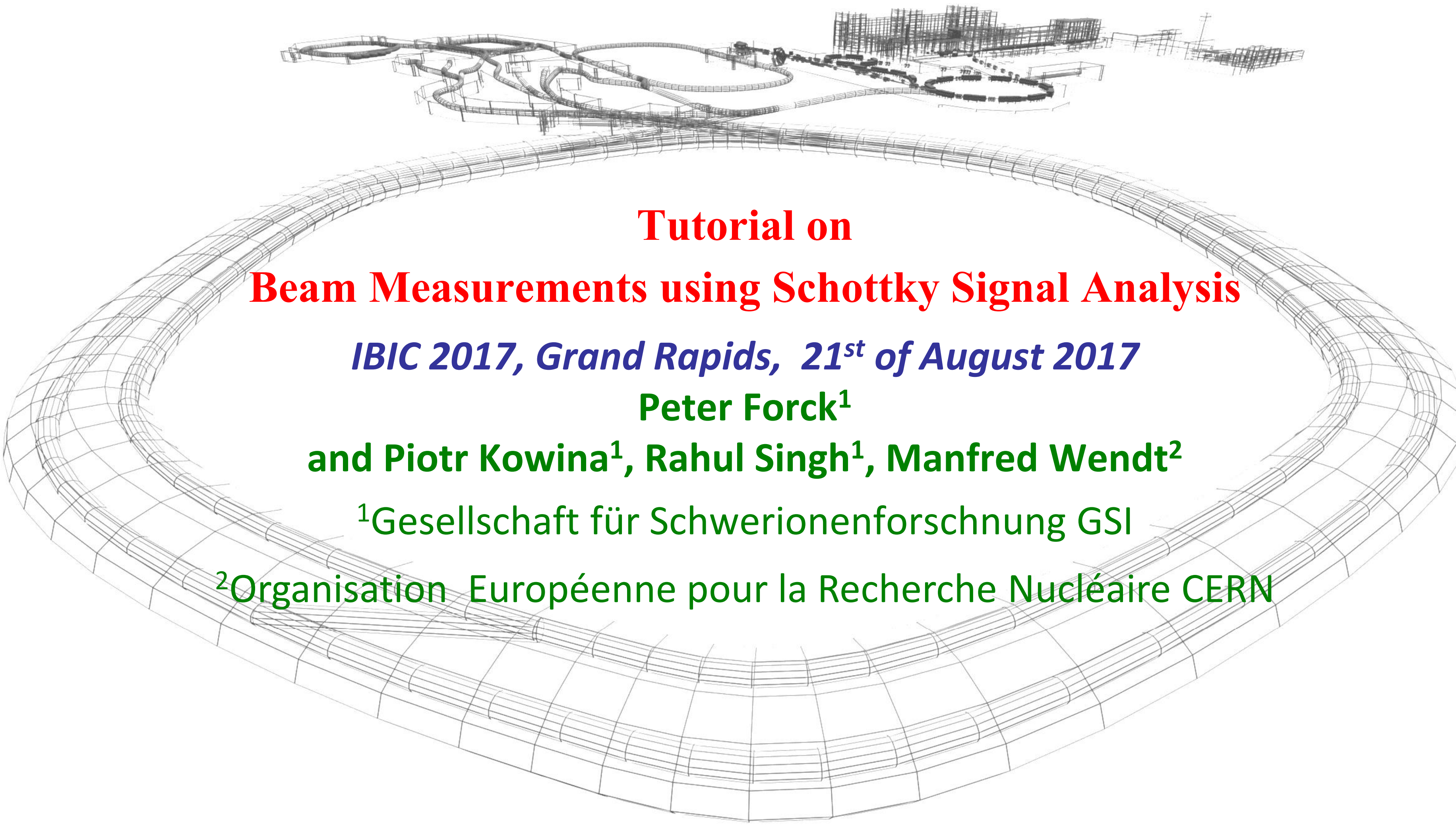

Tutorial on
Beam Measurements using Schottky Signal Analysis

IBIC 2017, Grand Rapids, 21st of August 2017

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Outline of the tutorial:

- Introduction to noise and fluctuations relevant for Schottky analysis
- Main part: Schottky signal generation and examples for the following case:
  - Longitudinal for coasting beams
  - Transverse for coasting beams
  - Longitudinal for bunched beams
  - Transverse for bunched beams
- Some further examples for exotic beam parameters
- Conclusion

Beam Parameters obtained from Schottky Signal Analysis

Longitudinal Schottky Spectrum delivers:

- Mean revolution frequency f_0, incoherent spread in revolution frequency $\Delta f\ /\ f_0$
  $\Rightarrow$ in accelerator physics: mean momentum p_0, momentum spread $\Delta p\ /\ p_0$
- For bunched beams: synchrotron frequency f_s
- Insight in longitudinal beam dynamics including non-linearities

Transverse Schottky Spectrum delivers:

- Tune Q i.e. number of betatron oscillations per turn
- Chromaticity ξ with $\frac{\Delta Q}{Q_0} = \xi \cdot \frac{\Delta p}{p_0}$ i.e. coupling between momentum and tune
- Transverse emittance (in most case in relative units)

For intense beams:

Modifications of the spectrum is used to probe beam models

Installed at nearly *every* proton, anti-proton & ion storage ring for coasting beams

Installed in many hadron synchrotrons for bunched beam investigations

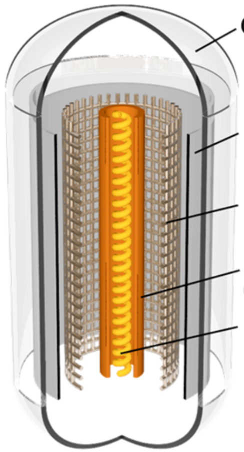
The basic ideas for standard applications & detection scheme are discussed!

Emission of electrons in a vacuum tube:

W. Schottky, ‘Spontaneous current fluctuations in various electrical conductors’, Ann. Phys. 57 (1918)
[original German title:‘Über spontane Stromschwankungen in verschiedenen Elektrizitätsleitern’]

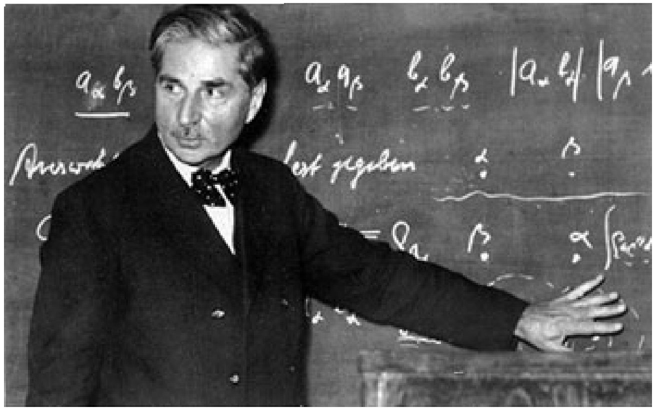
Emission of electrons follows statistical law like Brown’s law

Physical reason: Charge carrier of final mass and charge



Walter Schottky (1886 – 1976):

- German physicist at Universities Jena, Würzburg & Rostock and at company Siemens
- Investigated electron and ion emission from surfaces
- Design of vacuum tubes
- Super-heterodyne method i.e spectrum analyzer
- Solid state electronics e.g. metal-semiconductor interface
- **No** connection to accelerators



Source: Wikipedia

Shot Noise for free Charge Carriers (here Electrons)

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Emission of electrons follows statistical law like Brown's law

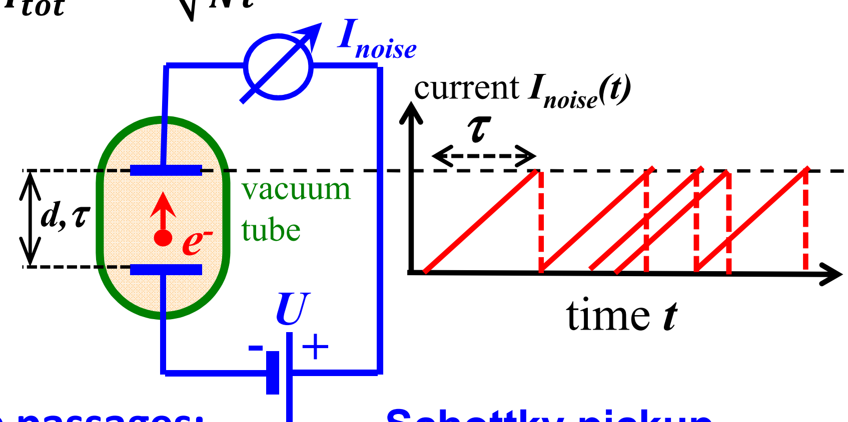
Assuming: charges of quantity e, N average charges per time interval and τ duration of travel

fluctuations as $I_{noise} = \sqrt{< I^2 >} = \sqrt{\frac{e^2 \cdot N}{\tau}} \Leftrightarrow \frac{I_{noise}}{I_{tot}} = \sqrt{\frac{1}{N\tau}}$, I_{tot} is total current

as caused by final charge & mass of charge carriers

This is white noise i.e. flat frequency spectrum

It is called **shot noise**



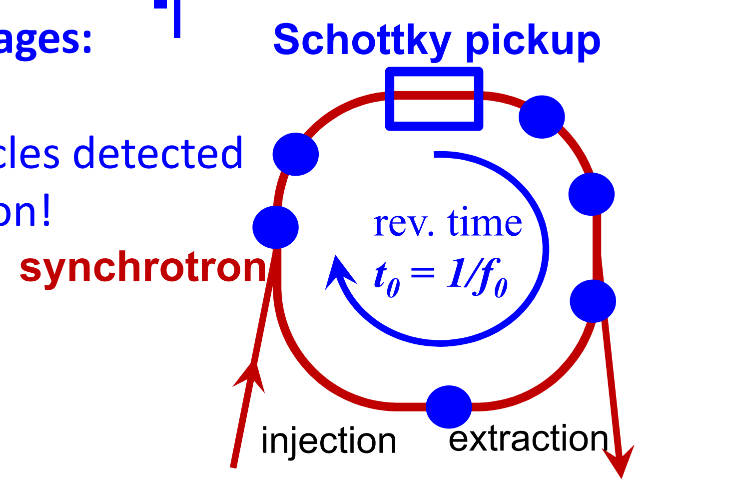
'Schottky signals' in circular accelerators of multiple passages:

This is **not** shot noise!

But the **fluctuations** caused by randomly distributed particles detected by the correlation of their **repeating** passage at one location!

$\Rightarrow$ The frequency spectrum has bands i.e. not flat

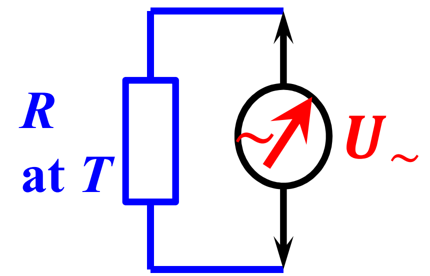
Schottky signal analysis: Developed at CERN ISR ≈ 1970th for operation of stochastic cooling



General Noise Sources of Electronics Devices

Any electronics is accompanied with noise due to:

- **Thermal noise** as given by the statistical movement of electrons described by Maxwell-Boltzmann distribution



Within **resistive matter** average cancels: $U_{mean} = \langle U \rangle = 0$

but standard deviation remains:

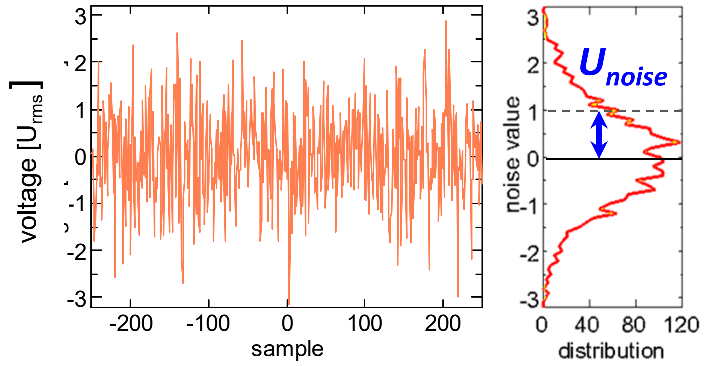
$U_{noise} = \sqrt{\langle U^2 \rangle} = \sqrt{4k_B \cdot T \cdot R \cdot \Delta f}$ this is **white noise** i.e. **no** frequency dependence,

k_B Boltzmann constant, T temperature, R resistivity, Δf bandwidth

⇔ Spectra noise for R = 50 Ω and T = 300 K: $U_{noise}/\sqrt{\Delta f} = \sqrt{4k_B T R} \approx 1\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$

⇔ spectral power density: $\frac{\Delta P_{noise}}{\Delta f} = \frac{1}{R} \cdot \left(\frac{U_{noise}}{\sqrt{\Delta f}}\right)^2 \approx -170\ \mathrm{dBm/Hz}$

Noise is the statistical fluctuations of a signal !



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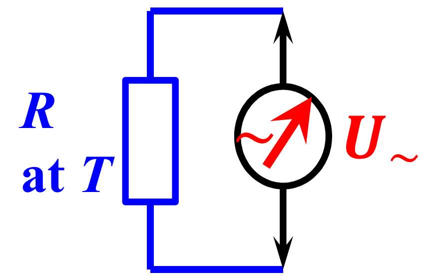

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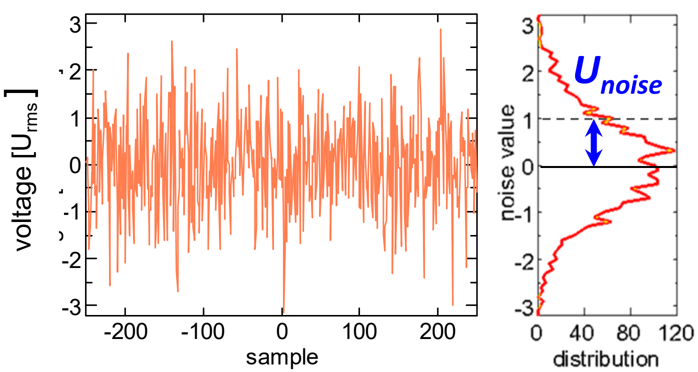
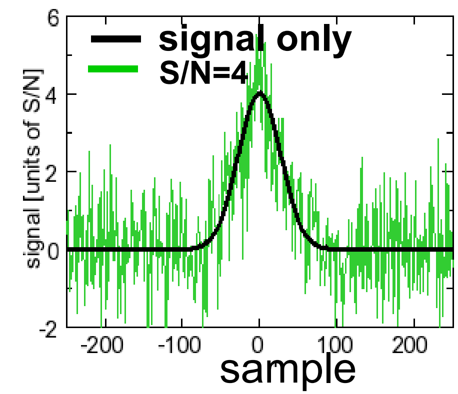


Typical challenge for 'regular' beam instrumentation:
Recovery signal from noise
i.e. fluctuations are disturbing



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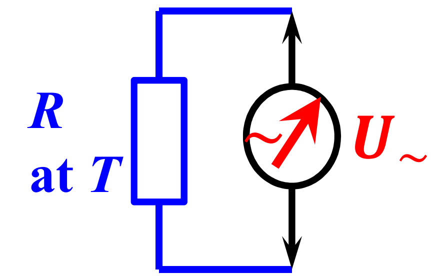

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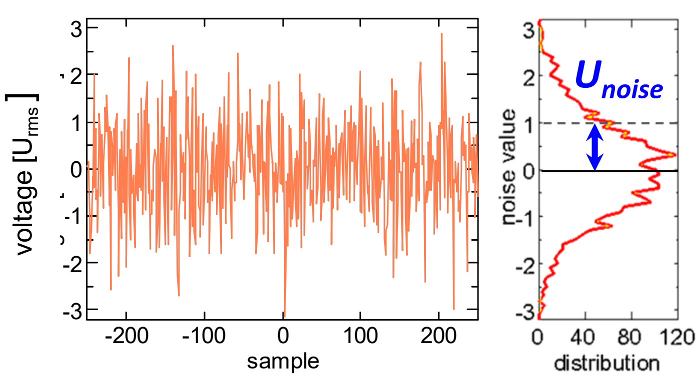

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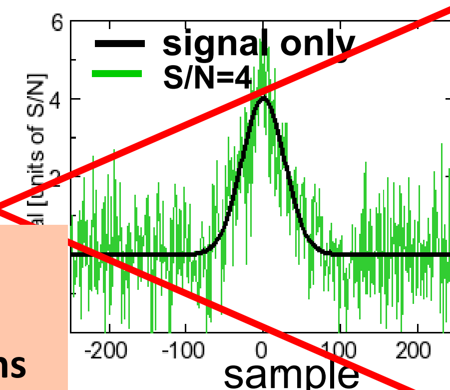
Schottky analysis has different point-of-view: Information from fluctuations



Simple Model of a Synchrotron

The important parameters of the reference particle:

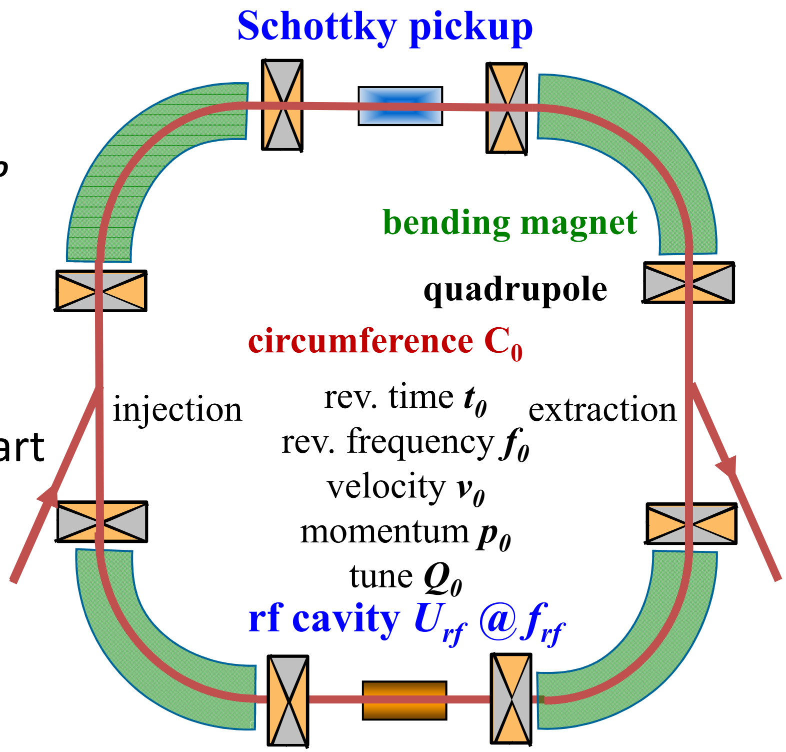
- Orbit C_o for reference particle (index 0)
- Revolution time t_o & revolution frequency $f_0 = 1/t_o$
  hadron synchrotron typ. 0.1 kHz < f_0 < 10 MHz
- Momentum p_0
- Tune Q_0 & betatron frequency $f_\beta = Q_0 \cdot f_0$
  can be decomposed in $Q_0 = n + q$, q non-integer part

For any other single particle e.g. t: $\Delta t = t - t_0$

For many particles: Δt is the width of the distribution

Coasting beam: particle randomly distributed along C

Bunched beam: Bunches by rf-cavity voltage



$\Rightarrow$ synchrotron oscillation with frequency $f_s \propto \sqrt{U_{rf}} \ll f_0$, typ. 1 MHz < f_{rf} < 20 MHz

For most considered cases (if not stated otherwise):

- No direct interaction of the particles, i.e. no incoherent effect like by space charge
- No significant contributions by induced wake field i.e. no coherent effects by impedances

Simple Model of a Synchrotron

Momentum compaction factor α:

A particle with a offset momentum $\rightarrow$ different orbit

$\Rightarrow$ orbit length C varies: $\frac{\Delta C}{C_0} = \alpha \cdot \frac{\Delta p}{p_0}$

Slip factor or frequency dispersion η:

A particle with offset momentum $\rightarrow$ diff. revolution frequency

$\Rightarrow$ rev. frequency varies: $\frac{\Delta f}{f_0} = \eta \frac{\Delta p}{p_0}$

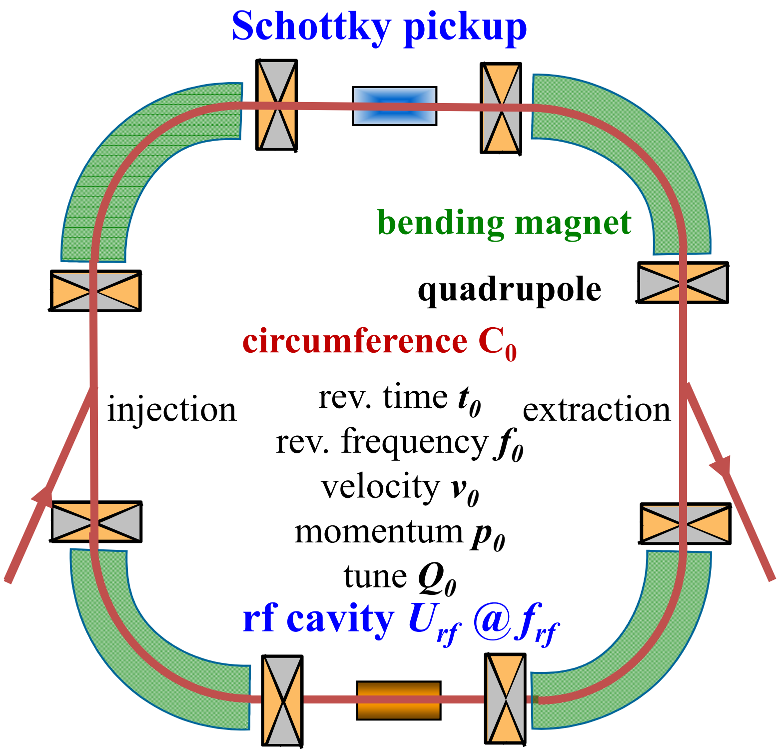
Chromaticity ξ:

