Coherence Limits of X-ray FEL Radiation

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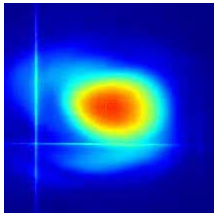
(DESY, Hamburg)

- General overview of coherence properties.
- Eigenmodes and eigenfunctions.
- Coherence properties of the radiation from optimized x-ray FEL.
- Photon beam pointing stability.
- Experience from FLASH.
- Seeding and self-seeding schemes.



Basis for studies of coherence phenomena

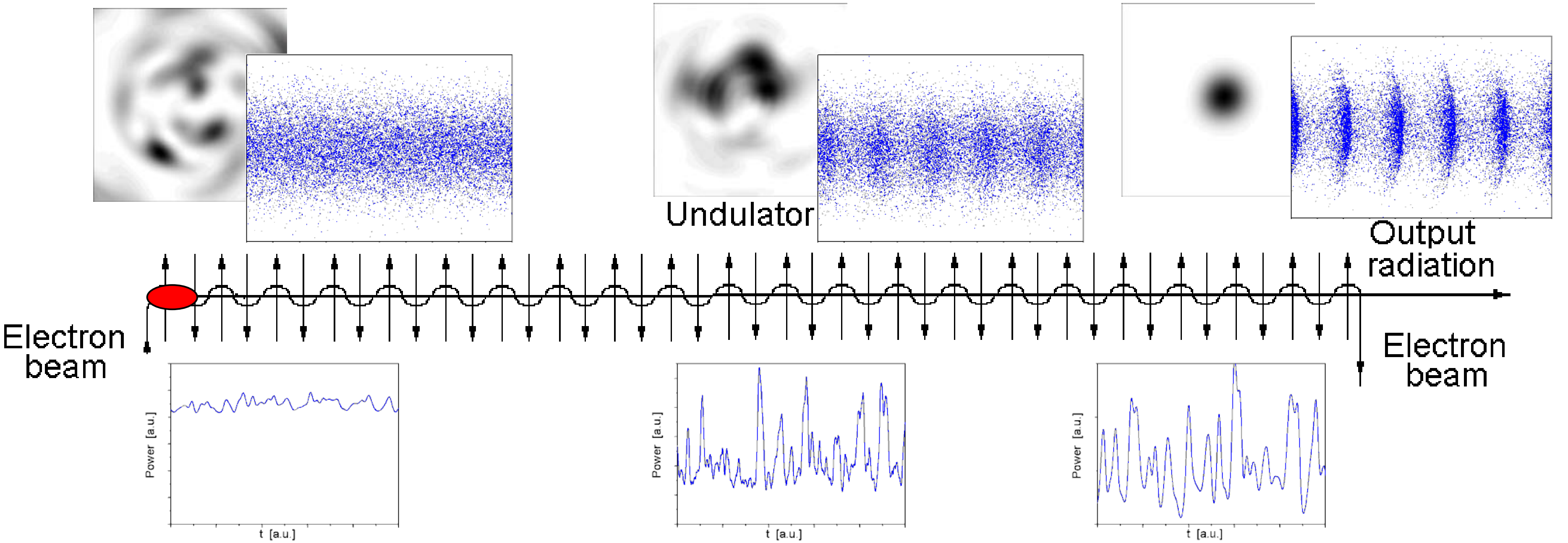
Description of coherence phenomena is the most complicated part of the FEL theory, and is essentially based on theoretical basis developed over last thirty years by numerous quantity of respectable scientists:

- Linear theory: solution of the eigenvalue and initial value problem for high gain FEL amplifier, studies of the optical guiding effect and mode selection, studies of the shot noise effects: P. Sprangle, A. Gover, N. Kroll P. Morton. N. Rosenbluth, D. Proznitz, J. Slater, D. Lowental, W. Colson, A. Kondratenko, E. Saldin, H. Haus, R. Bonifacio, C. Pellegrini, G. Moore, E. Scharlemann, A. Sessler, J. Wurtele, H. Freund, W. Fawley, M. Xie, K.J. Kim, S. Krinsky, J. Wang, L.H. Yu, Z. Huang, N. Vinokurov, O. Shevchenko, …
- Nonlinear theory: development of physical models, numerical simulation tools (steady-state and time-dependent), and numerical studies of FEL amplifiers: E. Scharlemann, W. Fawley, R. Jong, Z. Huang, W. Colson, T. Tran, J. Wurtele, P. Sprangle, H. Freund, S. Biedron, S. Milton, S. Reiche, L. Giannessi, P. Pierini, C. Penman, B. McNeil, and hundreds more individuals produced practical results …

One-page list can not accommodate all names, and detailed history of contributions can be found in the reviews by K.J. Kim, Z. Huang, C. Pellegrini, A. Gover, L. Giannessi …

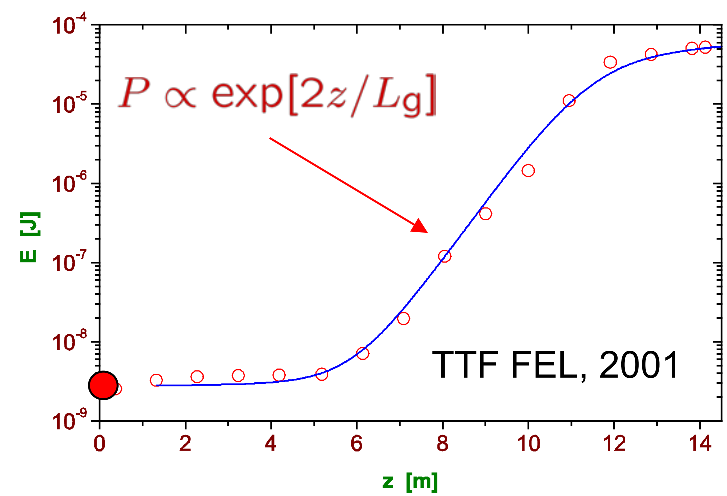
We started FEL studies in the USSR in early 80th, but joined international FEL community much later, with first open publications in the beginning of 90th. Start of collaboration with DESY in 1994 on TESLA , TTF FEL (FLASH), and x-ray FEL projects strongly stimulated us to launch detailed studies of SASE FEL, and after 20 years we are ready to describe this complicated phenomena in an elegant way. Key elements here are clear physical picture and application of similarity techniques. It is our pleasure to share this knowledge with you.

