

Time-resolved electron-bunch diagnostics using transverse wakefields

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Abstract

The development of future free electron lasers (FELs) requires reliable time-resolved measurement of variable ultra-short electron-bunch characteristics. A possible technique is to streak the bunch in the transverse direction by means of time-dependent external fields. In this paper we explore the possible use of self-generated electromagnetic fields [?]. A passive deflector, consisting of a dielectric-lined waveguide, is used to produce wakefields that impart a time-dependent transverse kick to the relativistic electron bunch passing off-axis. We investigate the technique and its performances and explore its possible application at the Fermilab Accelerator Science and Technology (FAST) facility.

Introduction

Techniques to tailor the electron beam phase space distribution by means of external and internal fields have come to play an increasingly important role in linear accelerators over the last decade. A wide of techniques has been developed to utilize the fields to influence the beam distribution. One of the manipulations operates within one degree of freedom, e.g., those based on the use of external and internal fields to control the distribution in one of three 2D phase-space planes: $(x - p_x)$, $(y - p_y)$, $(z - p_z)$.

The self-generated wakfields, as the internal fields, is used as a tool to provide the transverse kick on the beam so as to introduce a correlation between time and the transverse beam distribution. The transverse wakefield can be used as a passive deflector to provide time-dependent deflecting kick to a relativistic electron bunch. Such a capability could enable the development of new passive (and cheap) beam diagnostics [?]. The passive deflector does not need to be powered and it is easier to be manufactured compared to a rf transverse deflecting structure, thus, resulting in a considerable cost saving. We could use it to perform time-resolved measurements of a relativistic electron bunch based on the self-transverse wakefield interaction of the beam itself passing off-axis through a dielectric-lined tube and reconstruct the beam profile from the resulting image of the streaked beam on the downstream profile monitor.

Main Objectives

1. to provide an alternative method to time-resolved longitudinal profile of a relativistic beam bunch .
2. to reconstruct the longitudinal profile using simulation of beam by ELEGANT

Methods

The transverse coordinates of the system are x and y while the longitudinal laboratory coordinate along the straight beamline is z , the angular divergence as $x' \equiv \frac{p_x}{p_z}$ and $y' \equiv \frac{p_y}{p_z}$ and the relative momentum spread as $\delta \equiv \frac{p}{p}$ where $p^2 = p_x^2 + p_y^2 + p_z^2$.

In a drift space ($K = 0$) and assuming the beam energy remains unchanged $\gamma(s) = \gamma$ and $\frac{d\gamma}{dz} = 0$, the simplified equation is

$$\frac{d^2x(\zeta, z)}{dz^2} = \frac{r_0}{\gamma} \int_{\zeta}^{\infty} d\zeta' \rho(\zeta') w_{\perp}(\zeta - \zeta') x(\zeta', z) d\zeta'. \quad (1)$$

Taking the wakefield to be constantly applied over a length L the previous equation can be integrated to yield

$$\begin{aligned} x'(\zeta, z) &= \frac{dx(\zeta, z)}{dz} \\ &= \frac{Lr_0}{\gamma} \int_{\zeta}^{\infty} d\zeta' \rho(\zeta') w_{\perp}(\zeta - \zeta') x(\zeta', z) d\zeta'. \end{aligned} \quad (2)$$

The most-left equality is valid under the ultra-relativistic approximation $\gamma \gg 1$. Assuming that the slice position does not change during the interaction but only its divergence is affected (this is the so-called ‘‘impulse approximation’’) we can further simplify the previous equation into

$$\begin{aligned} x'(\zeta, z) &= \frac{dx(\zeta, z)}{dz} \\ &= x(\zeta, z) \frac{Lr_0}{\gamma} \int_{\zeta}^{\infty} d\zeta' \rho(\zeta') w_{\perp}(\zeta - \zeta') d\zeta'. \end{aligned} \quad (3)$$

This equation is the basis of transverse-wakefield calculation: knowing the longitudinal charge distribution $\rho(\zeta)$ and the transverse Green’s function describing the electromagnetic wake, one can infer the transverse displacement of longitudinal slices.

Given the description of the active deflection scheme, we can now modify the previous equations to apply them to the passive-deflection technique.

$$x = x_{\beta} + R_{12} x'_0(\zeta), \quad (4)$$

where $x'_0(\zeta) \equiv x'(\zeta, z = 0)$ with $z = 0$ corresponding to the position where the kick is applied (i.e. the center of the deflecting structure in the impulse approximation);

Also, using the charge conservation relation $f_{\zeta}(x) dx = \rho(\zeta) d\zeta$ that is

$$\rho(\zeta) = |R_{12}| \frac{dx'_0(\zeta)}{d\zeta} |f_{\zeta}(x)| \quad (5)$$

Here we can get the derivative $\frac{dx'_0(\zeta)}{d\zeta}$ from Eq 3.