A faster particle is less focused at a quadrupole

$\Rightarrow$ tune varies: $\frac{\Delta Q}{Q_0} = \xi \cdot \frac{\Delta p}{p_0}$

Remark:

The values of α η and ξ depend on the lattice setting
i.e. on the arrangement of dipoles and quadrupoles



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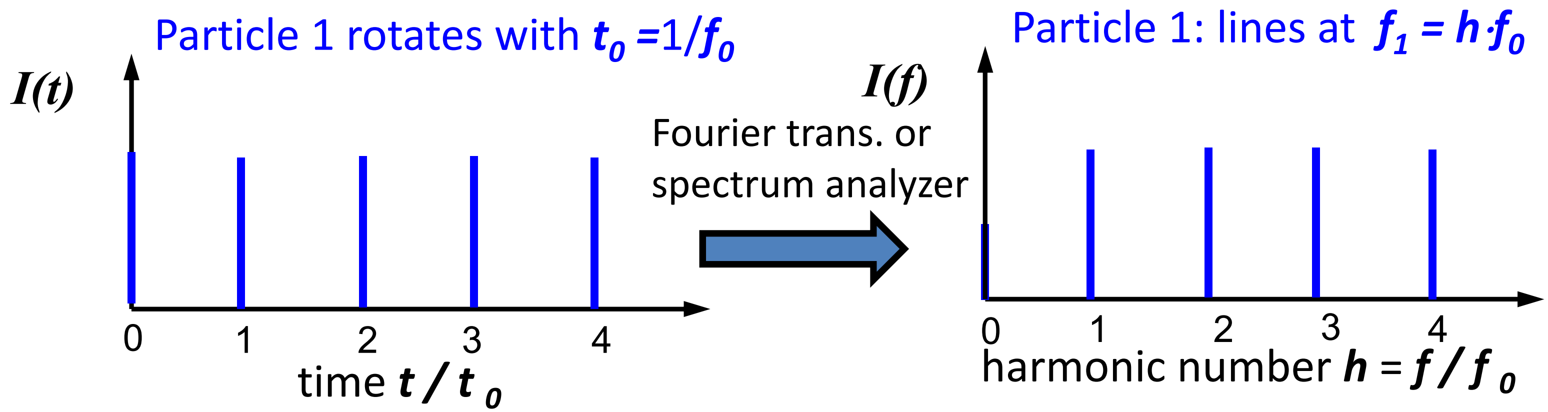
Longitudinal Schottky Analysis: 1st Step

Schottky noise analysis is based on the power spectrum
for consecutive passage of the **same** finite number of particles



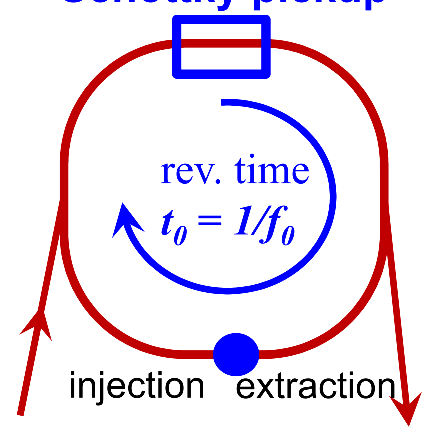
Particle 1 of charge e rotates with $t_1 = 1/f_0$ **:**

Current at pickup $I_1(t) = ef_0 \cdot \sum_{h=-\infty}^{\infty} \delta(t - ht_0)$

$$\Rightarrow I_1(f) = ef_0 + 2ef_0 \cdot \sum_{h=1}^{\infty} \delta(f - hf_0)$$

i.e. frequency spectrum comprise of δ-functions at $h \cdot f_0$

(This can formally be proven by **Fourier Series**)



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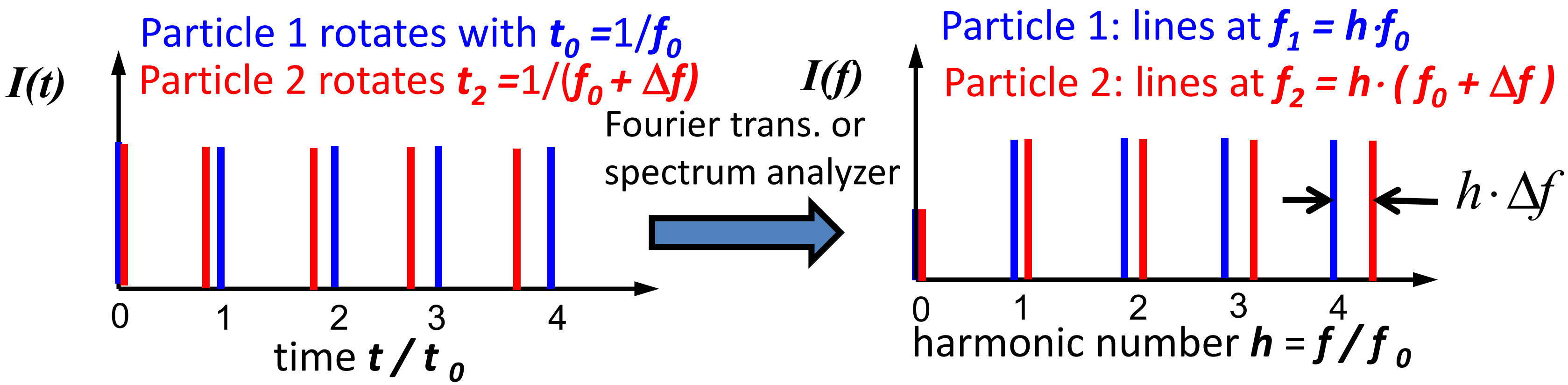

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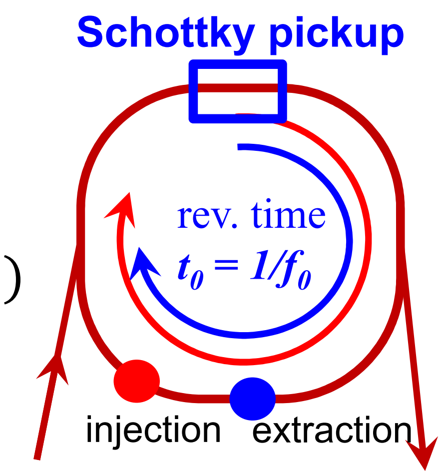
Particle 2 of charge e rotating with $t_2 = 1/(f_0 + \Delta f)$:

Current at pick-up $I_2(t) = ef_0 \cdot \sum_{h=-\infty}^{\infty} \delta(t - ht_2)$

$$\Rightarrow I_2(f) = ef_0 + 2ef_0 \cdot \sum_{h=1}^{\infty} \delta(f - h \cdot [\, f_0 + \Delta f \,])$$



Important result for 1st step:

- The **entire** information is available around all harmonics
- The distance in frequency domain scales with $h \cdot \Delta f$

Longitudinal Schottky Analysis: 2^{nd} Step

Averaging over many particles for a coasting beam:

Assuming ***N*** **randomly** distributed particles characterized by phase θ_1, θ_2, θ_3 ... θ_N
with **same** revolution time $t_0 = 1/f_0 \Leftrightarrow$ same revolution frequency f_0

The total beam current is: $$I(t) = ef_0 \sum_{n=1}^{N} \cos\theta_n + 2ef_0 \sum_{n=1}^{N} \sum_{h=1}^{\infty} \cos(2\pi f_0 ht + h\theta_n)$$

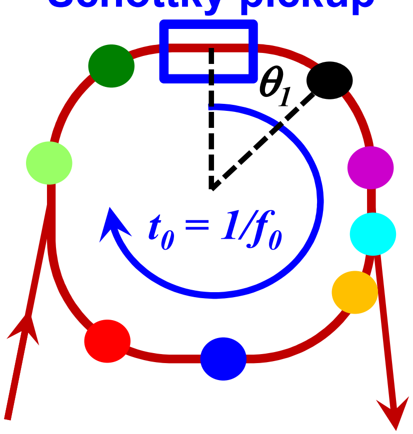
For observations much longer than one turn: average current $\langle I \rangle_h = 0$ for **each** harm. $h \neq 1$
but In a band around **each** harmonics ***h*** the *rms* current $I_{rms}(h) = \sqrt{\langle I^2 \rangle_h}$ remains:

$$\langle I^2 \rangle_h = \left(2ef_0 \sum_{n=1}^{N} \cos(h\theta_n) \right)^2 = (2ef_0)^2 \cdot (\cos h\theta_1 + \cos h\theta_2 + \ldots \cos h\theta_N)^2$$

$$\equiv (2ef_0)^2 \cdot N \langle \cos^2 h\theta_i \rangle = (2ef_0)^2 \cdot N \cdot \frac{1}{2} = 2\,e^2 f_0^2 \cdot N \quad \text{due to the random phases } \theta_n$$

The power at each harm. *h* is: $$P_h = Z_t \langle I^2 \rangle_h = 2\,Z_t\, e^2 f_0^2 \cdot N$$

measured with a pickup of transfer impedance Z_t



Important result for 2^{nd} step:

➢ The **integrated** power in each band is constant and $\propto$ ***N***

Remark: This random distribution is the connection
to shot noise as described by W. Schottky in 1918

Longitudinal Schottky Analysis: 3rd Step

Introducing a frequencies distribution for many particles:

The dependence of the distribution per band is: $\frac{dP_h}{df} = Z_t \cdot \frac{d}{df}\left\langle I^2 \right\rangle_h = 2\, Z_t e^2 f_0^2 N \cdot \frac{dN}{df}$

Inserting the acc. quantity $\frac{df}{f_0} = h\,\eta \cdot \frac{dp}{p_0}$ leads to $\frac{dP_h}{df} = 2\, Z_t e^2 f_0 p_0 \cdot N \cdot \frac{1}{h\eta} \cdot \frac{dN}{dp}$

Important results from 1st to 3rd step:

- The power spectral density $\frac{dP_h}{df}$ in **each** band

  reflects the particle's **momentum distribution**: $\frac{dP_h}{df} \propto \frac{dN}{dp}$
- The maxima of each band scales $\left.\frac{dP_h}{df}\right]_{max} \propto \frac{1}{h}$

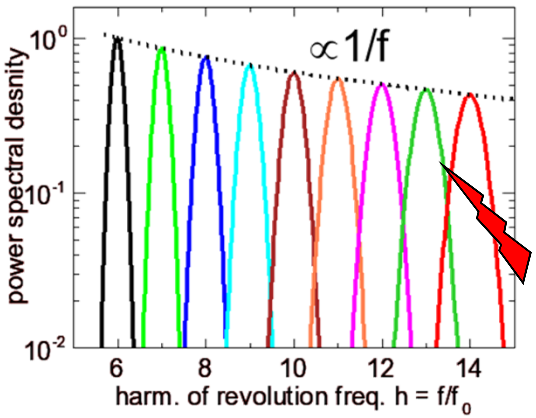
  due to $P_h \propto \langle I^2 \rangle_h = const.$ for each band as given in 2nd step

  Measurement: Low f preferred for good signal-to-noise ratio
- The width increase for each band: $\frac{dP_h}{df} \propto h$

  Measurement: High f preferred for good frequency resolution
- The power scales only as $\frac{dP_h}{df} \propto N$ due to random phases of particles

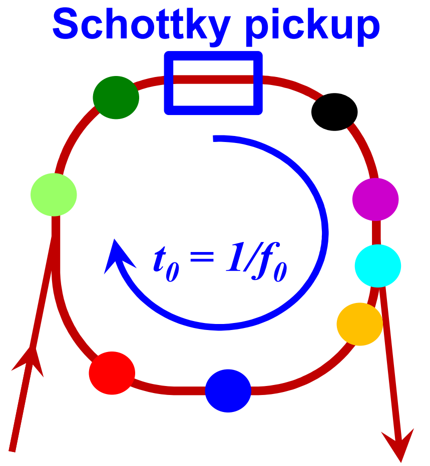
  i.e. <u>in</u>coherent single particles' contribution
- For ions A^{q+} the power scales $\frac{dP_h}{df} \propto q^2 \Rightarrow$ larger signals for ions

Remark: The 'power spectral density' $\frac{dP_h}{df}$ is called only 'power' P_h below



Example:
Gaussian
$\Delta p/p$ = 2%
η = 1

overlap

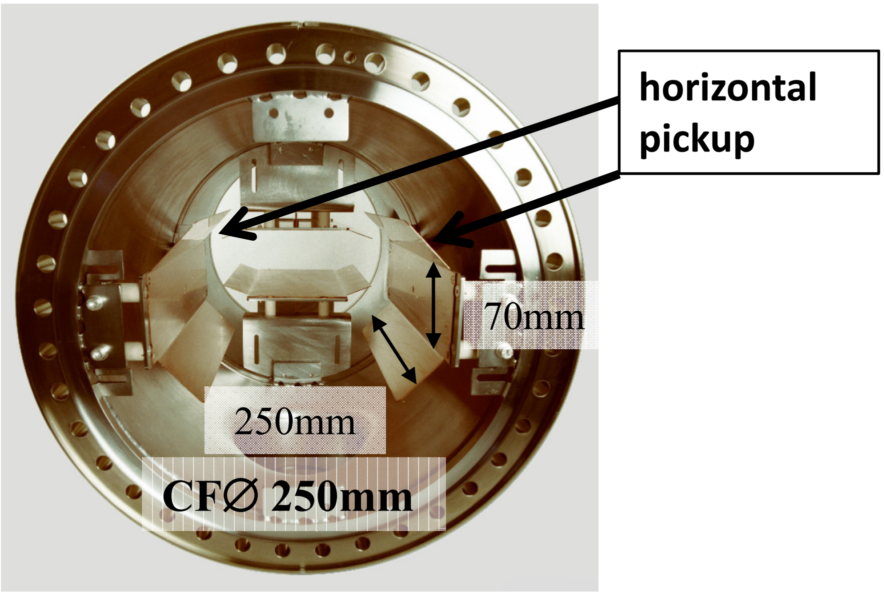


Pickup for Schottky Signals: Capacitive Pickup

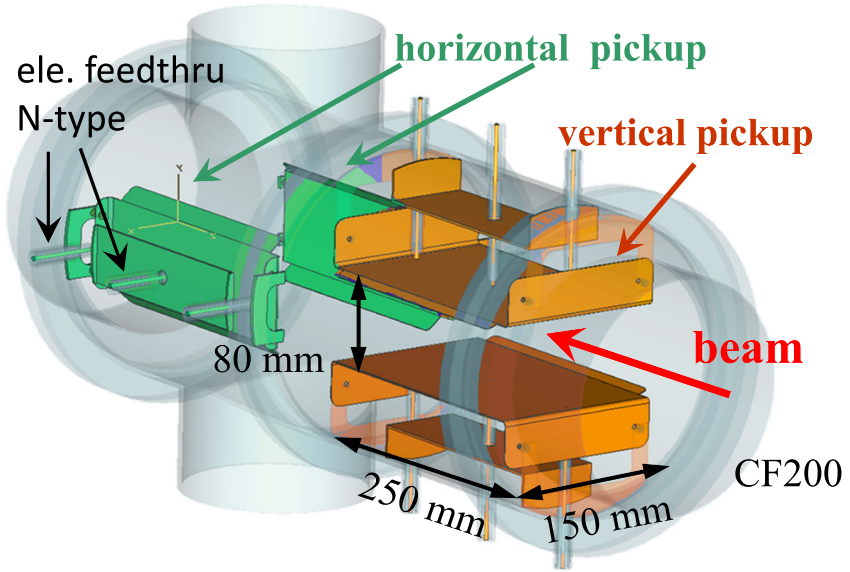
A Schottky pickup are e comparable to a capacitive BPM:

- Typ. 20 to 50 cm insertion length
- high position sensitivity for transverse Schottky
- Allows for broadband processing
- Linearity for position **not** important

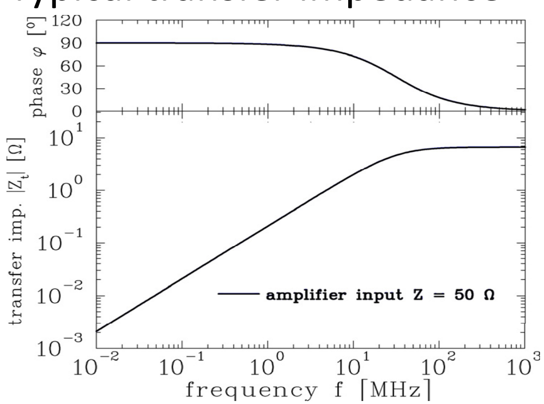
Example: Schottky pickup at GSI synhrotron



Example: Schottky for HIT, Heidelberg
operated as capacitive (mostly) or strip-line



Typical transfer impedance



Transfer impedance:

Coupling to beam $U_{signal} = Z_t \cdot I_{beam}$

Typically Z_t = 1 ...10 Ω, C = 30 ... 100 pF ⇒ $f_{cut} \approx$ 30 MHz

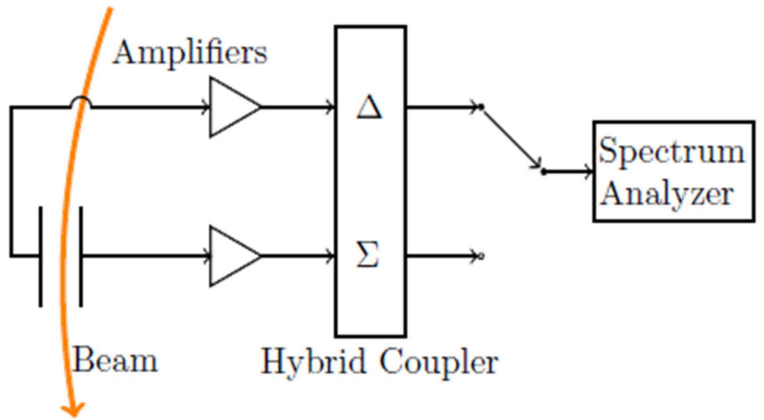
⇒ operation rang f = 30 ... 200 MHz

i.e. above f_{cut} but below signal distortion ≈ 200 MHz

Electronics for a typical broadband Pickup

Analog signal processing chain:

- Sensitive broadband amplifier
- Hybrid for sum or difference
- Evaluation by spectrum analyzer



Enhancement by external resonant circuit :

- Cable as $\lambda/2$ resonator
- Tunable by capacitive diode
- Typical quality factor $Q \approx 3 \ldots 10$

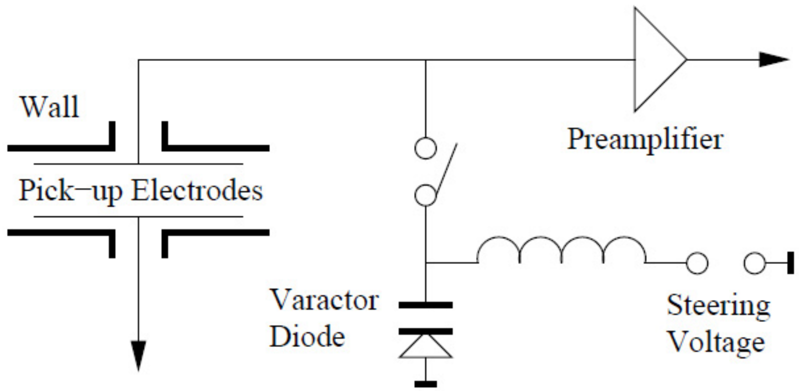
$\Rightarrow$ resonance must be broader than the beam's frequency spread

Challenge for a good design:

- Low noise amplifier required
- For multi stage amplifier chain: prevent for signal saturation

Choice of frequency range:

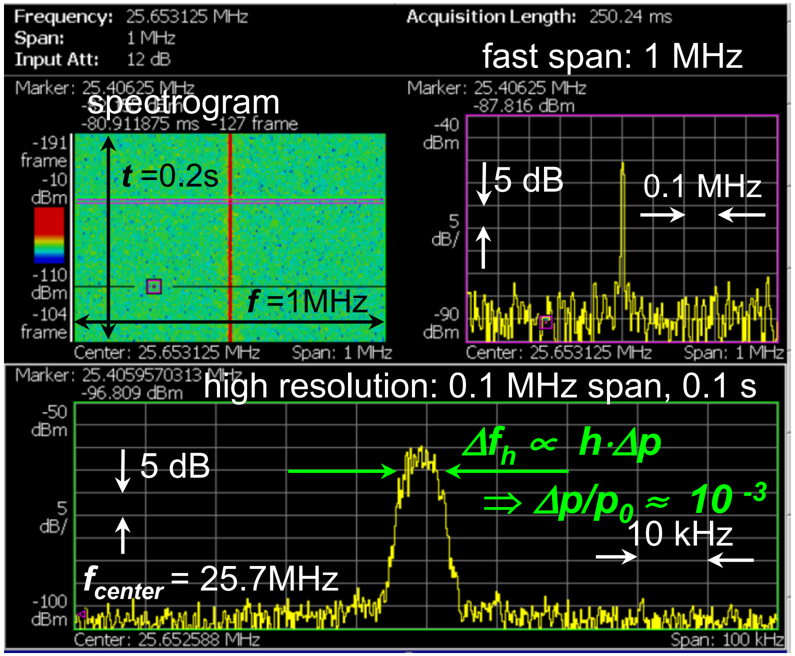
- At maximal pickup transfer impedance
- Lower $f \Rightarrow$ higher signal
- Higher $f \Rightarrow$ better resolution
- Prevent for overlapping of bands



