Microbunch Rotation and Coherent Undulator Radiation from a Kicked Electron Beam

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Preview

Motivation

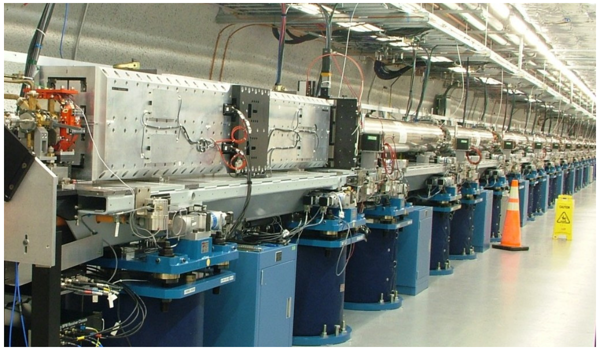
- Beam diversion:
  unexpectedly successful method of isolating circular x-rays

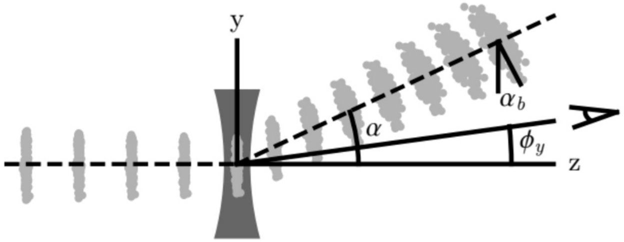
Rotation

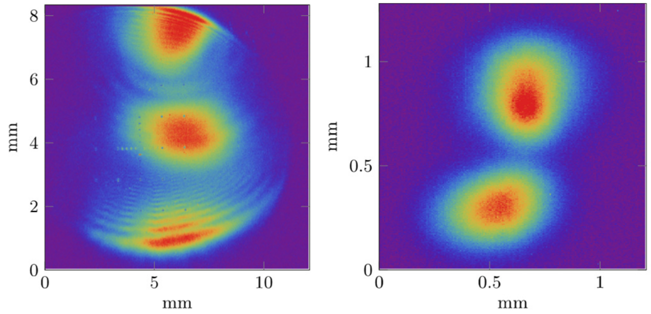
- Microbunch dynamics in a FODO lattice
- Comparison between LCLS data, FEL theory, Genesis, and Osiris

Extensions

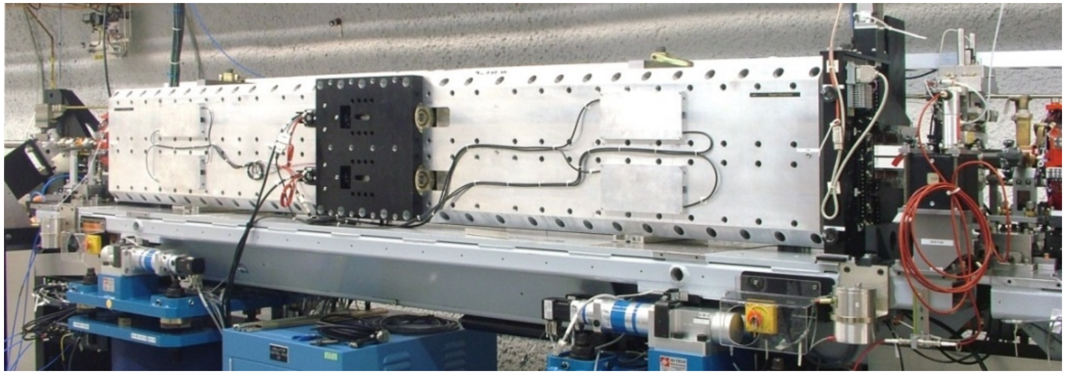
- Multiplexing at soft x-rays: 3 beams
- Multiplexing at hard x-rays: 2 beams
- Outcoupling from a cavity



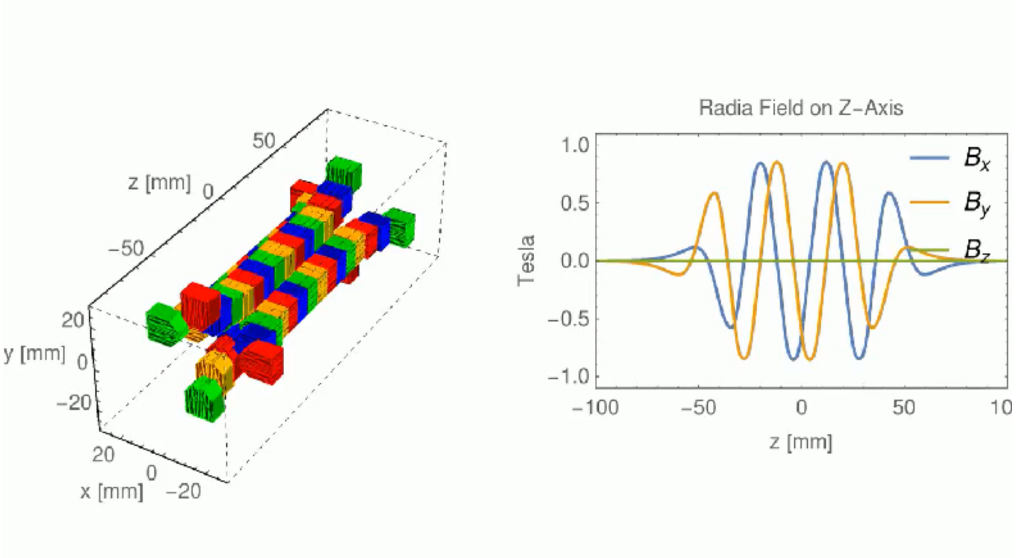




Delta Undulator for LCLS-I

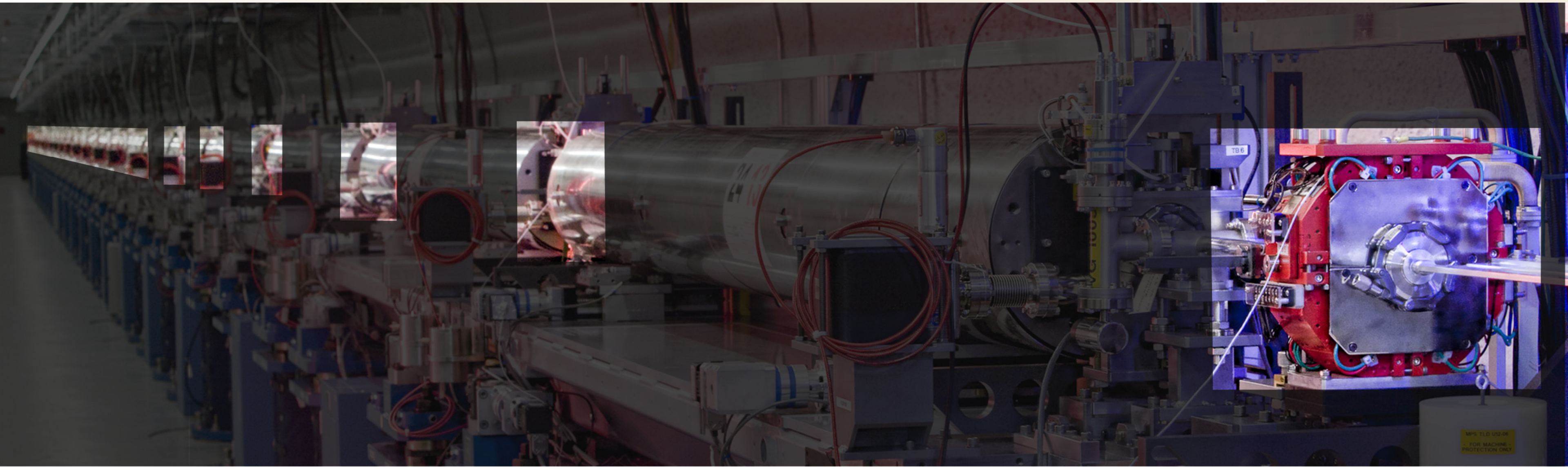


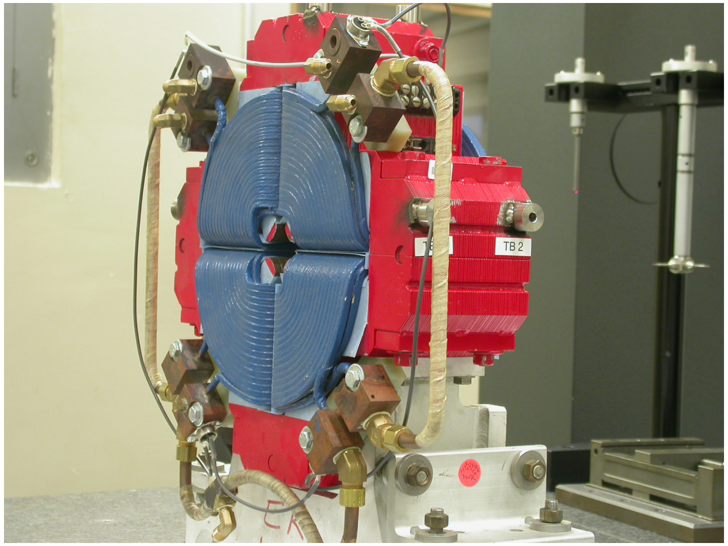
3.2 m



[Radia simulation]