The main optimized algorithm is the iteration:

Algorithm 1 Longitudinal charge distribution retrieval

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1: define  $\mathcal{G}$   $\triangleright$  gain for the adaptive loop
2: read  $f_{\zeta}^m(x)$   $\triangleright$  measured beam profile after deconvolution
3: initialize  $\rho_0(\zeta)$   $\triangleright$  initial (guessed) charge distribution
4: for  $i \in [0, N]$  do
5:    $x(\zeta) = \text{TransWake}[\text{Green}, \rho_i(\zeta)]$   $\triangleright$  compute deflecting kick for a given Green's function
6:    $f_{\zeta}(x) = \text{Streak}[\rho_i(\zeta), x(\zeta)]$   $\triangleright$  evaluate streaked profile
7:    $\rho_i^e(\zeta) = f_{\zeta}(x) \times |\frac{dx}{d\zeta}|$   $\triangleright$  estimated charge distribution from streaked profile
8:    $\rho_{i+1}(\zeta) = \rho_i + \mathcal{G} \times (\rho_i^e(\zeta) - \rho_i)$   $\triangleright$  successive approximation
9:    $\epsilon_i = \sum_x |f_{\zeta}(x) - f_{\zeta}^m(x)|$ 
10: end for
11: plot  $\rho_N(\zeta)$ 

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Figure 1: Adaptive loop algorithm commonly used in feedback control systems

Results

The algorithm is proved to be practical by using a generated super gaussian longitudinal distribution as in Fig. 2