Qualitative picture of coherence properties



- Longitudinal coherence is formed due to slippage effects. A figure of merit is relative slippage of the radiation with respect to the electron beam on a scale of the field gain length ➔ coherence time.

- Transverse coherence is formed due to diffraction effects. A figure of merit is ratio of the diffraction expansion of the radiation on a scale of the field gain length to the transverse size of the electron beam.

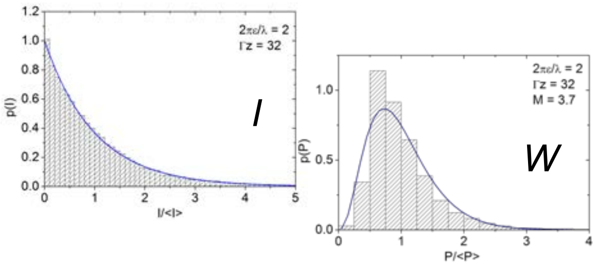


V. Ayvazyan et al., Phys. Rev. Lett. 88(2002)10482

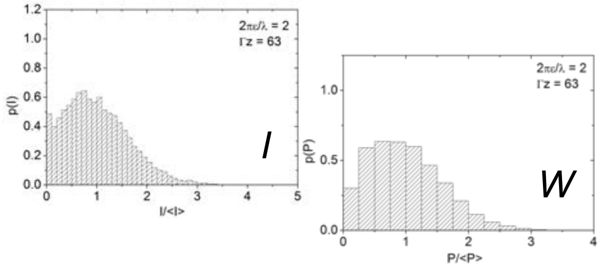
Statistical properties of the radiation from SASE FEL

Probability distributions of the radiation intensity and power

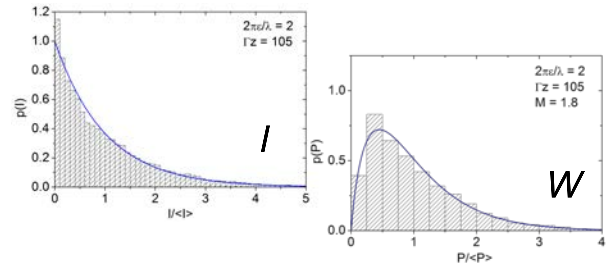
Linear regime



Saturation



Deep nonlinear regime



- Radiation from SASE FEL operating in the linear regime holds properties of completely chaotic polarized light, and is described with gaussian statistics. In particular, radiation intensity I fluctuates according to negative exponential distribution, and any integral of the power density W (radiation power, energy) fluctuates in accordance with the gamma distribution:

$$p(I) = \exp\left(-\frac{I}{\langle I \rangle}\right) , \qquad p(W) = \frac{M^M}{\Gamma(M)} \left(\frac{W}{\langle W \rangle}\right)^{M-1} \frac{1}{\langle W \rangle} \exp\left(-M\frac{W}{\langle W \rangle}\right) ,$$

where $\Gamma(M)$ is the gamma function with argument M,

$$M = \frac{1}{\sigma_{\mathrm{W}}^2} , \qquad \sigma_{\mathrm{W}}^2 = \langle (W - \langle W \rangle)^2 \rangle / \langle W \rangle^2 .$$

- The parameter M has clear physical interpretation - it is the average number of "degrees of freedom" or "modes". If the integral W is the radiation pulse energy, then parameter M is associated with the total number of modes in the radiation pulse, M_{tot} (longitudinal $\times$ transverse). If the integral is the radiation power, then parameter M is the number of transverse modes, $M_{\perp}$. Thus, the relative dispersion of the radiation power directly relates to the coherence properties of the SASE FEL operating in the linear regime.

E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Statistical properties of radiation from VUV and X-ray free electron laser, Opt. Commun. 148 (1998) 383-403

Strict definitions of statistical characteristics

- The first order time correlation function and coherence time:

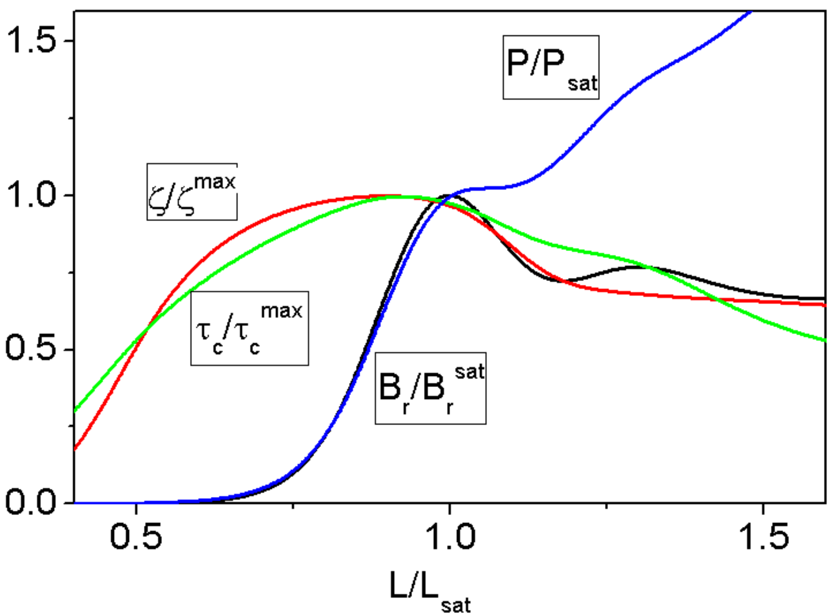
$$g_1(\vec{r}, t-t') = \frac{\langle \tilde{E}(\vec{r},t)\tilde{E}^*(\vec{r},t')\rangle}{\left[\langle \mid \tilde{E}(\vec{r},t) \mid^2\rangle\langle \mid \tilde{E}(\vec{r},t') \mid^2\rangle\right]^{1/2}} , \qquad \tau_c = \int\limits_{-\infty}^{\infty} |g_1(\tau)|^2 \, d\tau \, .$$

