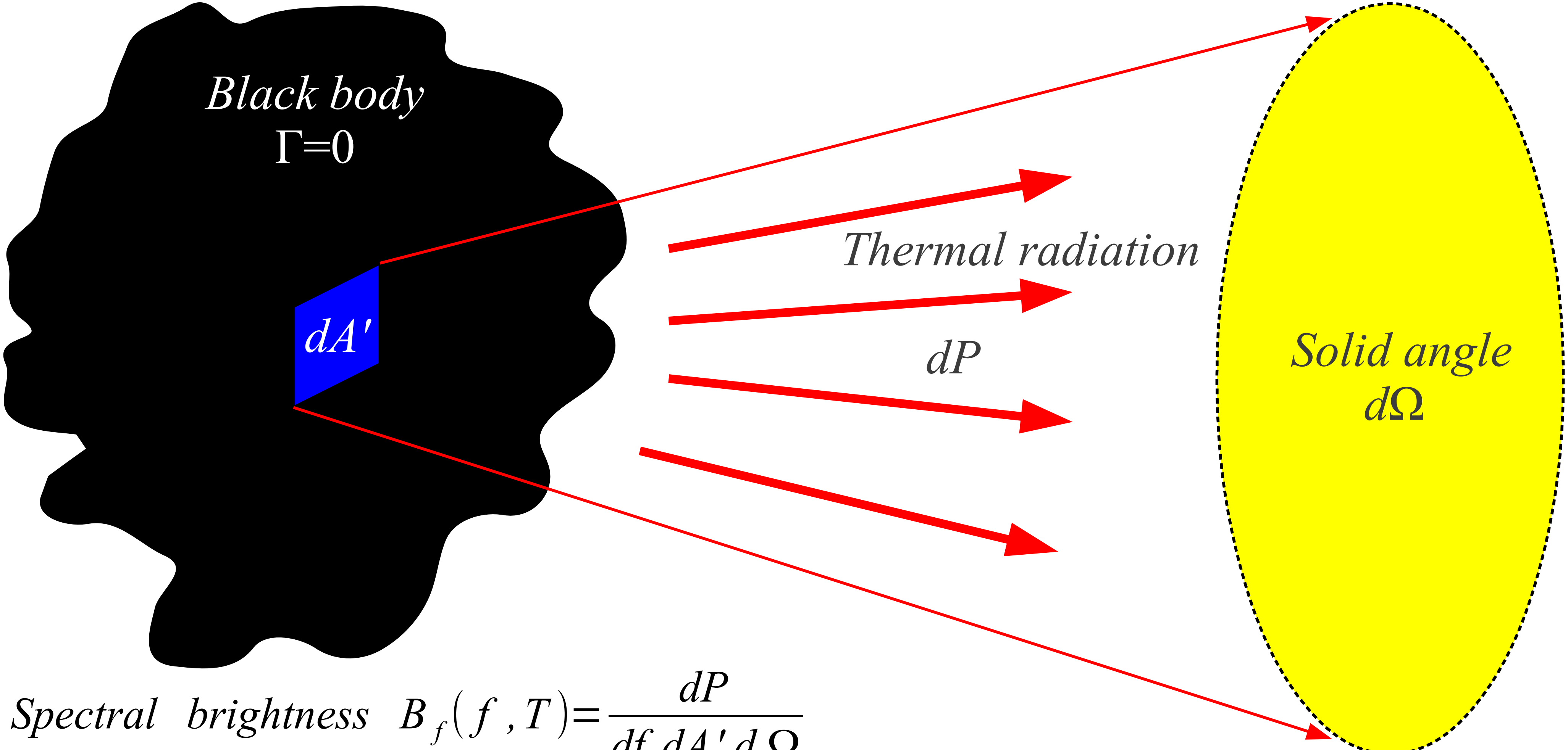
In telecommunications random signals are called noise. Noise impairs the performance of any communication link.

Noise is a macroscopic description of quantum effects!

1 - The dispute between famous scientists



2 - Noise spectral density



$$\textit{Spectral brightness}\quad B_f(f,T)=\frac{dP}{df\, dA'\, d\Omega}$$

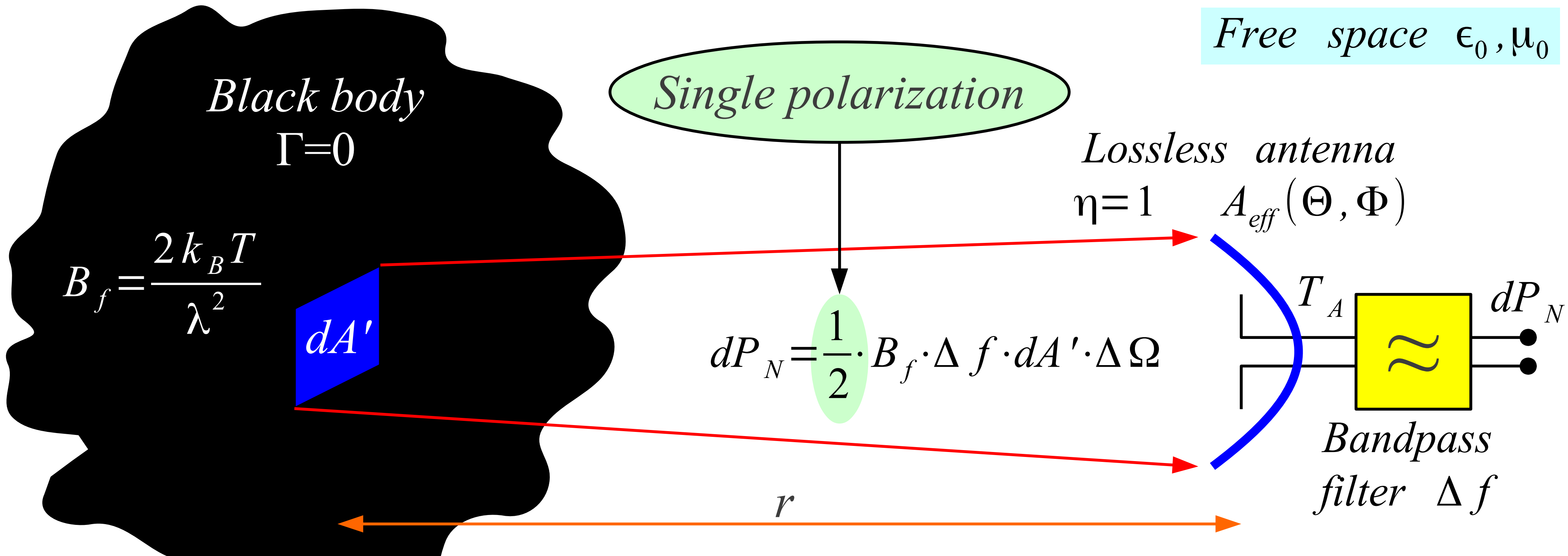
$$\textit{Planck law}\quad B_f(f,T)=\frac{2hf^3}{c_0^2}\cdot\frac{1}{e^{\frac{hf}{k_B T}}-1}$$

$$\textit{Free space}\quad \epsilon_0,\mu_0$$

$$c_0=299792458\,\mathrm{m/s}\approx 3\cdot 10^8\,\mathrm{m/s}$$

$$\textit{Radio}\quad hf\ll k_B T\ \rightarrow\ \textit{Rayleigh-Jeans approximation}\quad B_f(f,T)\approx\frac{2k_B T f^2}{c_0^2}=\frac{2k_B T}{\lambda^2}$$

3 – Black-body thermal radiation



$$\Delta\Omega = \frac{A_{eff}(\Theta,\Phi)}{r^2} = \frac{\lambda^2 D(\Theta,\Phi)}{4\pi r^2} = \frac{\lambda^2 |F(\Theta,\Phi)|^2}{r^2 \iint_{4\pi} |F(\Theta^*,\Phi^*)|^2 d\Omega^*}$$

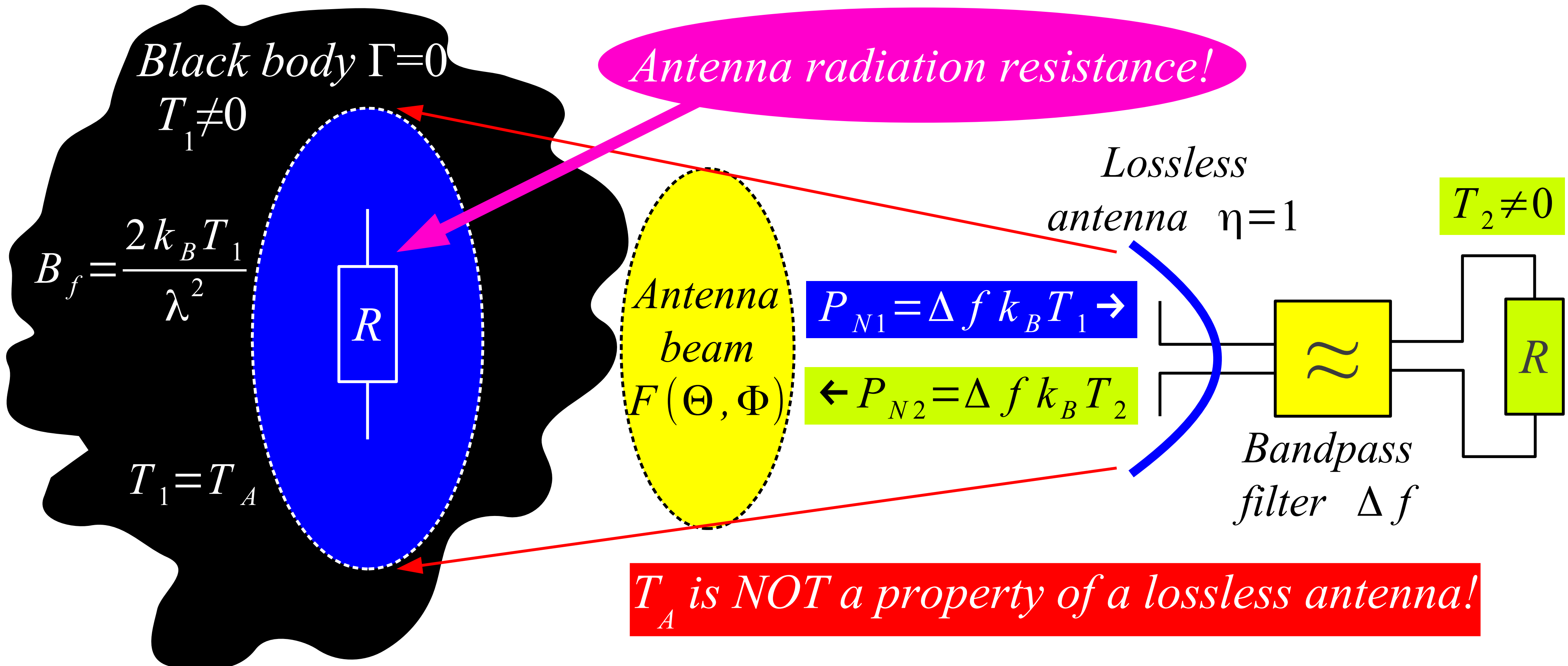
$$dA' = r^2 d\Omega$$

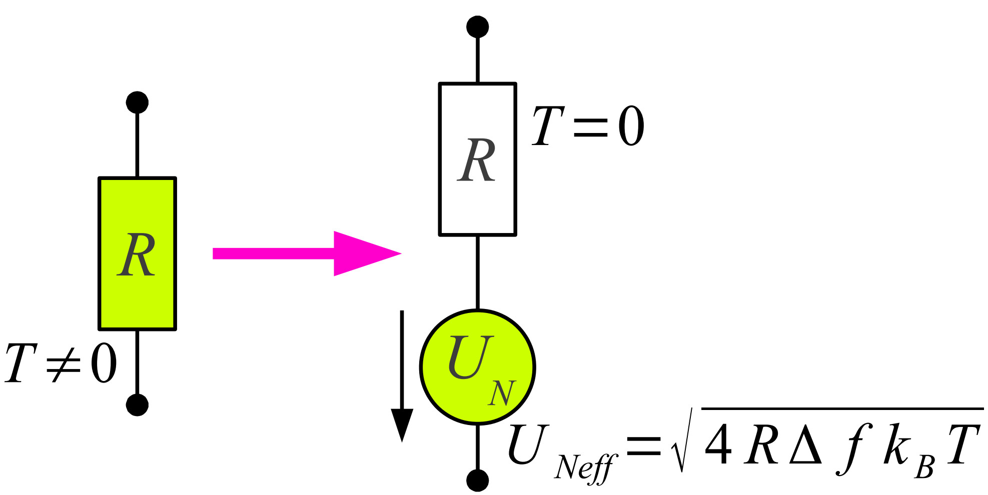
$$P_N = \iint_{A'} \frac{1}{2} \cdot B_f \cdot \Delta f \cdot dA' \cdot \Delta\Omega = \iint_{4\pi} \frac{1}{2} \cdot \frac{2 k_B T(\Theta,\Phi)}{\lambda^2} \cdot \Delta f \cdot r^2 d\Omega \cdot \frac{\lambda^2 |F(\Theta,\Phi)|^2}{r^2 \iint_{4\pi} |F(\Theta^*,\Phi^*)|^2 d\Omega^*}$$

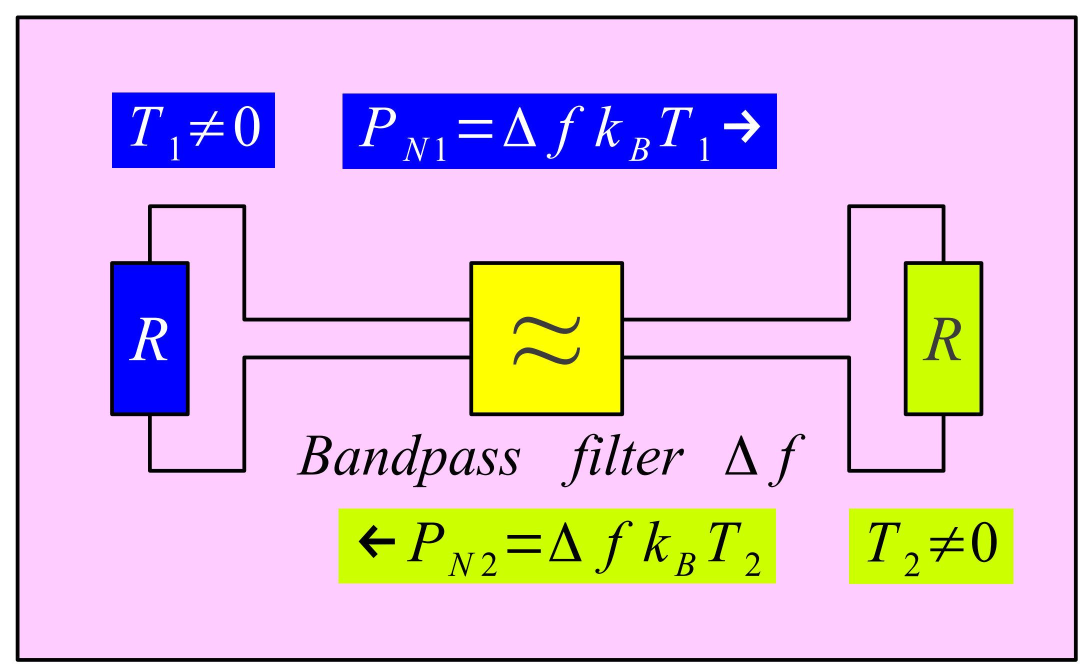
$$P_N = \Delta f \, k_B \frac{\iint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega} = \Delta f \, k_B T_A$$

$$T_A = \frac{\iint_{4\pi} T(\Theta,\Phi) |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