Example of longitudinal Schottky Analysis for a coasting Beam

Example: **Coasting** beam at GSI synchrotron at injection

E_{kin} = 11.4 MeV/u ⇔ β = 15.5 %, harmonic number h = 119

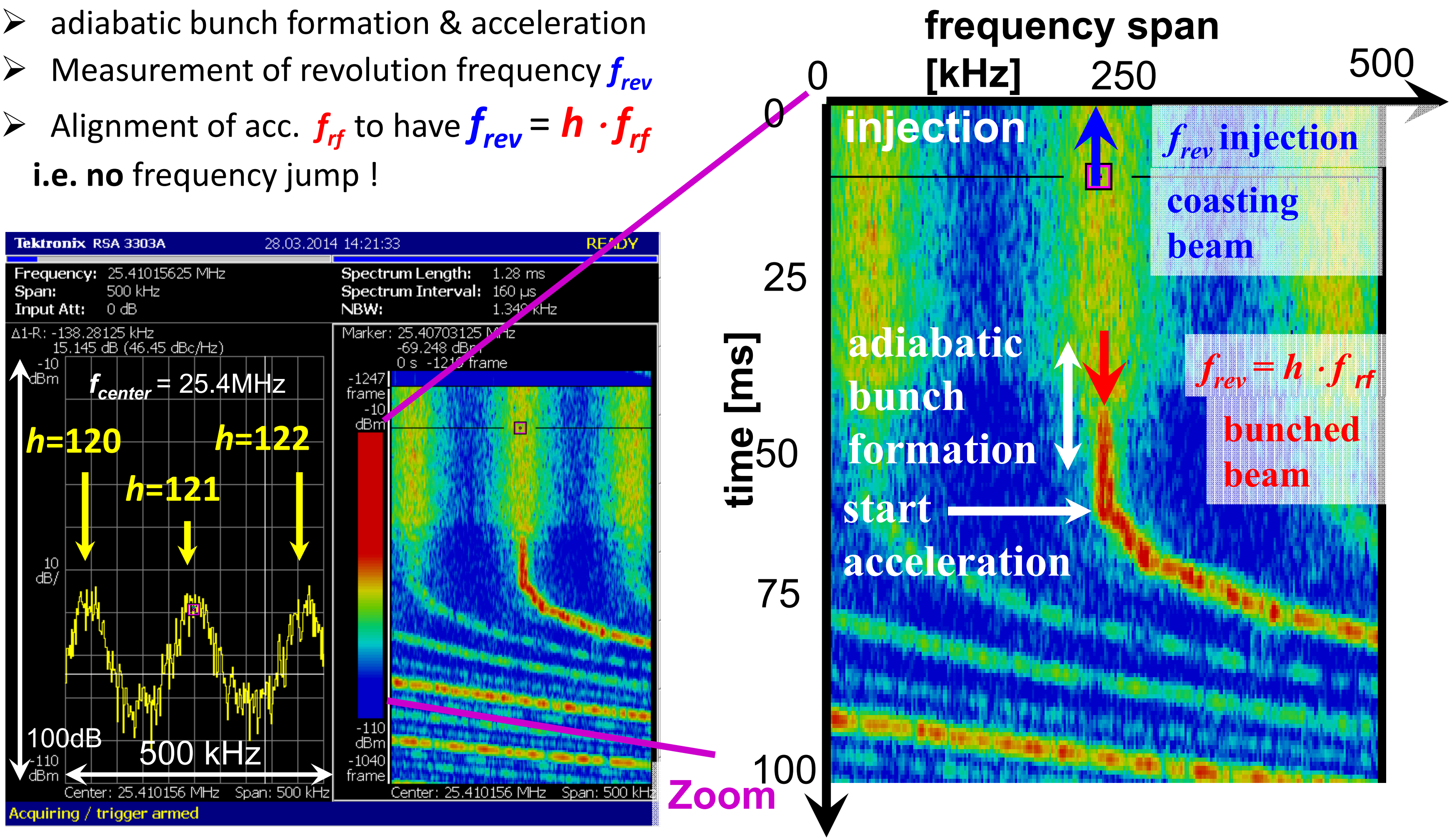


Application for coasting beam diagnostics:

- Injection: momentum spread via $\frac{\Delta p}{p_0} = -\frac{1}{\eta} \cdot \frac{\Delta f_h}{h\, f_0}$ as influenced by re-buncher at LINAC
- Injection: matching i.e. f_{center} stable at begin of ramp
- Dynamics during beam manipulation e.g. cooling
- Relative current measurement for low current below the dc-transformer threshold of ≈ 1µA

Example for longitudinal Schottky spectrum to check proper acceleration frequency:

- Injection energy given by LINAC settings, here E_{kin} =11.4 MeV/u ⇔ β = 15.5 %, $\Delta p/p \approx 10^{-3}$ (1σ)
- multi-turn injection & de-bunching within ≈ ms
- adiabatic bunch formation & acceleration
- Measurement of revolution frequency f_{rev}
- Alignment of acc. f_{rf} to have $f_{rev} = h \cdot f_{rf}$
  i.e. no frequency jump !



Longitudinal Schottky for Momentum Spread $\Delta p/p_0$ Analysis

Momentum spread $\Delta p/p_0$ measurement after multi-turn injection & de-bunching of t < 1ms duration to stay within momentum acceptance during acceleration

Method: Variation of buncher voltage

i.e. sheering in phase space

$\rightarrow$ minimizing of momentum spread $\Delta p/p_0$

$\rightarrow$ $\Delta p/p_0$ preserves after de-bunching

Example: 10^{10} U^{28+} at 11.4 MeV/u

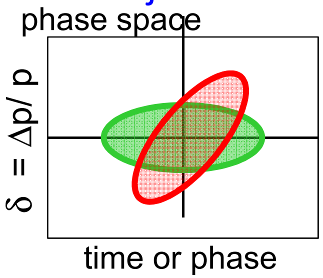
injection plateau 150 ms, η = 0.94

Longitudinal Schottky at harmonics h = 117

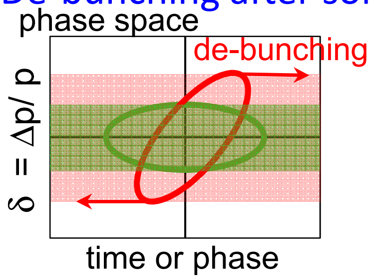
Momentum spread variation:

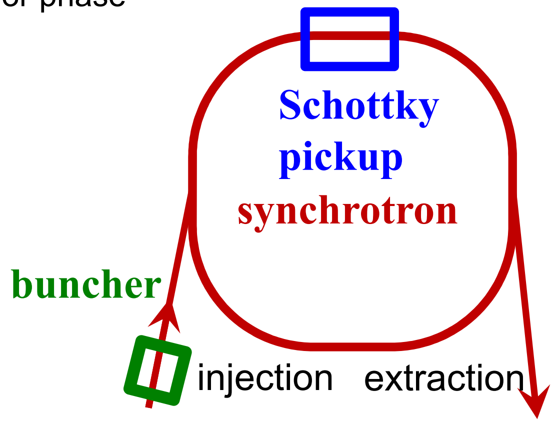
$\Delta p/p \approx (0.6 ... 2.5) \cdot 10^{-3}$ (1σ)

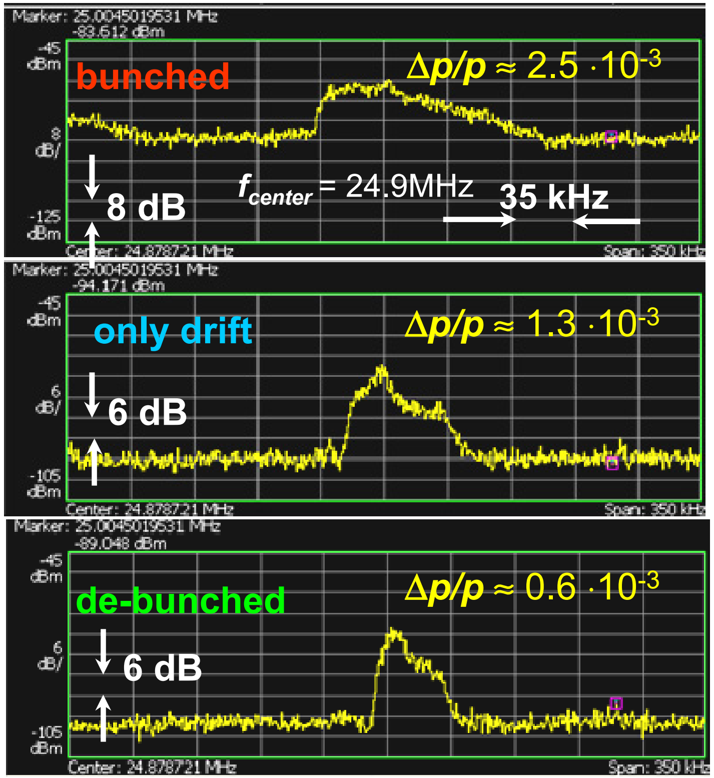
At injection:



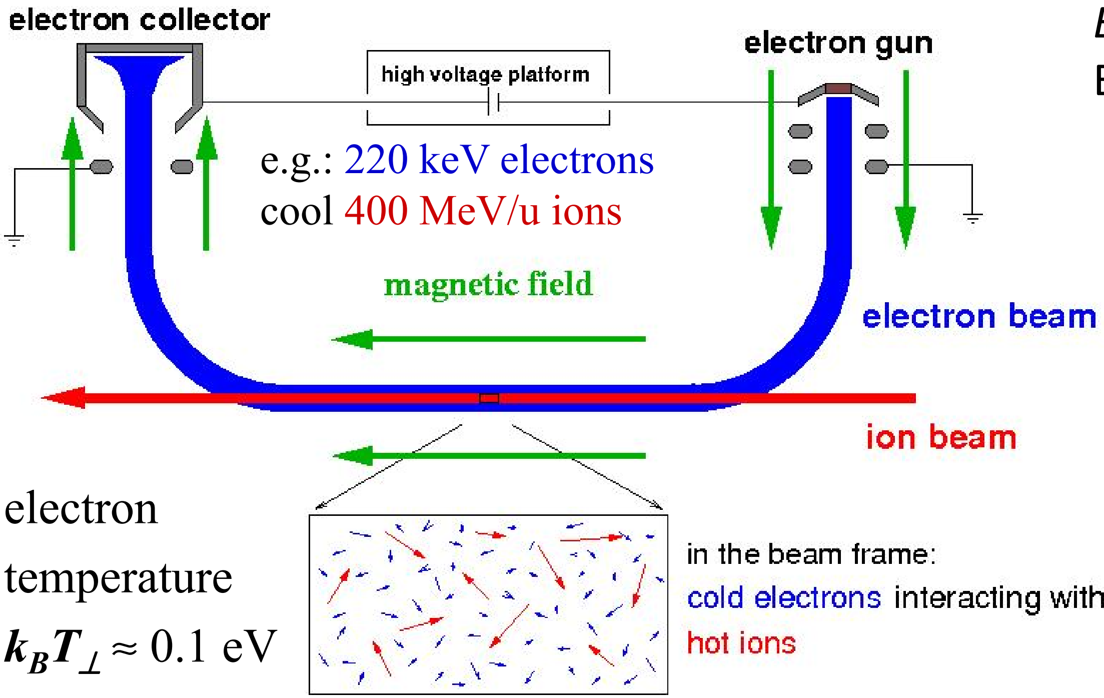
De-bunching after some ms:







Electron cooling: Superposition ion and cold electron beams with the same



electron temperature

$k_B T_\perp \approx 0.1$ eV

$k_B T_\parallel \approx 0.1 - 1$ meV

Example:

Electron cooler at GSI, U_{max} = 300 kV

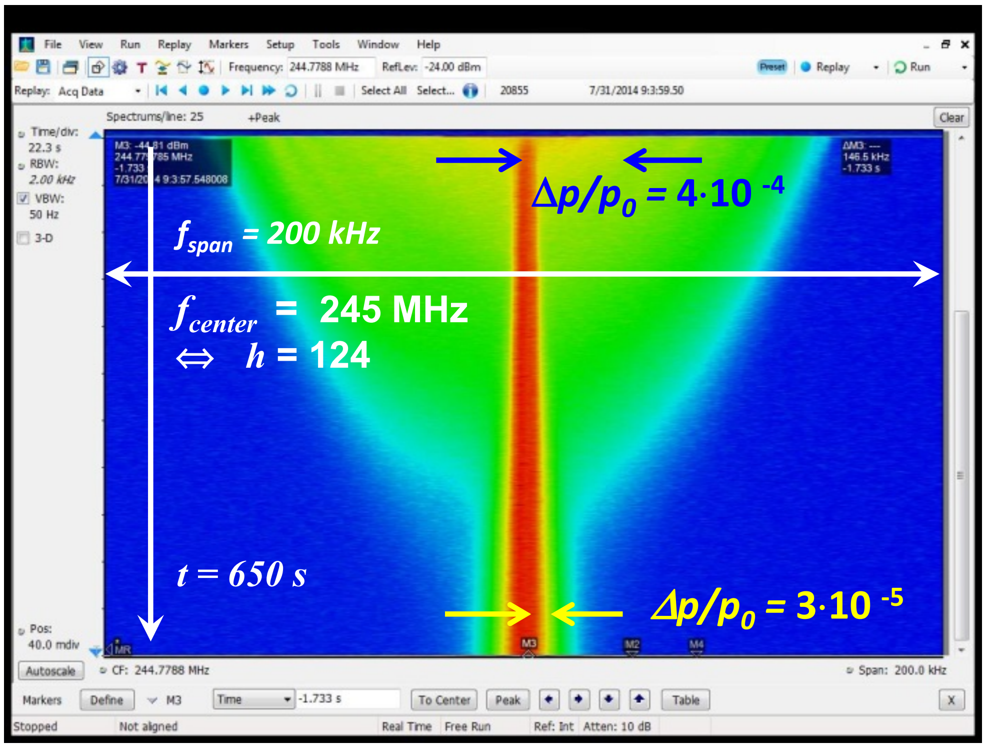
Physics:

- Momentum transfer by Coulomb collisions
- Cooling force results from energy loss in the cold, co-moving electron beam

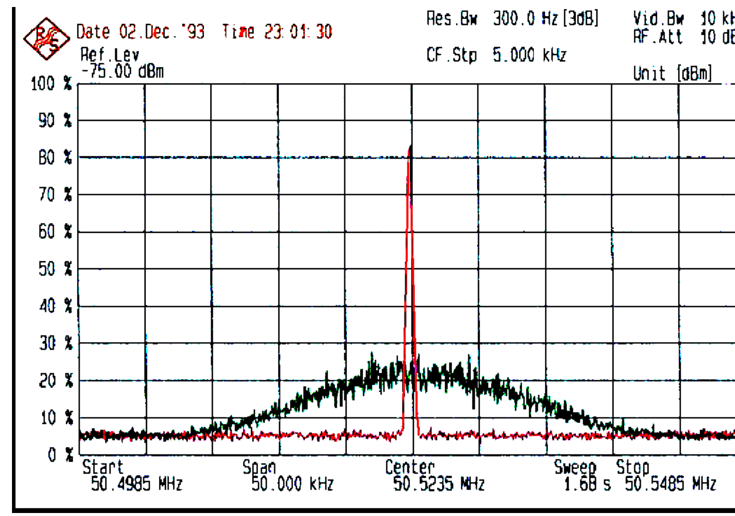
Cooling time: 0.1 s for low energy highly charged ions, 1000 s for high energy protons

Electron Cooling: Monitoring of Cooling Process

Example: Observation of cooling process at GSI storage ring
Ion beam: 10^8 protons at 400 MeV
Electron beam $\boldsymbol{I_{ele}}$ = 250 mA
Momentum spread (1σ): $\Delta p/p = 4 \cdot 10^{-4} \rightarrow 3 \cdot 10^{-5}$ within 650 s



Beam: 10^8 Ar^{18+} at 400 MeV
Electron beam $\boldsymbol{I_{el}}$ = 250 mA
$\Delta p/p_0 = 4 \cdot 10^{-4} \rightarrow 1 \cdot 10^{-5}$



Application:
- Alignment of cooler parameter and electron-ion overlap
- Determination of cooling forces and intra-beam scattering acting as a counteract

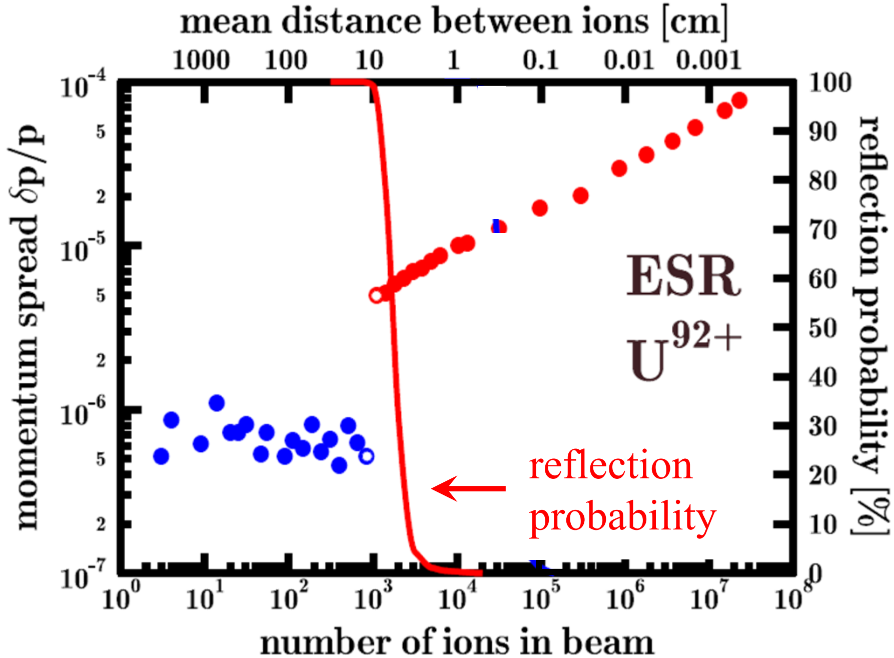
J. Roßbach et al., Cool 2015, p. 136 (2015)

Electron Cooling: Linear Chain by Minimal Momentum Spread

Example: Observation of longitudinal momentum at GSI storage ring

- Ion beam: U^{92+} at 360 MeV/u applied to electron cooling with I_{ele} = 250 mA
- Variation of stored ions by lifetime of $\tau \approx 10$ min i.e. total store of several hours
- Longitudinal Schottky spectrum with 30 s integration every 10 min

$\Rightarrow$ Momentum spread (1σ): $\Delta p/p = 10^{-4} \rightarrow$ below 10^{-6} when reaching an intensity threshold



Interpretation:

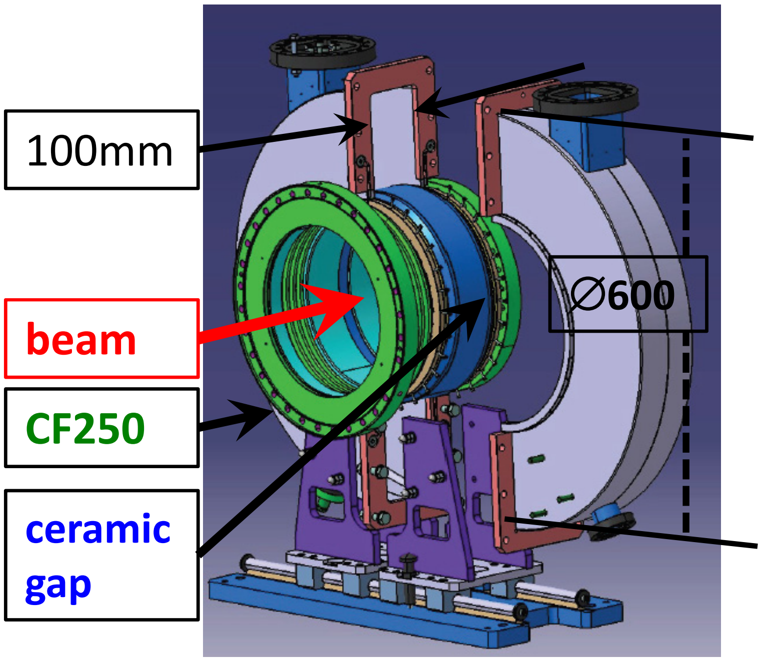
- Intra beam scattering as a heating mechanism is suppressed below the threshold
- Ions can't overtake each other, but building a 'linear chain' (transverse size σ_x < 30 µm)
- Momentum spread is basically given by stability of power suppliers

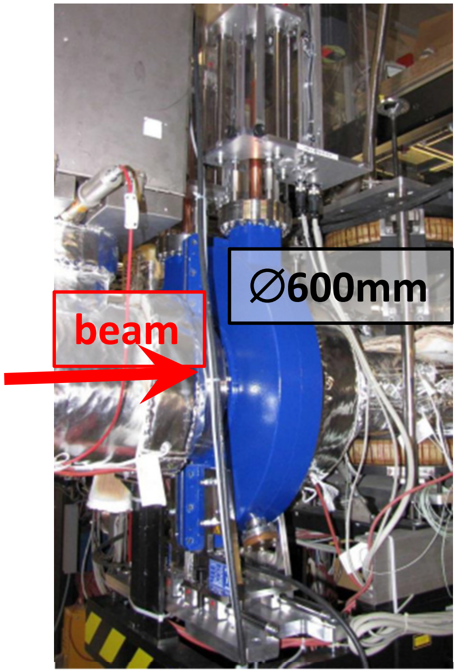
M. Steck et al., Phys. Rev. Lett 77, 3803 (1996), R.W. Hasse, EPAC 00, p. 1241 (2000)