- The first-order transverse correlation function and degree of transverse coherence:

$$\gamma_1(\vec{r}_\perp, \vec{r'}_\perp, z, t) = \frac{\langle \tilde{E}(\vec{r}_\perp, z, t)\tilde{E}^*(\vec{r'}_\perp, z, t)\rangle}{\left[\langle|\tilde{E}(\vec{r}_\perp, z, t)|^2\rangle\langle|\tilde{E}(\vec{r'}_\perp, z, t)|^2\rangle\right]^{1/2}} , \qquad \zeta = \frac{\int\int |\gamma_1(\vec{r}_\perp, \vec{r'}_\perp)|^2 \langle I(\vec{r}_\perp)\rangle\langle I(\vec{r'}_\perp)\rangle \, d\vec{r}_\perp \, d\vec{r'}_\perp}{\left[\int \langle I(\vec{r}_\perp)\rangle \, d\vec{r}_\perp\right]^2} \, .$$

- Degeneracy parameter and peak brilliance:

$$\delta = \dot{N}_{ph}\tau_c\zeta \, , \qquad B_r = \frac{\omega \, d\dot{N}_{ph}}{d\omega} \frac{\zeta}{\left(\frac{\lambda}{2}\right)^2} = \frac{4\sqrt{2}c}{\lambda^3} \delta \, .$$



- **Radiation power** continues to grow along the undulator length.
- **Brilliance** reaches maximum value in the saturation point.
- **Degree of transverse coherence** and **coherence time** reach their maximum values in the end of exponential gain regime.

1-D theory: main properties of SASE FEL

- Main properties of SASE FEL in the saturation can be quickly estimated in terms of FEL parameter ρ and number of electrons in coherence volume $N_{\mathrm{c}} = I/(e\rho\omega)$:

The field gain length and saturation length : $$L_{\mathrm{g}} \sim \frac{\lambda_{\mathrm{w}}}{4\pi\rho}\,, \qquad L_{\mathrm{sat}} \sim \frac{\lambda_{\mathrm{w}}}{4\pi\rho}\left[3 + \frac{\ln N_{\mathrm{c}}}{\sqrt{3}}\right]$$

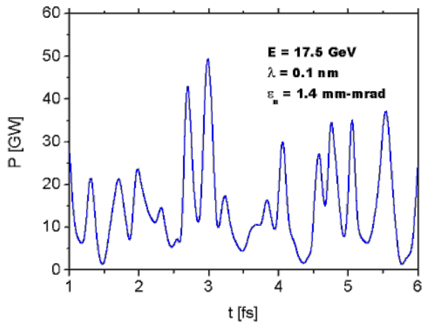
Effective power of shot noise and saturation efficiency : $$\frac{P_{\mathrm{sh}}}{\rho P_{\mathrm{b}}} \simeq \frac{3}{N_{\mathrm{c}}\sqrt{\pi \ln N_{\mathrm{c}}}}\,, \qquad \frac{P_{\mathrm{rad}}}{\rho P_{\mathrm{b}}} \simeq \rho$$

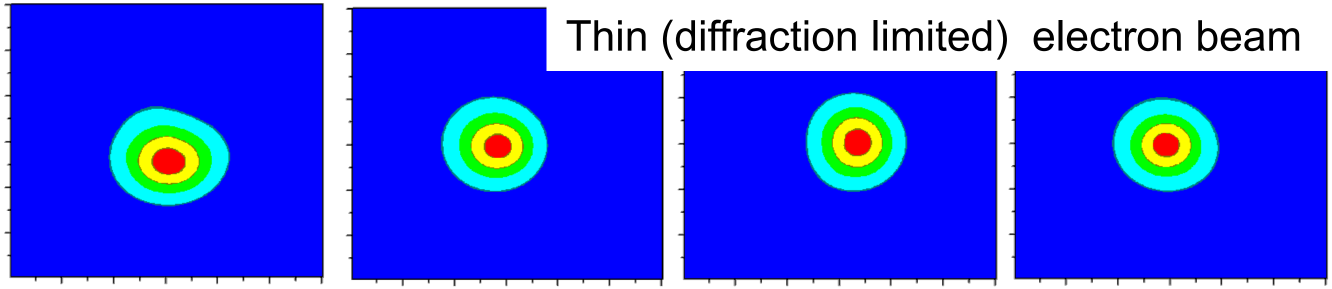
Coherence time and spectrum bandwidth at saturation : $$\tau_{\mathrm{c}} \simeq \frac{1}{\rho\omega}\sqrt{\frac{\pi \ln N_{\mathrm{c}}}{18}}\,, \qquad \sigma_{\omega} = \sqrt{\pi}/\tau_{\mathrm{c}}$$

- Odd harmonics in planar undulator:
  - Contributions of the higher odd harmonics to the FEL power for SASE FEL operating at saturation are universal functions of the undulator parameter K.
  - Power of higher harmonics is subjected to larger fluctuations than that of the fundamental one. Probability distributions of the instantaneous power of higher harmonics in saturation regime is close to the negative exponential distribution.
  - The coherence time in saturation falls inversely proportional to harmonic number: $\tau_c \propto 1/h$.
  - Relative spectrum bandwidth remains constant with harmonic number.

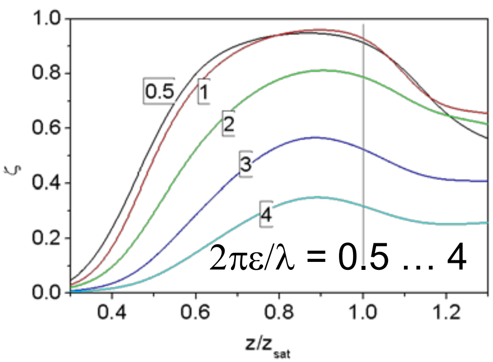
K.J. Kim, Nucl. Instrum. and Methods A 250(1986)396.
J.M. Wang and L.H. Yu, Nucl. Instrum. and Methods A 250(1986)484.
R. Bonifacio, L. De Salvo, P. Pierini, N. Piovella, and C. Pellegrini, Phys. Rev. Lett. 73 (1994) 70.
E.L. Saldin, E.A. Schneidmiller and M.V. Yurkov, Phys. Rev. ST Accel. Beams 9(2006)030702.