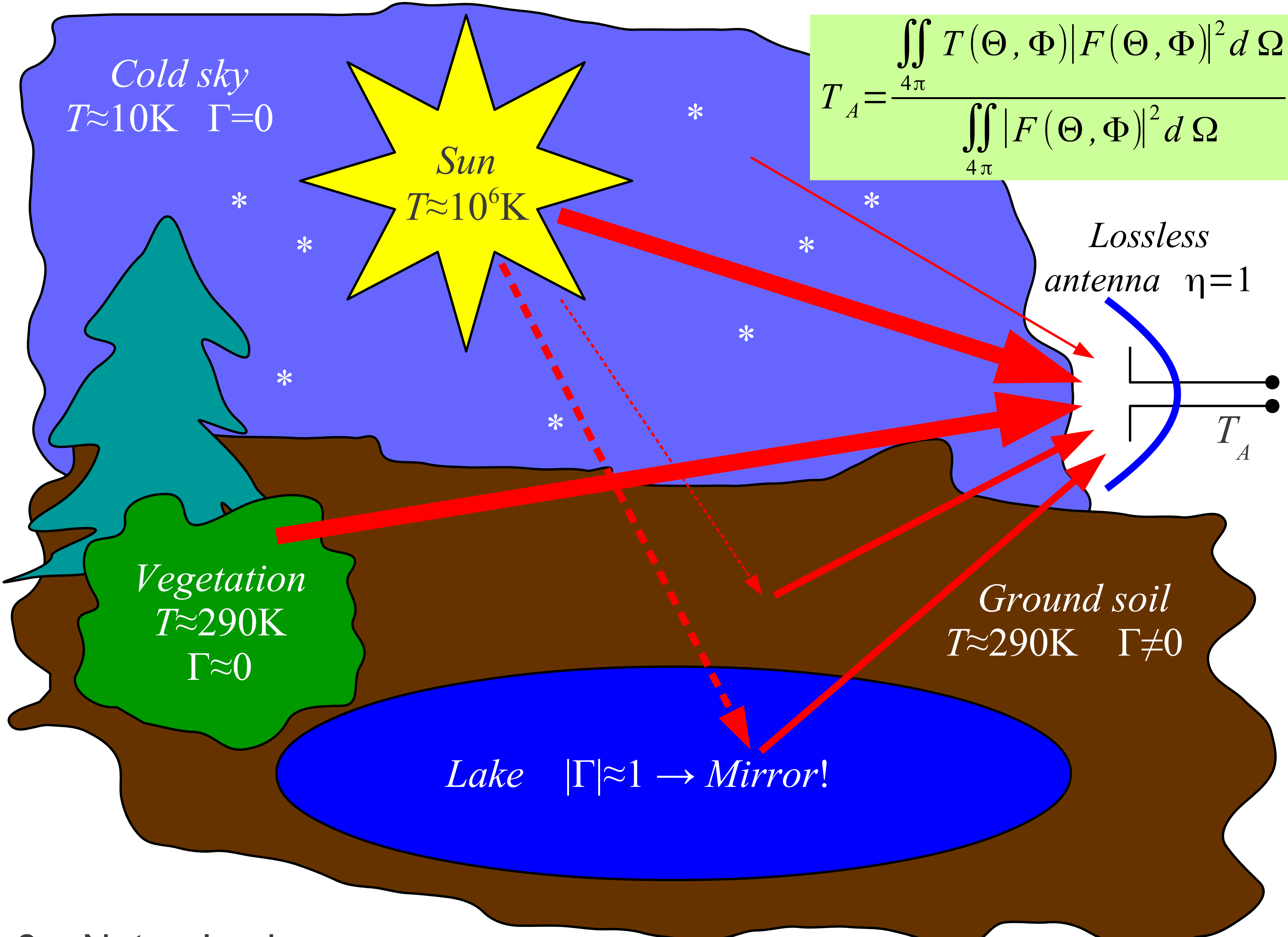
4 – Received thermal-noise power





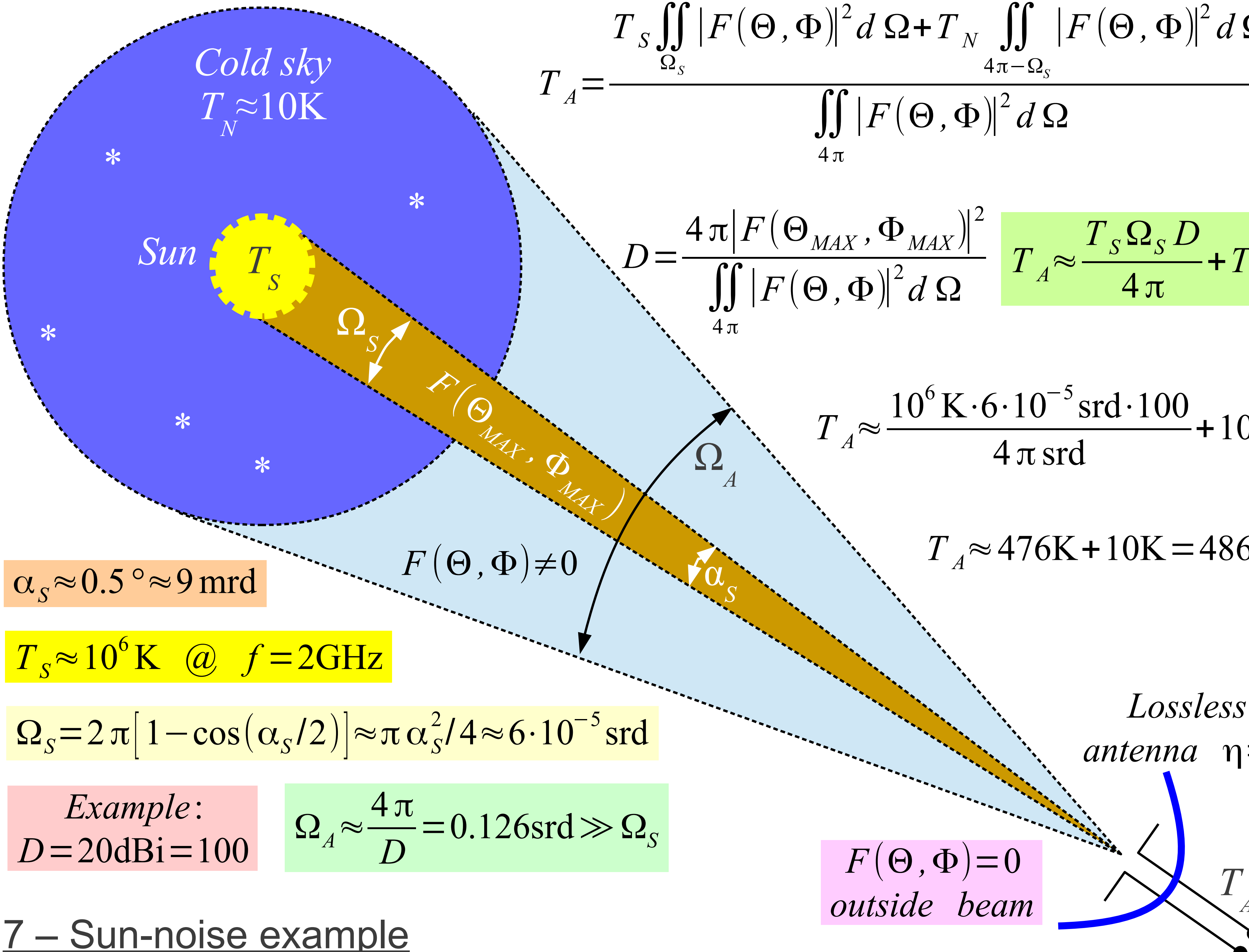


5 – Thermal equilibrium



$$T_A = \frac{\iint_{4\pi} T(\Theta,\Phi)|F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

6 – Natural noise sources



$$T_A = \frac{T_S \iint_{\Omega_S} |F(\Theta,\Phi)|^2 d\Omega + T_N \iint_{4\pi-\Omega_S} |F(\Theta,\Phi)|^2 d\Omega}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$D = \frac{4\pi |F(\Theta_{MAX}, \Phi_{MAX})|^2}{\iint_{4\pi} |F(\Theta,\Phi)|^2 d\Omega}$$

$$T_A \approx \frac{T_S \Omega_S D}{4\pi} + T_N$$

$$T_A \approx \frac{10^6\,\mathrm{K} \cdot 6 \cdot 10^{-5}\,\mathrm{srd} \cdot 100}{4\pi\,\mathrm{srd}} + 10\mathrm{K}$$

$$T_A \approx 476\mathrm{K} + 10\mathrm{K} = 486\mathrm{K}$$

$\alpha_S \approx 0.5° \approx 9\,\mathrm{mrd}$

$T_S \approx 10^6\,\mathrm{K}$ @ $f = 2\mathrm{GHz}$

$\Omega_S = 2\pi[1 - \cos(\alpha_S/2)] \approx \pi \alpha_S^2/4 \approx 6 \cdot 10^{-5}\,\mathrm{srd}$

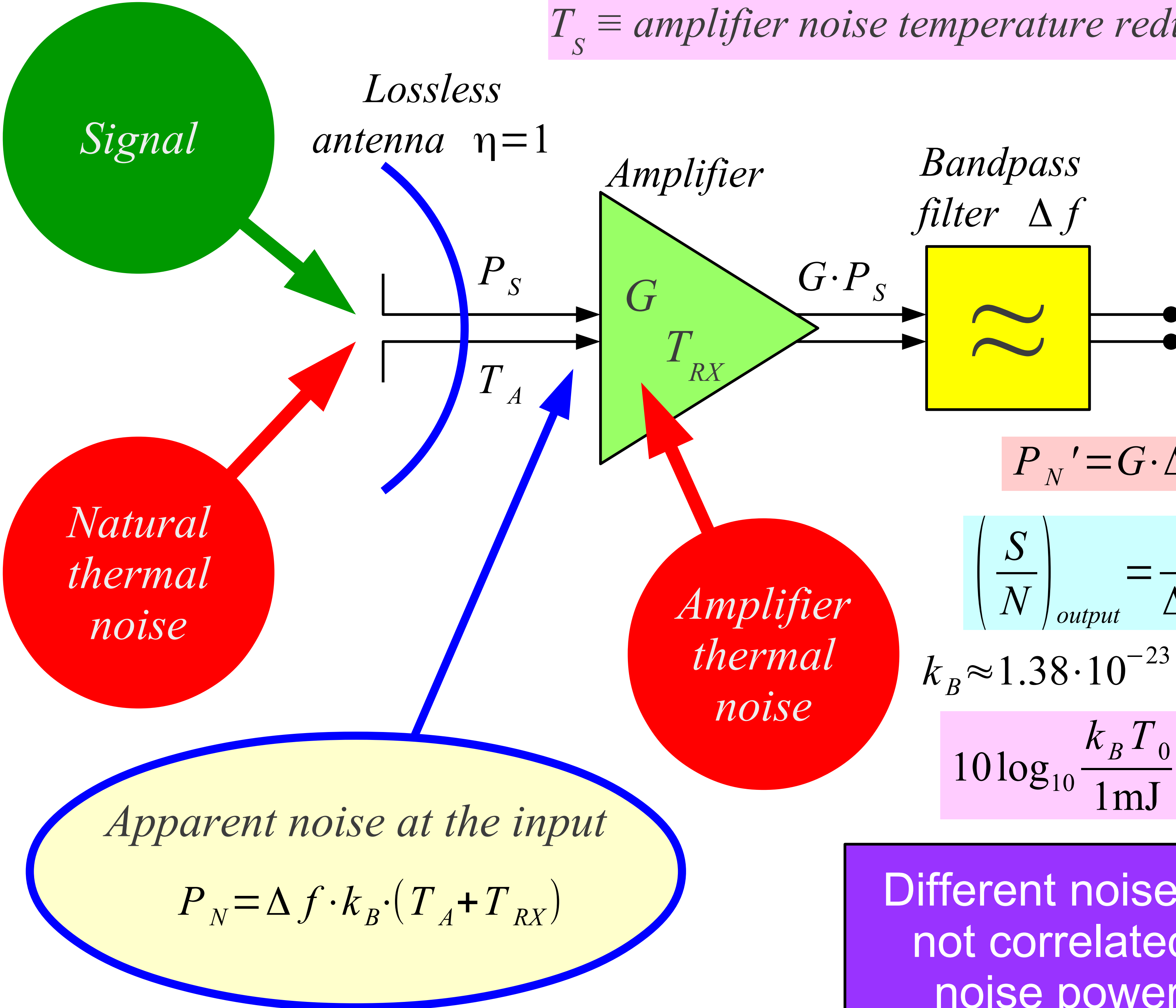
Example:
$D = 20\mathrm{dBi} = 100$

$\Omega_A \approx \frac{4\pi}{D} = 0.126\mathrm{srd} \gg \Omega_S$

$F(\Theta, \Phi) = 0$
outside beam

7 – Sun-noise example

$T_S \equiv$ *amplifier noise temperature reduced to the input!*



$$P_S' = G \cdot P_S$$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_S'}{P_N'}$$

$$P_N' = G \cdot \Delta f \cdot k_B \cdot (T_A + T_{RX})$$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_S}{\Delta f \cdot k_B \cdot (T_A + T_{RX})}$$

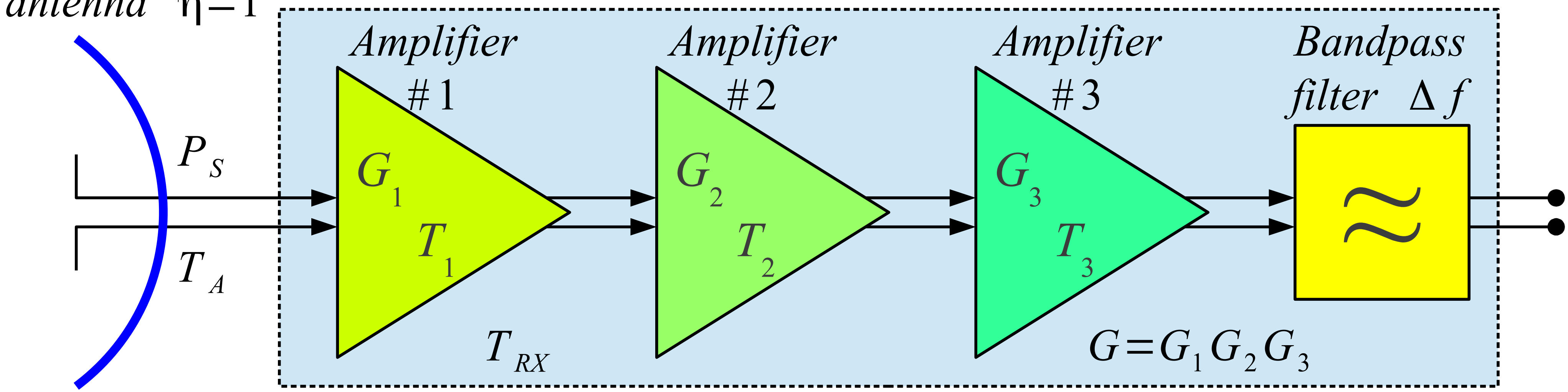
$$k_B \approx 1.38 \cdot 10^{-23}\ \mathrm{J/K} \qquad T_0 = 290\mathrm{K}$$

$$10 \log_{10} \frac{k_B T_0}{1\mathrm{mJ}} \approx -174\ \mathrm{dBm/Hz}$$

Different noise sources are not correlated, therefore noise powers or noise temperatures are summed!