LCLS Undulator Magnetic Lattice

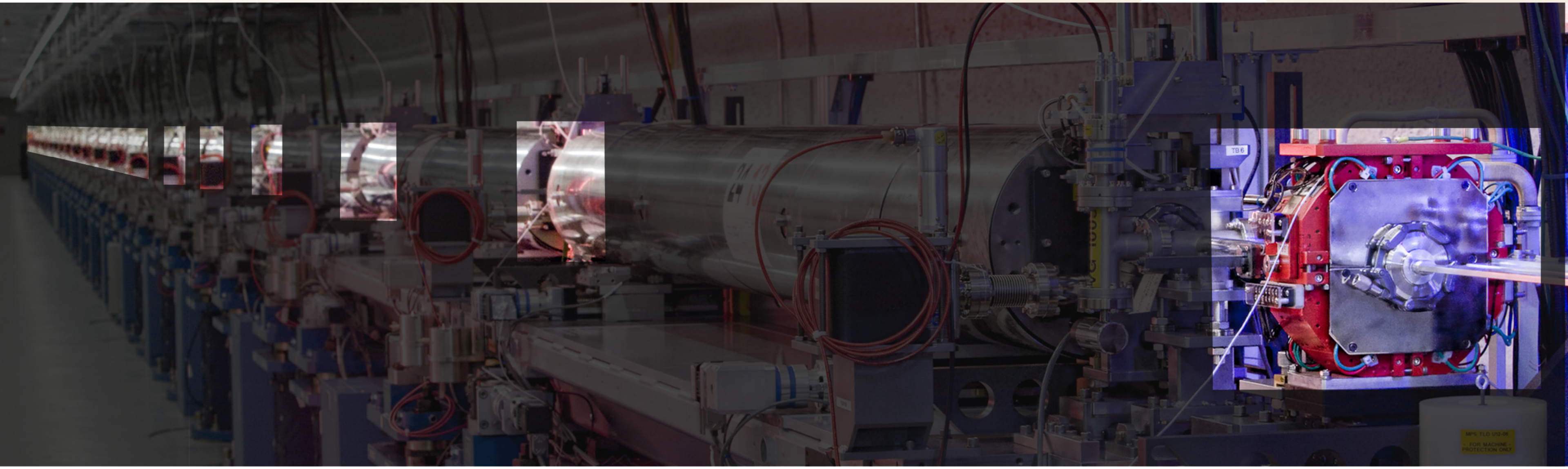


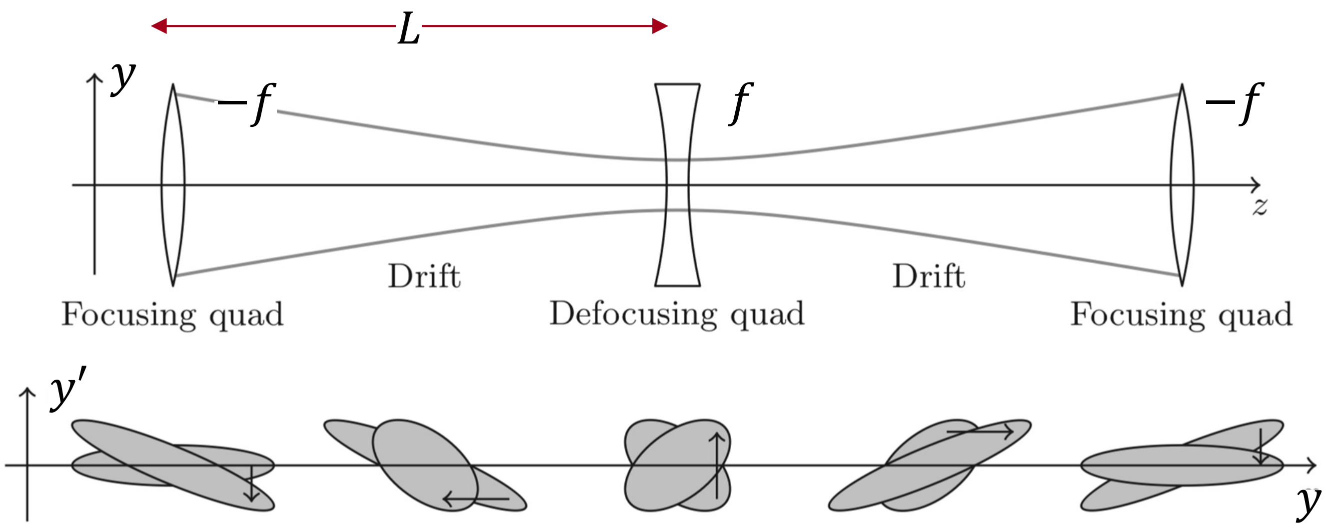


Magnetic Quadrupole & Corrector

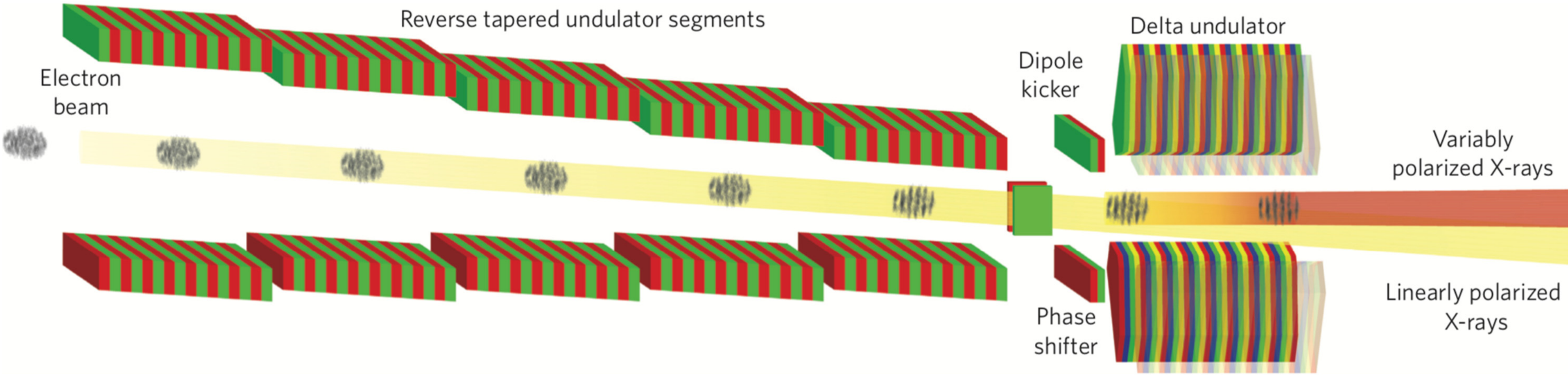
- Tunable -40 kG to 40 kG quadrupole
  - Focal length ~ 4 meters for 500 eV operation
- Tunable -6 G·m to +6 G·m dipole
  - Max kick ~ ±55 uRad for 500 eV operation
- Quadrupole may also be offset to provide additional dipole kick

LCLS Undulator Magnetic Lattice





Isolated Circular Beam



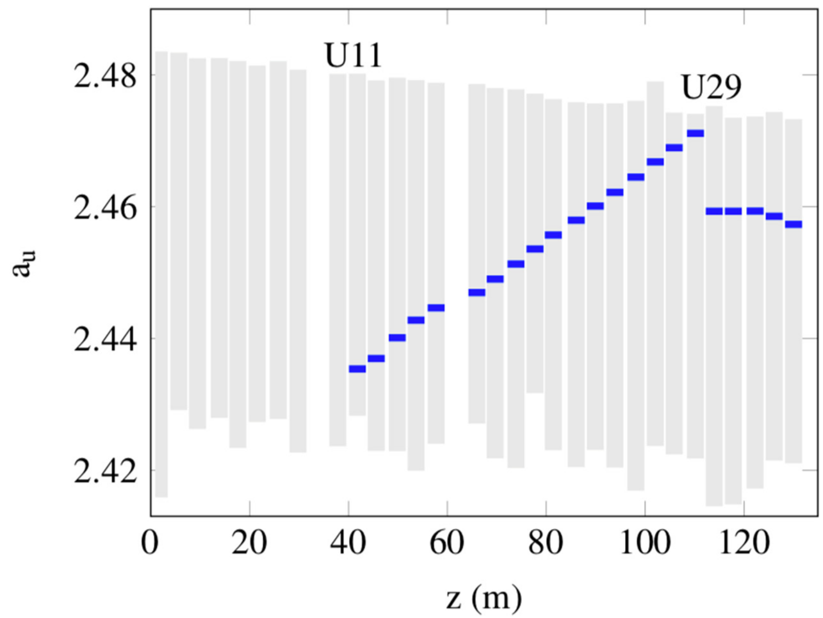
Step 1: reverse taper

- *Schneidmiller & Yurkov, PRSTAB* ***16****, 110702 (2013)*

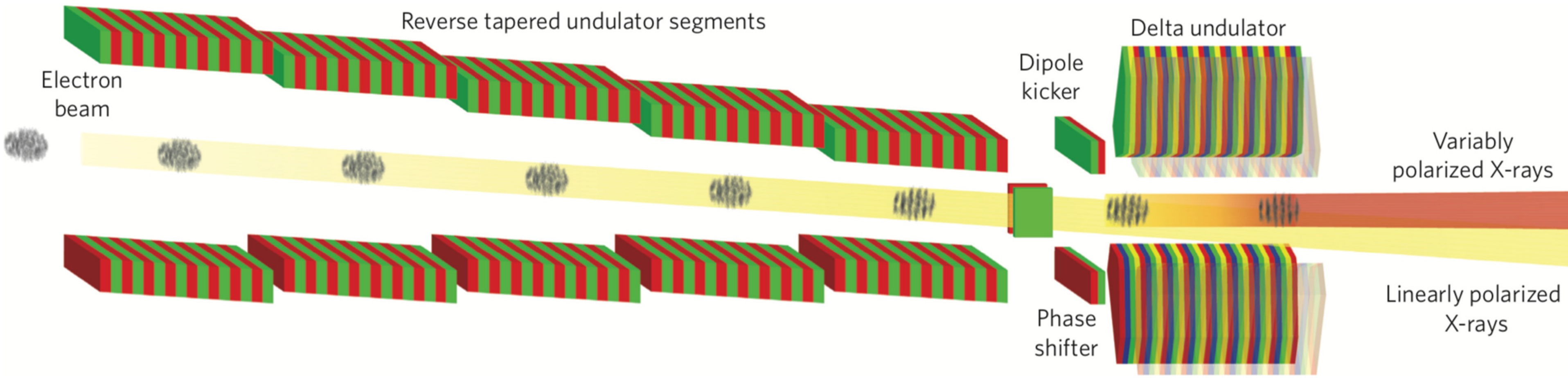
$$\left|\frac{b_\nu}{a_\nu}\right| \approx \Delta\nu \propto \Delta K / K_0$$

- Energy spread constraint (*MacArthur et al., WEP004 FEL 2015*)

$$\left|\frac{b_\nu}{a_\nu}\right| \approx \Delta\nu \leq \frac{\rho^2}{(\Delta\eta)^2}$$