Qualitative look at the transverse coherence





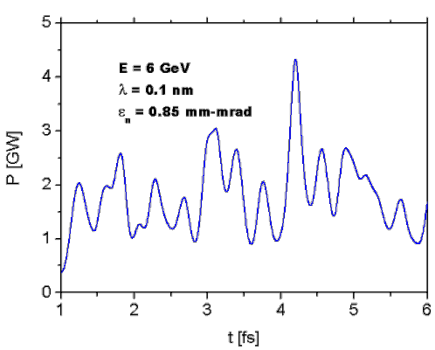
Thin (diffraction limited) electron beam

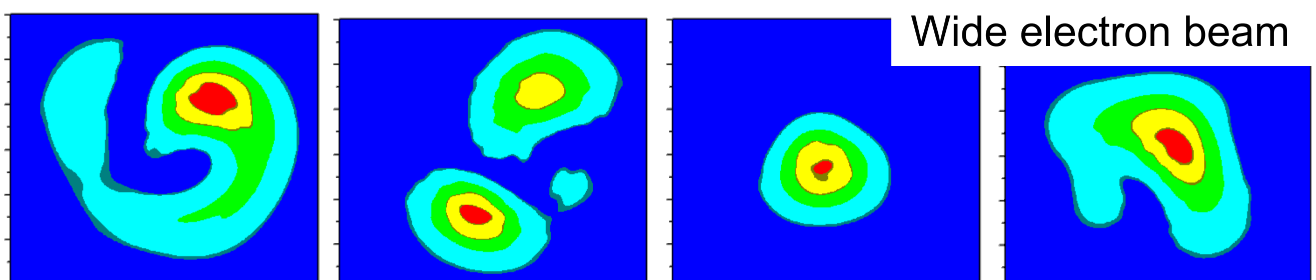


- In the linear high-gain limit the radiation emitted by the electron beam in the undulator can be represented as a set of modes:

$$E_{\mathrm{x}} + iE_{\mathrm{y}} = \int \mathrm{d}\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r, \omega) \exp[\Lambda_{nk}(\omega)z + in\phi] \, .$$

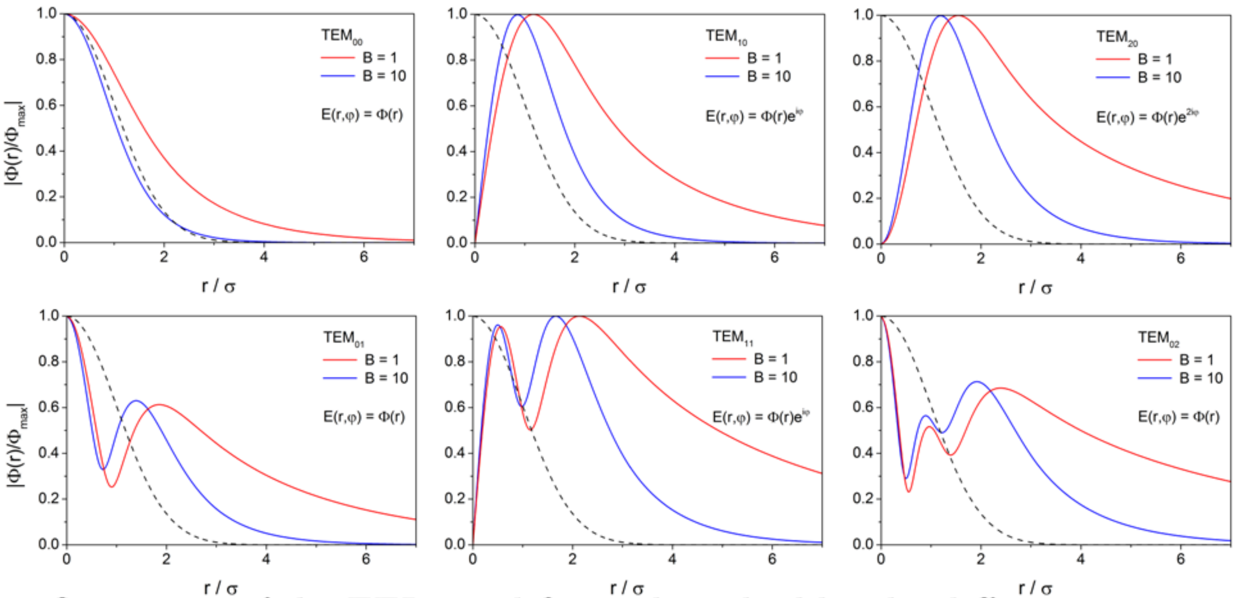
- A large number of transverse radiation modes are excited when the electron beam enters the undulator.
- Undulator length to saturation is limited , 9 to 10 field gain length for X-ray FELs.
- In the case of wide electron beam (with transverse size larger than diffraction expansion of the radiation on the scale of the field gain length) , the degree of transverse coherence degrades due to poor mode selection.

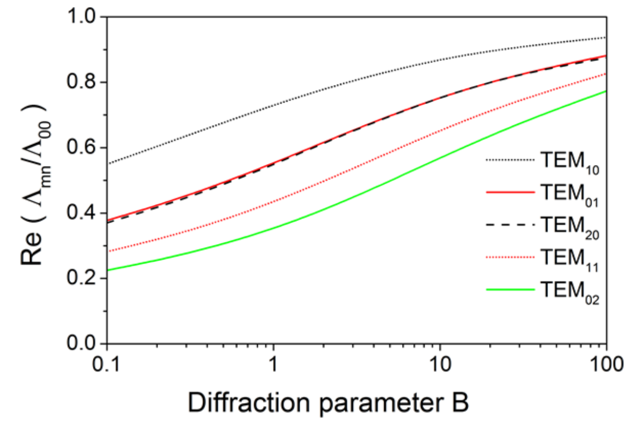




Wide electron beam

Self-reproducing FEL radiation modes
Mode degeneration





E.A. Schneidmiller, M.V. Yurkov, J. Mod. Optics (2015), DOI: 10.1080/09500340.2015.1066456.

- Operation of the FEL amplifier is described by the diffraction parameter B, the energy spread parameter $\hat{\Lambda}^2_{\mathrm{T}}$, and the betatron motion parameter $\hat{k}_\beta$:

$$B = 2\Gamma\sigma^2\omega/c \, , \qquad \hat{k}_\beta = 1/(\beta\Gamma) \, , \qquad \hat{\Lambda}^2_{\mathrm{T}} = (\sigma_{\mathrm{E}}/\mathcal{E})^2/\rho^2 \, ,$$

with the gain parameter $\Gamma = 4\pi\rho/\lambda_{\mathrm{u}}$.