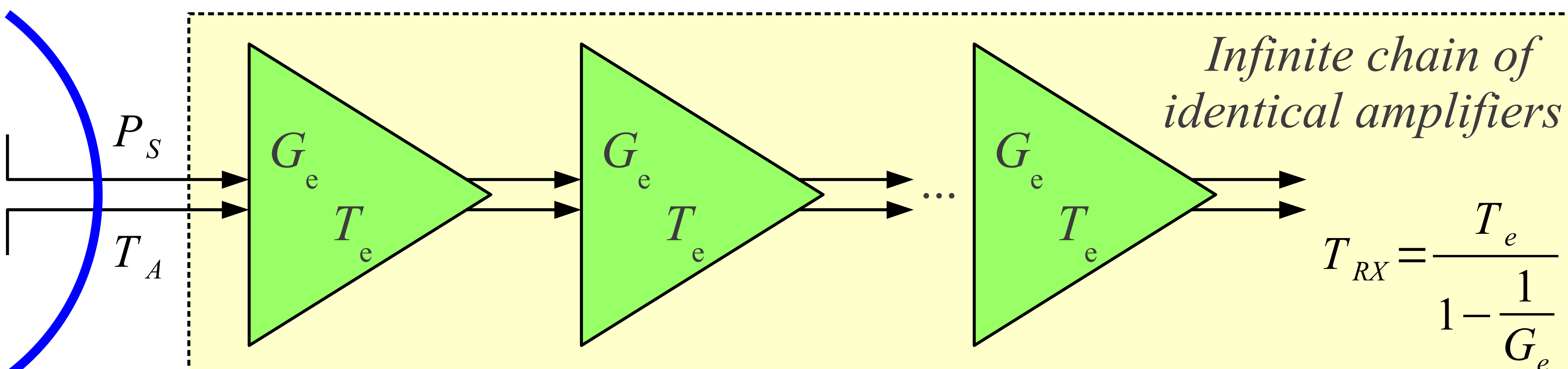
8 – Receiver signal-to-noise ratio

$$P_S' = G_3 G_2 G_1 P_S$$

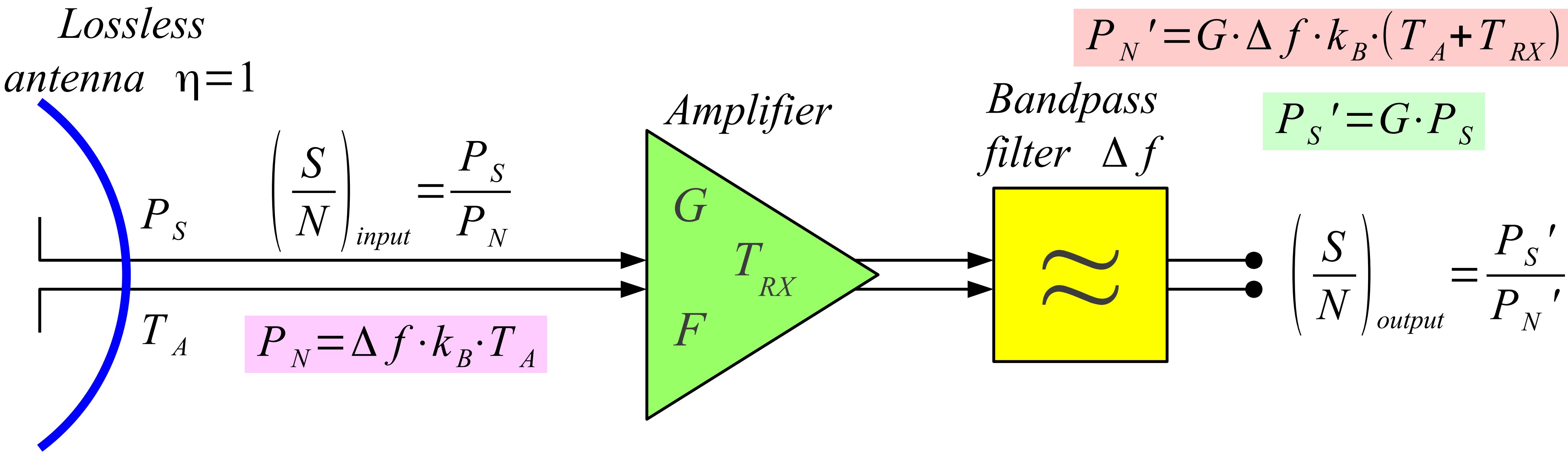


$$P_N' = \Delta f\, k_B \left[G_3 G_2 G_1 (T_A + T_1) + G_3 G_2 T_2 + G_3 T_3 \right]$$

$$P_N' = G_3 G_2 G_1 \Delta f\, k_B (T_A + T_{RX}) \quad \rightarrow \quad T_{RX} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots$$



9 – Chain noise temperature



Nonsense definition of the noise figure:

$$F=\frac{\left(\frac{S}{N}\right)_{input}}{\left(\frac{S}{N}\right)_{output}}=\frac{\dfrac{P_S}{\Delta f\, k_B T_A}}{\dfrac{G\,P_S}{G\,\Delta f\, k_B (T_A+T_{RX})}}=\frac{T_A+T_{RX}}{T_A}=1+\frac{T_{RX}}{T_A}$$

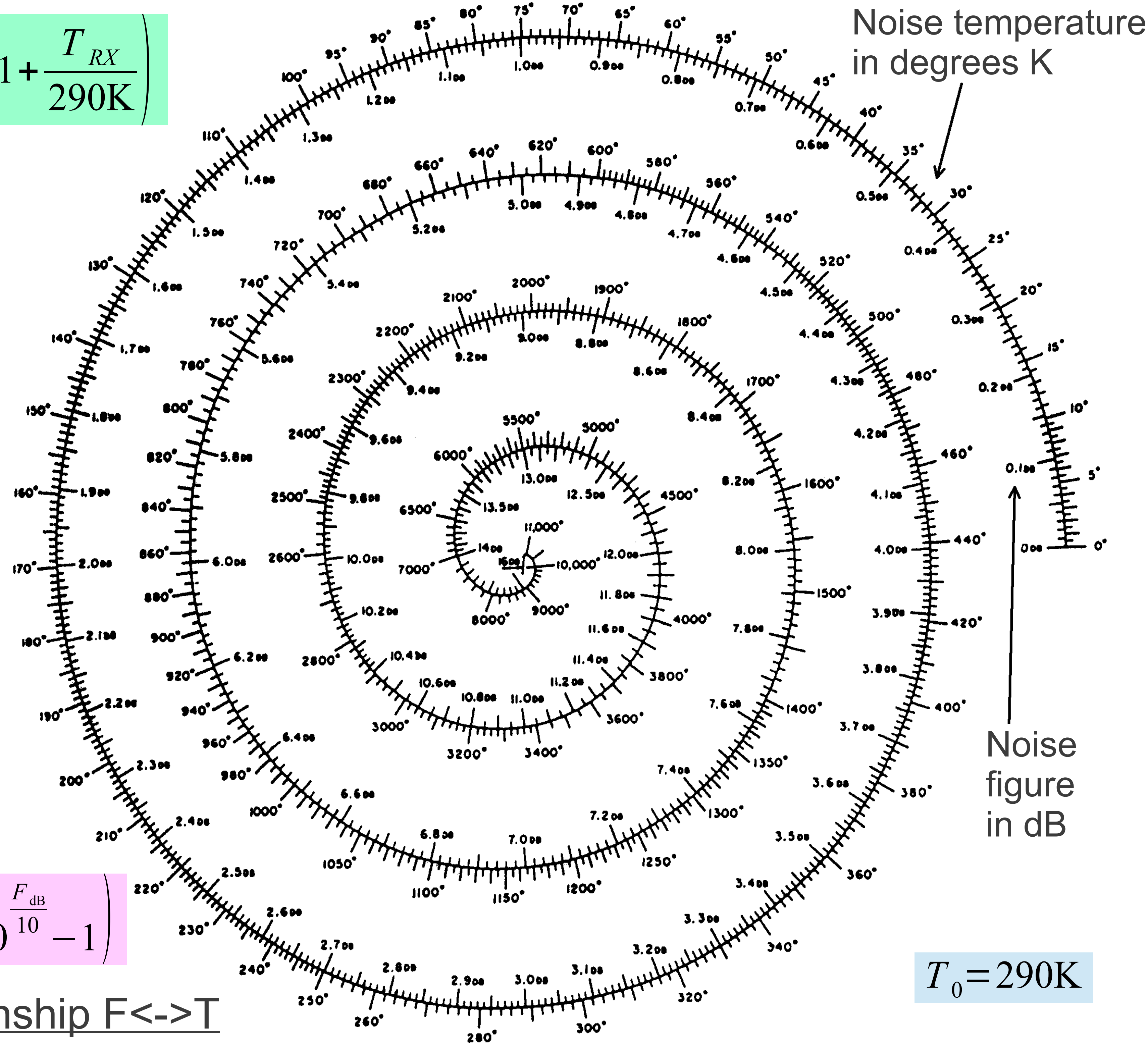
A property of an amplifier can not be a function of T_A!

Sensible definition $\quad F=1+\frac{T_{RX}}{T_0}$ @ $T_0=290\mathrm{K} \quad \leftrightarrow \quad T_{RX}=T_0(F-1)$

Logarithmic units $\quad F_{\mathrm{dB}}=10\log_{10}F=10\log_{10}\left(1+\frac{T_{RX}}{T_0}\right) \quad \leftrightarrow \quad T_{RX}=T_0\left(10^{\frac{F_{\mathrm{dB}}}{10}}-1\right)$

10 – Amplifier noise figure

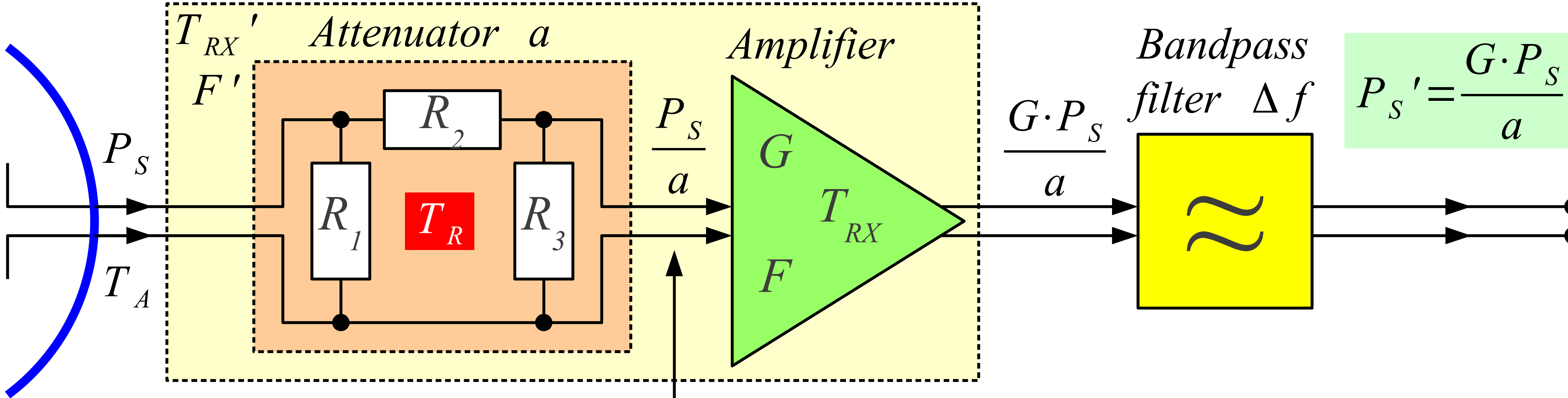
$$F_{\mathrm{dB}} = 10\log_{10}\left(1 + \frac{T_{RX}}{290\mathrm{K}}\right)$$



$$T_{RX} = 290\mathrm{K}\left(10^{\frac{F_{\mathrm{dB}}}{10}} - 1\right)$$

$$T_0 = 290\mathrm{K}$$

11 – Relationship F<->T



$$P_S' = \frac{G \cdot P_S}{a}$$

Lossless antenna $\eta = 1$

$$\frac{T_A}{a} + T_R\left(1 - \frac{1}{a}\right)$$

$$P_N' = G \cdot \Delta f \cdot k_B \cdot \left[\frac{T_A}{a} + T_R\left(1 - \frac{1}{a}\right) + T_{RX}\right]$$

$$T_{RX}' = T_R(a-1) + a\,T_{RX}$$

$$\left(\frac{S}{N}\right)_{output} = \frac{P_S'}{P_N'} = \frac{P_S}{\Delta f \cdot k_B \cdot \left[T_A + T_R(a-1) + a\,T_{RX}\right]}$$

$$F' = 1 + \frac{T_{RX}'}{T_0} = 1 + \frac{T_R}{T_0}(a-1) + a\frac{T_{RX}}{T_0}$$

Frequent case $T_R \approx T_0 = 290\text{K}$

$$F' \approx a + a\frac{T_{RX}}{T_0} = a\left(1 + \frac{T_{RX}}{T_0}\right) = a \cdot F$$

$$F_{\text{dB}}' \approx a_{\text{dB}} + F_{\text{dB}}$$

Attenuator examples $T_R \approx T_0 = 290\text{K}$

$F' \approx a \cdot F \qquad F_{\text{dB}}' \approx a_{\text{dB}} + F_{\text{dB}}$