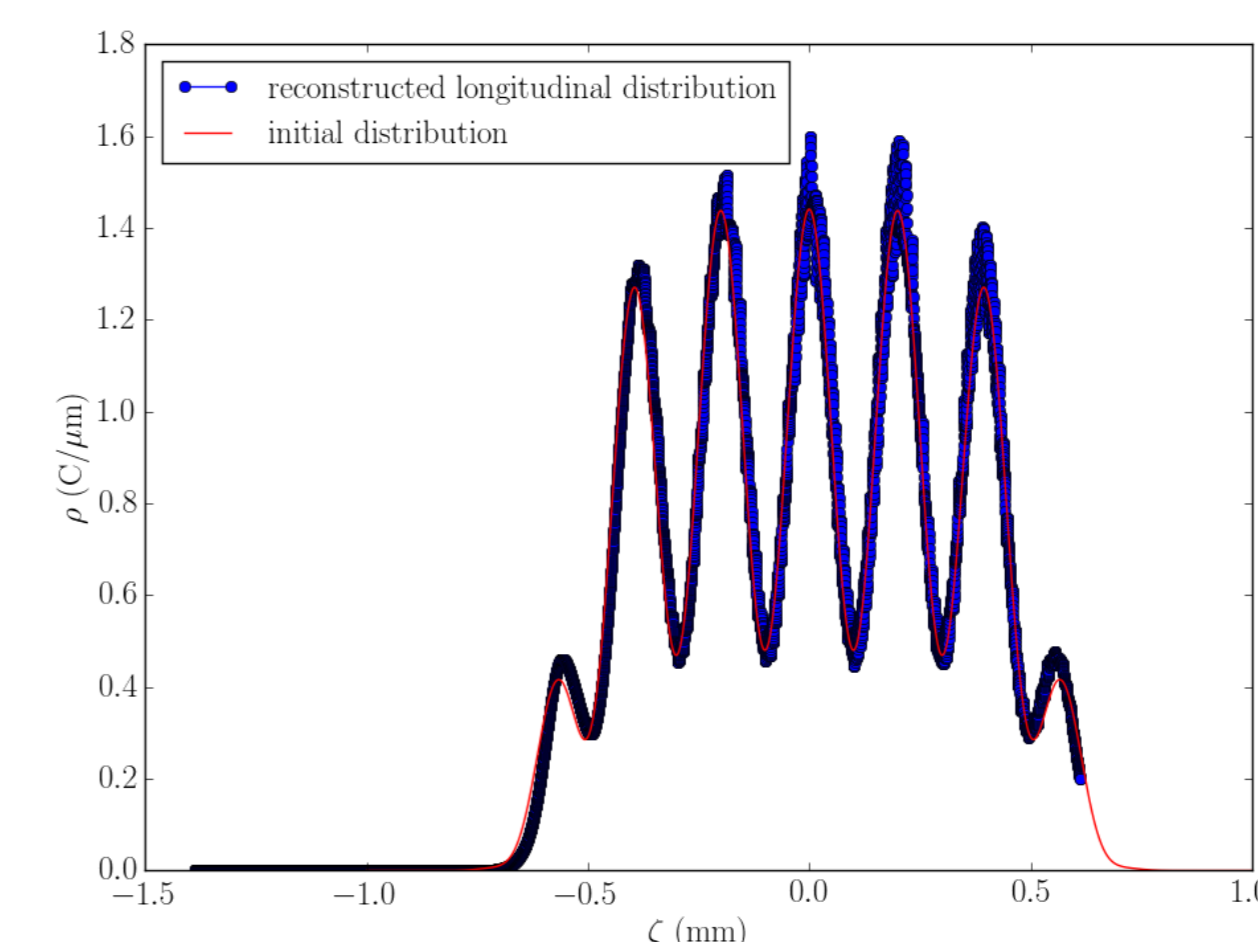


Figure 2: Comparison between the initial longitudinal distribution and the reconstructed distribution

After the generated beam passes through the waveguide with an offside, the beam is deflected by the self-generated wake field as in Fig. 3

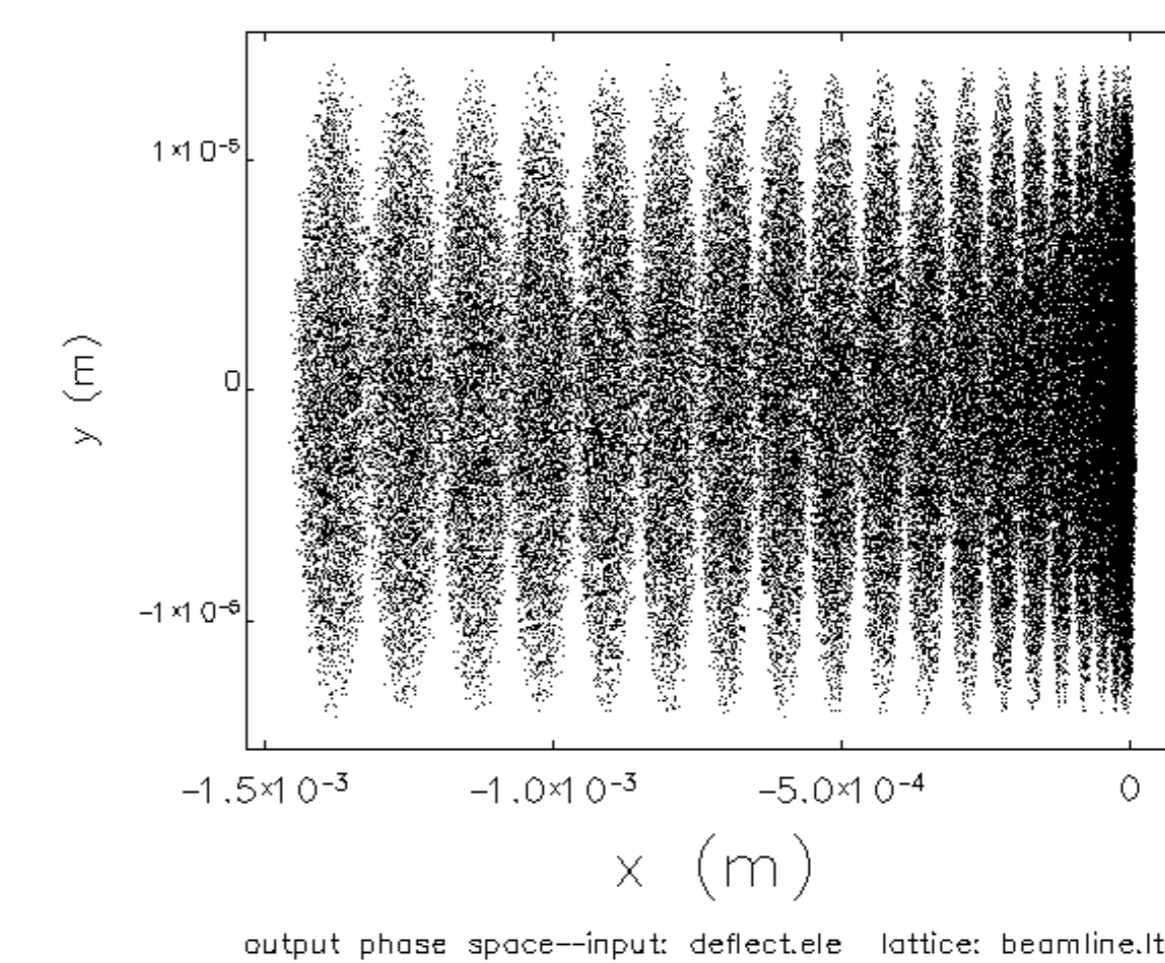


Figure 3: Deflected beam projected on the monitor $x - y$ direction

We then test the proposed method to reconstruct the longitudinal profile of the beam bunch, using the density distribution of the deflected beam and undeflected beam along x on the $x - y$ monitor.