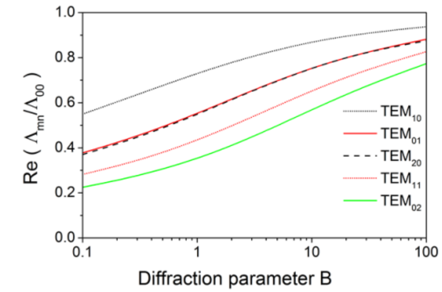
- An effect of the mode degeneration takes place for large values of the diffraction parameter B (wide electron beam):

$$\Lambda_{mn}/\Gamma \simeq \frac{\sqrt{3}+\mathrm{i}}{2B^{1/3}} - \frac{(1+\mathrm{i}\sqrt{3})(1+n+2m)}{3\sqrt{2}B^{2/3}}$$

- The strongest higher order spatial modes are azimuthally nonsymmetric modes TEM_{10} and TEM_{20} .

E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Output power and degree of transverse coherence of X-ray free electron lasers, Opt. Commun. 281 (2008) 4727-4734.

Self-reproducing FEL radiation modes
Mode degeneration



- The length of the undulator to saturation is 9 to 11 field gain length only, and significant separation of the gain required for higher spatial modes to be suppressed

$$E_x + iE_y = \int d\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r,\omega) \exp[\Lambda_{nk}(\omega)z + in\phi] \, .$$

- Parameter space of long wavelength x-ray FELs corresponds to the case of diffraction limited electron beams (small value of diffraction parameter). Radiation modes in this case are well separated in the gain, and it is possible to obtain a high degree of transverse coherence in the saturation point.

- Parameter space of short wavelength XFELs correspond to the case of the wide electron beam (large value of diffraction parameter), and the mode degeneration effect prevents suppression of higher spatial modes. The consequences are:
  - Degradation of transverse coherence;
  - Complicated and essentially non-gaussian field distributions across the slices of the radiation pulse which happens due to interference of many statistically independent spatial modes;
  - Poor pointing stability.

- These effects stem from fundamental origin - start-up of the amplification process from the shot noise in the electron beam.

Optimized x-ray FEL

- Typical procedure of optimization of short wavelength SASE FEL consists in optimization for the maximum gain of the fundamental (TEM_{00}) beam radiation mode. This case is referred as optimized x-ray FEL.

- Gain and optimum beta function in the case of small energy spread:

$$L_{\mathrm{g}} \simeq 1.67 \left(\frac{I_A}{I}\right)^{1/2} \frac{(\epsilon_n \lambda_{\mathrm{u}})^{5/6}}{\lambda^{2/3}} \frac{(1+K^2)^{1/3}}{K A_{JJ}} \, , \qquad \beta_{\mathrm{opt}} \simeq 11.2 \left(\frac{I_A}{I}\right)^{1/2} \frac{\epsilon_n^{3/2} \lambda_{\mathrm{u}}^{1/2}}{\lambda K A_{JJ}} \, .$$

- Application of similarity techniques to the FEL equations gives elegant result: characteristics of SASE FEL written down in the normalized form are functions of two parameters, ratio of geomtrical emittance to the wavelength, and the number of electrons in the volume of coherence:

$$\hat{\epsilon} = 2\pi\epsilon/\lambda \, , \qquad N_{\mathrm{c}} = I L_{\mathrm{g}} \lambda/(e \lambda_{\mathrm{u}} c) \, .$$

- Dependence of the FEL characterestics on N_{c} is very slow, and with logarithmic accuracy they depend on $\hat{\epsilon}$ only.

- The diffraction parameter B and the betatron oscillation parameter k_β are:

$$B = 2\Gamma\sigma^2\omega/c \simeq 13 \times \hat{\epsilon}^{5/2} \, , \qquad k_\beta = 1/(\beta\Gamma) \simeq 0.154/\hat{\epsilon}^{3/2}$$

Note that very fast growth of the diffraction parameter with emittance takes place.

E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Design formulas for short-wavelength FELs, Opt. Commun. 235 (2004)415-420;
Coherence properties of the radiation from X-ray free electron laser, Opt. Commun. 235 (2004)1179-1188.

Optimized XFEL at saturation

Saturation length:

$$\hat{L}_{\rm sat} = \Gamma L_{\rm sat} \simeq 2.5 \times \hat{\epsilon}^{5/6} \times \ln N_{\rm c} \ ,$$

FEL efficiency:

$$\hat{\eta} \ = \ P/(\bar{\rho} P_{\rm b}) \simeq 0.17/\hat{\epsilon} \ ,$$

Coherence time and rms spectrum width:

$$\hat{\tau}_{\rm c} \ = \ \bar{\rho}\omega\tau_{\rm c} \simeq 1.16 \times \sqrt{\ln N_{\rm c}} \times \hat{\epsilon}^{5/6} \ , \qquad \sigma_{\omega} \simeq \sqrt{\pi}/\tau_{\rm c} \ .$$

Degree of transverse coherence:

$$\zeta_{\rm sat} \ \simeq \ \frac{1.1\hat{\epsilon}^{1/4}}{1 + 0.15\hat{\epsilon}^{9/4}} \ ,$$

Degeneracy parameter:

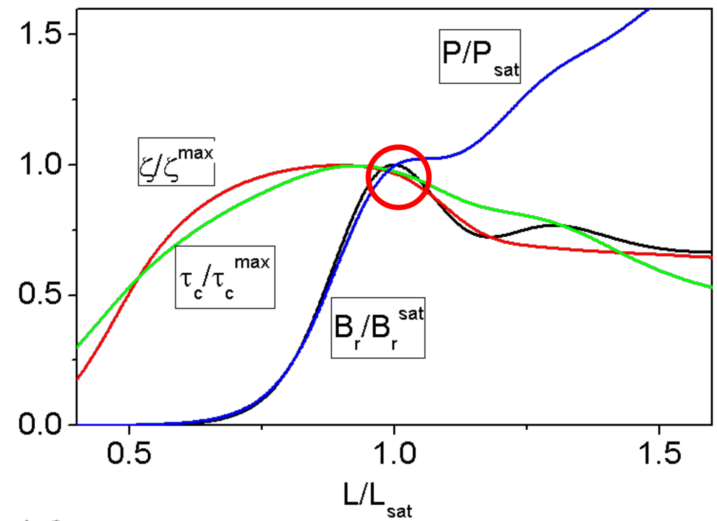
$$\hat{\delta} \ = \ \hat{\eta}\zeta\hat{\tau}_{\rm c}$$