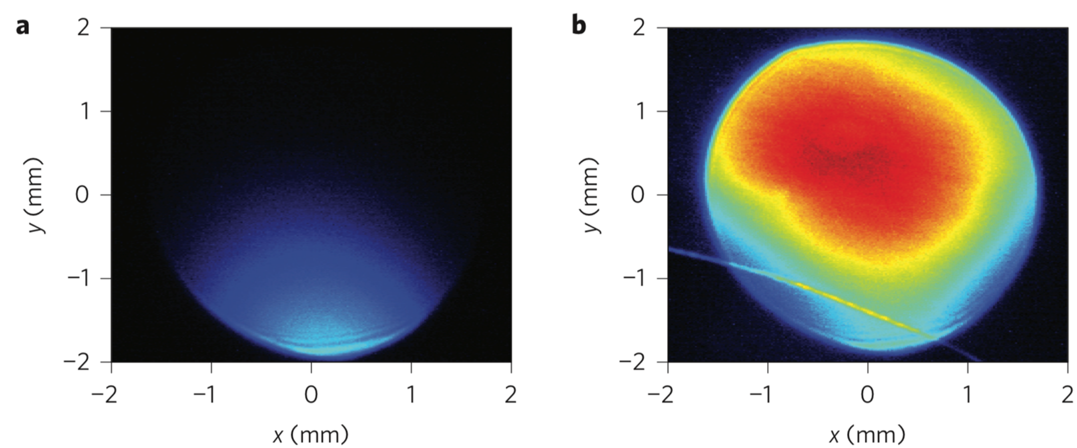


Isolated Circular Beam

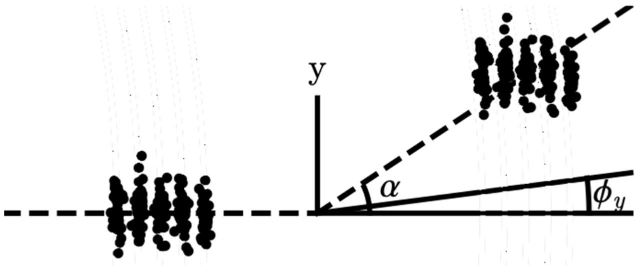


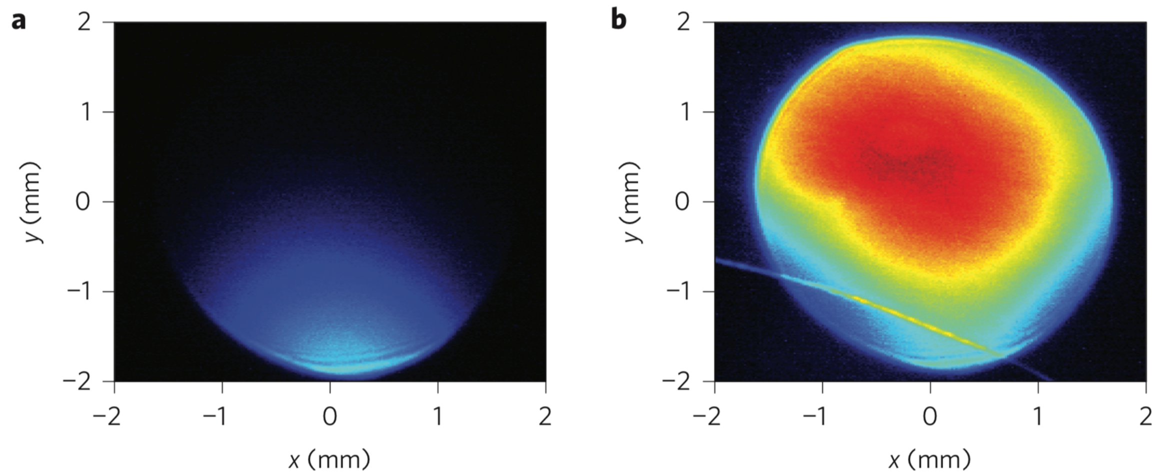
Step 2: beam diversion:

- Transverse kick before Delta
- Results in *Lutman, MacArthur et al., Nat. Photonics* ***10*** *468 (2016)*

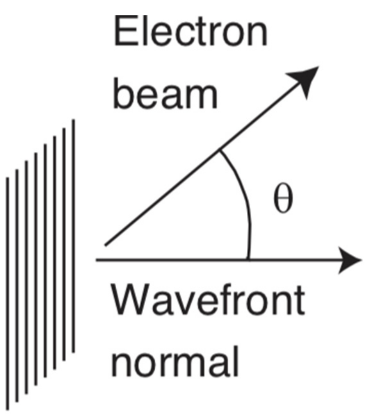


An Unexpected Success

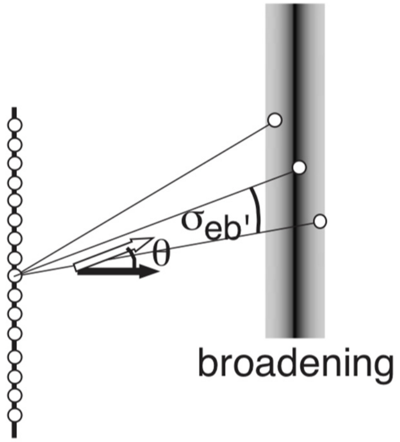




Radiation Suppression

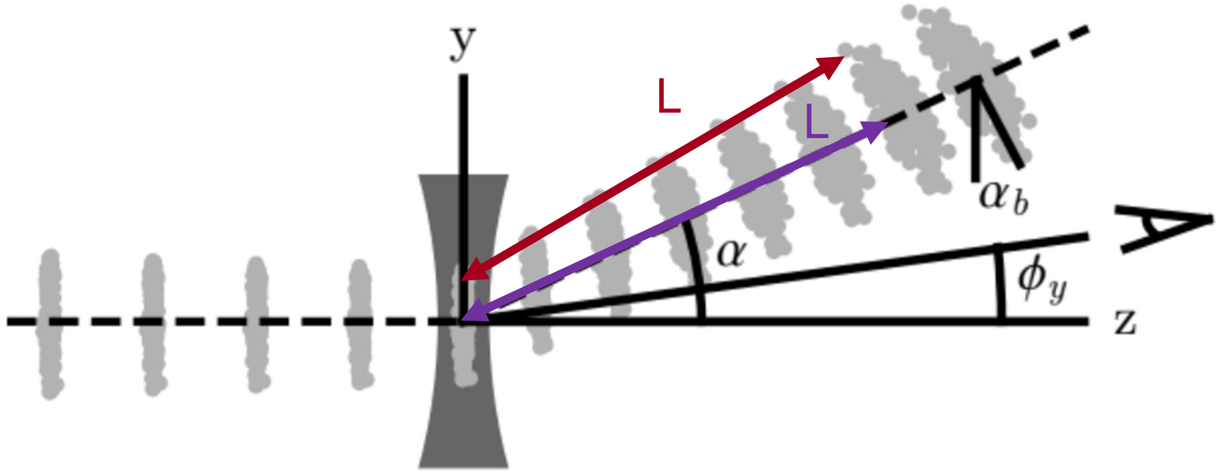


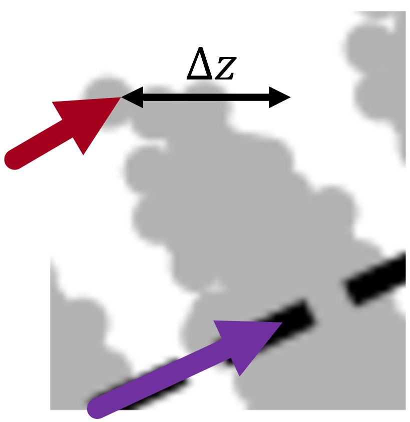
Microbunch Smearing



[T. Tanaka et al., NIMA **528** 172 (2004)]

Intuitive Picture: The Quadrupole Matters

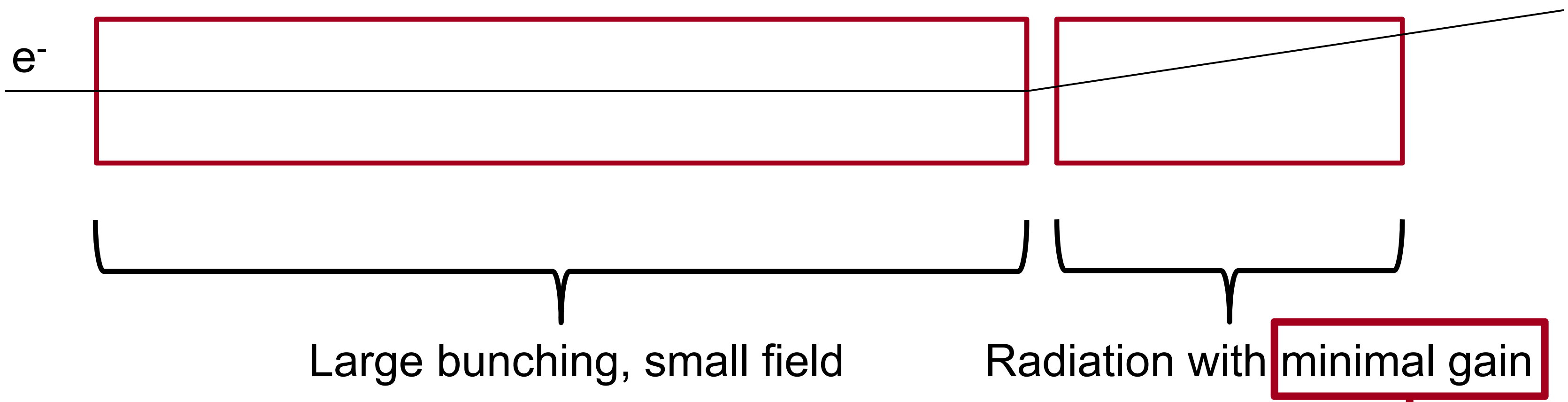




$$\Delta z = L\cos\left(y_0' + y_0/2f + \alpha\right) - L\cos\alpha$$

$$\approx \underbrace{\frac{\alpha L}{2f}}_{\text{tilt }(\alpha_b)} y_0 + \underbrace{\alpha L y_0'}_{\text{smearing}} + \underbrace{\frac{L}{2}\left(\frac{1}{2f}y_0 + y_0'\right)^2}_{\text{curvature, other}}$$

A More Mathematica Approach



Field growth:

$$\left[\frac{\partial}{\partial z} + i\Delta\nu k_u + \frac{ik}{2}\phi^2\right]\tilde{E}_\nu = -\kappa_1 n_e \int d\mathbf{x}' d\eta \tilde{F}_\nu$$