(1) lossy antenna $a_{\text{dB}} = -10\log_{10}\eta$

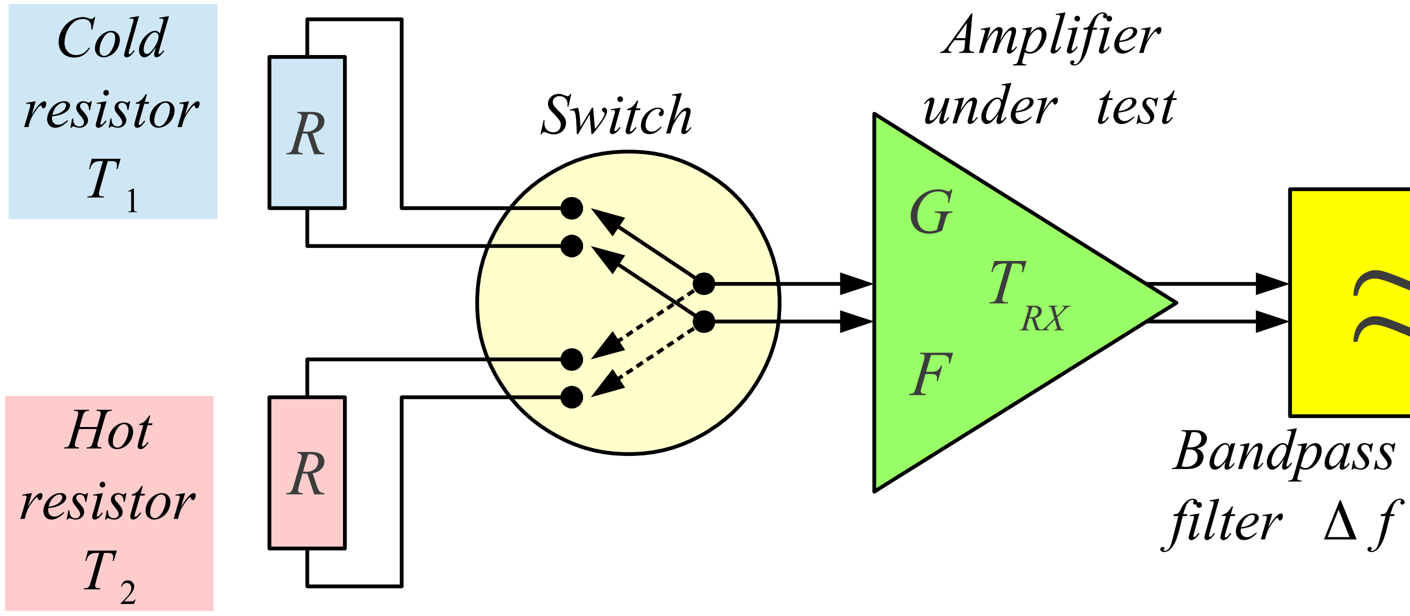
(2) lossy transmission line a_{dB}

(3) lossy bandpass filter a_{dB}

(4) passive−mixer loss a_{dB}

12 – Attenuator noise

Amplifier device	Gain G [dB]	Noise temperature T_{RX} [K]	Noise figure F_{dB} [dB]
Vacuum tube with control grid (triode, pentode)	10↔20	1600↔9000	8↔15
Vacuum tube with speed modulation (klystron, TWT)	20↔50	3000↔30000	10↔20
Parametric amplifier (room temperature)	10↔15	75↔300	1↔3
Si BJT, JFET or MOSFET (room temperature)	10↔20	75↔300	1↔3
GaAs FET or HEMT (room temperature)	10↔15	20↔120	0.3↔1.5
GaAs FET or HEMT (liquid-nitrogen 77K)	10↔15	7↔35	0.1↔0.5
Si or GaAs MMIC amplifier	10↔25	170↔1600	2↔8
Operational amplifier	40↔100	10^4↔10^9	16↔66



$$P_1 = G \cdot \Delta f \cdot k_B \cdot (T_1 + T_{RX})$$

$$P_2 = G \cdot \Delta f \cdot k_B \cdot (T_2 + T_{RX})$$

The unknowns $G \cdot \Delta f \cdot k_B$ *cancel in the Y ratio!*

$$Y = \frac{P_2}{P_1} = \frac{T_2 + T_{RX}}{T_1 + T_{RX}}$$

$$T_{RX} = \frac{T_2 - Y \cdot T_1}{Y - 1}$$

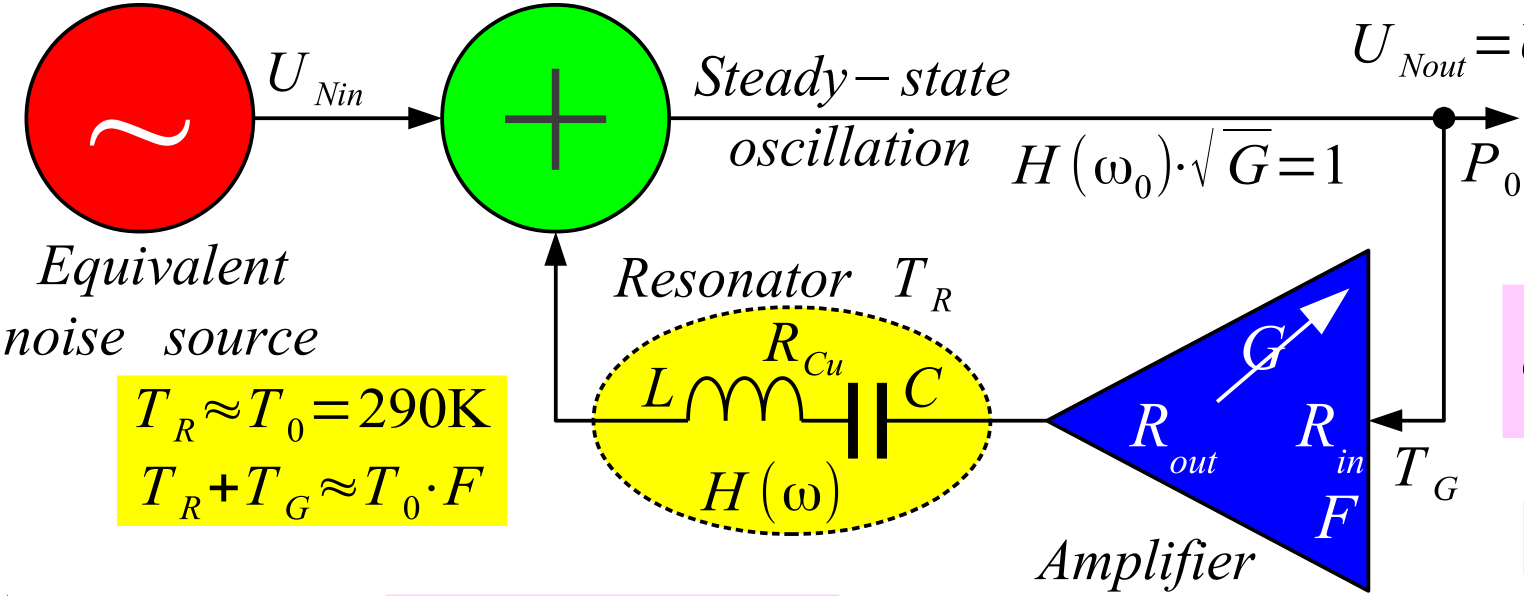
$$T_0 = 290\text{K}$$

$$F_{dB} = 10 \log_{10}\left[1 + \frac{T_2 - Y \cdot T_1}{(Y-1) \cdot T_0}\right]$$

Resistor type	Temperature
Antenna into cold sky	~20K
Liquid N_2 cooled R	~77K
Antenna into absorber	~290K
R at room temperature	~290K
Light-bulb filament as R	~2000K
Ionized gas as R	~10^4K
Avalanche breakdown	~10^6K

14 – Hot/cold method

$$U_{Nout} = U_{Nin} + H(\omega)\cdot\sqrt{G}\cdot U_{Nout}$$

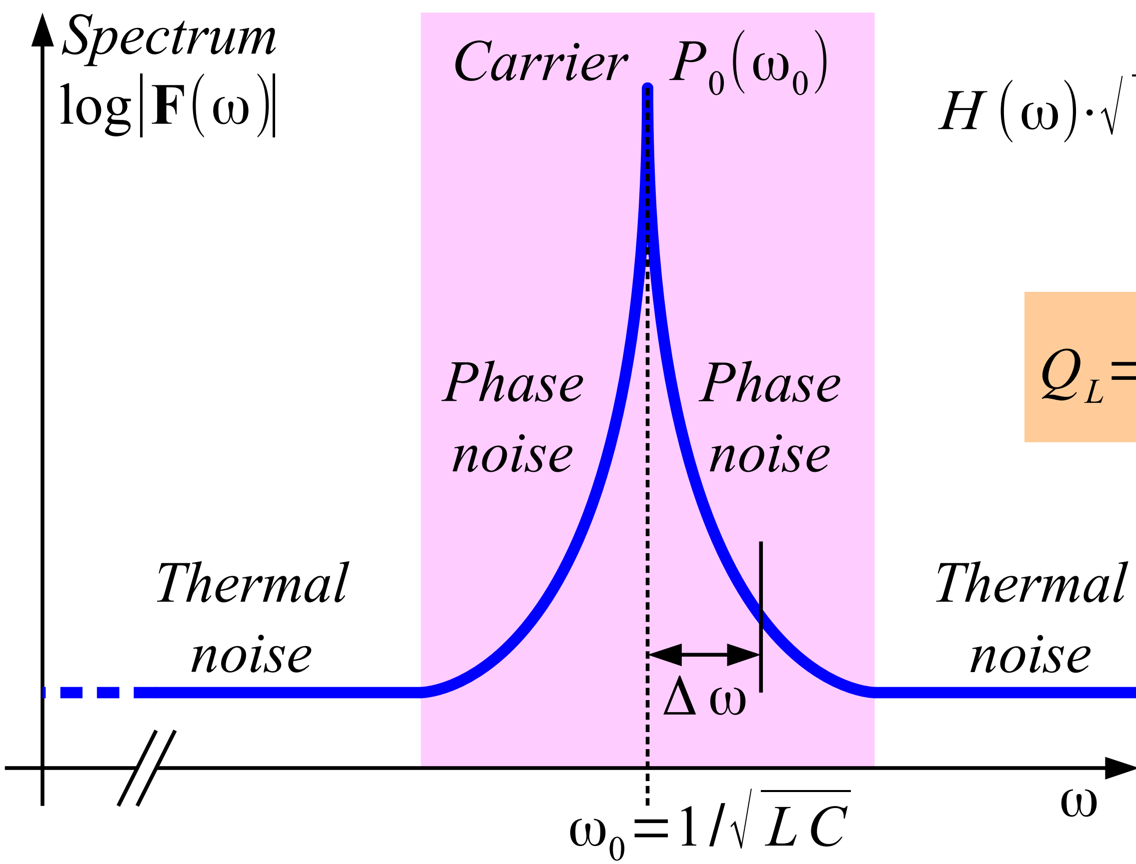


Valid at $P_{Nout} \ll P_0$

$$U_{Nout} = \frac{U_{Nin}}{1 - H(\omega)\cdot\sqrt{G}}$$

$$\Sigma R = R_{out} + R_{Cu} + R_{in}$$

$$H(\omega)\cdot\sqrt{G} = \frac{\Sigma R}{\Sigma R + j\omega L + \frac{1}{j\omega C}} \approx \frac{1}{1 + j2Q_L\frac{\Delta\omega}{\omega_0}}$$



$$Q_L = \frac{\omega_0 L}{\Sigma R}$$

$$U_{Nout} \approx U_{Nin}\cdot\left(1 + \frac{\omega_0}{j2Q_L\Delta\omega}\right)$$

$$P_{Nout} \approx P_{Nin}\cdot\left[1 + \left(\frac{\omega_0}{2Q_L\Delta\omega}\right)^2\right]$$

Amplitude and phase noise

$$P_{Nout} \approx P_{Nin}\cdot\left[1 + \left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]$$

15 – Oscillator phase noise

Normalized phase−noise spectral density $\log L(\Delta f)$ $[\mathrm{dBc/Hz}]$

Saturation removes amplitude noise $P_\phi = P_{Nout}/2$

$$\frac{dP_{Nin}}{df} = N_0 = k_B(T_R + T_G) \approx k_B T_0 F$$

$$L(\Delta f) = \frac{1}{P_0}\cdot\frac{dP_\phi}{df} = \frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)\quad[\mathrm{Hz}^{-1}]$$