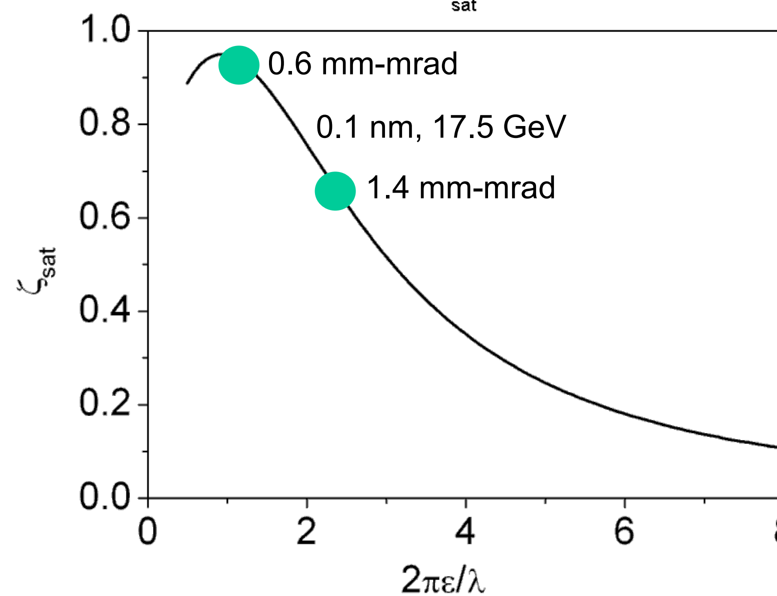
Brilliance:

$$B_r \ = \ \frac{\omega d\dot{N}_{ph}}{d\omega} \frac{\zeta}{\left(\frac{\lambda}{2}\right)^2} = \frac{4\sqrt{2}c}{\lambda^3} \frac{P_{\rm b}}{\hbar\omega^2}\hat{\delta} \ .$$

Normalizing parameters:

$$\Gamma \ = \ \left[\frac{I}{I_{\rm A}} \frac{8\pi^2 K^2 A_{\rm JJ}^2}{\lambda\lambda_{\rm w}\gamma^3}\right]^{1/2} \ , \qquad \bar{\rho} = \frac{\lambda_{\rm w}\Gamma}{4\pi} \ .$$

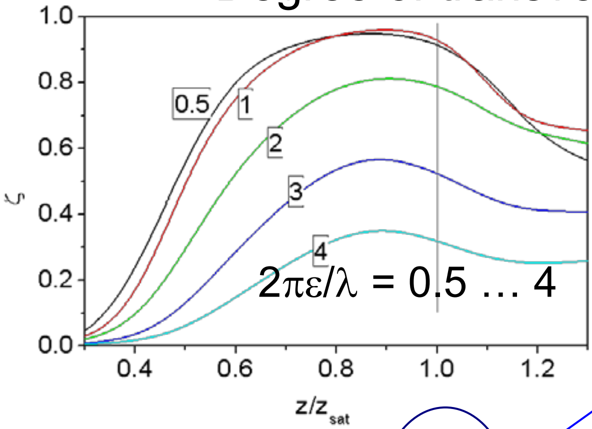


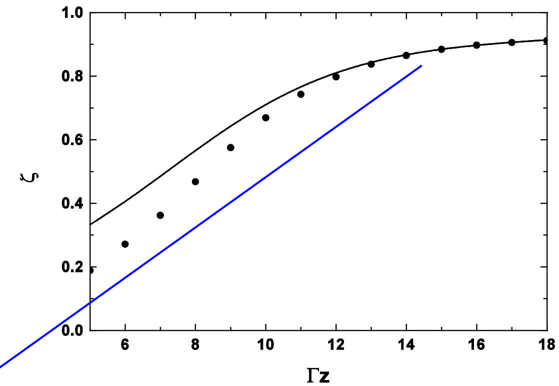


E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 281(2008)1179; New J. Phys. 12 (2010) 035010

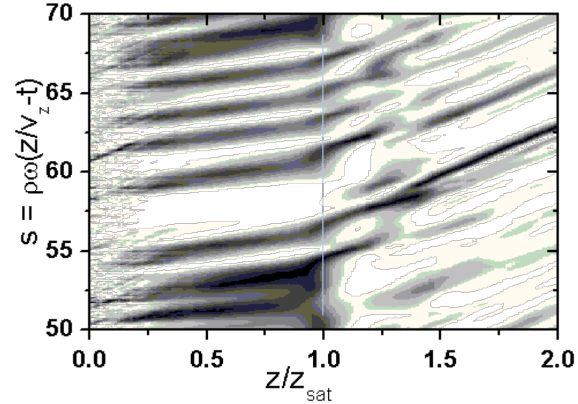
Limitations on transverse coherence for diffraction limited beam

Degree of transverse coherence





z-s intensity distribution



$$E_x + iE_y = \int d\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r,\omega) \exp[\Lambda_{nk}(\omega)z + in\phi]$$

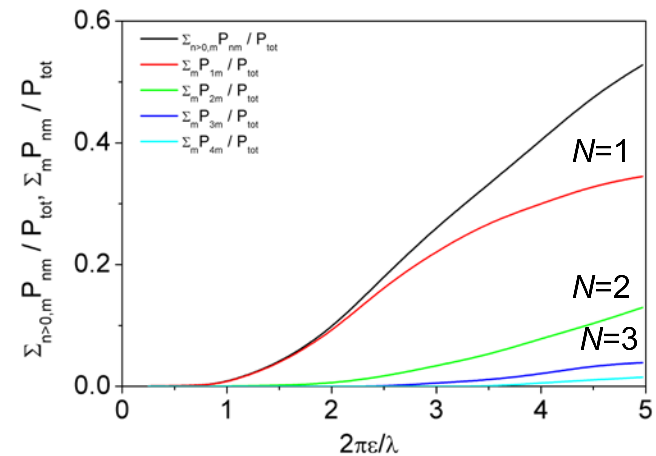
- In the case of large emittance the degree of transverse coherence degrades due to poor mode selection.
- For small emittances the degree of transverse coherence visibly differs from unity. Moreover,it falls down at very small emittance values. This happens due to poor longitudinal coherence: radiation spikes move forward along the electron beam, and interact with those parts of the beam which have different amplitude/phase.
- Longitudinal coherence develops slowly with the undulator length thus preventing full transverse coherence. In fact, within framework of the linear theory, the degree of transverse coherence asymptotically tends to the unity not exponentially, but proportionally to $1/z^{1/2}$, in the same way as the radiation spectrum of SASE FEL is shrinking.