Microbunch evolution:

$$\left[\frac{d}{dz} + i\left(2\nu\eta k_u - \frac{k}{2}\mathbf{x}'^2\right)\right] F_\nu = -\chi_1 E_\nu \frac{\partial \bar{F}}{\partial \eta},$$

~0

MacArthur et al., PRX **8**, 041036 (2018)

Result #1: The Bunching Rotation Angle

With no field interaction and the trajectories given before, the bunching has an analytic expression. It's ugly…

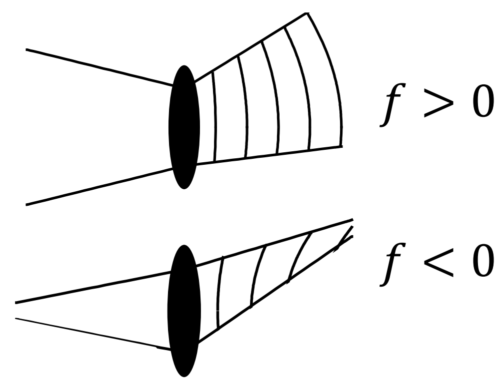
$$\left[\frac{d}{dz} + i\left(2\nu\eta k_u - \frac{k}{2}\mathbf{x}'^2\right)\right] F_\nu = -\chi_1 E_\nu \frac{\partial \bar{F}}{\partial \eta},$$

~0

$$\frac{b_\nu(\phi_x = 0, \phi_y; z)}{b_\nu(\phi_x = 0, \phi_y; 0)} = \frac{e^{-2(k_u \sigma_\eta \nu z)^2 + \frac{|f| k (i\psi + \zeta)}{\sqrt{1 - \hat{L}^2 - 2i\hat{\epsilon}|\hat{z}|}}}}{i + \frac{2\hat{\epsilon}|\hat{z}|}{\sqrt{1 - \hat{L}^2}}}$$

$|b(\phi)|$ is maximized at an angle α_b that matches the geometric prediction $\left(\hat{z} = z/2f, \hat{L} = L_u/2f, \hat{\epsilon} = k\epsilon\right)$

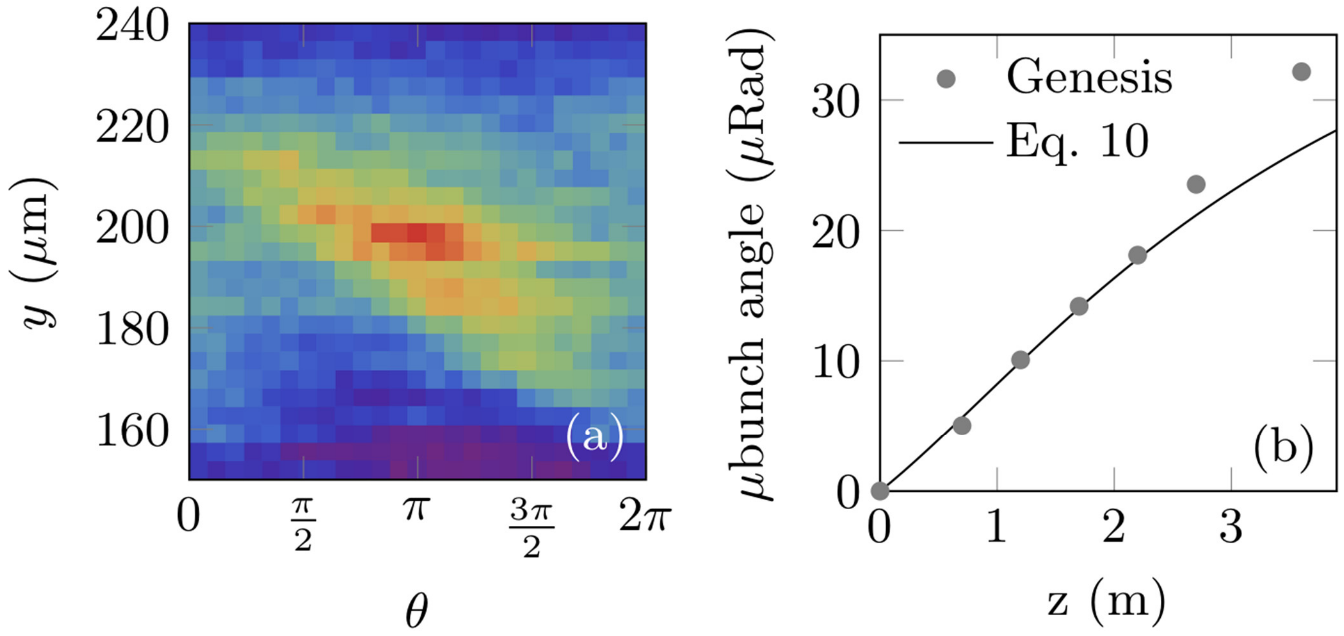
$$\hat{\epsilon}^2 \hat{L}^2 \ll 1 - \hat{L}^2 \Rightarrow \boxed{\alpha_b(\hat{z} = \hat{L}) \approx \alpha \hat{L} = \frac{\alpha L_u}{2f}}$$



Genesis Comparison

The previous result can be compared with Genesis simulations

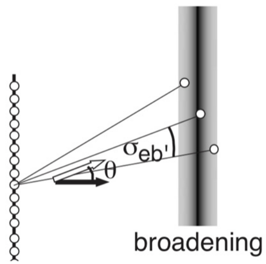
$$\alpha_b(\hat{z} = \hat{L}) \approx \alpha \hat{L} = \frac{\alpha L_u}{2f}$$



Result #2: A New Critical Angle

Tanaka showed that the bunching is suppressed when kicked,

$$|b_\nu(\phi_x = 0, \phi_y = \alpha_b; z)|^2 \propto e^{-\alpha^2/\phi_c^2}$$



A larger critical angle when bunches rotate:

$$\phi_c^2 = \sigma_{y'}^2 + \frac{\sigma_y^2}{\epsilon_y^2 k^2 z^2} \quad \Rightarrow \quad \phi_c^2 = s_{y'}^2 + \frac{s_y^2}{\epsilon_y^2 k^2 z^2}$$

Tanaka critical angle | New critical angle

The RMS beam size and divergence after a drift z replace the beam size and divergence in the quad:

$$s_y^2 = (1 + z/2f)^2 \, \sigma_y^2 + \sigma_{y'}^2 z^2$$
$$s_{y'}^2 = \sigma_{y'}^2 + \sigma_y^2/(2f)^2$$

Result #3: The Field Produced By A Rotating Bunch

Plug in expression for microbunch dynamics to field equation

$$\left[\frac{\partial}{\partial z} + i\Delta\nu k_u + \frac{ik}{2}\phi^2\right]\tilde{E}_\nu = -\kappa_1 n_e \int d\mathbf{x}' d\eta \tilde{F}_\nu$$

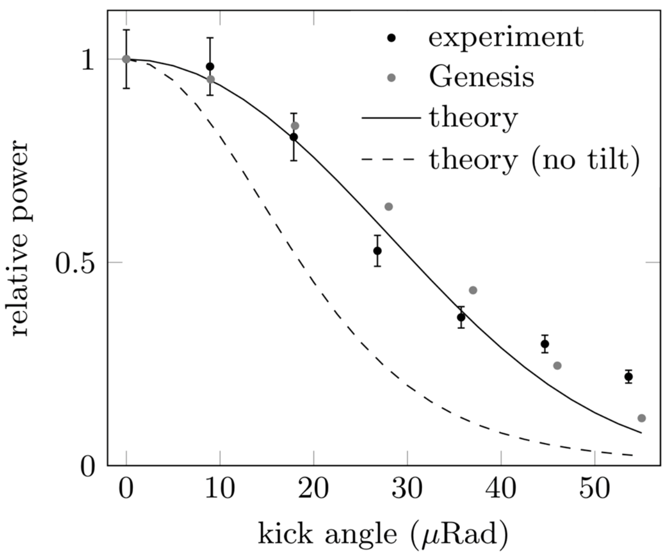
$$\left[\frac{d}{dz} + i\left(2\nu\eta k_u - \frac{k}{2}\mathbf{x}'^2\right)\right] F_\nu = -\chi_1 E_\nu \frac{\partial \bar{F}}{\partial \eta},$$

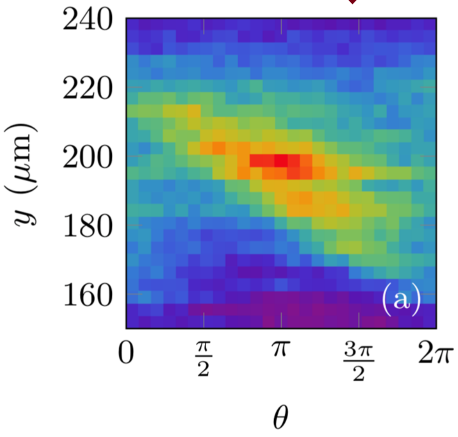
Result: equation showing the field grows with z:

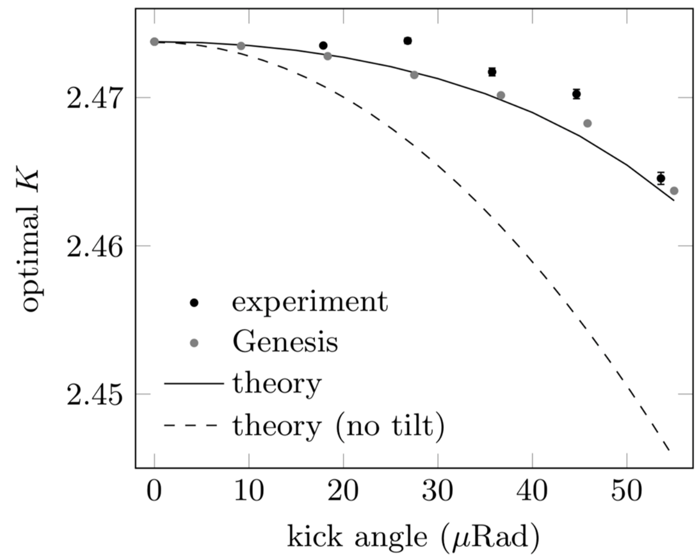
$$\frac{\partial \tilde{E}_\nu}{\partial z} \propto e^{-\frac{\alpha^2}{2\phi_c^2} - \frac{1}{2}\left(k^2 z \epsilon \phi_c (\phi_y - \alpha_b)\right)^2 + i\left(k_u \Delta\nu + k\frac{(\alpha-\phi_y)^2}{2}\right)z}$$