Valid at $L(\Delta f)\cdot\Delta f \ll 1$

Phase noise only

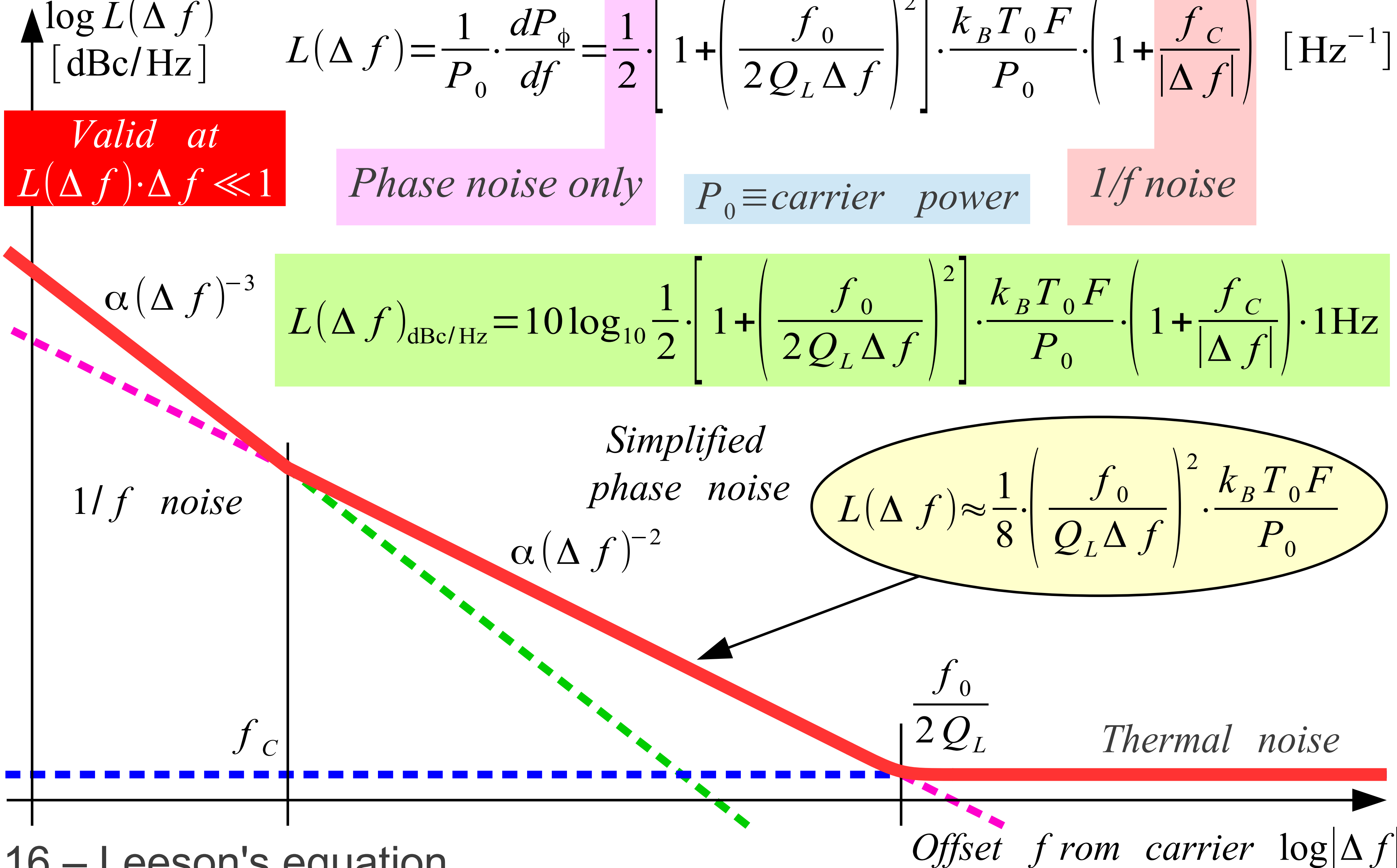
$P_0 \equiv$ *carrier power*

1/f noise

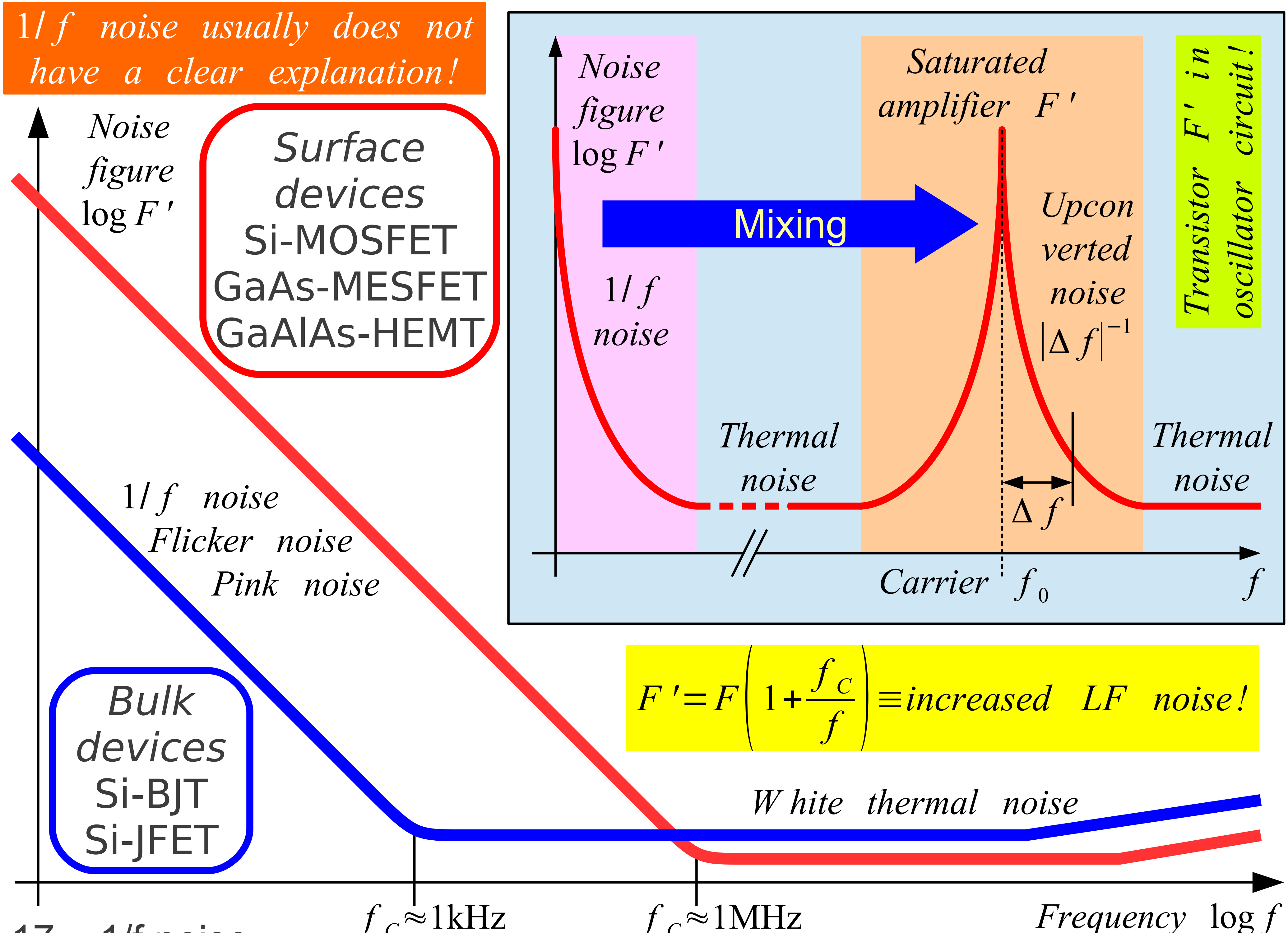
$$L(\Delta f)_{\mathrm{dBc/Hz}} = 10\log_{10}\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)\cdot 1\mathrm{Hz}$$

Simplified phase noise

$$L(\Delta f) \approx \frac{1}{8}\cdot\left(\frac{f_0}{Q_L\Delta f}\right)^2\cdot\frac{k_B T_0 F}{P_0}$$



16 – Leeson's equation



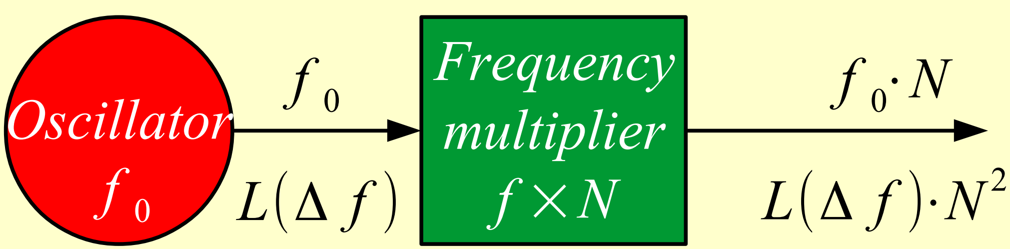
1/ f noise usually does not have a clear explanation!
Noise figure log F '
Surface devices
Si-MOSFET
GaAs-MESFET
GaAlAs-HEMT
1/ f noise
Flicker noise
Pink noise
Bulk devices
Si-BJT
Si-JFET
Noise figure log F '
Saturated amplifier F '
Transistor F ' in oscillator circuit!
Mixing
1/ f noise
Upcon verted noise $|\Delta f|^{-1}$
Thermal noise
Thermal noise
Δf
Carrier f_0
f
$F' = F\left(1 + \frac{f_C}{f}\right) \equiv$ increased LF noise!
White thermal noise
$f_C \approx 1$kHz
$f_C \approx 1$MHz
Frequency log f

17 – 1/f noise

The loaded resonator quality Q_L defines the oscillator phase noise!

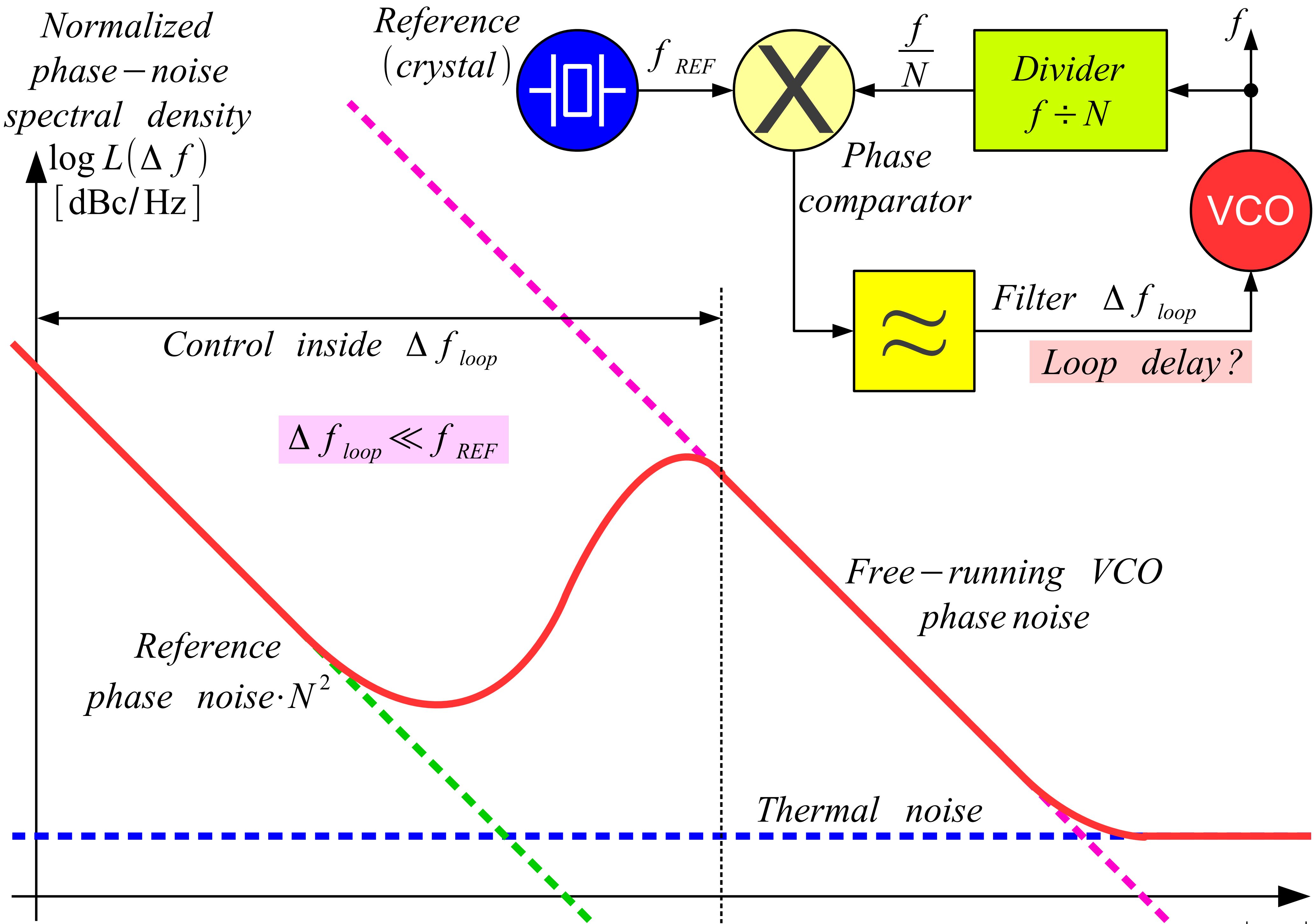
$$L(\Delta f)=\frac{1}{2}\cdot\left[1+\left(\frac{f_0}{2Q_L\Delta f}\right)^2\right]\cdot\frac{k_B T_0 F}{P_0}\cdot\left(1+\frac{f_C}{|\Delta f|}\right)$$

Variable-frequency oscillators	Q_L
RC VCO	~1
BWO tube	~1
Varactor-tuned LC VCO	10↔30
YIG ($Y_3Fe_5O_{12}$) oscillator	300↔1000

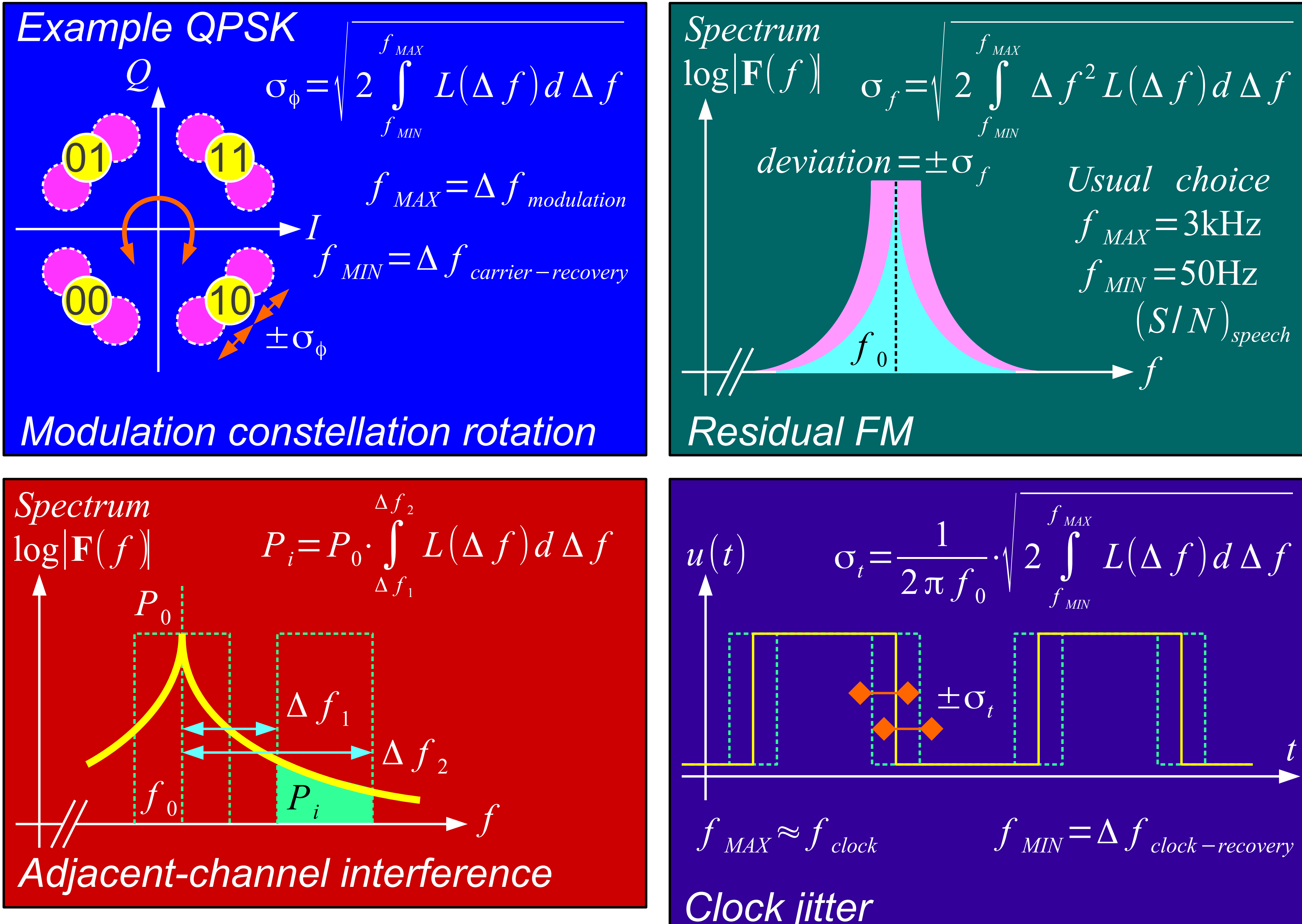


The phase noise multiplies with the square of the frequency multiplication factor!