Pillbox Cavity for vey low Detection Threshold

Enhancement of signal strength by a cavity

Example: Pillbox cavity at GSI and Lanzhou storage ring for with variable frequency

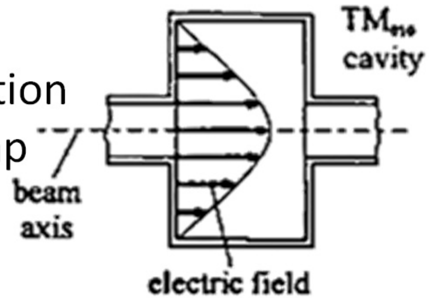




Outer $\varnothing_{out}$	600 mm
Beam pipe $\varnothing_{in}$	250 mm
Mode (monopole)	TM_{010}
Res. freq. f_{res} Variable by plunger	≈ 244 MHz ± 2 MHz
Quality factor Q_0	≈ 1200
Loaded Q_l	≈ 500
R/Q_0	≈ 30 Ω
Coupling	Inductive loop

Advantage:

- Sensitive down to single ion observation
- Part of cavity in air due to ceramic gap
- Can be sort-circuited to prevent for wake-field excitation

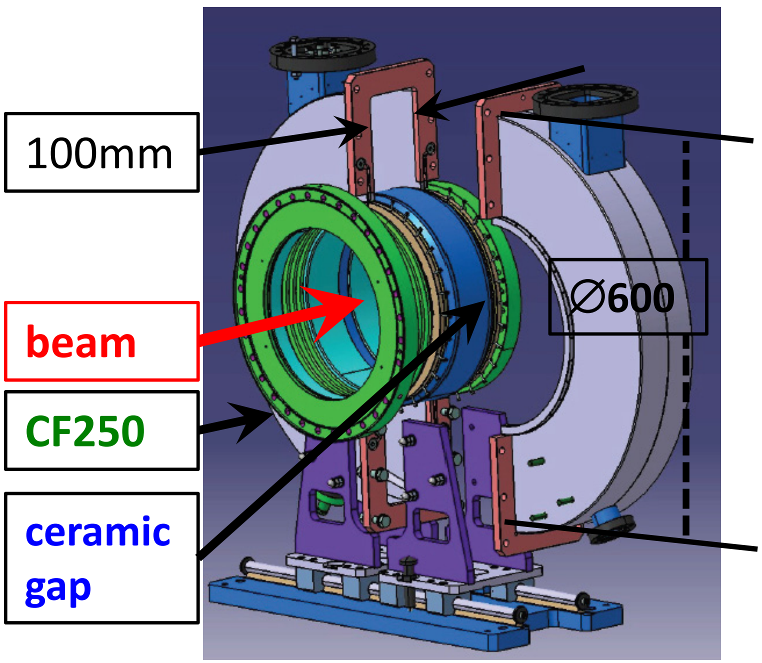


F. Nolden et al., NIM A 659, p.69 (2011), F. Nolden et al., DIPAC'11, p.107 (2011), F. Suzaki et al., HIAT'15, p.98 (2015)
For RHIC design: W. Barry et al., EPAC'98, p. 1514 (1998), K.A. Brown et al., Phys. Rev. AB, 12, 012801 (2009)

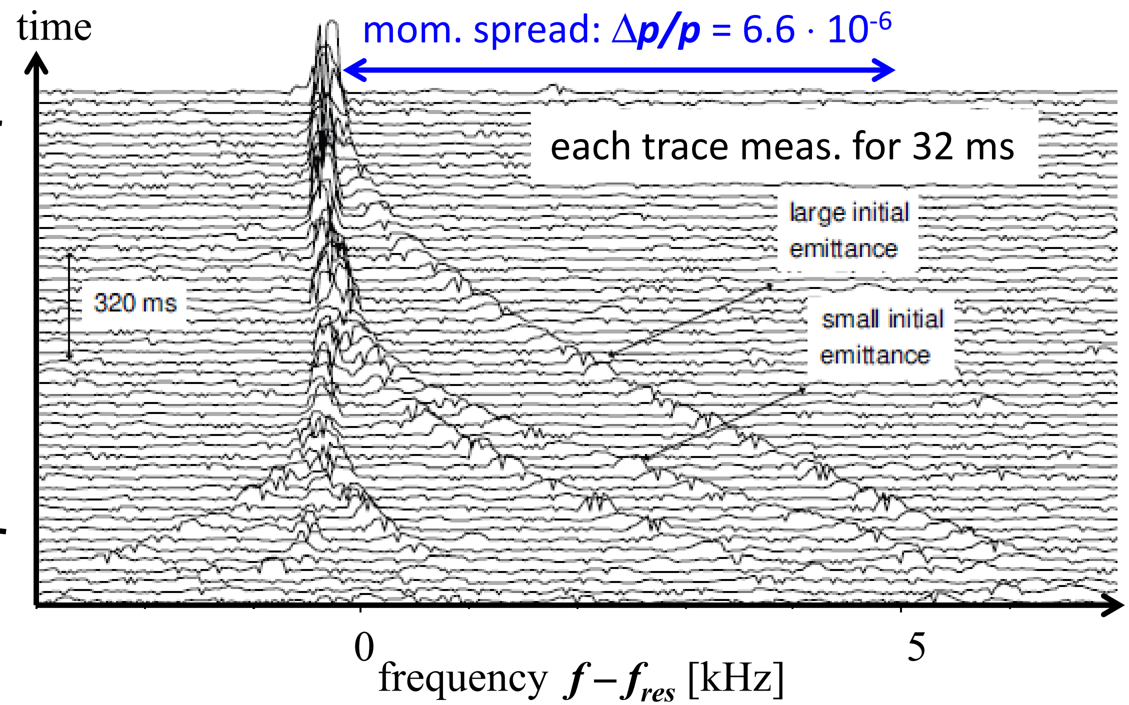
Pillbox Cavity for single Ion Detection

Observation of *single* ions is possible:

Example: Storage of **six** $^{142}Pm^{59+}$ at 400 MeV/u during electron cooling



f_{res} = 244.965 MHz



Application:

- Single ion observation for basic accelerator research
- Observation of radio-active nuclei for life time and mass measurements

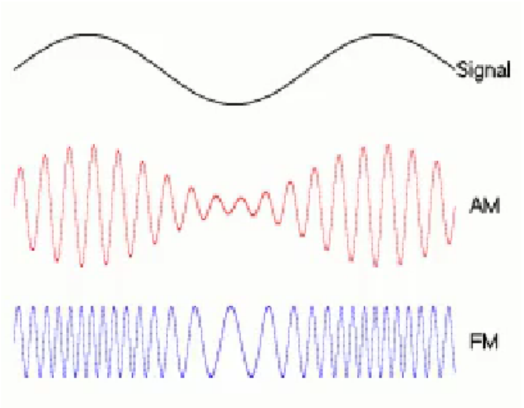
F. Nolden et al., NIM A 659, p.69 (2011), F. Nolden et al., DIPAC'11, p.107 (2011), F. Suzaki et al., HIAT'15, p.98 (2015)

Outline of the tutorial:

- Introduction to noise and fluctuations relevant for Schottky analysis
- Main part: Schottky signal generation and examples for the following case:
  - Longitudinal for coasting beams
  - **Transverse for coasting beams**
  - Longitudinal for bunched beams
  - Transverse for bunched beams
- Some further examples for exotic beam parameters
- Conclusion

Composition of two waves:

- **Carrier:** For synchrotron→ revolution freq. $\boldsymbol{f_0 = 1/t_0}$
  $U_c(t) = \hat{U}_C \cdot \cos(2\pi f_0 t)$
- **Signal:** For synchrotron→ betatron frequency $\boldsymbol{f_\beta = q \cdot f_0}$
  $\boldsymbol{q} < 1$ non-integer part of tune $\boldsymbol{Q = n + q}$
  $\boldsymbol{m_\beta(t)} = m_\beta \cdot \cos(2\pi q f_0 t)$



Source: wikipedia

Principle of Amplitude Modulation

Composition of two waves:

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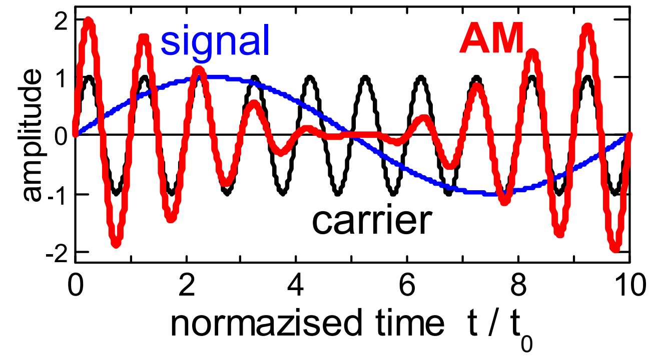
Example: $q = 0.1$, $m_\beta = 1$



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  $\boldsymbol{q < 1}$ non-integer part of tune $\boldsymbol{Q = n + q}$

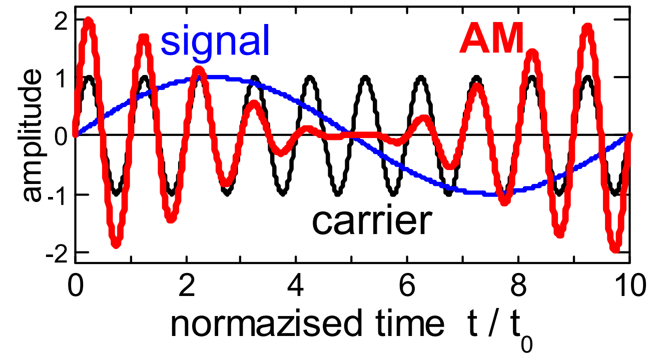
  $\boldsymbol{m_\beta(t)} = m_\beta \cdot \cos(2\pi q f_0 t)$

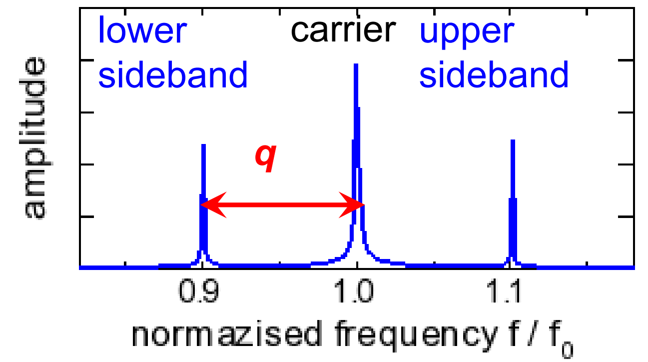
Multiplication of both for modulation $m_\beta = \dfrac{\hat{U}_\beta}{\hat{U}_C} = 1 \Rightarrow$

$$\boldsymbol{U_{tot}(t)} = \hat{U}_C \cdot m_\beta \cdot \cos(2\pi f_0 t) \cdot \cos(2\pi q f_0 t)$$
$$= {}^1/_2\, \hat{U}_C \cdot m_\beta \cdot [\, \cos(2\pi[1-q]f_0 t) + \cos(2\pi[1+q]f_0 t)\,]$$

Using: $\cos(x) \cdot \cos(y) = {}^1/_2\, [\cos(x-y) + \cos(x+y)]$

Example: $\boldsymbol{q} = 0.1$, $\boldsymbol{m_\beta} = 1$





Remark:

100 % modulation means the beam well centered in pickup

Transverse Spectrum for a coasting Beam: Single Particle

Observation of the difference signal of two pickup electrodes:

Betatron motion by a single particle 1 at Schottky pickup:

Displacement: $\boldsymbol{x_1(t)} = A_1 \cdot \cos(2\pi q f_0 t)$

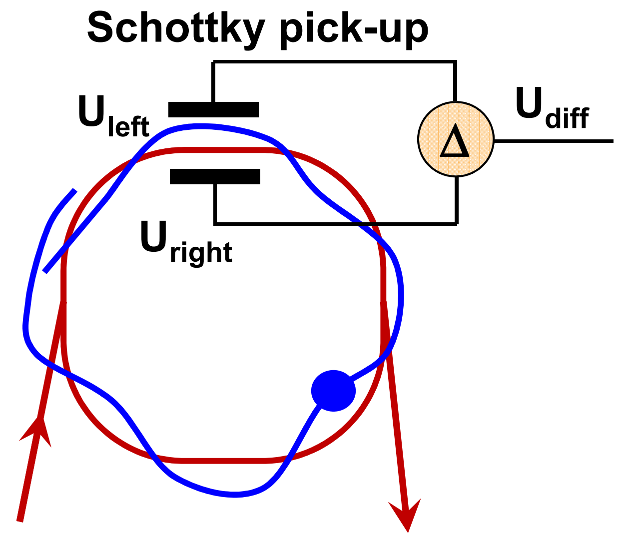
***A*₁**: single particle trans. amplitude

q: non-integer part of tune

Dipole moment: $\boldsymbol{d_1(t)} = x_1(t) \cdot I(t)$

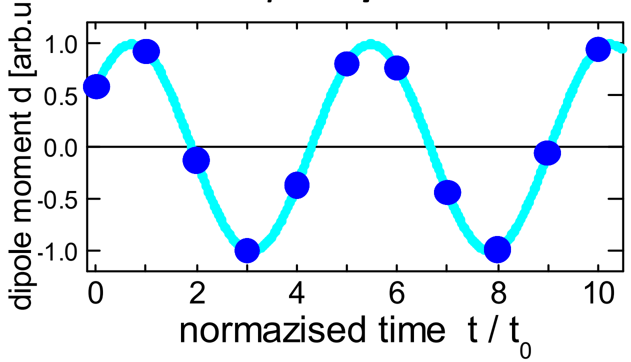
transverse part equals 'signal'

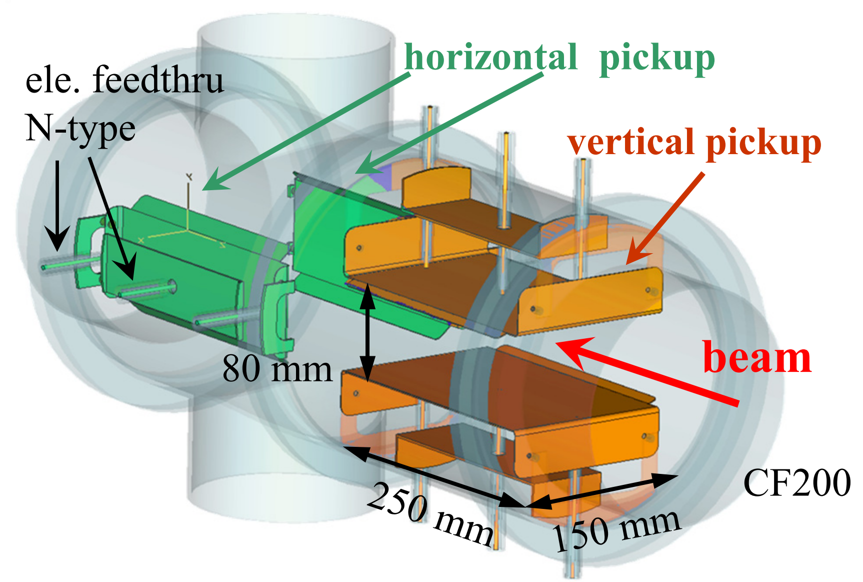
longitudinal part equals 'carrier'



Pickup voltage: $\boldsymbol{U_1(t)} = Z_\perp \cdot d_1(t)$

Example: ***q*** = 0.21





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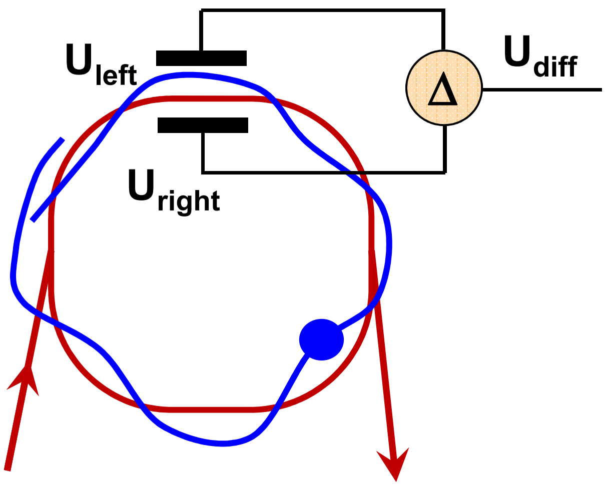
Displacement: $\boldsymbol{x_1(t)} = A_1 \cdot \cos(2\pi q f_0 t)$

A_1: single particle trans. amplitude

q: non-integer part of tune



Dipole moment: $\boldsymbol{d_1(t)} = x_1(t) \cdot I(t)$

transverse part equals 'signal'

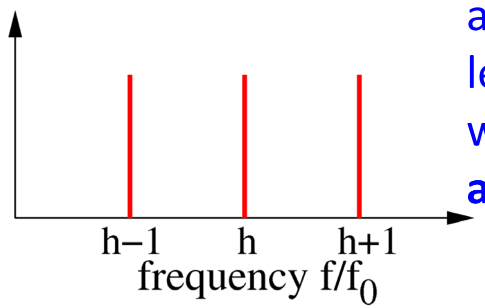
longitudinal part equals 'carrier'

Inserting longitudinal Fourier series: $\boldsymbol{d_1(f)} =$

$$e f_0 \cdot A_1 + 2 e f_0 A_1 \cdot \sum_{h=1}^{\infty} \cos(2\pi q f_0 t) \cdot \cos(2\pi h f_0 t)$$

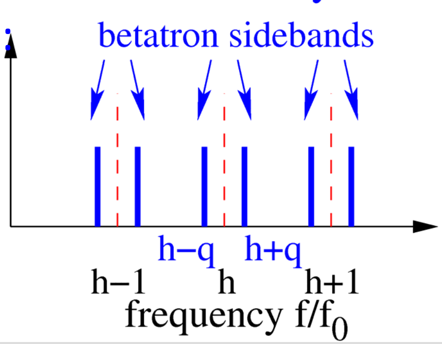
$$= e f_0 \cdot A_1 + e f_0 A_1 \cdot \sum_{h=1}^{\infty} \cos(2\pi [h-q] f_0 t) \cdot \cos(2\pi [h+q] f_0 t)$$

longitudinal Schottky



amplitude modulation:
left & right sideband
with distance ***q***
at each harmonics

transverse Schottky



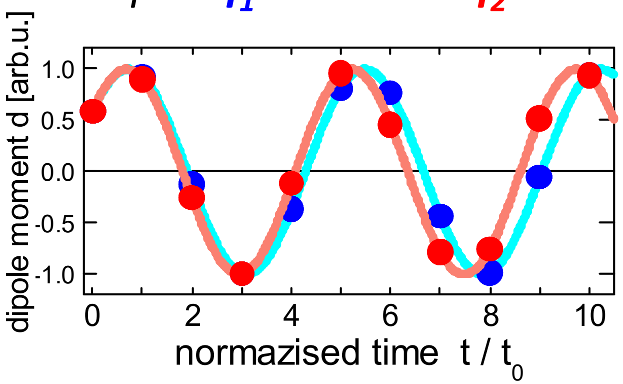
Observation of the difference signal of two pickup electrodes:

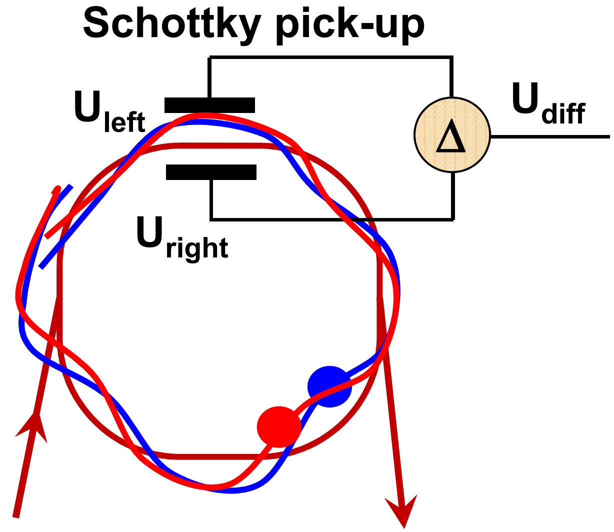
Betatron motion by two particles at pickup:

Displacements: $x_1(t) = A_1 \cdot \cos(2\pi q_1 f_0 t)$

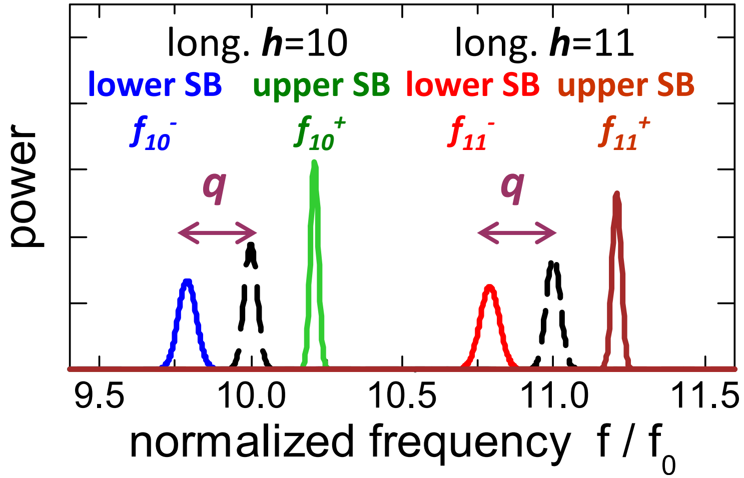
: $x_2(t) = A_2 \cdot \cos(2\pi q_2 f_0 t)$

Example: $q_1 = 0.21$ & : $q_2 = 0.26$





Example: $Q = 4.21$, $\Delta p/p_0 = 2 \cdot 10^{-3}$, $\eta = 1$, $\xi = -1$



Transverse Schottky band for a distribution:

- Amplitude modulation of longitudinal signal (i.e. 'spread of carrier')
- Two sideband centered at $f_h^{\pm} = (h \pm q) \cdot f_0$
  $\Rightarrow$ tune measurement
- The width is unequal for both sidebands (see below)
- The integrated power is constant (see below)

Example of a transverse Schottky spectrum:

- Wide scan with lower and upper sideband
- Tune from central position of both sidebands

$$q = h \cdot \frac{f_h^+ - f_h^-}{f_h^+ + f_h^-}$$

- Sidebands have different shape
- Tune measurement without beam influence

$\Rightarrow$ usage during regular operation

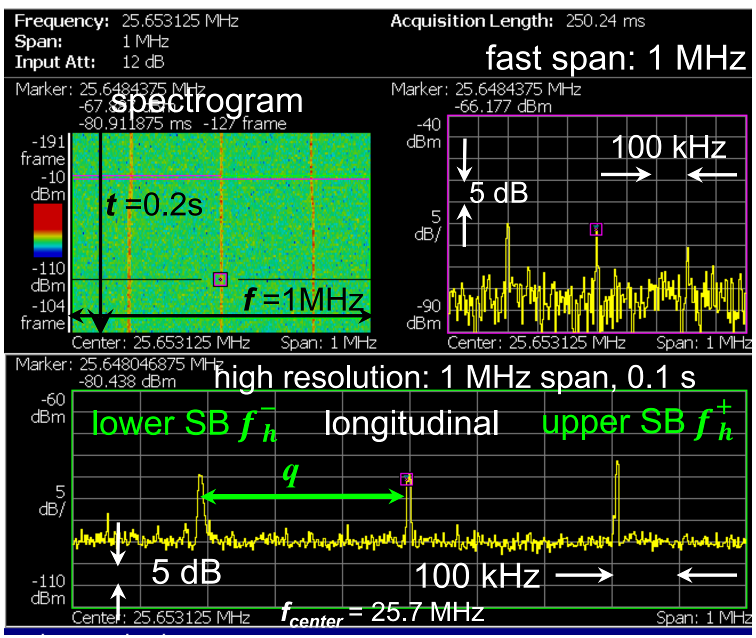
Example: Horizontal tune Q_h = 4.161 $\rightarrow$ 4.305

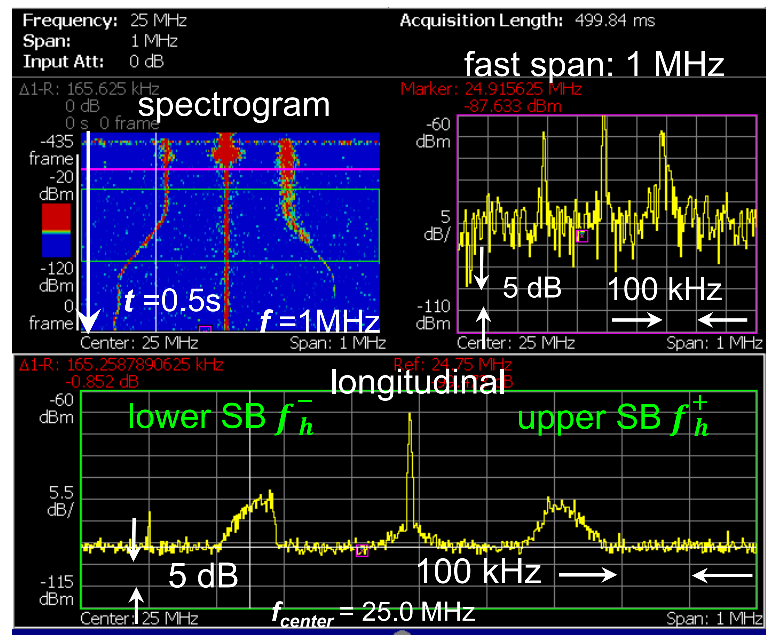
within 0.3 s for preparation of slow extraction

Beam Kr^{33+} at 700 MeV/u,

f_0 = 1.136 MHz $\Leftrightarrow$ h = 22

Characteristic movements of sidebands visible





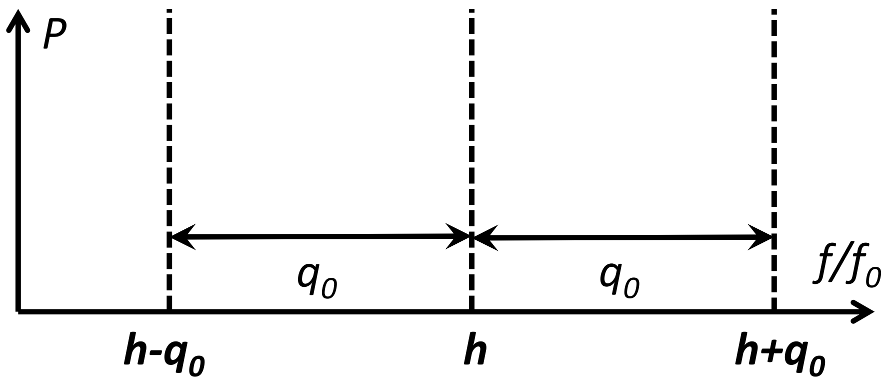
Sideband Width for a coasting Beam

Calculation of the sideband width:

The sidebands at $f_h^{\pm} = (h \pm q) \cdot f_0$ comprises of

- Longitudinal spread expressed via momentum $\frac{\Delta f}{f_0} = \eta \frac{\Delta p}{p_0}$ (η: freq. dispersion)
- Transverse tune spread $\Delta Q = \Delta q$ for low current dominated by the chromatic effect $\frac{\Delta Q}{Q_0} = \xi \frac{\Delta p}{p_0}$

Depictive Example: $\eta = 1$, $\xi = -1$



Reference particle: tune q_0

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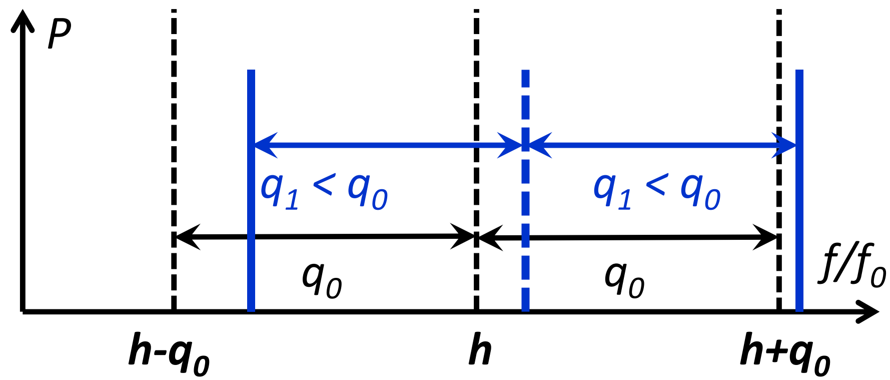

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Particle 1 with $p_1 > p_0 \Rightarrow q_1 = q_0 - |\xi| \cdot \Delta p_1 / p_0 < q_0$

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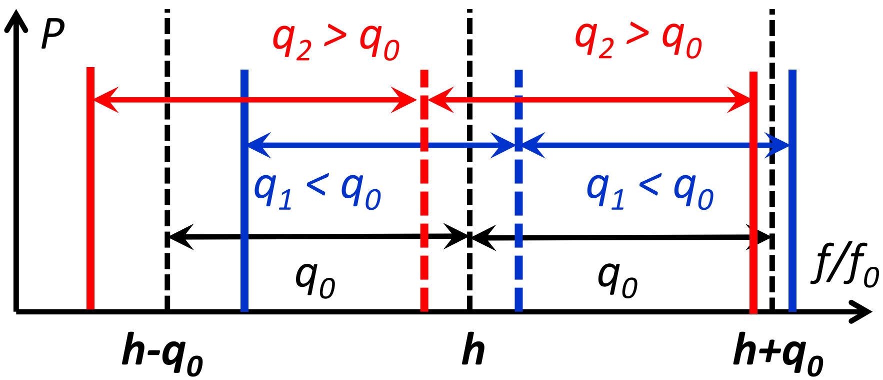

Reference particle: tune q_0

Particle 1 with $p_1 > p_0 \Rightarrow q_1 = q_0 - |\xi| \cdot \Delta p_1 / p_0 < q_0$

Particle 2 with $p_2 < p_0 : q_2 = q_0 + |\xi| \cdot \Delta p_2 / p_0 < q_0$

Calculation of the sideband width:

The sidebands at $f_h^{\pm} = (h \pm q) \cdot f_0$ comprises of

- Longitudinal spread expressed via momentum $\frac{\Delta f}{f_0} = \eta \frac{\Delta p}{p_0}$ (η: freq. dispersion)
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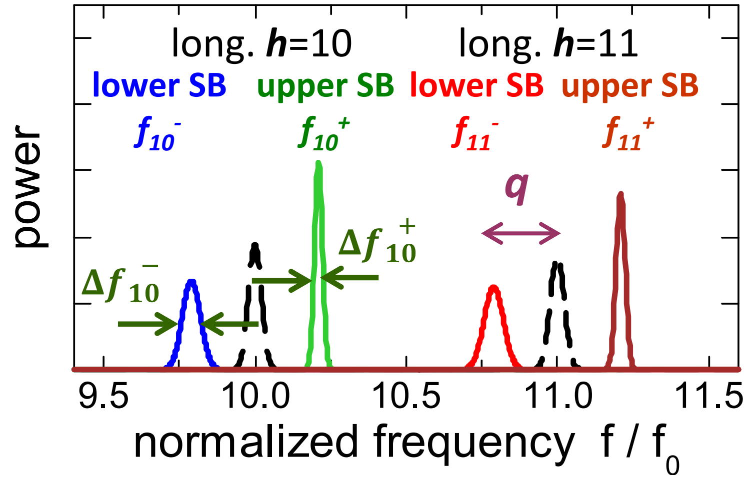


Using $f_h^{\pm} = (h \pm q) \cdot f_0$

$\Rightarrow$ lower sideband : $\Delta f_h^- = (h - q) \cdot \Delta f_h - \Delta q \cdot f_0 = \eta \frac{\Delta p}{p_0} \cdot f_0 \left(h - q - \frac{\xi}{\eta} Q_0 \right)$

$\Rightarrow$ upper sideband: $\Delta f_h^+ = (h + q) \cdot \Delta f_h + \Delta q \cdot f_0 = \underbrace{\eta \frac{\Delta p}{p_0} \cdot f_0 \left(h\right.}_{\text{long. part}} + \underbrace{q}_{\text{trans.}} + \underbrace{\left.\frac{\xi}{\eta} Q_0 \right)}_{\text{coupling}}$

Results:

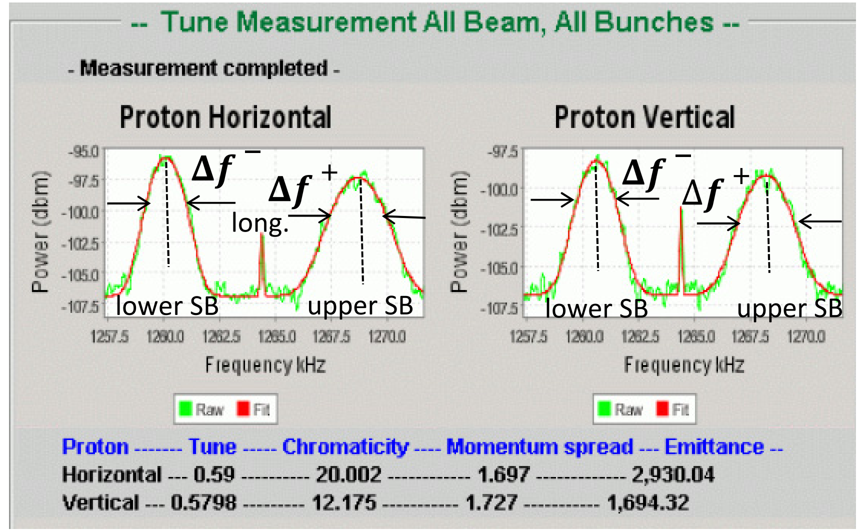
- Sidebands have different width in dependence of Q_0, η and ξ 'chromatic tune' i.e. 'longitudinal $\pm$ transverse $\pm$ coupling' $\Rightarrow$ 'chromatic tune'
- The width measurement can be used for chromaticity ξ measurements
- for high harmonics typically $h > 100$ the difference in width decreases

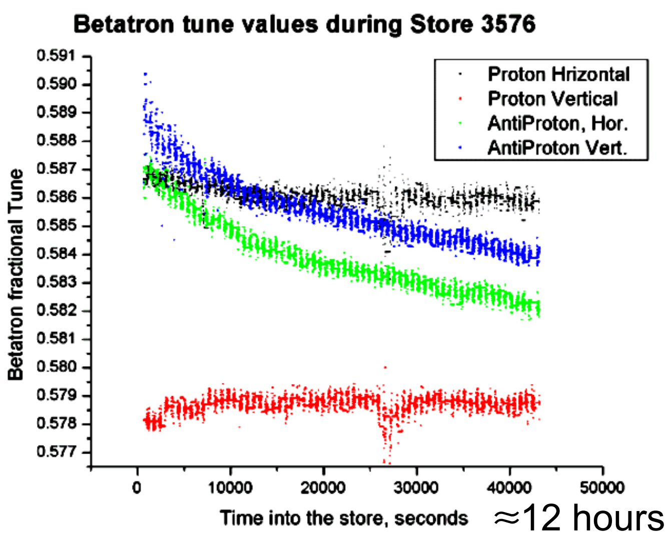
It is: $\Delta f_h^+ + \Delta f_h^- = 2\eta \frac{\Delta p}{p_0} \cdot h f_0 \Rightarrow \frac{\Delta p}{p_0}$ and $\Delta f_h^+ - \Delta f_h^- = 2\eta \frac{\Delta p}{p_0} \cdot h f_0 (q + \frac{\xi}{\eta} Q_0) \Rightarrow \xi$

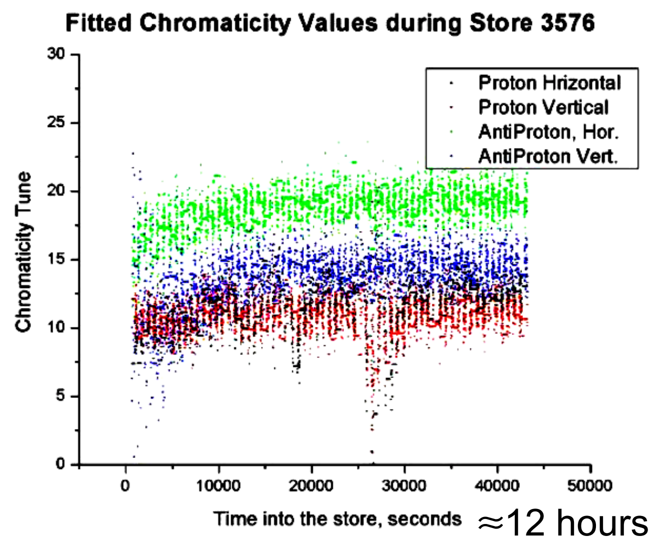
Example of Chromaticity Measurement at Tevatron

Permanent chromaticity monitoring at Tevatron:

- Sidebands around 1.7 GHz i.e. $\boldsymbol{h} \approx 36{,}000$
  with slotted waveguide, see below for CERN type
- Gated, down-mixing & filtered
  by analog electronics
- Gaussian fit of sidebands
  Center $\rightarrow$ tune $\boldsymbol{q}$
  Width $\rightarrow$ chromaticity ξ via Δf^{+}- Δf^{-}
  $\rightarrow$ momentum spread $\Delta p/p$ via Δf^{+}+ Δf^{-}







Remark: Spectrum measured with bunched beam and gated signal path, see below

A. Jansson et al., EPAC'04, p. 2777 (2004) & R. Pasquinelli, A. Jansson, Phys. Rev AB **14**, 072803 (2011)

Power per Band for a coasting Beam & transverse *rms* Emittance

Dipole moment for a harmonics ***h*** for a particle with betatron amplitude $\boldsymbol{A_n}$:

$$\boldsymbol{d_n(hf)} = 2ef_0A_n \cdot \cos(2\pi q f_0 t + \theta_n) \cdot \cos(2\pi h f_0 t + \varphi_n)$$

Averaging over betatron phase $\boldsymbol{\theta_n}$ and spatial distribution for the ***n = 1…N*** particles:

$$\Rightarrow \langle d^2 \rangle = e^2 {f_0}^2 \cdot N/2 \cdot \langle A^2 \rangle \cdot N/2$$

with $\langle A^2 \rangle \equiv {x_{rms}}^2 = \varepsilon_{rms}\beta$ square of average transverse amplitudes

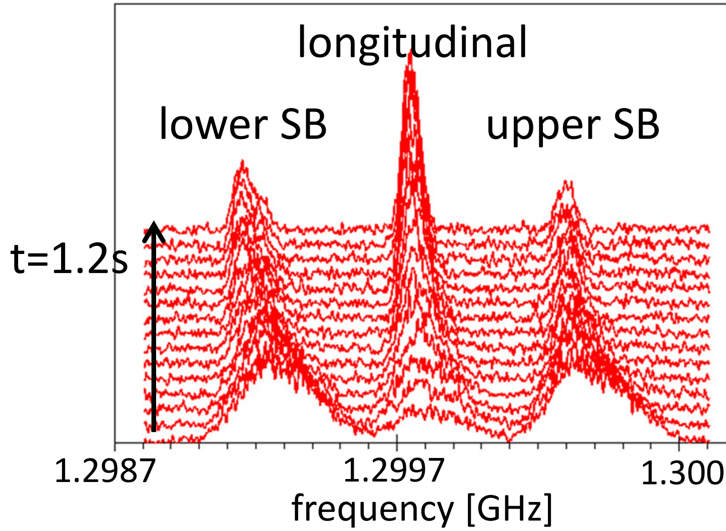
$\Rightarrow {P_h}^{\pm} \propto \langle d^2 \rangle = e^2 {f_0}^2 \cdot \frac{N}{2} \cdot \varepsilon_{rms}\beta$ with $\boldsymbol{\varepsilon_{rms}}$ transvers emittance and $\boldsymbol{\beta}$-function at pickup

Results:

- Power ${P_h}^{\pm}$ is the same at each harmonics ***h***
- Power decreases for lower emittance beams (due to decreasing modulation power)

$\Rightarrow$ measurement of rms emittance is possible.

Example: Transverse Schottky at GSI during cooling



Example:

Emittance shrinkage during stochastic cooling

Sideband behavior:

- width: smaller due to longitudinal cooling
- high: ≈ constant due to transverse cooling
- Power ${P_h}^{\pm}$ decreases $\Rightarrow$Emittance determination, but requires normalization by profile monitor

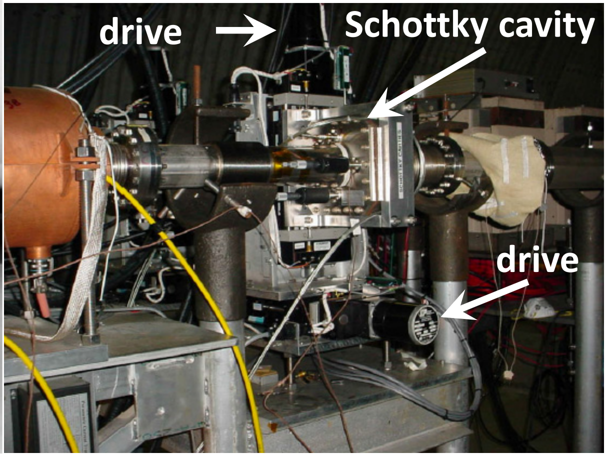
F. Nolden , DIPAC'01, p. 6 (2001)

Transverse rms Emittance Determination by Schottky Analysis at RHIC

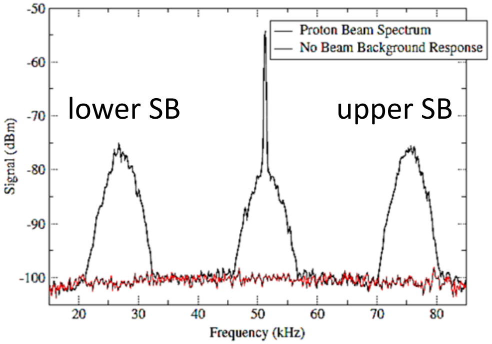
The integrated power in a sideband delivers the rms emittance $P_h^{\pm} \propto \langle d^2 \rangle \propto \varepsilon_{rms} \cdot \beta$

Example: Schottky cavity operated at dipole mode TM_{120} @ 2.071 GHz & TM_{210} @ 2.067 GHz i.e. a beam with offset excites the mode (like in cavity BPMs)

Peculiarity: The entire cavity is movable $\Rightarrow$ the stored power delivers a calibration ***P(x)***



power in long band depend on beam position ***x***



Result: rms emittances coincide with IPM measurement within the 20 % error bars

TABLE II. Results of Schottky emittance scan and comparison to RHIC IPM. Emittance values are normalized.