Result #3: More Radiation Off-Axis, Less Detune

$$\frac{\partial \tilde{E}_\nu}{\partial z} \propto e^{-\frac{\alpha^2}{2\phi_c^2} - \frac{1}{2}\left(k^2 z \epsilon \phi_c (\phi_y - \alpha_b)\right)^2 + i\left(k_u \Delta\nu + k\frac{(\alpha-\phi_y)^2}{2}\right)z}$$







Validity of FEL Equation Approach

SLAC

- Genesis & FEL theory approach solve the same equations, are they correct?
- Are relativistic kinematic effects included in the simple kicked beam analysis?
- E. Saldin suggests Wigner rotation may change the resonant condition:

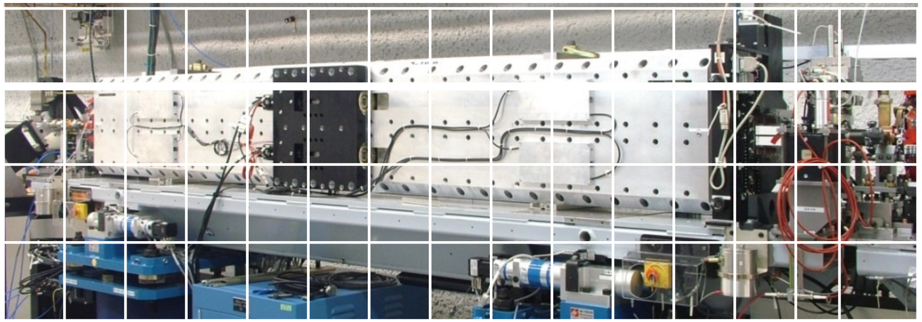
Relativity and Synchrotron Radiation: Critical Reexamination of Existing Theory

Evgeny Saldin [a]

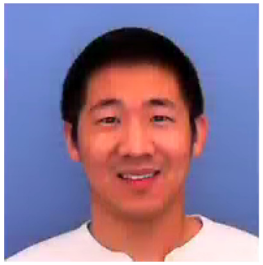
arXiv:1903.07452v1

A PIC Code with Fewer Assumptions

- Osiris is a general purpose PIC code (see also WarpX)
- Problem:
  - $\lambda_u = 3\ cm$
  - $\lambda_r = 3\ nm$
  - $L_u = 3\ m$

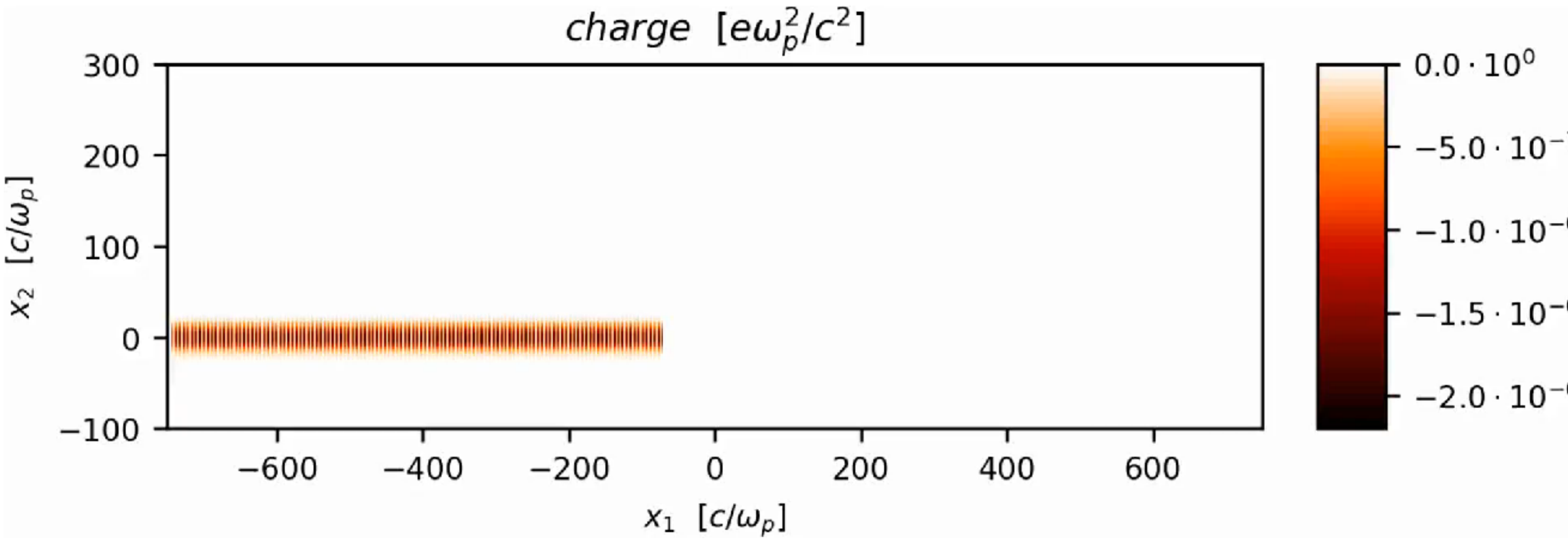


- Solution: transform to the average rest frame of the electron beam (credit to to X. Xu)

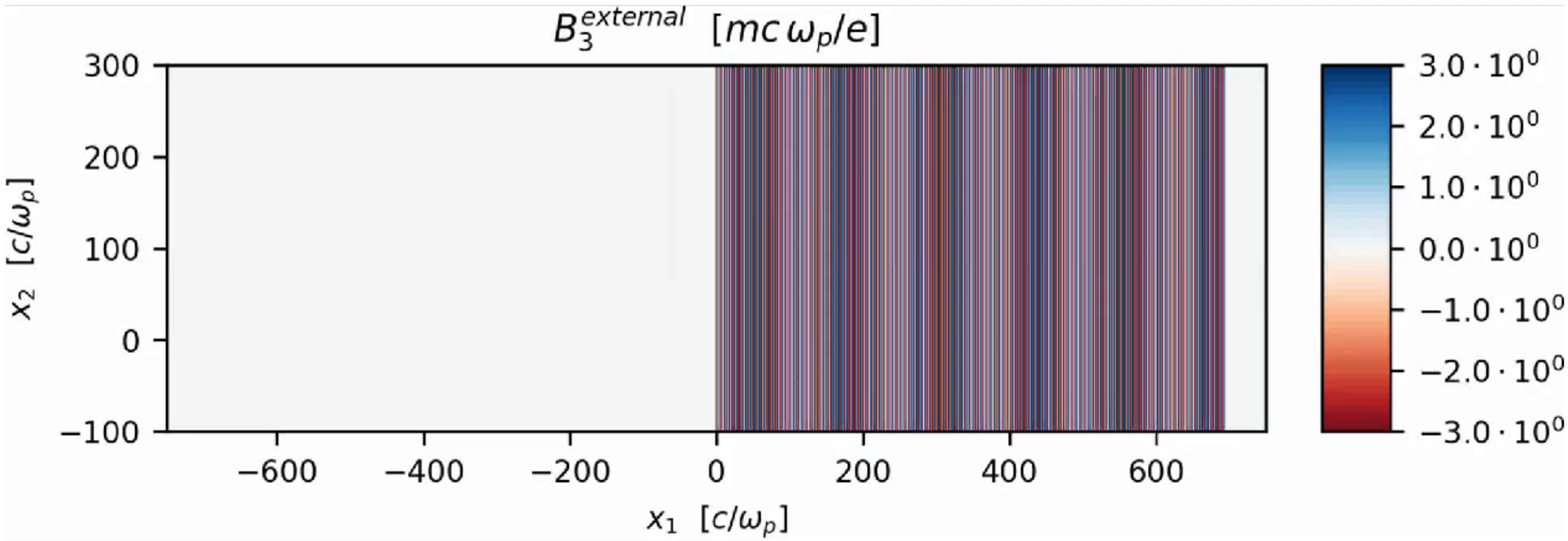


- Relevant parameters:

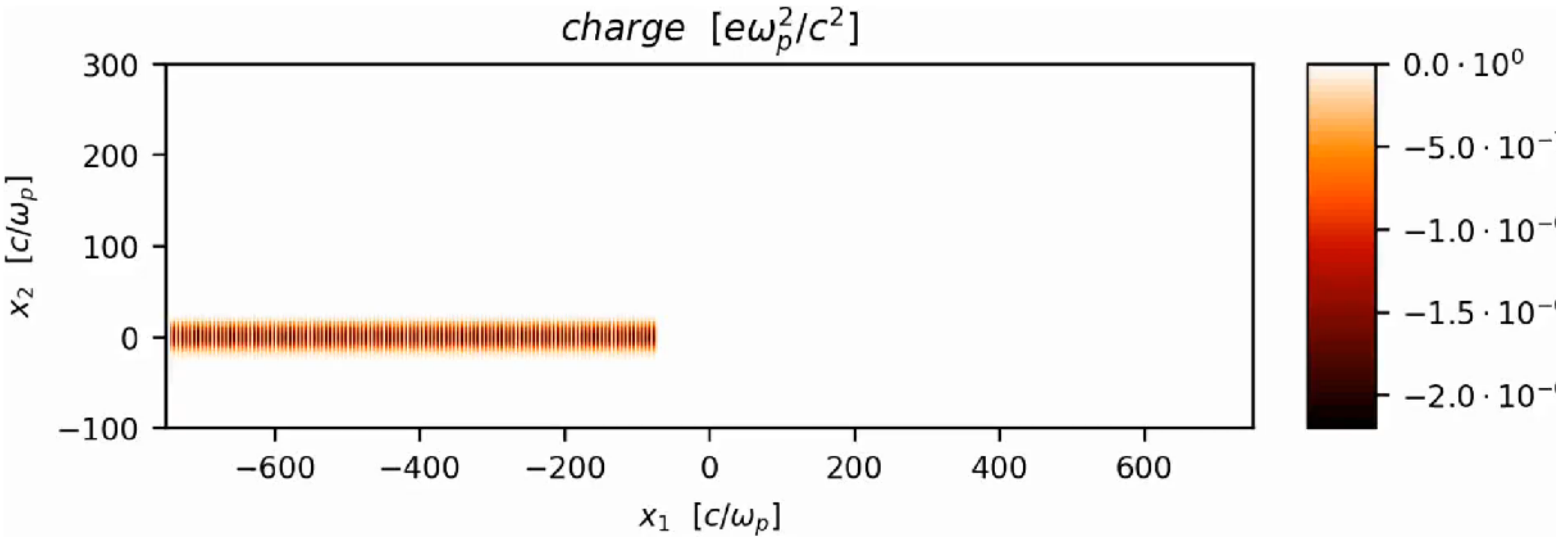
$$\gamma_z = \frac{\gamma}{\sqrt{1+a_u^2}} \approx 2500 \qquad \lambda_u \rightarrow \lambda_u' = \frac{\lambda_u}{\gamma_z} \approx 12\ um \qquad \lambda_{rad} \rightarrow \lambda_{rad}' \approx 2\ \gamma_z \lambda_{rad} = \lambda_u'$$