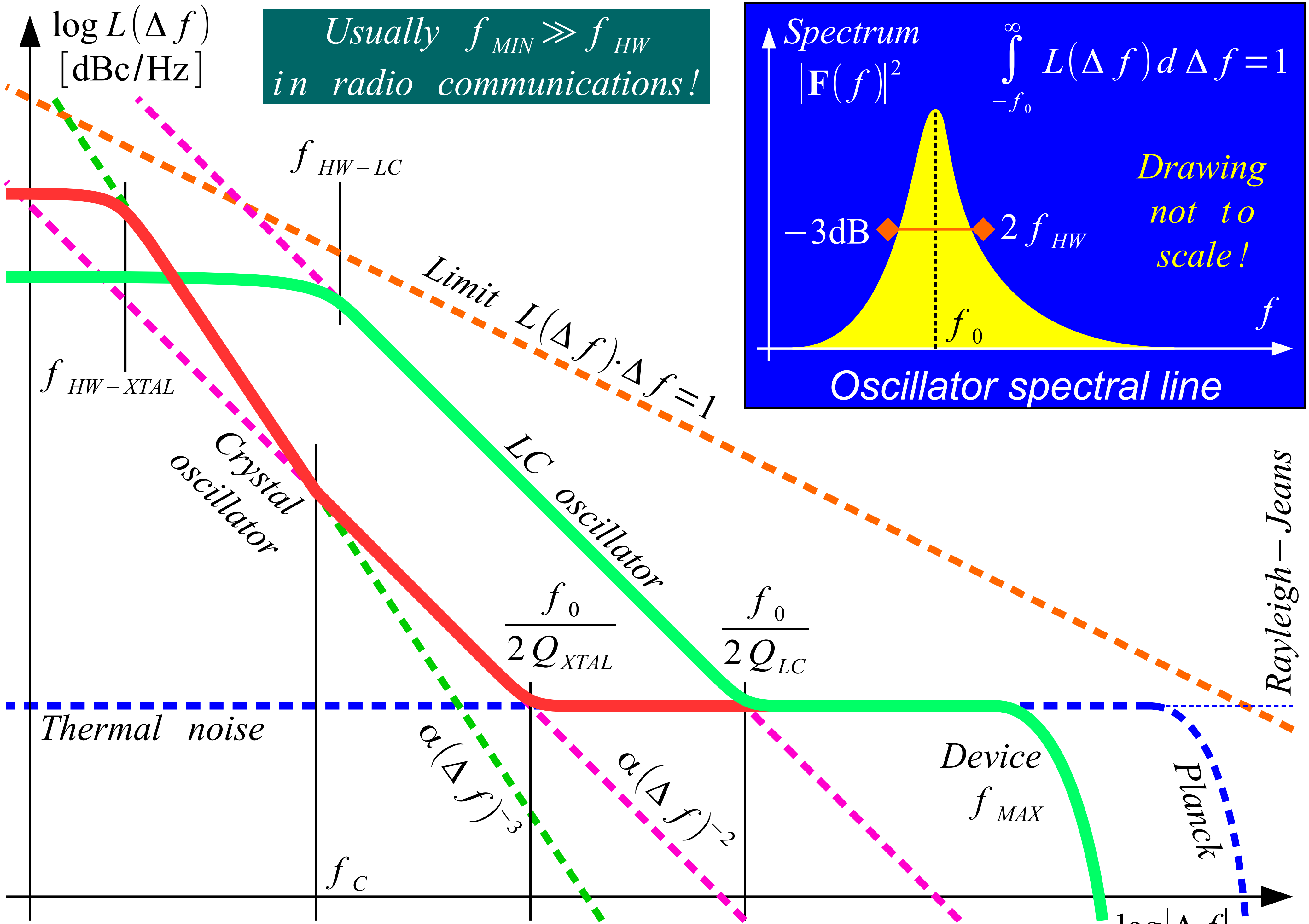
Fixed-frequency oscillators	Q_L
RC multivibrator	~1
LC resonator	30↔100
Cavity resonator	1000↔3000
Ceramic dielectric resonator	1000↔3000
AT-cut quartz crystal (fundamental mode)	3000↔10000
AT-cut quartz crystal (third/fifth overtone)	10000↔30000
Electro-optical delay line ($)	~10^6 (noisy!)
Sapphire dielectric resonator ($$$)	~3.10^5
Red HeNe LASER	~10^8



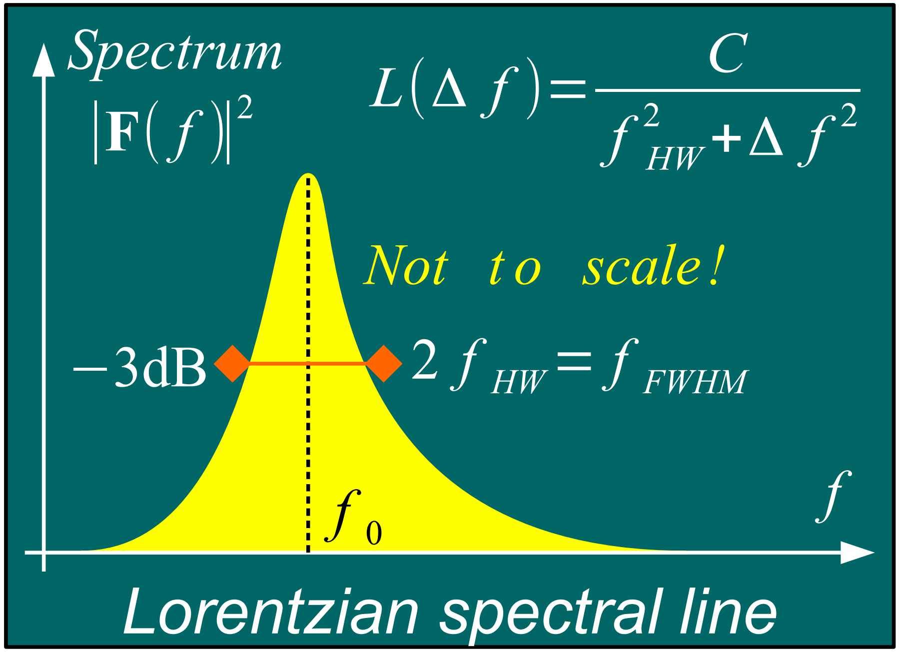
19 – Phase-Locked Loop (PLL)



20 – Effects of phase noise



21 – Phase noise without approximations



Flat thermal noise can be neglected: device f_{MAX} or Planck law

$LC-$oscillator $1/f$ noise can be neglected

$$L(\Delta f) = \frac{1}{8} \cdot \left(\frac{f_0}{Q_L}\right)^2 \cdot \frac{1}{\boxed{f_{HW}^2} + \Delta f^2} \cdot \frac{k_B T_0 F}{P_0}$$

Lorentzian line in Leeson's equation

$$\int_{-f_0}^{\infty} L(\Delta f)\, d\Delta f = 1 \approx \int_{-\infty}^{\infty} L(\Delta f)\, d\Delta f = \frac{1}{8} \cdot \left(\frac{f_0}{Q_L}\right)^2 \cdot \frac{k_B T_0 F}{P_0} \int_{-\infty}^{\infty} \frac{1}{f_{HW}^2 + \Delta f^2}\, d\Delta f =$$

$$= \frac{1}{8} \cdot \left(\frac{f_0}{Q_L}\right)^2 \cdot \frac{k_B T_0 F}{P_0} \cdot \left[\frac{1}{f_{HW}} \cdot \arctan \frac{\Delta f}{f_{HW}}\right]_{\Delta f = -\infty}^{\Delta f = \infty} = \frac{k_B T_0 F}{8 P_0} \cdot \left(\frac{f_0}{Q_L}\right)^2 \cdot \frac{\pi}{f_{HW}}$$

$$f_{HW} = \frac{\pi k_B T_0 F}{8 P_0} \cdot \left(\frac{f_0}{Q_L}\right)^2$$

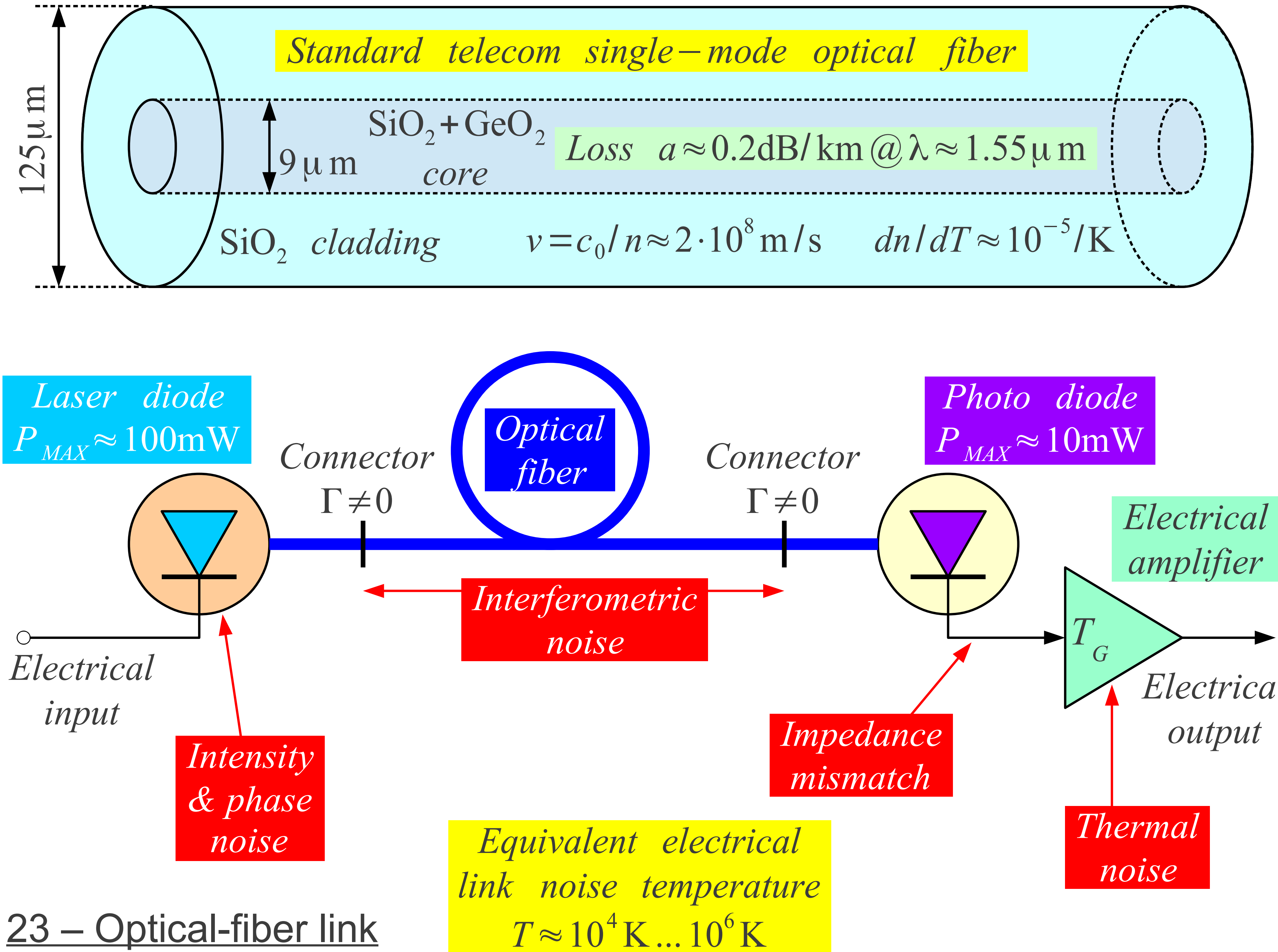
Example $f_0 = 3\text{GHz}$ $Q_L = 10$
$P_0 = 0.1\text{mW}$ $F = 10\text{dB}$
$f_{HW} = 14\text{Hz}$ $f_{FWHM} = 28\text{Hz}$

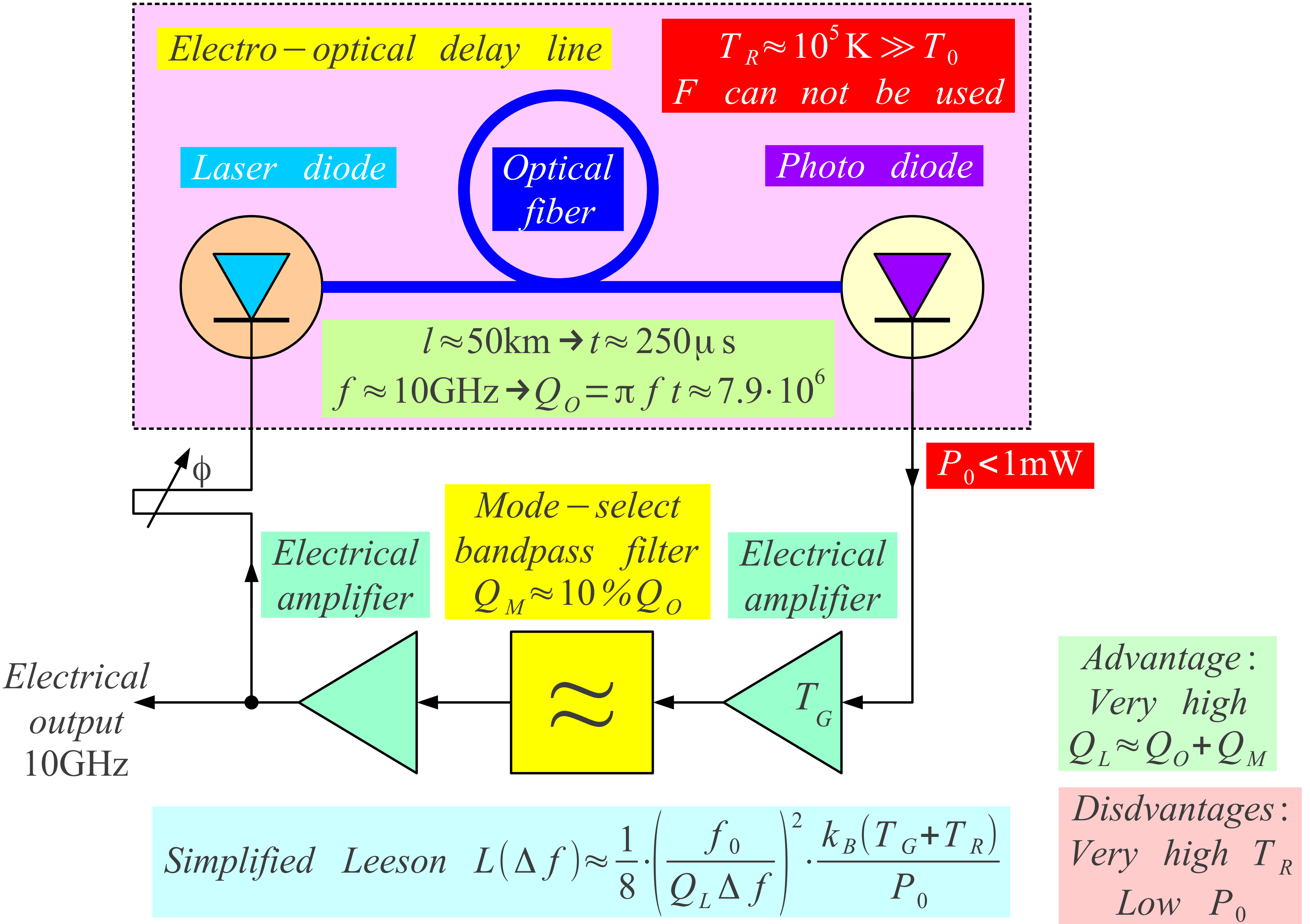
$$C = \frac{k_B T_0 F}{8 P_0} \cdot \left(\frac{f_0}{Q_L}\right)^2 = \frac{f_{HW}}{\pi}$$