Ring and plane	Schottky β function (m)	Schottky rms beam size (mm)	Schottky emittance (π μm, 95%)	IPM emittance (π μm, 95%)
Blue horizontal	28 ± 4	1.04 ± 0.1	23 ± 5	24 ± 5
Blue vertical	27 ± 4	0.95 ± 0.1	20 ± 4	23 ± 3
Yellow horizontal	27 ± 4	0.99 ± 0.1	22 ± 4	19 ± 4
Yellow vertical	30 ± 5	1.15 ± 0.1	26 ± 5	28 ± 4

K.A. Brown et al., Phys. Rev. AB, 12, 012801 (2009), W. Barry et al., EPAC’98, p. 1514 (1998)

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  - Transverse for coasting beams
  - **Longitudinal for bunched beams**
  - Transverse for bunched beams
- Some further examples for exotic beam parameters
- Conclusion

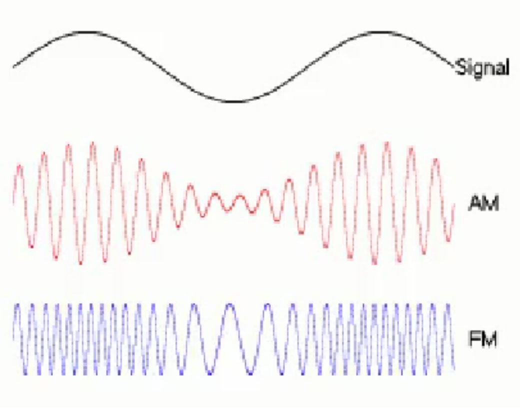
Principle of Frequency Modulation

Frequency modulation by composition of two waves:

- **Carrier:** here with revolution frequency $\boldsymbol{f_0 = 1/t_0}$
  $U_c(t) = \hat{U}_C \cdot \cos(2\pi f_0 t)$
- **Signal:** here with synchrotron frequency $\boldsymbol{f_s = Q_s \cdot f_0}$
  $\boldsymbol{Q_s} \ll 1$ synchrotron tune i.e. long. oscillations per turn
  $\boldsymbol{\tau_s(t)} = \hat{\boldsymbol{\tau}}_s \cdot \cos(2\pi f_s t)$

Frequency modulation is: $\boldsymbol{U_{tot}(t)} = \hat{U}_C \cdot \cos\left(2\pi f_0 t + m_s \cdot \int_0^t \tau_s(t')dt'\right)$

$$= \hat{U}_C \cdot \cos\left(2\pi f_0 t + \frac{m_s \hat{\tau}_s}{2\pi f_s} \cdot \sin(2\pi f_s t)\right)$$



Source: wikipedia

Frequency Modulation: General Investigations

Frequency modulation by composition of two waves:

- **Carrier:** For synchrotron → revolution freq. $f_0 = 1/t_0$

  $U_c(t) = \hat{U}_C \cdot \cos(2\pi f_0 t)$
- **Signal:** For synchrotron → synchrotron freq. $f_s = Q_s \cdot f_0$

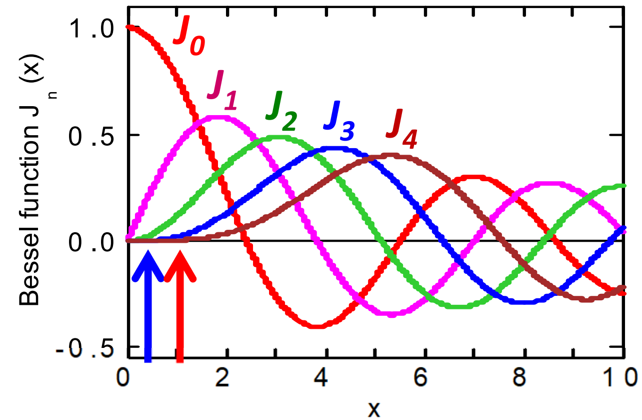
  $Q_s << 1$ synchrotron tune i.e. long. oscillations per turn

  $\tau_s(t) = \hat{\tau}_s \cdot \cos(2\pi f_s t)$

Frequency modulation is: $U_{tot}(t) = \hat{U}_C \cdot$

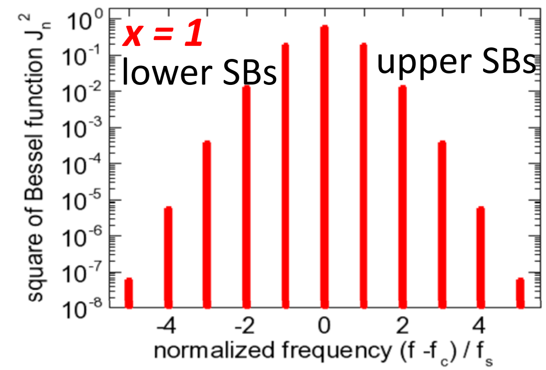
$\cos\left(2\pi f_0 t + m_s \cdot \int_0^t \tau_s(t')dt'\right)$

$$= \hat{U}_C \cdot \cos\left(2\pi f_0 t + \frac{m_s \hat{\tau}_s}{2\pi f_s} \cdot \sin(2\pi f_s t)\right)$$



Frequency domain representation:

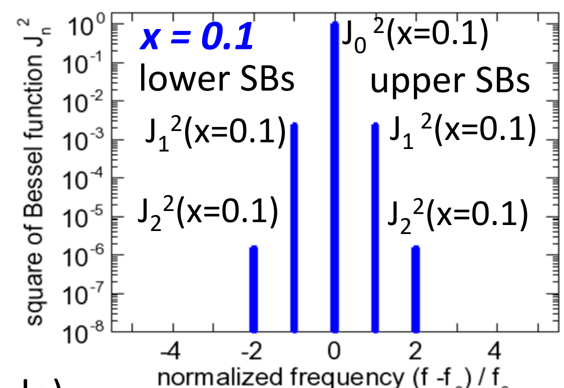
Bessel functions $J_p(x)$ with modulation index $x = \frac{m_s \hat{\tau}_s}{2\pi f_s}$



$$U_{tot}(t) = \hat{U}_c \cdot J_0(x)\cos(2\pi f_0 t) \qquad \text{central peak}$$

$$+\sum_{p=1}^{\infty}(-1)^p \hat{U}_c \cdot J_p(x)\cos(2\pi(f_0 - pf_s)t) \qquad \text{lower sidebands}$$

$$+\sum_{p=1}^{\infty} \hat{U}_c \cdot J_p(x)\cos(2\pi(f_0 + pf_s)t) \qquad \text{upper sidebands}$$



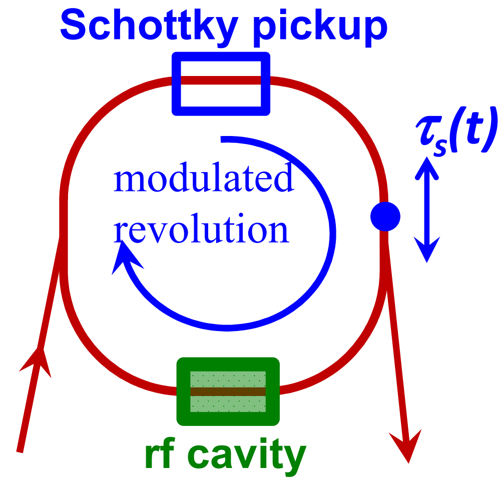
⇒ infinite number of satellites,
but only few are above a detectable threshold (Carson bandwidth rule)

Single particle of a bunched beam→ modulation of arrival by synchrotron oscillation:

Synchrotron frequency $f_s = Q_s \cdot f_0$

$Q_s << 1$ synchrotron tune i.e. long. oscillations per turn

$$\tau_s(t) = \hat{\tau}_s \cdot \cos(2\pi f_s t + \psi)$$



Modification of coasting beam case for a frequency modulation:

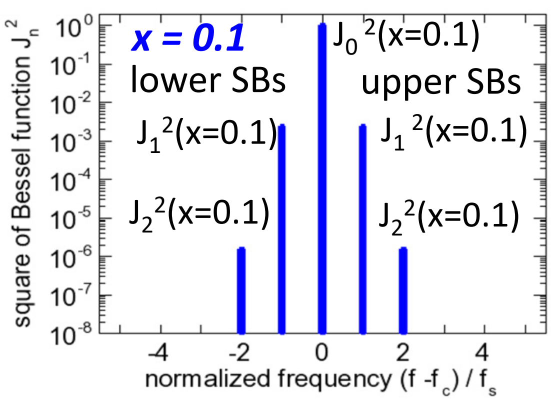
$$I_1(t) = ef_0 + 2ef_0 \sum_{h=0}^{\infty} \cos\{2\pi h f_0 [t + \hat{\tau}_s \cdot \cos(2\pi f_s t + \psi)]\}$$

Each harmonics h comprises of lower and upper sidebands:

$$\sum_{p=-\infty}^{\infty} J_p(2\pi h f_0 \hat{\tau}_s) \cdot \cos(2\pi h f_0 t + 2\pi p f_s t + p\psi)$$

For *each* revolution harmonics h the longitudinal is split

- Central peak at hf_0 with height $J_0(2\pi \cdot hf_0 \cdot \hat{\tau}_s)$
- Satellites at $hf_0 \pm pf_s$ with height $J_p(2\pi \cdot hf_0 \cdot \hat{\tau}_s)$

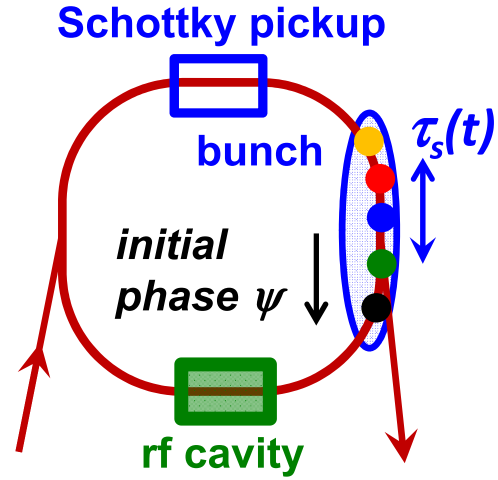


Note:

- The argument of Bessel functions contains amplitude of synchrotron oscillation $\hat{\tau}_s$ & harmonics h
- Distance of sidebands are independent on harmonics h

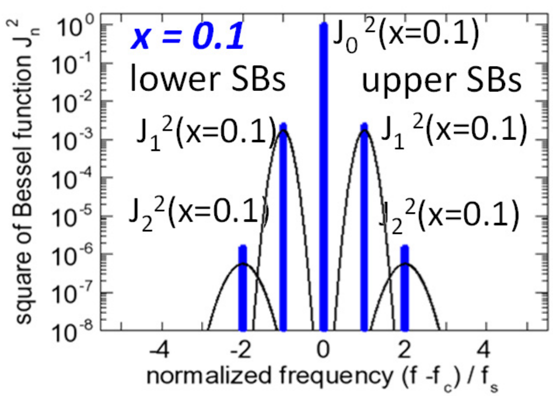
Bunched Beam: Longitudinal Schottky Spectrum for many Particles

Particles have different amplitudes $\hat{\tau}_s$ and initial phases ψ
$\Rightarrow$ averaging over initial parameters for *n = 1...N* particles:



Results:

- **Central peak *p = 0*:** No initial phase for single particles
  $U_0(t) \propto J_0(2\pi \cdot hf_0 \cdot \hat{\tau}) \cdot \cos(2\pi hf_0 t)$
  $\Rightarrow$ **Total power $P_{tot}(p=0) \propto N^2$**
  i.e. contribution from 1...***N*** particles add up **coherently**
  $\Rightarrow$ **Width: $\sigma_{p=0} = 0$** (ideally without power supplier ripples etc.)
  Remark: This signal part is used in regular BPMs
- **Satellites *p* ≠ *0*:** initial phases ***ψ*** appearing
  $U_p(t) \propto J_p(2\pi \cdot hf_0 \cdot \hat{\tau}) \cdot \cos(2\pi hf_0 t + 2\pi p f_s t + p\psi)$
  $\Rightarrow$ **Total power $P_{tot}(p \neq 0) \propto N$**
  i.e. contribution from 1...***N*** particles add up **incoherently**
  $\Rightarrow$ **Width: $\sigma_{p\neq 0} \propto p \cdot \Delta f_s$** lines getting wider
  due to mom. spread $\Delta p / p_0$ &
  possible spread of synchrotron frequency Δf_s

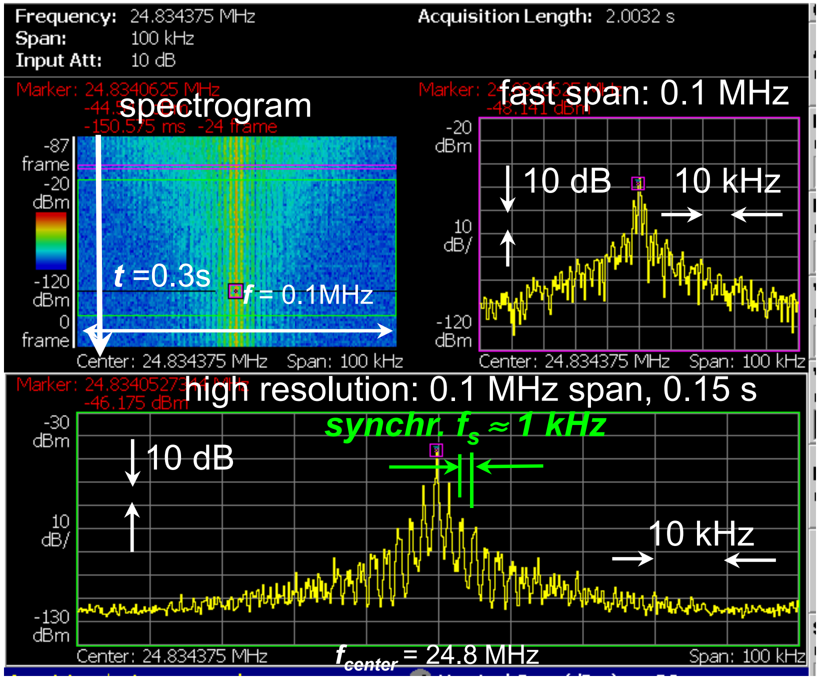


Example for scaling of power:
If ***N*** = *10*[10] *then* $P_{tot}(p=0) \approx 100\text{dB} \cdot P_{tot}(p \neq 0)$
Remarks: The central peak is coherent $P_{tot}(p=0) \propto N^2$
$\Rightarrow$ applying a **stringent** definition this is **not** a Schottky line

Example of longitudinal Schottky Analysis for a bunched Beam

Example: **Bunched** beam at GSI synchrotron

Beam: Injection $\boldsymbol{E_{kin}}$ = 11. 4 MeV/u harm. $\boldsymbol{h}$ = 120



Application for 'regular' beams:

- Determination of synchrotron frequency f_s
- Determination of momentum spread:
  - envelop does **not** represent directly coasting beam
    $\Rightarrow$ **not** directly usable for daily operation
  - but can be extracted with detailed analysis
    due to the theorem $\sum_{p=-\infty}^{\infty} J_p(x) = 1$ for all x
    $\Rightarrow$ for each band ***h***: $\int P_{bunch}\, df = \int P_{coasting}\, df$

Application for intense beams:

- The sidebands reflect the distribution $\boldsymbol{P(f_s)}$ of the synchrotron freq. due to their incoherent nature
  see e.g. E. Shaposhnikova et al., HB'10, p. 363 (2010) & PAC'09, p. 3531 (2009), V. Balbecov et al., EPAC'04, p. 791 (2004)
- However, the spectrum is significantly deformed amplitude $\hat{\tau}_s$dependent synchrotron freq. $f_s\,(\hat{\tau}_s)$
  see e.g. O. Boine-Frankenheim, V. Kornilov., Phys. Rev. AB 12. 114201 (2009)

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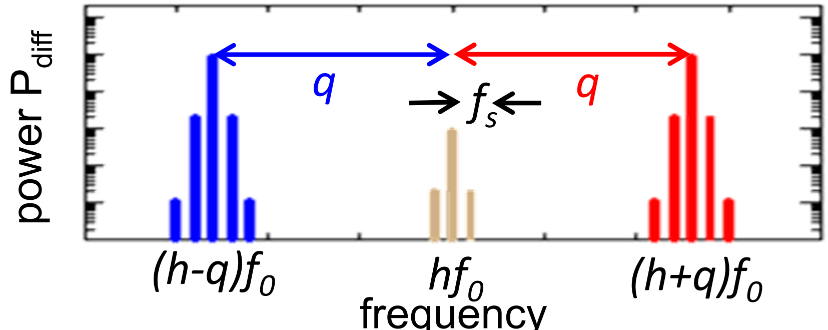
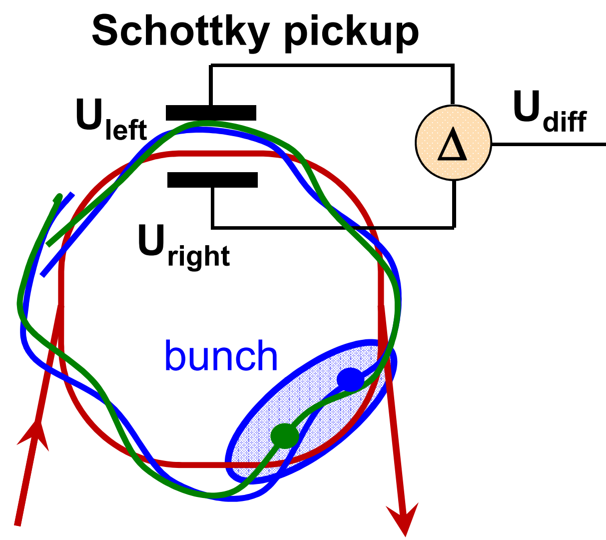
Transverse Schottky Analysis for bunched Beams

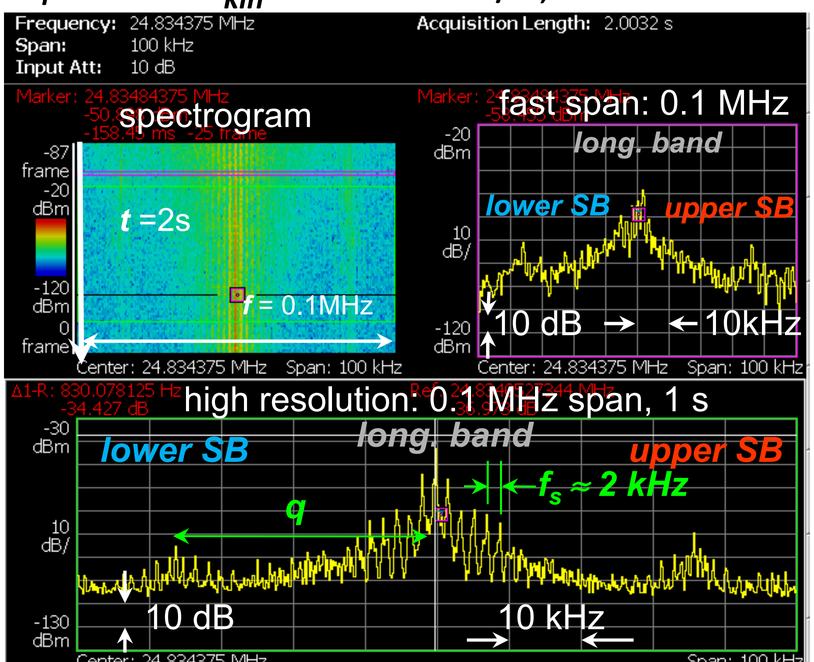
Transverse Schottky signals are understood as

- **amplitude modulation of the longitudinal signal**
- ***convolution by transverse sideband***





Example: GSI E_{kin} = 11.4 MeV/u, harmonics h = 119



Structure of spectrum:

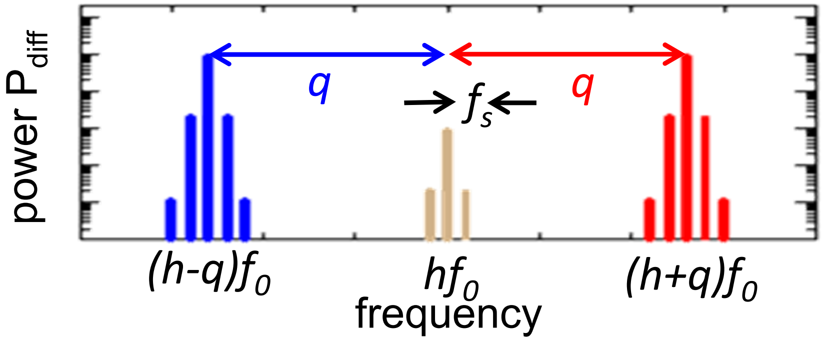
- **Longitudinal** peak with synchrotron SB
  - central peak $P_0 \propto N^2$ called coherent
  - sidebands $P_p \propto N$ called incoherent
- **Transverse** peaks comprises of
  - replication of coherent long. structure
  - incoherent base might be visible

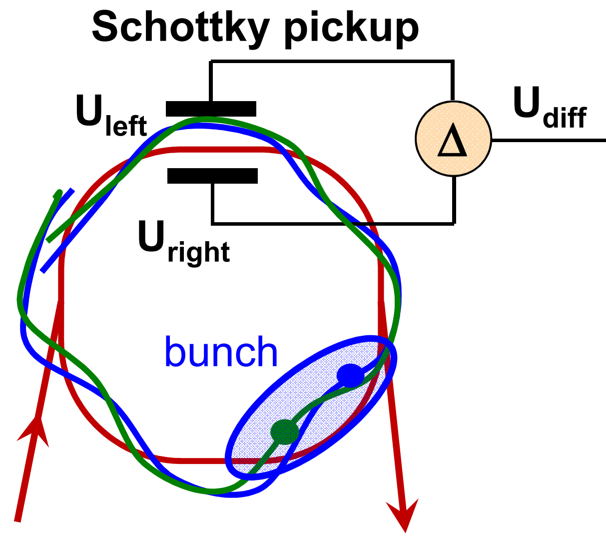
Remark: Spectrum can be described by lengthy formula see e.g. S. Chattopadhay, CERN 84-11 (1984)

Remark: Height of long. band depends geometrical center of the beam in the pick-up