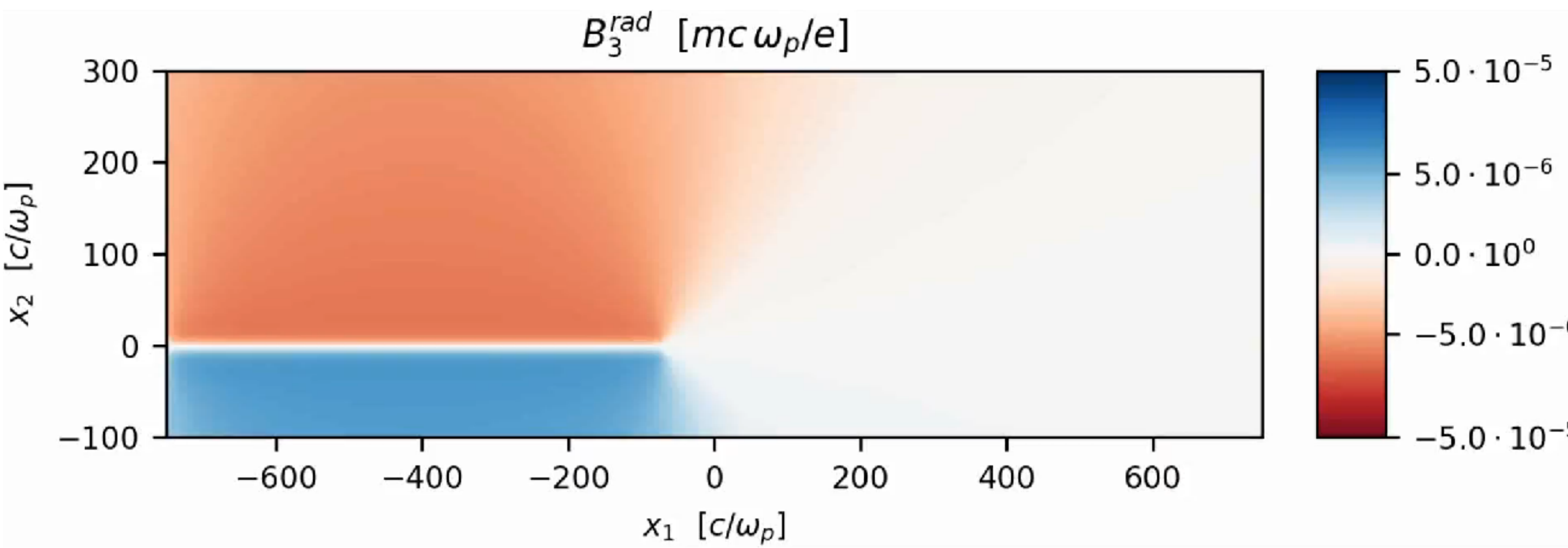


charge [eω_p²/c²]
300
200
100
0
−100
−600
−400
−200
0
200
400
600
x₁ [c/ω_p]
x₂ [c/ω_p]
0.0 · 10⁰
−5.0 · 10⁻⁷
−1.0 · 10⁻⁶
−1.5 · 10⁻⁶
−2.0 · 10⁻⁶

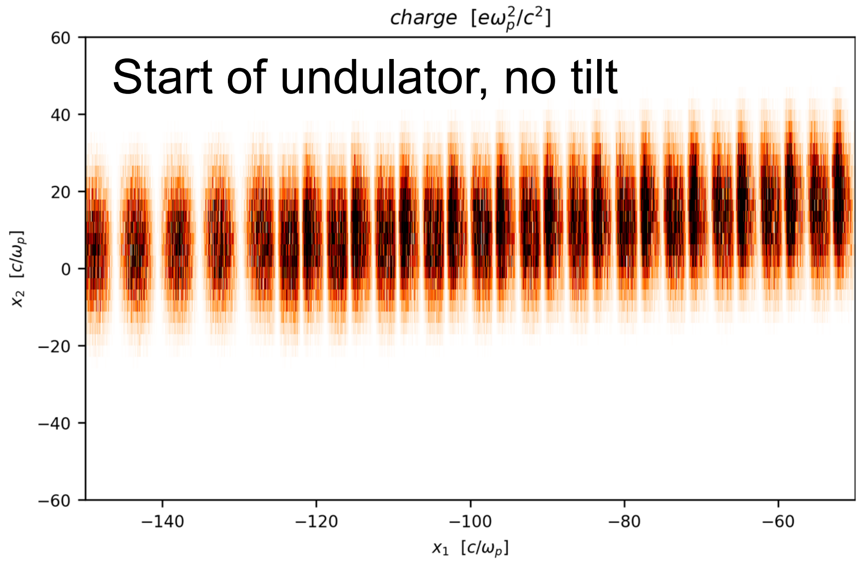


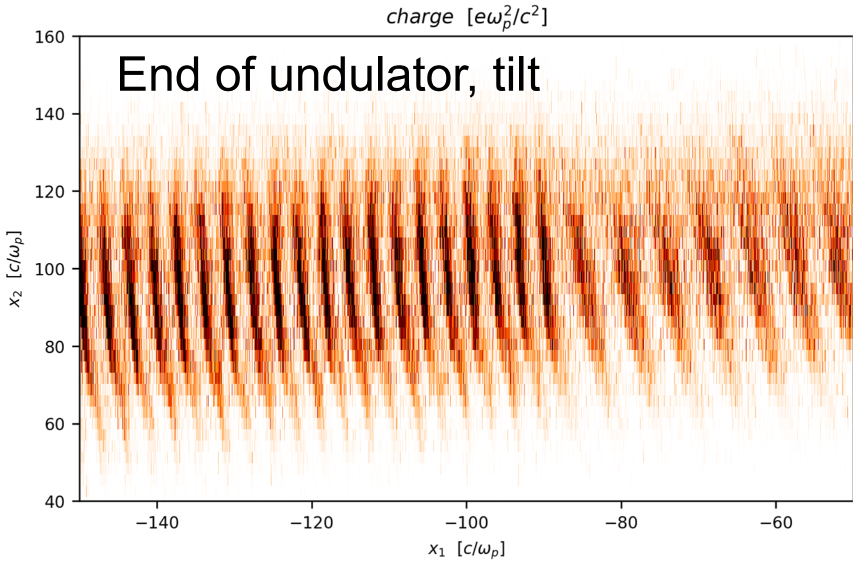
B₃^external [mc ω_p/e]
300
200
100
0
−100
−600
−400
−200
0
200
400
600
x₁ [c/ω_p]
x₂ [c/ω_p]
3.0 · 10⁰
2.0 · 10⁰
1.0 · 10⁰
0.0 · 10⁰
−1.0 · 10⁰
−2.0 · 10⁰
−3.0 · 10⁰

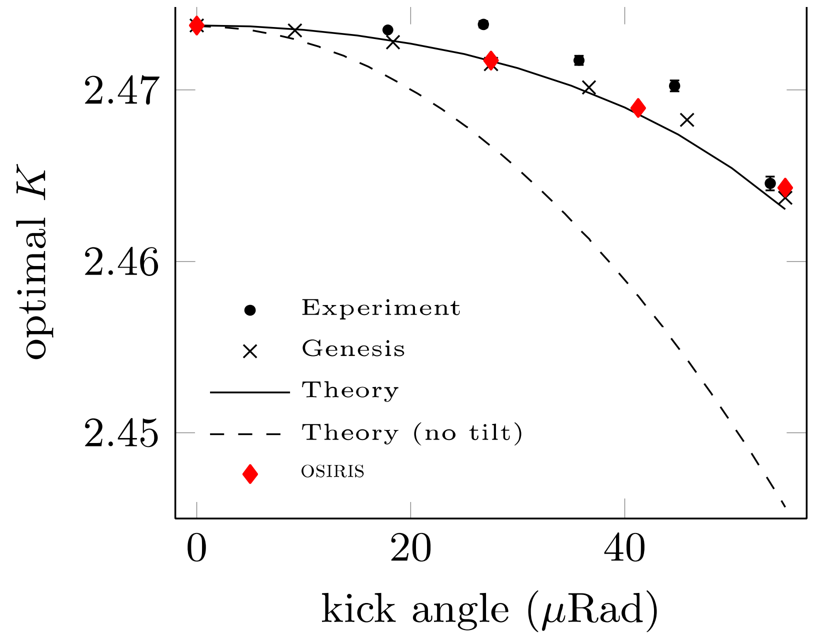

$charge \ [e\omega_p^2/c^2]$
$x_2 \ [c/\omega_p]$
300
200
100
0
−100
−600
−400
−200
0
200
400
600
$x_1 \ [c/\omega_p]$
$0.0\cdot10^{0}$
$-5.0\cdot10^{-7}$
$-1.0\cdot10^{-6}$
$-1.5\cdot10^{-6}$
$-2.0\cdot10^{-6}$


$B_3^{rad} \ [mc\,\omega_p/e]$
$x_2 \ [c/\omega_p]$
300
200
100
0
−100
−600
−400
−200
0
200
400
600
$x_1 \ [c/\omega_p]$
$5.0\cdot10^{-5}$
$5.0\cdot10^{-6}$
$0.0\cdot10^{0}$
$-5.0\cdot10^{-6}$
$-5.0\cdot10^{-5}$