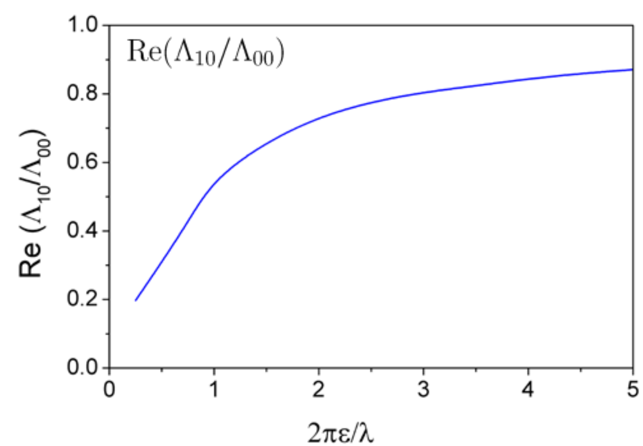
E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 186(2000)185

Optimized x-ray FEL: mode degeneration

Features of optimized x-ray FEL for parameter space $2\pi\epsilon/\lambda > 1$:

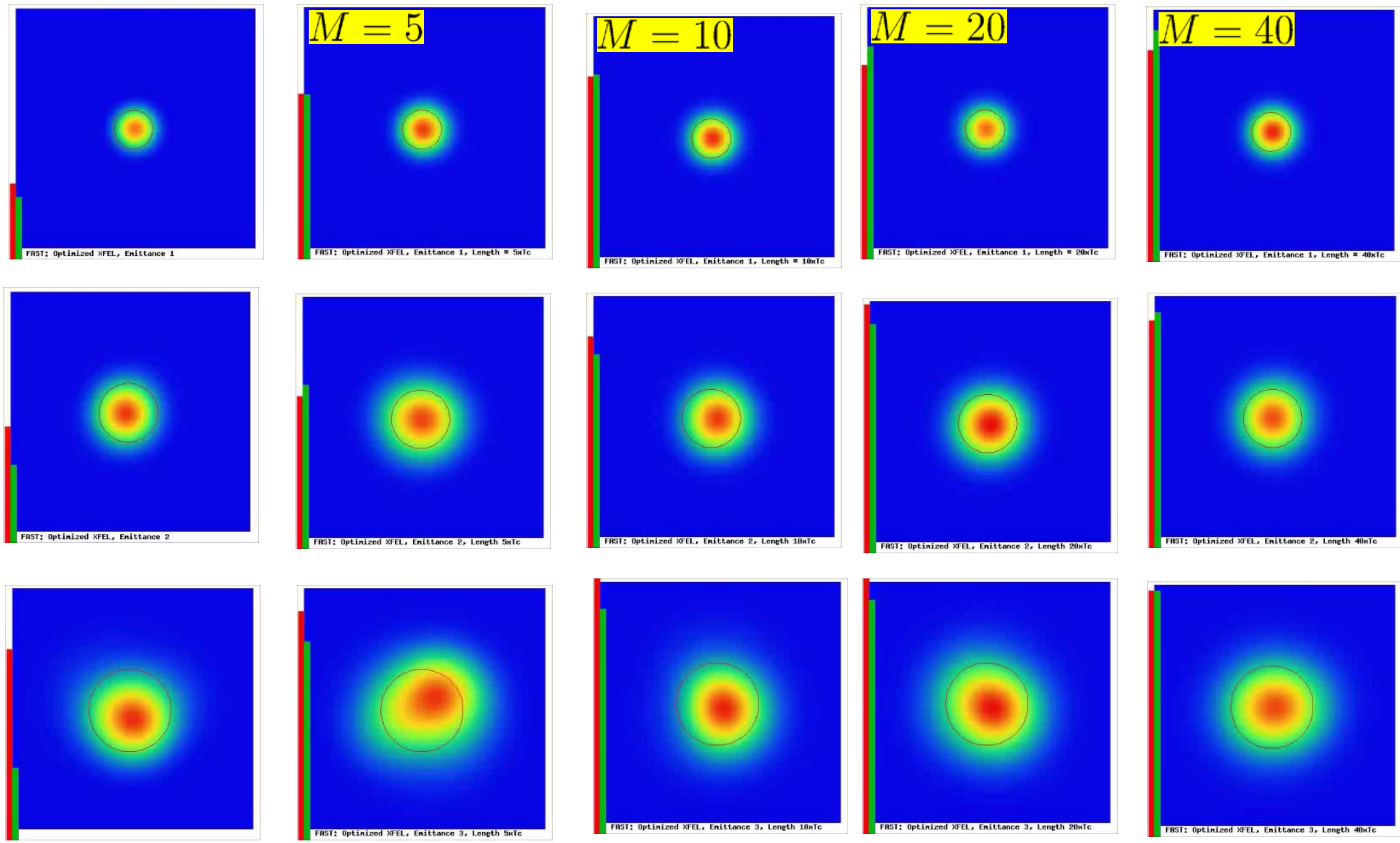
- Large values of the diffraction parameter.
- Mode degeneration effect takes place.
- Significant contribution of higher azimuthal radiation modes.
- Poor spatial coherence.
- Complicated and essentially non-gaussian field distributions across slices of the radiation pulse.
- Poor pointing stability of the radiation.