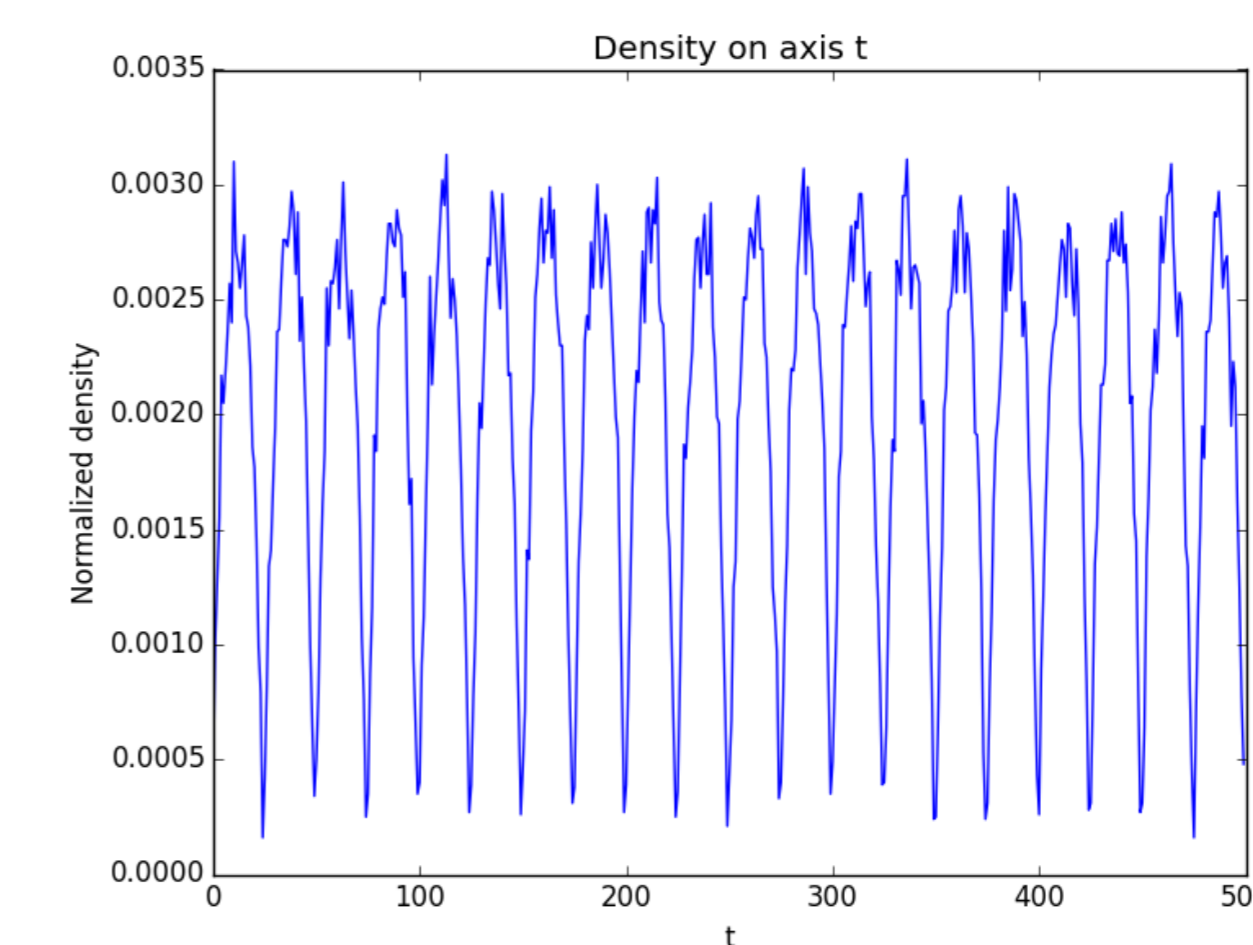


Figure 4: Normalized density along binned t

Conclusions

- Thus, in this paper, we proved the deconvolution and iteration method to extract the longitudinal profile of a ultra-short beam in simulation. The result basically agrees with the measurement of the simulation. Thus, the result of the time-resolved measurement based on the passive deflector of a relativistic beam is valid. Also, other methods need to be explored and tested, and further experiment should be verified.

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