$$L(\Delta f) = \frac{f_{HW}/\pi}{f_{HW}^2 + \Delta f^2}$$

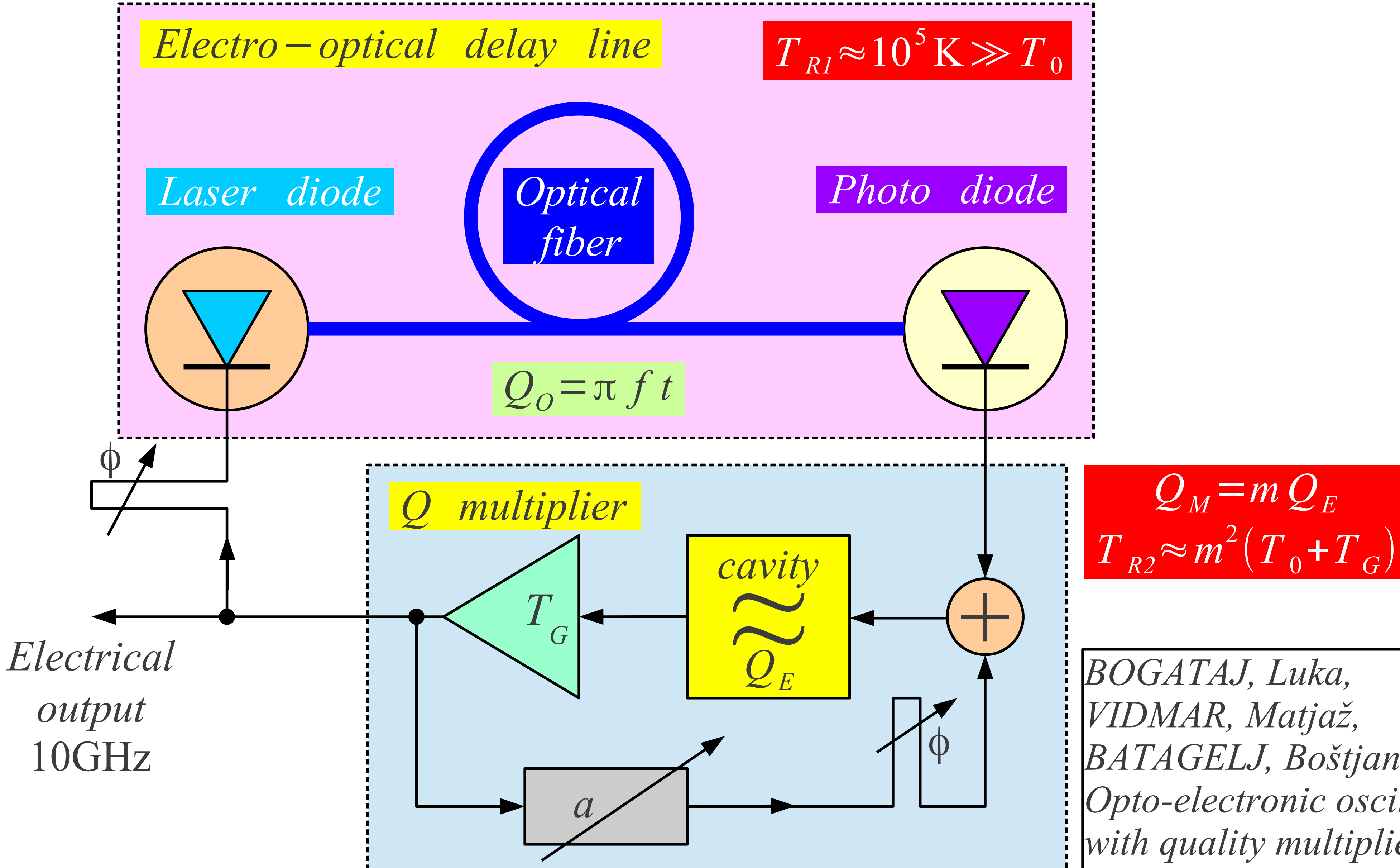
22 – Lorentzian spectral-line width

23 – Optical-fiber link





24 – Opto-electronic oscillator

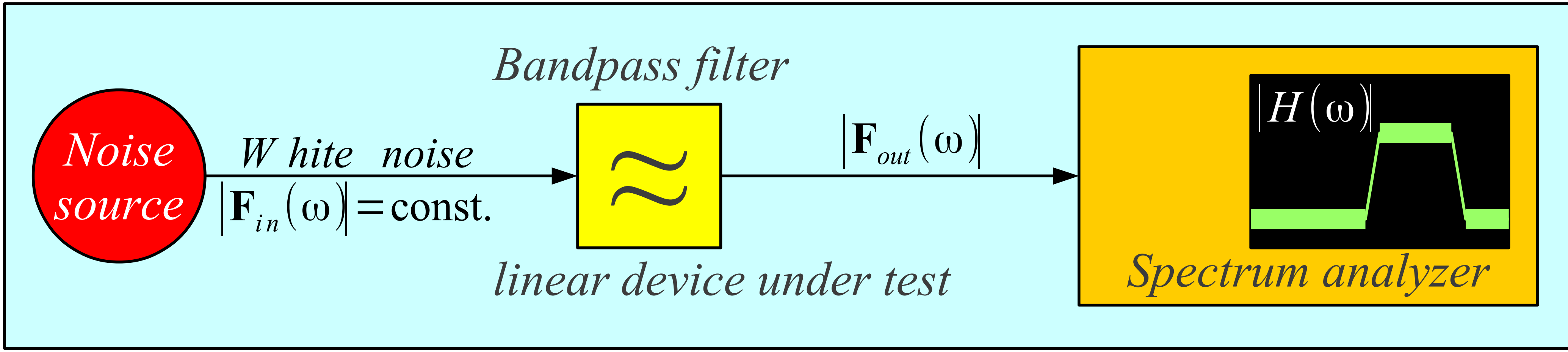


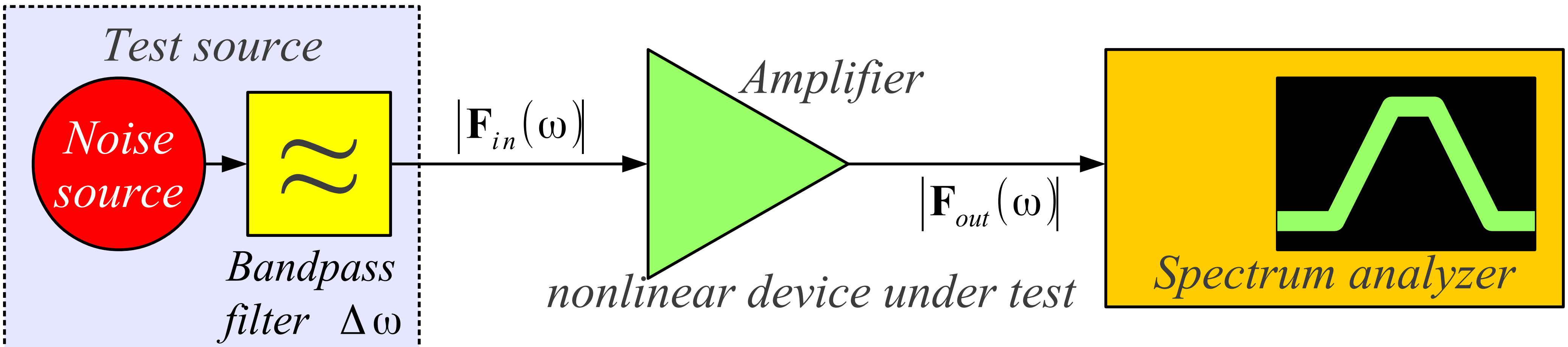
Simplified Leeson $L(\Delta f) \approx \frac{1}{8} \cdot \left(\frac{f_0}{Q_L \Delta f} \right)^2 \cdot \frac{k_B (T_{R1} + T_{R2})}{P_0}$

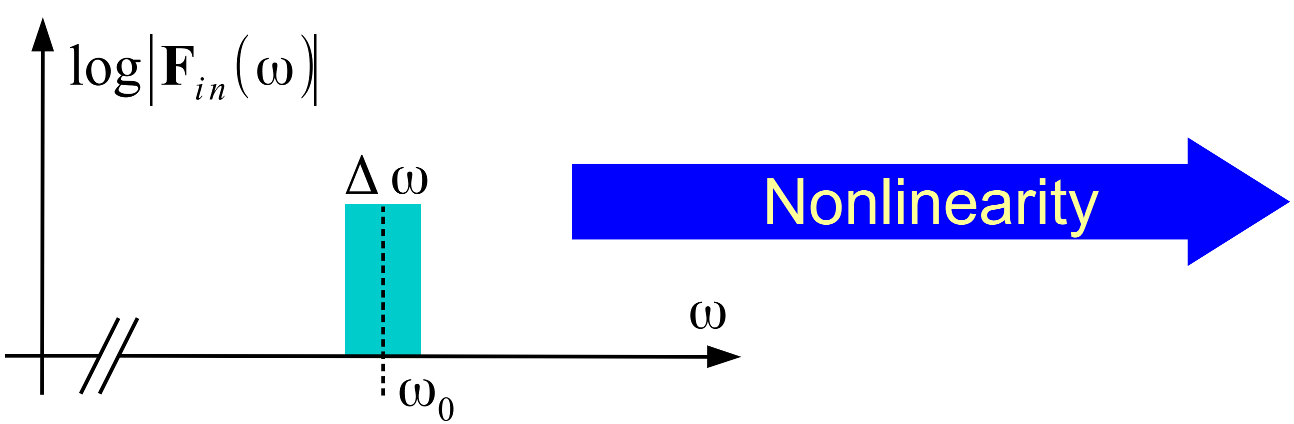
$Q_L \approx Q_O + Q_M$

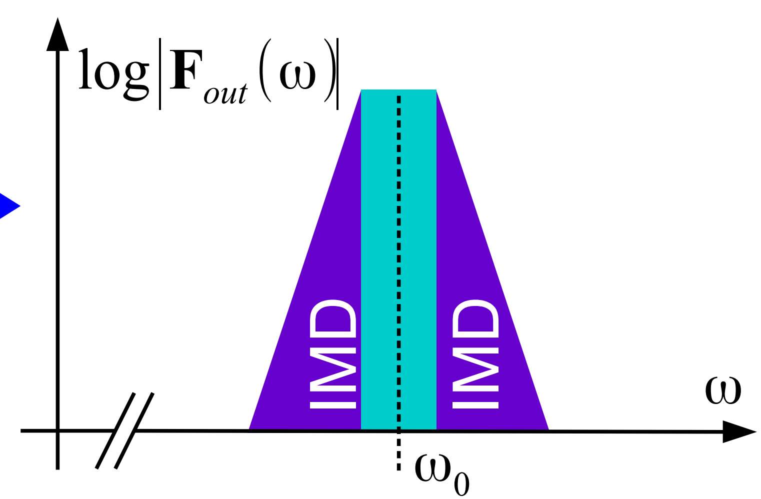
BOGATAJ, Luka,
VIDMAR, Matjaž,
BATAGELJ, Boštjan:
Opto-electronic oscillator with quality multiplier,
IEEE transactions on microwave theory and Techniques,
ISSN 0018-9480.
Feb. 2016, vol. 64, no. 2, pp. 663-668.