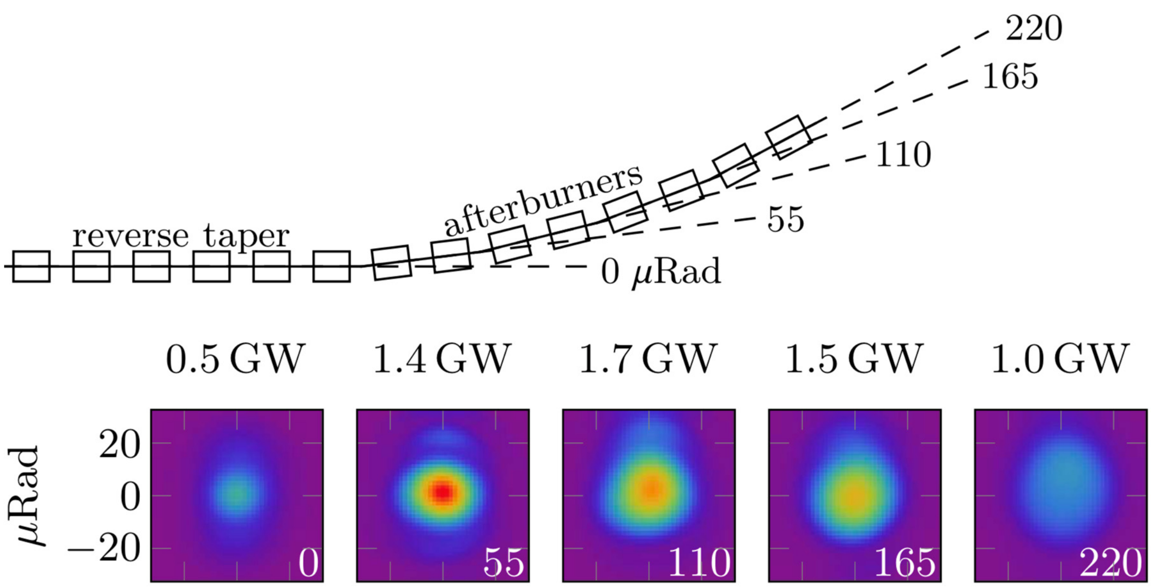
Rotation, Comparison With FEL Theory





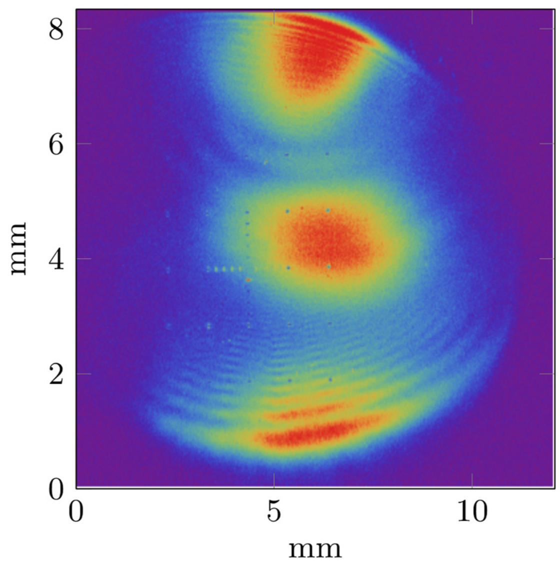


Soft X-ray Multiplexing (530 eV)

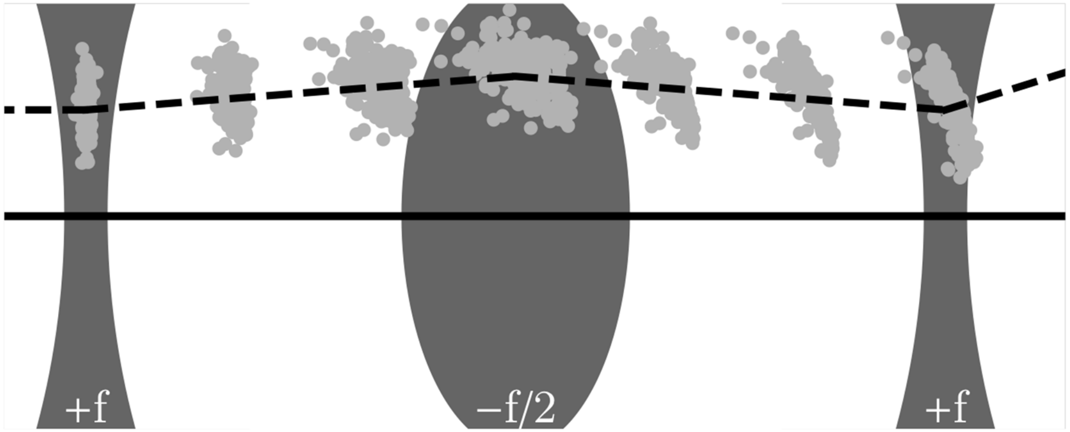


Simulation, 530 eV

Experiment, 500 eV, 90 uJ Total



Rotation at Hard X-ray is… Hard

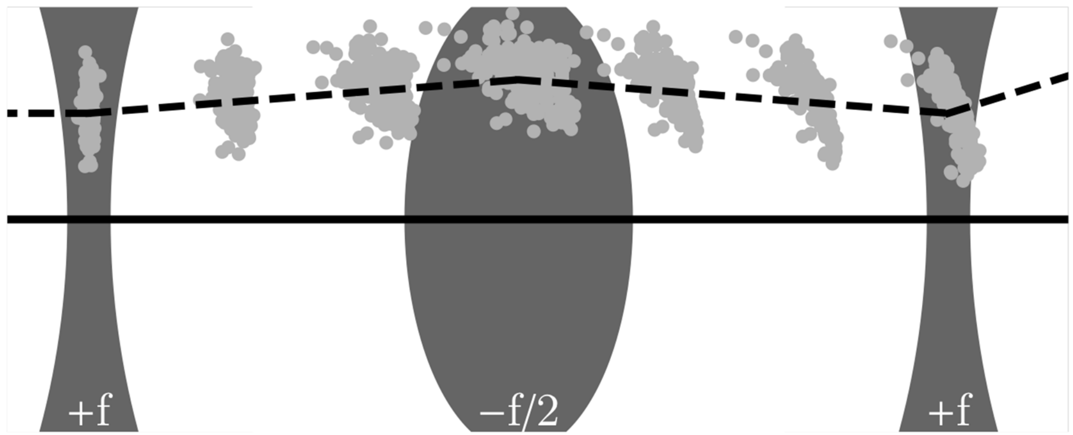


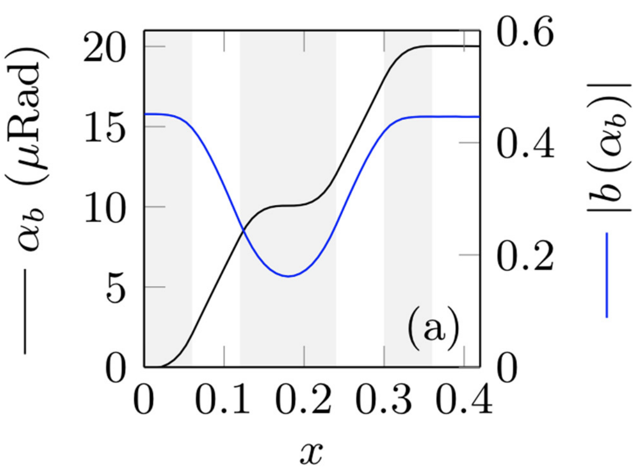
$$\Delta z \approx \frac{\alpha L}{2f} y_0 + \boxed{\alpha L y_0'} + \boxed{\frac{L}{2}\left(\frac{1}{2f} y_0 + y_0'\right)^2}$$