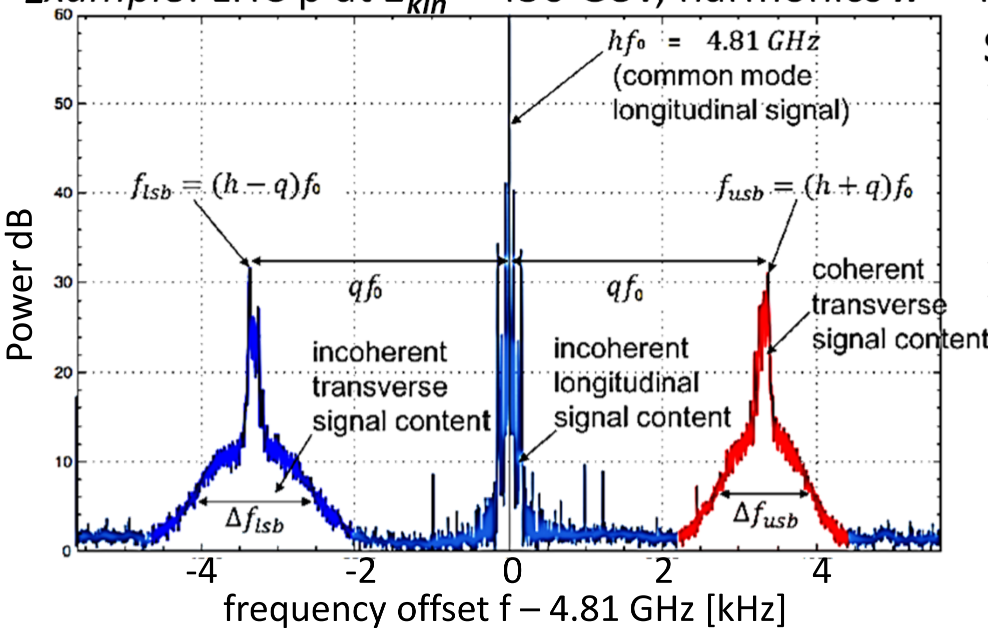
Transverse Schottky signals are understood as

- amplitude modulation of the longitudinal signal
- convolution by transverse sideband





Example: LHC p at E_{kin} = 450 GeV, harmonics $h \approx 4 \cdot 10^5$



M. Wendt et al. IBIC'16, p. 453 (2016), M. Betz et al., NIMA submitted

Structure of spectrum (LHC f_s < 150 Hz):

- **Longitudinal** peak with synchrotron SB
  - central peak $P_0 \propto N^2$ called coherent
  - sidebands $P_p \propto N$ called incoherent
- **Transverse** peaks comprises of
  - replication of coherent long. structure
  - incoherent base
    dominated by chromatic tune spread

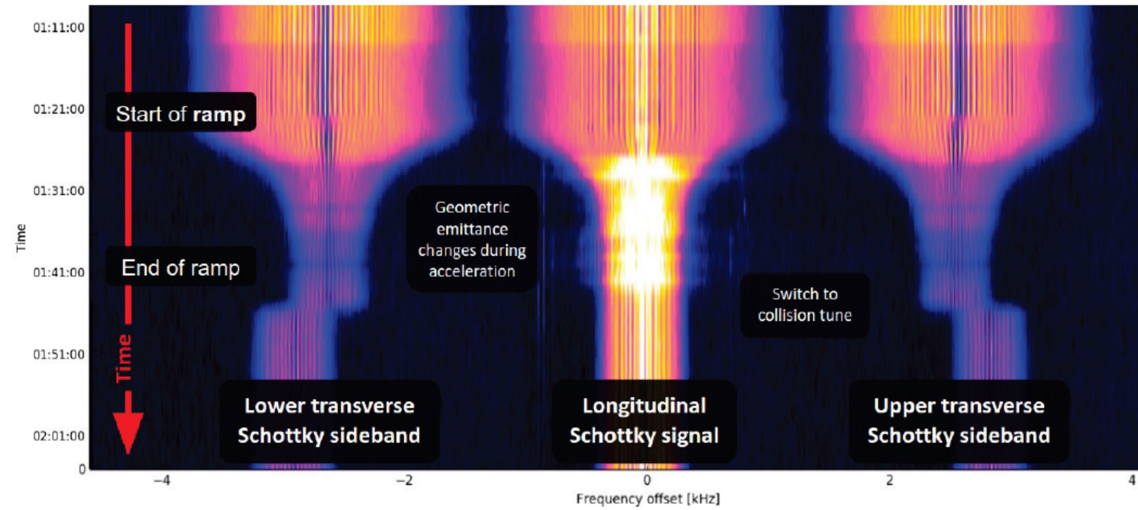
$$\Delta f_h^{\pm} = \eta \frac{\Delta p}{p_0} \cdot f_0 \left(h \pm q \pm \frac{\xi}{\eta} Q_0 \right)$$

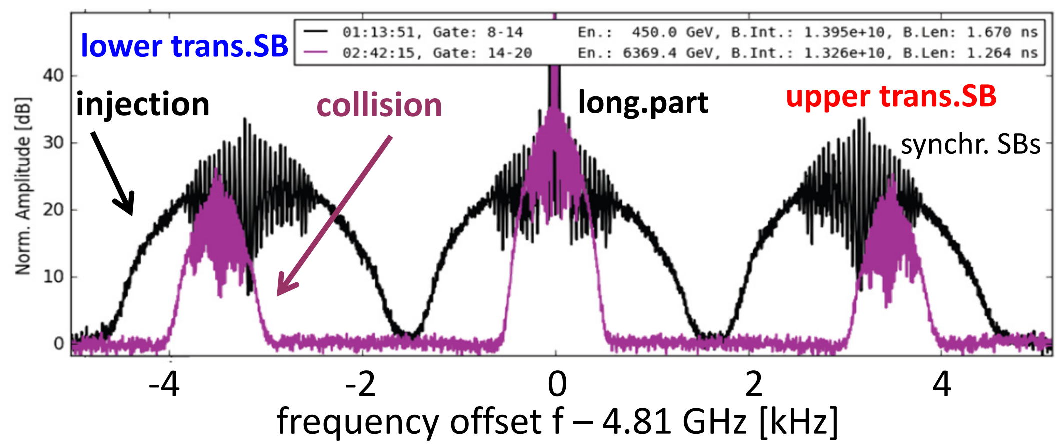
Schottky spectrogram during LHC ramp and collision:

The interesting information is in the ***in****coherent part of the spectrum (i.e. like for coasting beams)*

- **Longitudinal part**
  - ***Width:*** → momentum spread
    momentum spread decreases
- **Transverse part**
  - **Center:** → tune
    shift for collision setting
  - **Width**: → chromaticity
    difference of lower & upper SB
  - **Integral** : → emittance
    reduction of geometric emittance

Example: LHC nominal filling with Pb^{82+}
→ acceleration & collisional optics within ≈ 50 min





M. Betz et al. IPAC'16, p. 226 (2016)

LHC 4.8 GHz Schottky: Tune and Chromaticity Measurement

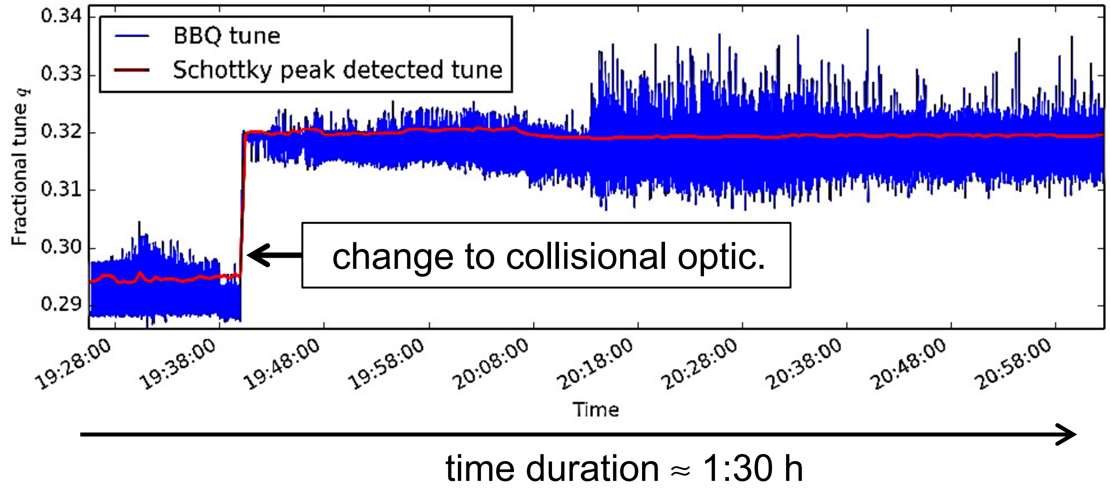
Tune from position of sideband:

Permanent monitoring of tune

- Without excitation
- High accuracy down to 10^{-4} possible
- Time resolution here 30 s

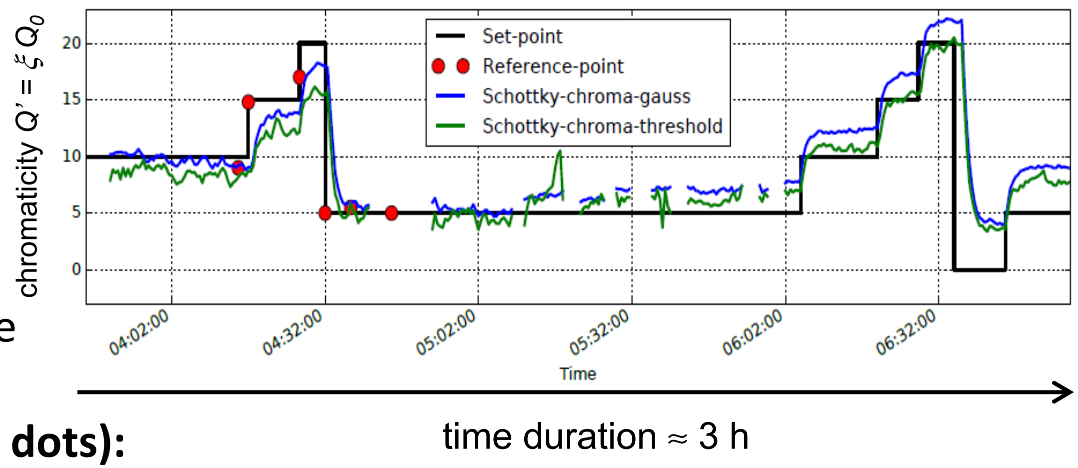
Comparison to BBQ system based on:

- Transverse (gentle) excitation
- Bunch center detection
- Time resolution here 1 s



Chromaticity from width of sidebands of incoherent part:

- Two different offline algorithms
- Satisfactory accuracy
- Time resolution here 30 s
- Performed at MD time, breaks are due to experimental realignments



Comparison to traditional method (red dots):

- Change of bunching frequency $\Rightarrow \delta p = p_{actual} - p_0$
- Tune measurement and fit $\Delta Q / Q_0 = \xi \cdot \delta p / p_0$

M. Betz et al. IPAC'16, p. 226 (2016),
M. Betz et al., NIM A submitted

LHC 4.8 GHz Schottky: Technical Design of slotted Waveguide

Challenge for bunched beam Schottky:
Suppression of broadband sum signal to prevent for saturation of electronics

Design consideration:

Remember scaling: width $\Delta f \propto h$, power $P \propto 1/h$

- Low sum signal i.e. outside of bunch spectrum (LHC: acceleration by f_{acc} = 25 MHz)
- Avoiding overlapping Schottky bands
- Sufficient bandwidth to allow switching

Technical choice:

- Narrow band pickup by two wave guide for TE_{10} mode, cut-off at 3.2 GHz
- Coupling slots for beam's TEM mode

$\Rightarrow$ center f_c =4.8 GHz $\Leftrightarrow$ harm. $h \approx 4 \cdot 10^5$ & $BW \approx$0.2 GHz

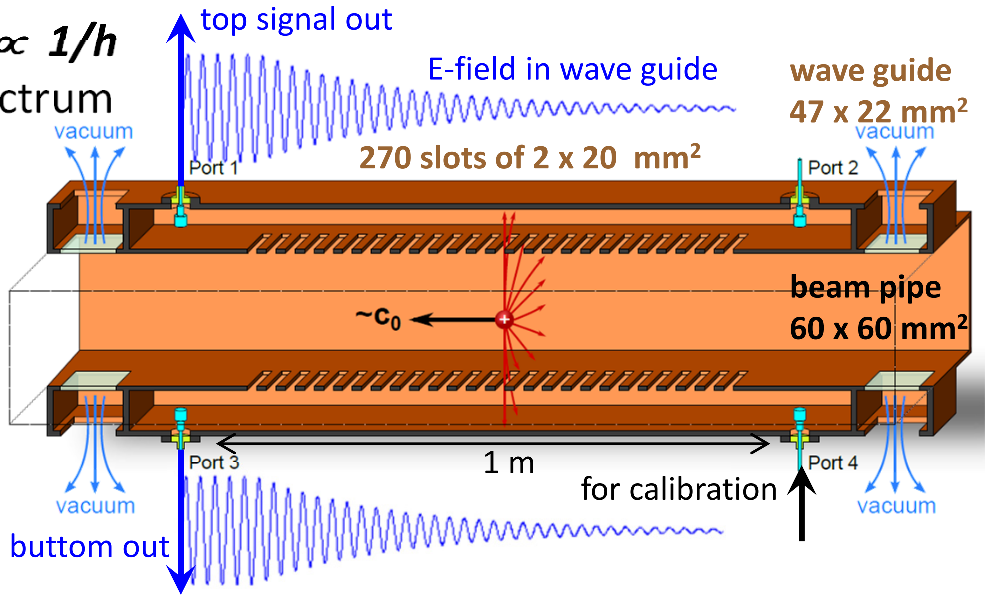
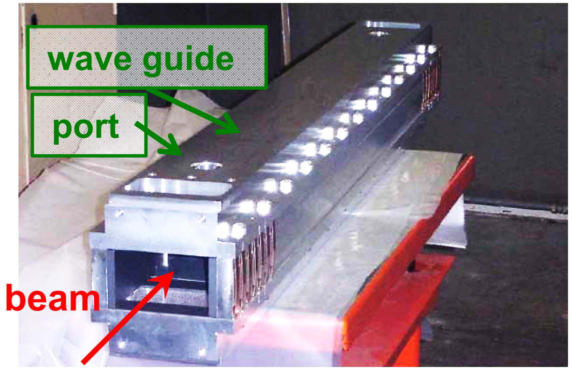
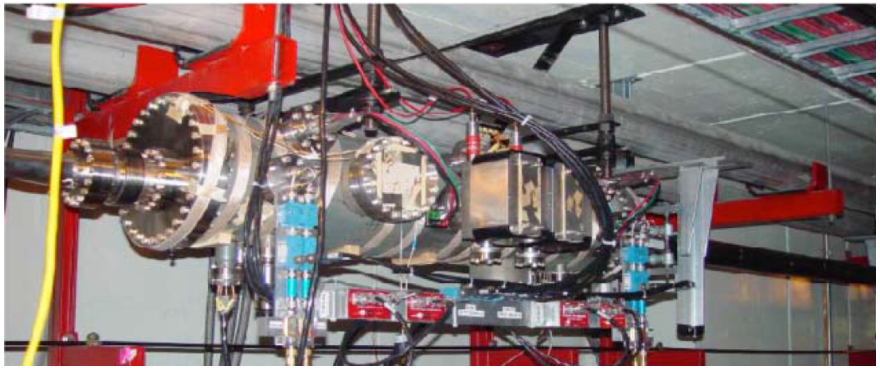


Photo of 1.8 GHz Schottky pickup at FNAL recycler





CERN: M. Wendt et al. IBIC'16, p. 453 (2016), M. Betz, NIM A submitted

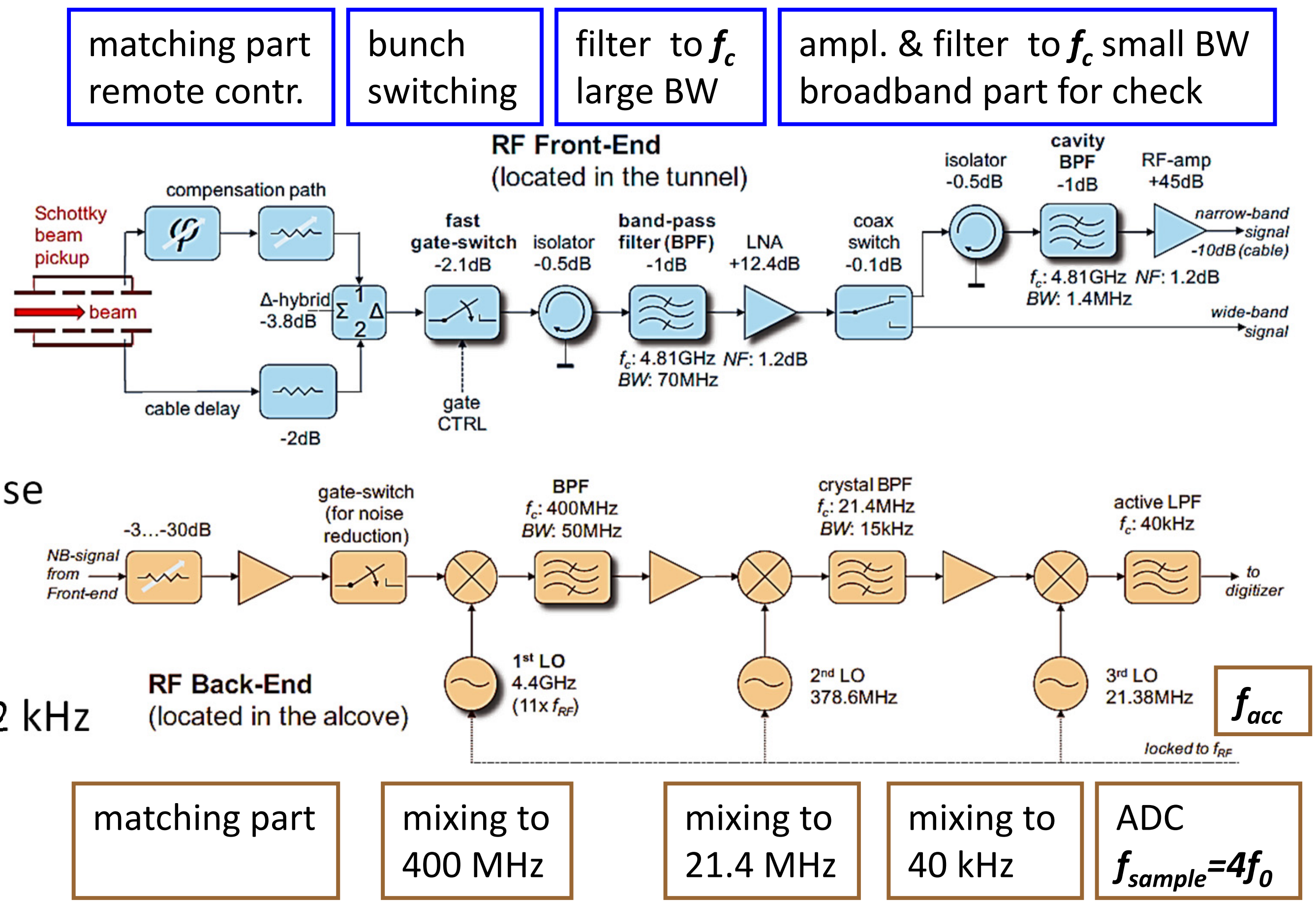
FNAL: R. Pasquinelli et al., PAC'03, p. 3068 (2003) & R. Pasquinelli, A. Jansson, Phys. Rev AB **14**, 072803 (2011).