25 – Q multiplier

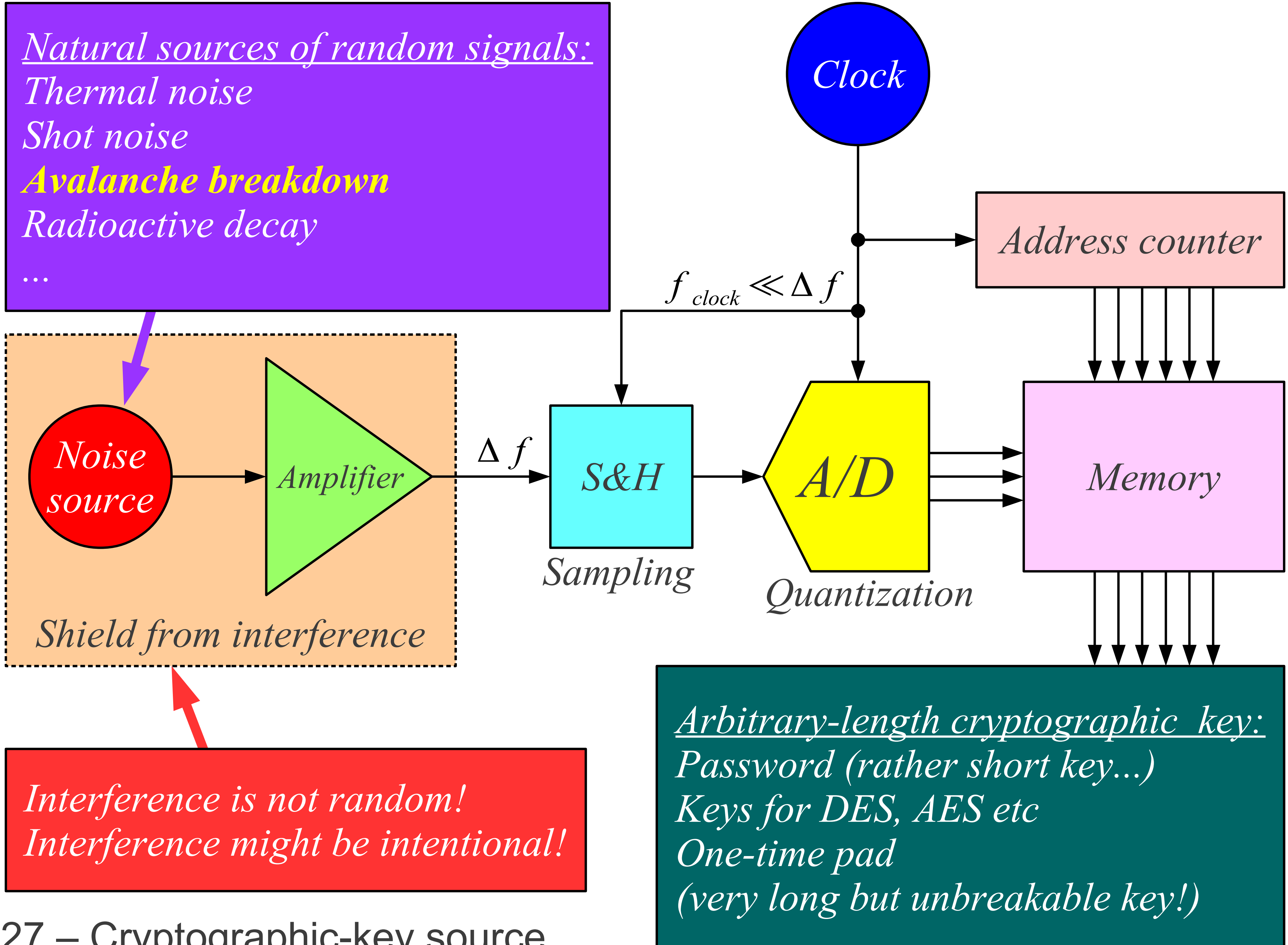




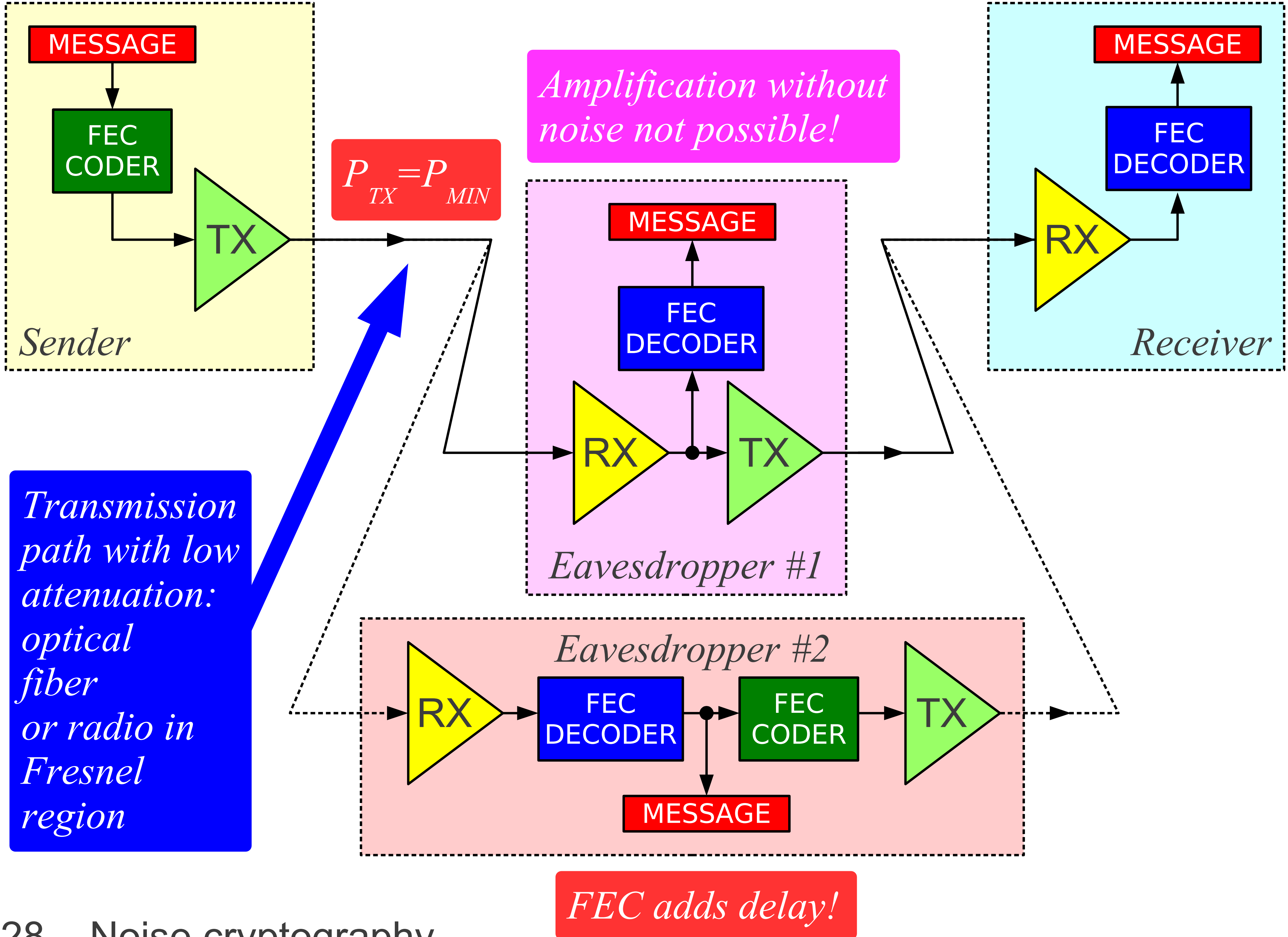




26 – Noise as test signal

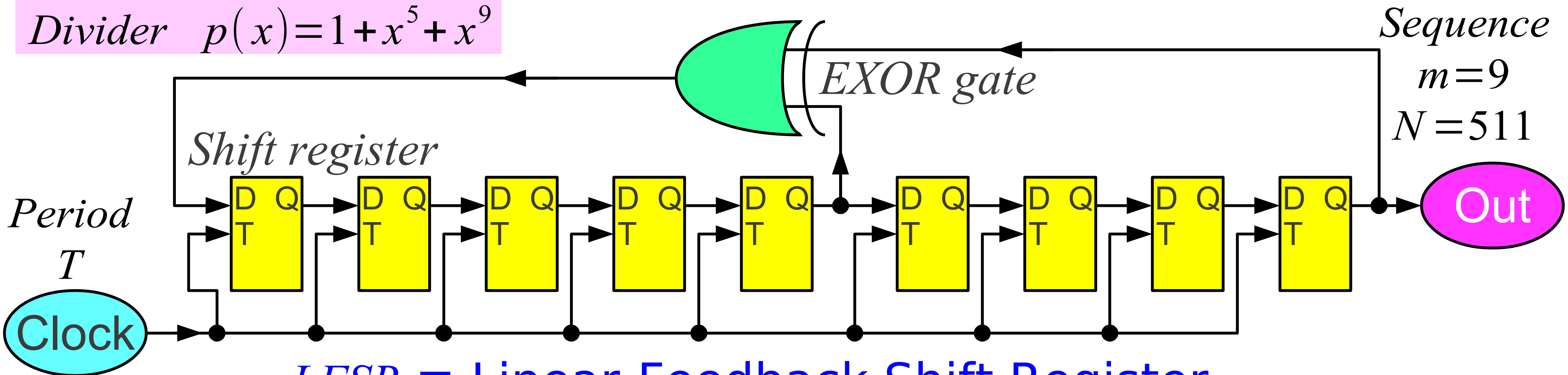


27 – Cryptographic-key source



28 – Noise cryptography

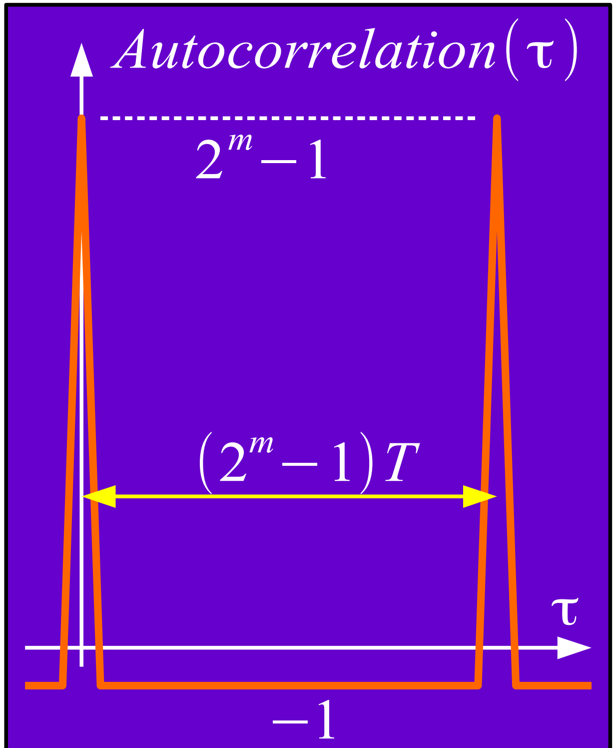
Divider $p(x)=1+x^5+x^9$

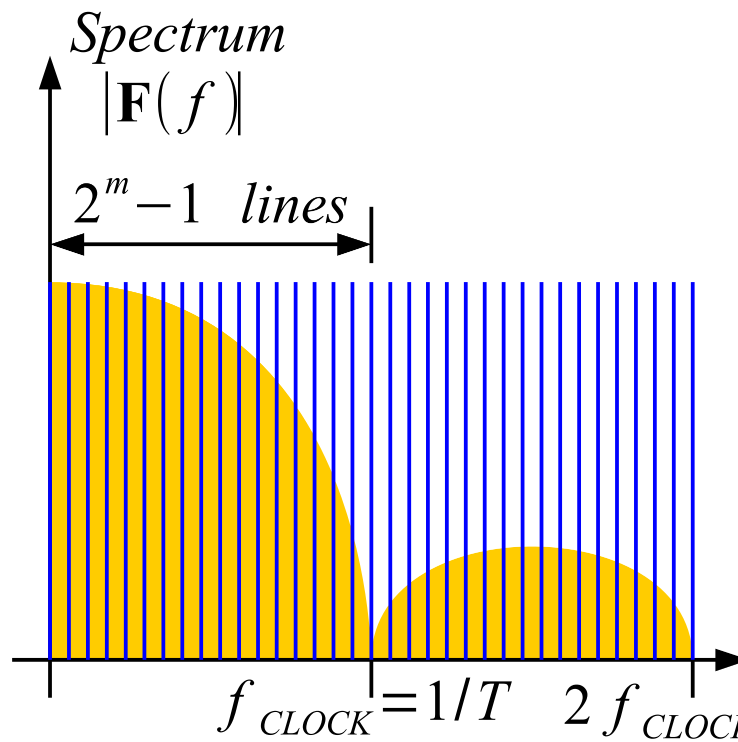


LFSR ≡ Linear-Feedback Shift Register

Primitive polynomial $p(x)=1+x^l+x^m$ → *max sequnce length* $N=2^m-1$

2^{m-1} ones and 2^{m-1}-1 zeros arranged in groups of
1X m ones, m-1 zeros
1X m-2 ones and zeros
2X m-3 ones and zeros
4X m-4 ones and zeros
....
2^{m-5} groups 111 and 000
2^{m-4} groups 11 and 00
2^{m-3} individual 1 and 0





29 – LFSR pseudo-random sequences

Sound and appear as white noise!

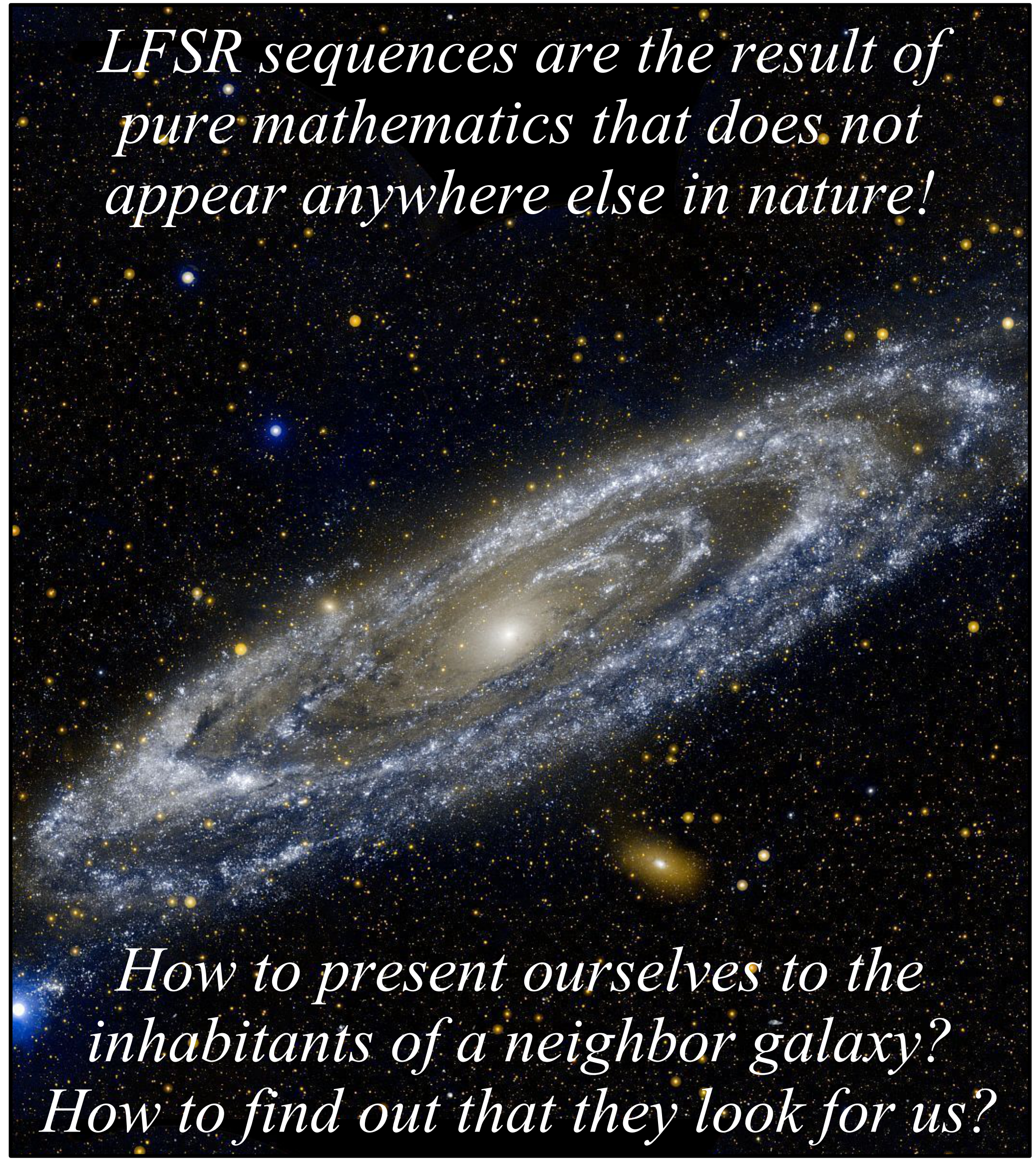
Two-valued autocorrelation with a single very pronounced peak:
- synchronization headers for data frames
- spreading sequences in CDMA
- accurate time transfer in radio navigation (GPS, GLONASS)

Perfect frequency spectrum of uniformly-spaced lines and simple generation/checking:
- test sequences for all kinds of telecommunication links
- data scrambling (randomization) as part of line coding

Peak-to-average power ratio:

$$LFSR: \quad \frac{P_{MAX}}{\langle P \rangle} \approx 1 \qquad\qquad Noise: \quad \frac{P_{MAX}}{\langle P \rangle} \to \infty$$

LFSR pseudo-random sequences are of NO cryptographic value: Berlekamp-Massey algorithm 1969



30 – Use of pseudo-random sequences