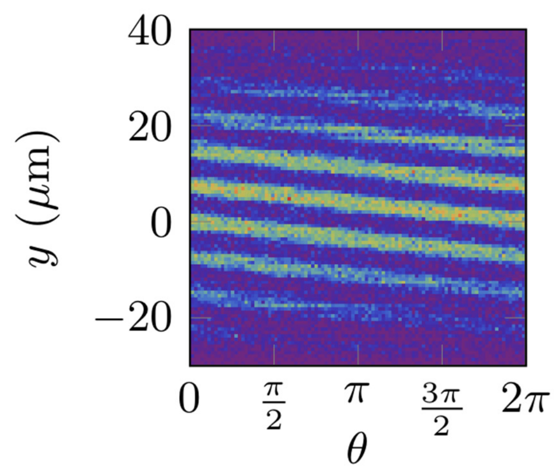
Cancelled in second drift

Small for small z/f

10x Rotation, Minimal Degradation

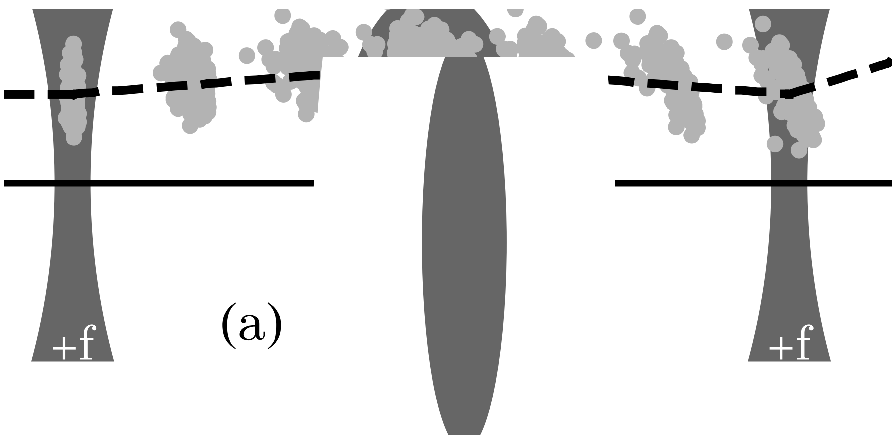




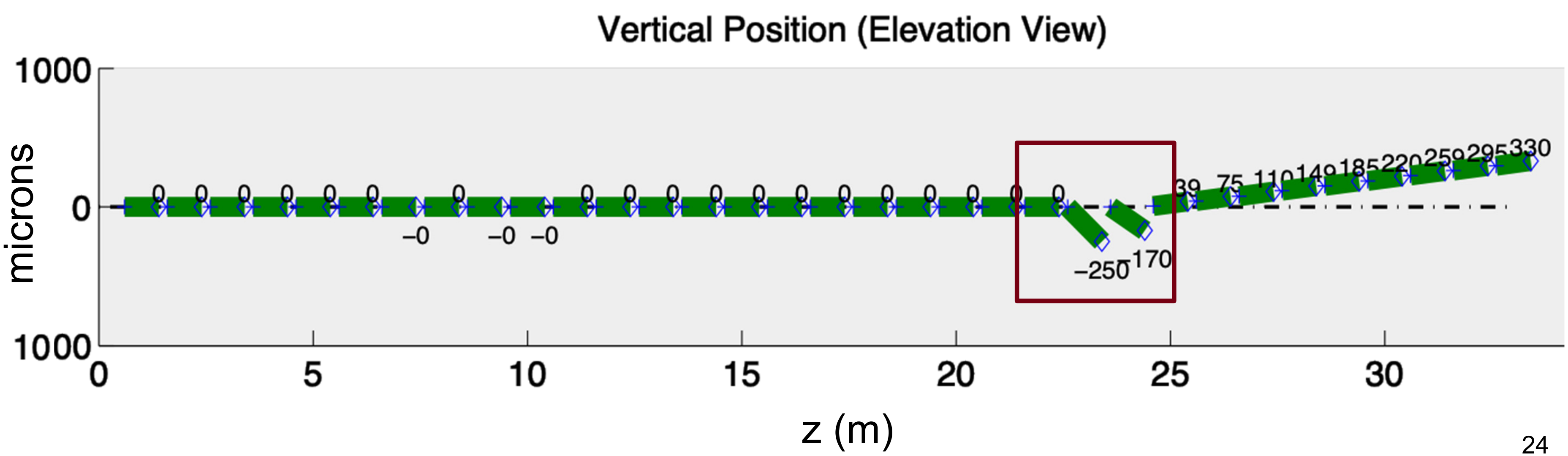


~20 uRad rotation, 10x critical angle

Girder Gymnastics

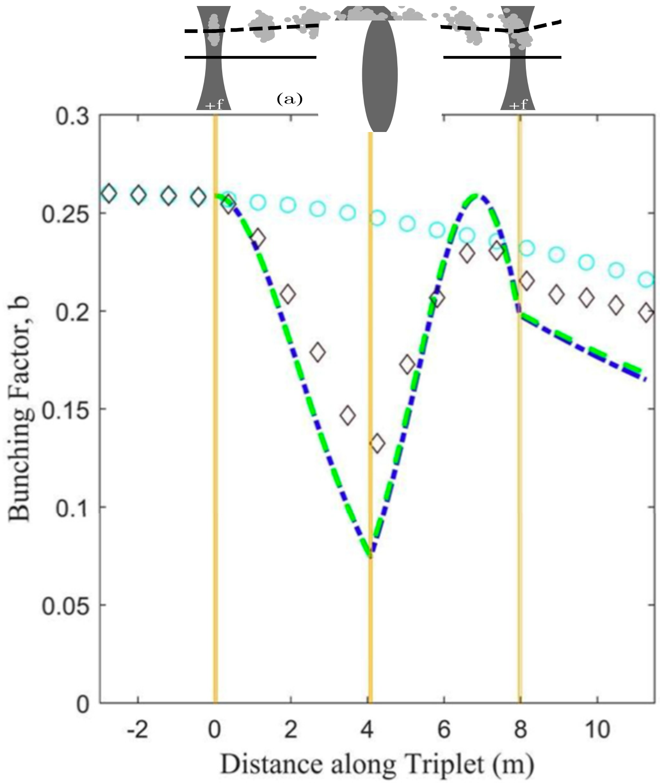


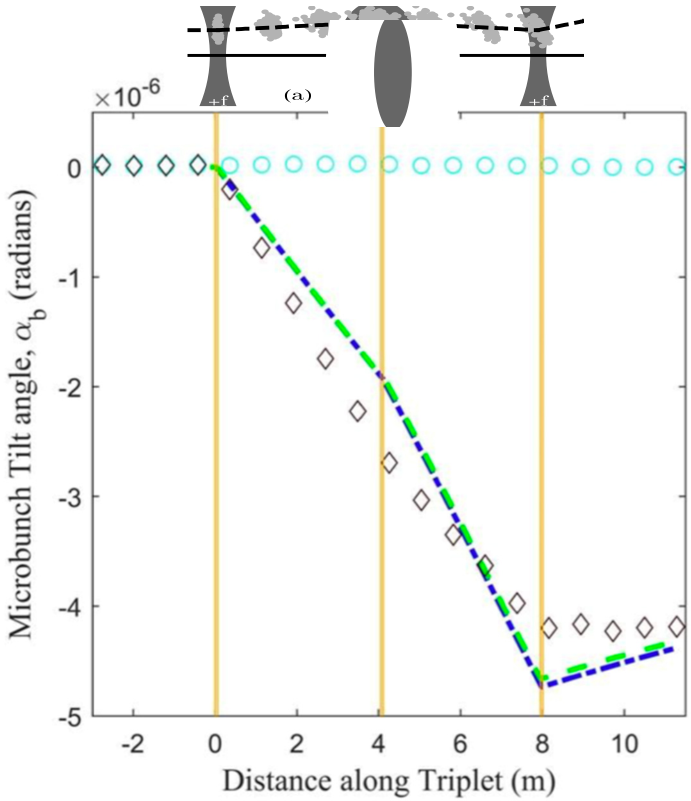
U22 end: Beam kicked up with dipole corrector. Beam defocused.
U23 end: Beam kicked down with mis-aligned quad. Beam focused.
U24 end: Beam kicked along new u-bunch direction with quad. Beam defocused



Bunch Evolution

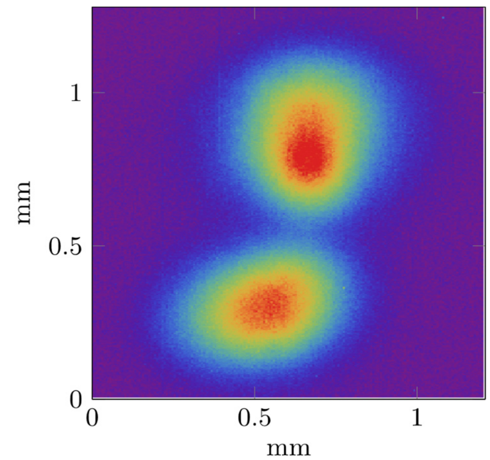
- R. Margraf and X.J. Deng have developed a matrix optics approach to understand the offset quadrupole triplet system.
- See Rachel's poster, **THP036**, for details.





Hard X-Ray Microbunch Rotation Experiment

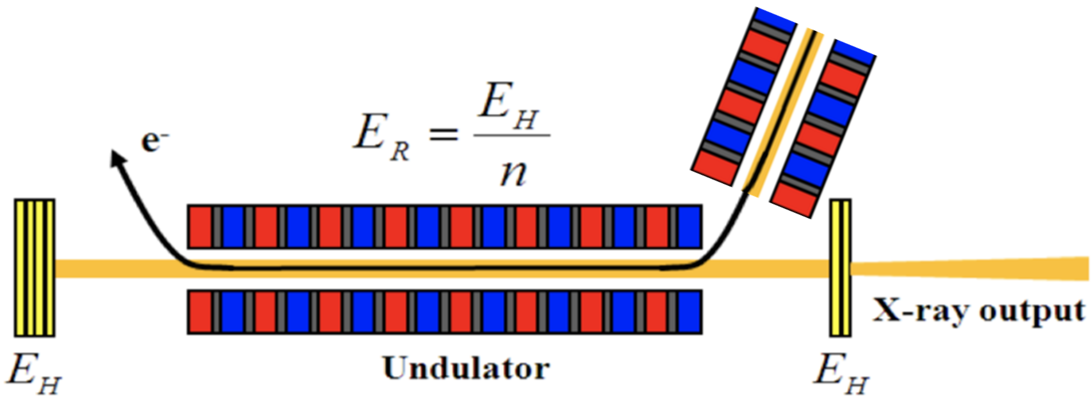
- Dec 19, 2018 (last day of LCLS-I)
- Two 9.5 keV spots produced
- 850 uJ, split equally between spots
- ~5 uRad separation

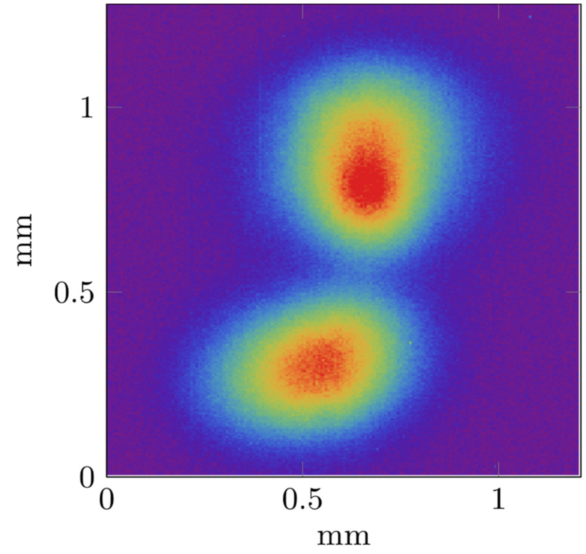


- Ongoing work:
  - R. Margraf (1st year grad student) is analyzing this problem from a beam-optics & simulation perspective – more rotation may be possible!
  - E. Fiadonu (summer student) is analyzing the detector and orbit data from the experiment

Oscillator Application

- Another potential (but speculative) application: outcoupling from an oscillator
- See "*Cavity-Based Free-Electron Laser Research and Development: A Joint Argonne National Laboratory and SLAC National Laboratory* Collaboration," G. Marcus, **TUD04**, for oscillator discussion.

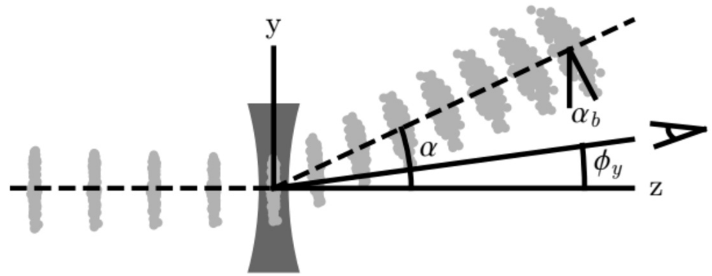




Summary

Rotation

- Microbunches rotate in response to a kick
- FEL theory agrees with Genesis, Osiris, and experimental data



Extensions

- Multiplexing at soft x-rays: 3 beams
- Multiplexing at hard x-rays: 2 beams