Challenge for bunched beam Schottky:
Suppression of broadband sum signal to prevent for saturation of electronics

Design considerations:

- Careful matching
- Switching
  during bunch passage
  switching time ≈ 1 ns
  ⇒ one bunch per turn
- Filtering of low signals
  without deformation
  to increase signal-to-noise
- Down-mixing
  locked to acc. rf
- ADC sampling with $\boldsymbol{4 \cdot f_0}$
  revolution freq. f_0 = 11.2 kHz

Requirements: low noise & large dynamic range



M. Wendt et al. IBIC’16, p. 453 (2016), & M. Betz et al., NIM A submitted

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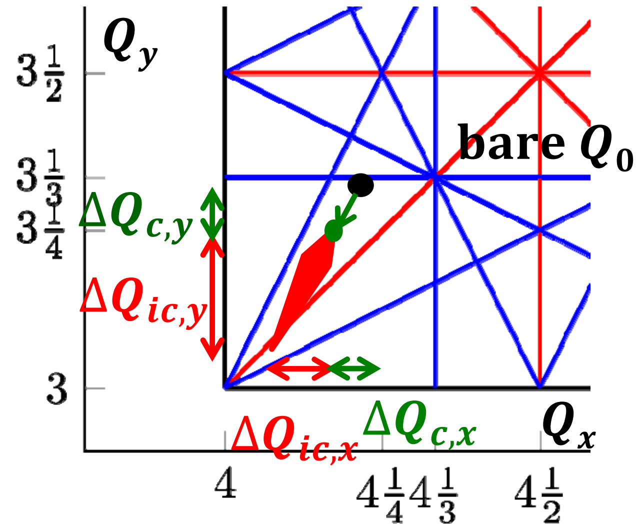
Remarks to intense Beams

Remark to intense beams: (just as a hint for a complex subject)

- **Coherent tune shift** is significant e.g. ΔQ_c > ≈ 0.05 caused by image current at the wall or wake-fields
- **Incoherent tune spread** is significant e.g. ΔQ_{ic} > ≈ 0.05 caused by the space charge of the beam

⇒ Realized for higher currents of low & medium energy protons up to ≈ 10 GeV/u (γ≈11) and cooled beams

⇒ **Not** realized for high energies e.g. LHC and many electron synchrotrons



Coherent tune shift: $\Delta Q_c \propto \frac{q^2}{A} \cdot \frac{1}{\beta^3\gamma^3} \cdot \frac{1}{b^2} \cdot N$

beam pipe

Incoh. tune spread: $\Delta Q_{ic} \propto \frac{q^2}{A} \cdot \frac{1}{\beta^3\gamma^2} \cdot \frac{1}{\varepsilon} \cdot N$

importance for ions | energy scaling | tr. emittance | # particles

q ion charge, ***A*** ion mass
β velocity, ***γ*** Lorentz factor
b pipe radius
ε transverse emittance
N number of ions

Deformed Schottky Spectra for high Intensity coasting Beams

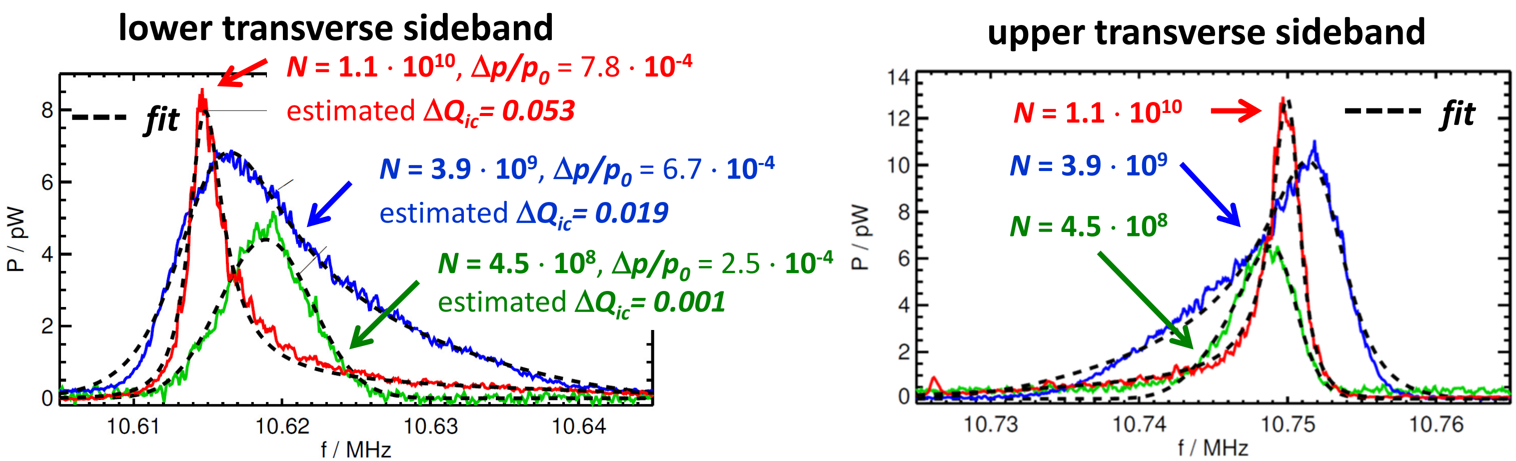
Transverse spectra can be deformed even at 'moderate' intensities for lower energies

Remember: Transverse sidebands were introduced as **coherent** amplitude modulation

Goal: Modeling of a possible deformation leading to correct interpretation of spectra

Extracting parameters like tune spread ΔQ_{ic} by comparison to detailed simulations

Example: Coasting beam GSI synchrotron Ar^{18+} at 11.4 MeV/u, harm. h = 40, coherent $\Delta Q_c \approx 0$



Method:

- Calculation of space charge & impedance modification using sophisticated models
- Calculation of beam's frequency spectrum
- Fit to the experimental results

$\Rightarrow$ Model delivers reliable beam parameters , spectra can be explained

Schottky diagnostics:

- Spectra do not necessarily represents the distribution, but parameter can be extracted

O. Boine-Frankenheim et al., Phys. Rev. AB 12, 114201 (2009) , S. Paret et al., Phys. Rev. AB 13, 022802 (2010)

Longitudinal Schottky: Modification for very cold Beams

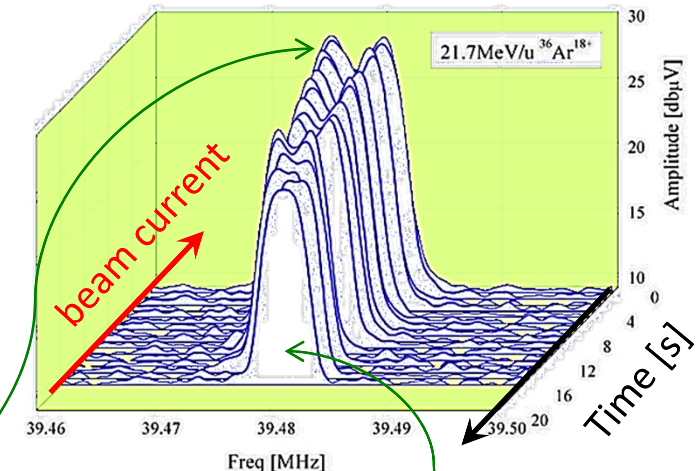
Very high phase space density leads to modification of the longitudinal Schottky spectrum

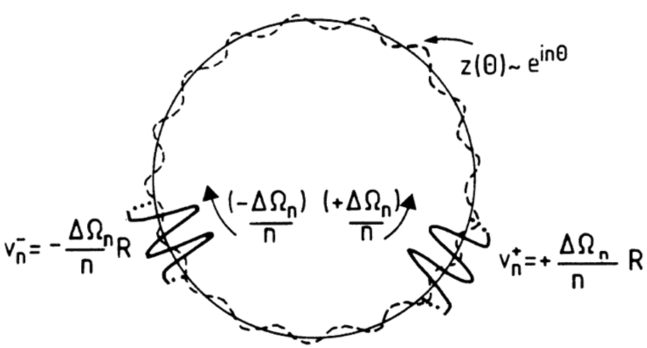
Low energy electron cooler ring:
High long. & trans. phase space density
$\Rightarrow$ Strong coupling between the ions
$\Rightarrow$ Excitation of co-&counter propagation plasma waves by wake-fields (beam impedance)

This collective density modulation is a coherent effect!

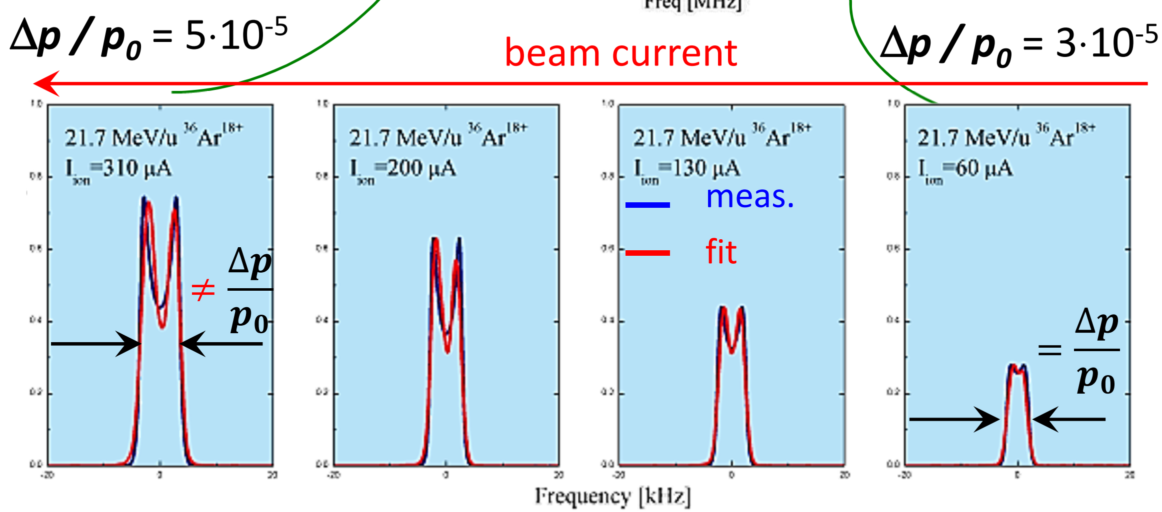
$\Rightarrow$ Schottky spectrum comprises then
coherent part with power scaling $P \propto N^2$
+ the regular **incoherent** part with $P \propto N$
$\Leftrightarrow$ Schottky **doesn't** represent distribution e.g. $\boldsymbol{\sigma \neq \Delta p/p_0}$
but $\Delta p / p_0$ can be gained from model fit

Example: at CSRe cooler ring in Lanzhou, China
Beam: Ar^{18+} at E_{kin} = 21 MeV/u, harm. h = 100





S. Chattopadhay, CERN 84-11 (1984)



L.J. Mao et al. IPAC'10, p. 1946 (2010)

Schottky Spectrum at Synchrotron Light Sources

Hadron synchrotron: most beams non-relativistic or $\gamma < 10$ (exp. LHC) $\Rightarrow$ **no** synch. light emission
$\Leftrightarrow$ stationary particle movement $\Rightarrow$ turn-by-turn correlation
Electron synchrotrons relativistic $\gamma \approx 5000$ $\Rightarrow$ synchrotron light emission
$\Leftrightarrow$ break-up of turn-by-turn correlation **?**

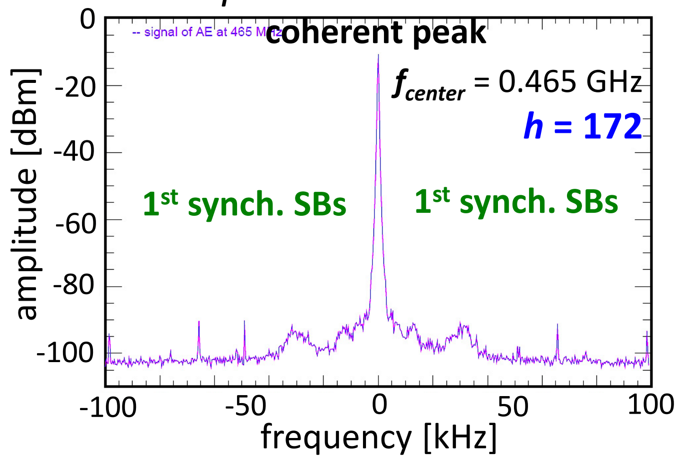
Test of longitudinal Schottky at ANKA (Germany):
Goal: determination of momentum spread $\Delta p / p_0$
Ring shaped electrode as broadband detector

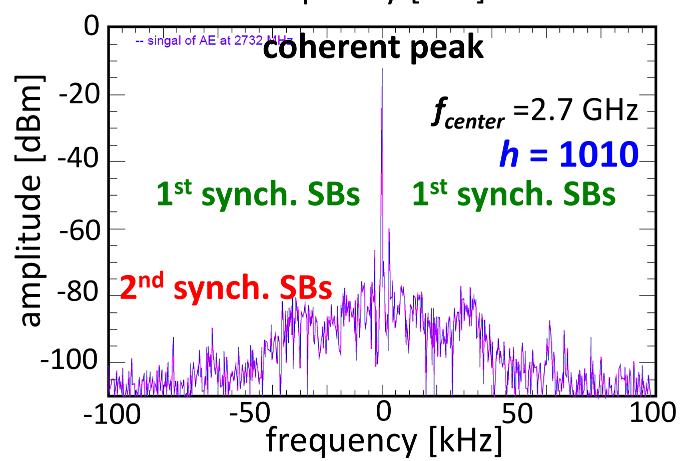
Results:

- Narrow coherent central peak
- Synchrotron sidebands clearly observed
- Sideband wider as central peak
  $\Rightarrow$ incoherent cntribution
- Ratio of power $P_{central} / P_{SB}$ as expected

$\Rightarrow$ Attempt started, feasibility shown!
Further investigations are ongoing

Example: ANKA at 2.5 GeV





K.G. Sonnad et al., PAC’09, p. 3880 (2009)

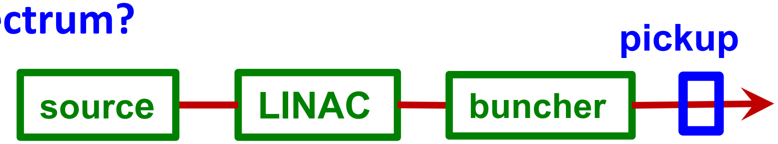
Longitudinal Schottky at a LINAC ???

Is it possible to measure the momentum spread at a single pass accelerator i.e. is there an incoherent contribution to the bunch spectrum?

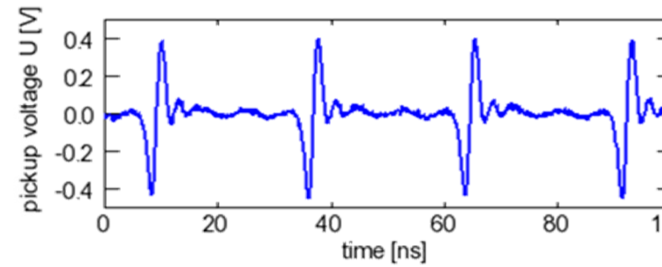
Experiment at GSI: broadband pickup & oscilloscope

Advantage: Spectra for different 'harmonics' $h \cdot f_{acc}$

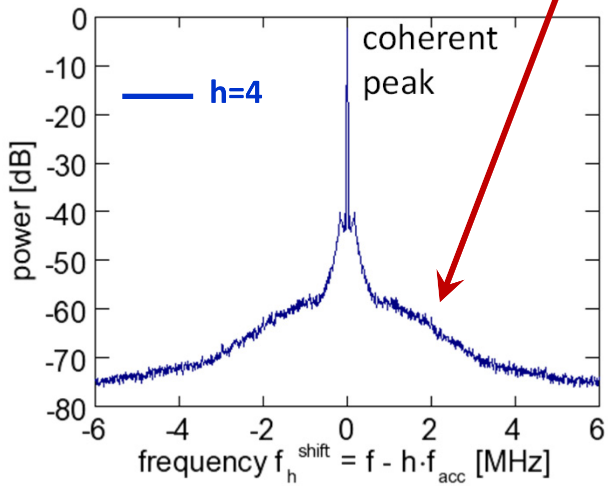
Schottky in synchr.: Incoherent width $\Delta f_h \propto h$



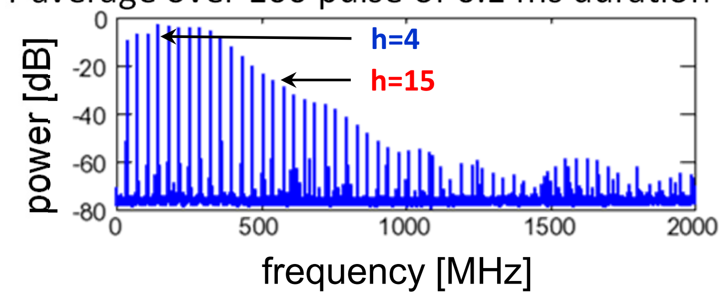
Beam: U^{28+} at 11.4 MeV/u, f_{acc} = 36 MHz

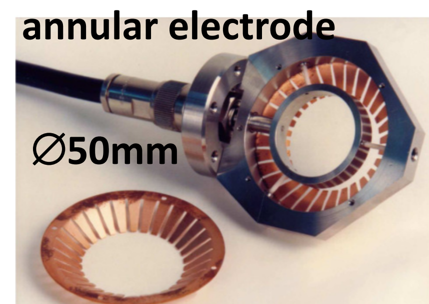


Is this the incoherent frequency spread $\propto \Delta p / p_0$?



FFT average over 100 pulse of 0.1 ms duration





P. Kowina et al., HB'12, p. 538 (2012)

Longitudinal Schottky at a LINAC ??? → Result: Probably not possible

Is it possible to measure the momentum spread at a single pass accelerator
i.e. is there an incoherent contribution to the bunch spectrum?

Experiment at GSI: broadband pickup & oscilloscope

Advantage: Spectra for different 'harmonics' $h \cdot f_{acc}$

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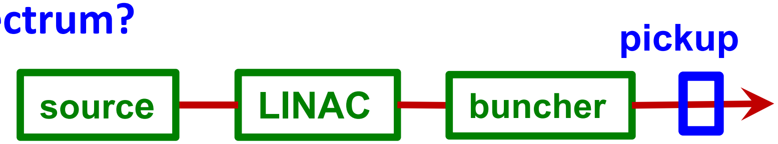
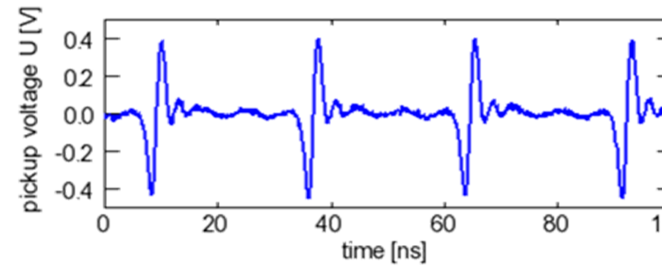
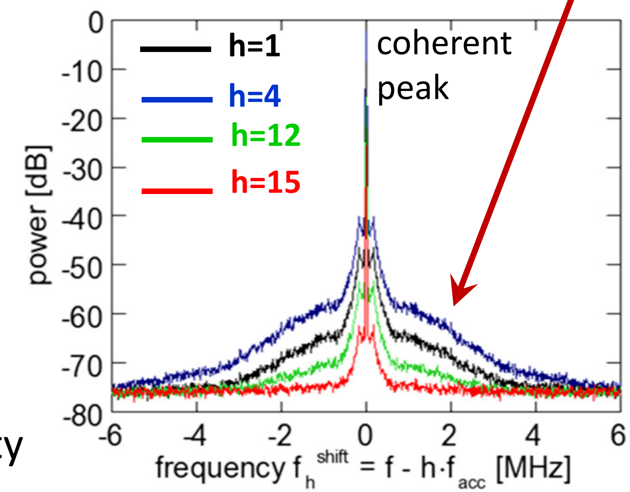


Beam: U^{28+} at 11.4 MeV/u, f_{acc} = 36 MHz

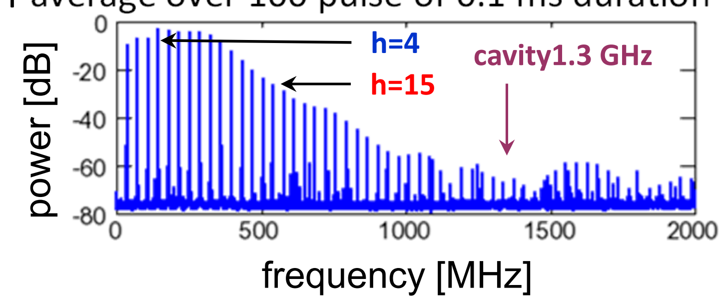


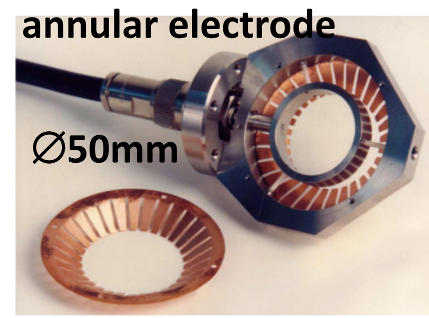
Result:
Peak structure does **not** change for different 'harmonics' h:
⇒ no incoherent Schottky part!
Supported by spectra recorded with a cavity @ 1.3 GHz of high h and sensitivity

Is this the incoherent frequency spread $\propto \Delta p / p_0$?
No, but bunches' amplitude variation!



FFT average over 100 pulse of 0.1 ms duration





Interpretation:
Schottky signals require the periodic passage of the **same** particle to ensure the correlation to build up.

P. Kowina et al., HB'12, p. 538 (2012)

Usage of the Schottky method at hadron synchrotrons

Beam	Measurement	My personal assessment
Coasting long.	f_0 , $\Delta p/p_0$, matching, stacking and cooling	OP: Basic daily operation tool 'It just works!'
Coasting trans.	Q_0 , ξ , ε_{trans}	OP: Very useful tool for Q_0 for ξ & ε indirect method MD: For ξ & ε sometimes used requires some evaluation
Bunched long.	f_s , $\Delta p/p_0$	OP: Seldom used MD: Important
Bunched trans.	Q_0, ξ , ε_{trans}	OP: Online monitoring for Q_0 very useful MD: Important tool

High intensity beam investigations:
Schottky spectrum is well suited to given access to parameter like to spread ΔQ_{ic}
Frequency spectrum of the beam $\Rightarrow$ characteristic modifications $\Rightarrow$ model verification

OP: operation, MD Machine Development

Summary

Schottky signals are based on modulations and fluctuations:

Modulations: Coherent quantities $\rightarrow$ measurement of $\boldsymbol{f_0}$, $\boldsymbol{Q_0}$ & $\boldsymbol{f_s}$ with high precision

Fluctuations: Incoherent quantities $\rightarrow$ measurement of $\Delta \boldsymbol{p/p_0}$ & ξ from peak width

$\rightarrow$ scaling of width $\Delta \boldsymbol{f(h)} \propto \boldsymbol{h}$ and signal power $\boldsymbol{P(h)} \propto \boldsymbol{1/h}$

Signal spectrum: Partly complex, but computable

Detection: Recordable with wide range of pickups, measurement possible in each harmonics, electronics for very weak signals must be matched to the application

High intensity beams: Characteristic modifications, important for model verification

For valuable discussion I like to thank:

- Co-authors: P. Kowina GSI, R. Singh GSI, M. Wendt CERN
- M. Betz LBL (formally CERN), O. Boine-Frankenheim GSI, P. Hülsmann GSI , A. Jansson ESS (formally FNAL), A.S. Müller KIT, M. Steck GSI, J. Steinmann KIT and many others

Thank you for your attention!