Optimized x-ray FEL: pointing stability, slice and projected

Slice — Full photon pulse (projection of all slices)



$2\pi\epsilon/\lambda = 1$

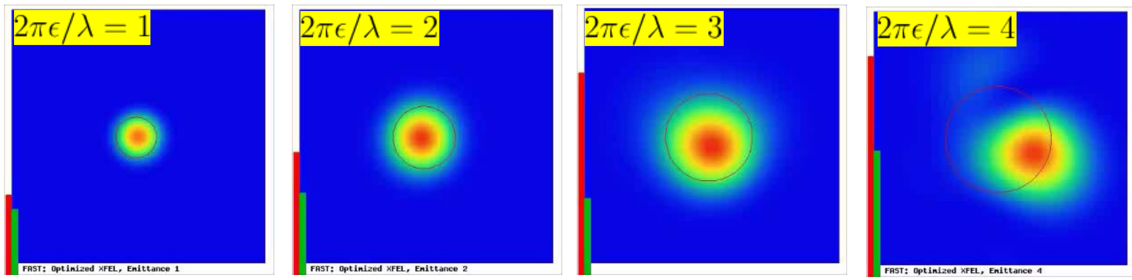
$2\pi\epsilon/\lambda = 2$

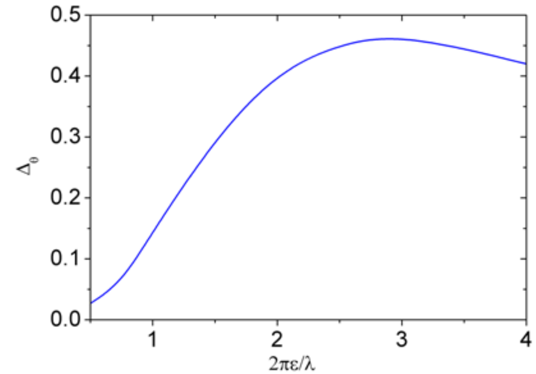
$2\pi\epsilon/\lambda = 3$

Slice and projected single shot photon beam images for the pulse durations $T = M \times \tau_c$ for $M = 5$, 10, 20, and 40, and emittances $2\pi\epsilon/\lambda = 1$, 2, and 3.

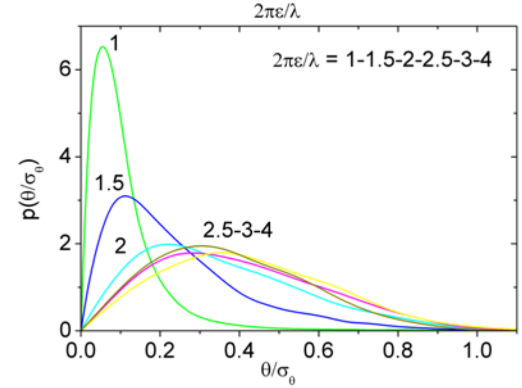
Optimized x-ray FEL: pointing stability (slice)

- Slice intensity distributions in the far zone.
- rms deviation of the photon beam center of gravity.
- Probability distribution of the photon beam center of gravity.



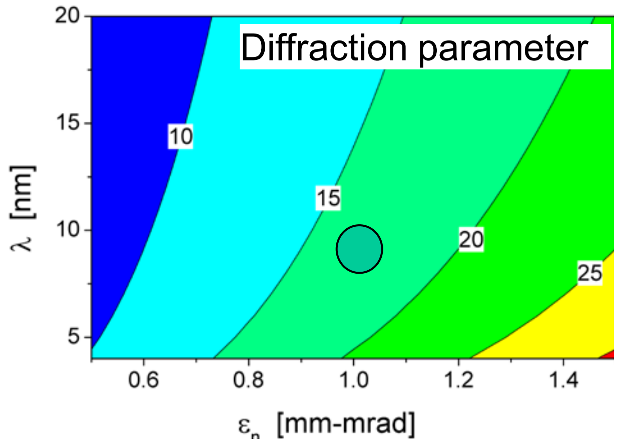


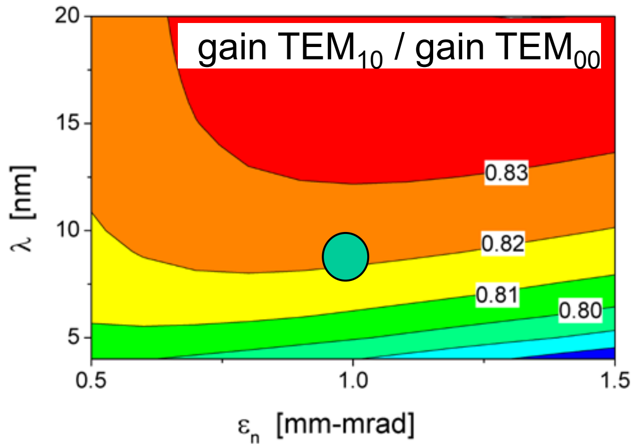
	LCLS	SACLA	EXFEL	SWISS FEL	PAL XFEL
Energy [GeV]	13.6	8.0	17.5	5.8	10
Wavelength [A]	1.5	0.6	0.5	0.7	0.6
ϵ_n [mm-rad]	0.4	0.4	0.4	0.4	0.4
$\hat{\epsilon} = 2\pi\epsilon/\lambda$	1	2.7	1.5	3.4	2.1



- Complicated and essentially non-gaussian field distributions across the slices of the radiation pulse caused by interference of many statistically independent spatial modes.
- Probaility of the radiation intensity to have exact pointing in the center is equal to zero.
- Poor pointing stability. Starting from $2\pi\epsilon/\lambda \gtrsim 1$ rms deviation of the photon beam center of gravity exceeds 10%, and reach maximum value of 45% at $2\pi\epsilon/\lambda \simeq 3$. Then pointing stability is improved due to significant growth of the number of transverse modes.
- These effects are fundamental and stem from start-up of the amplification process from the shot noise in the electron beam.

FLASH: Transverse coherence and pointing stability. Experiment and theory





FLASH: experiment

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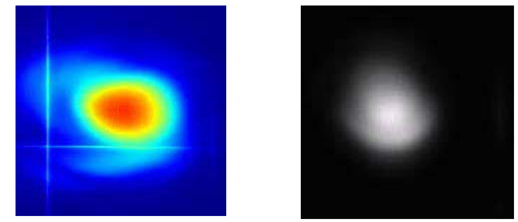
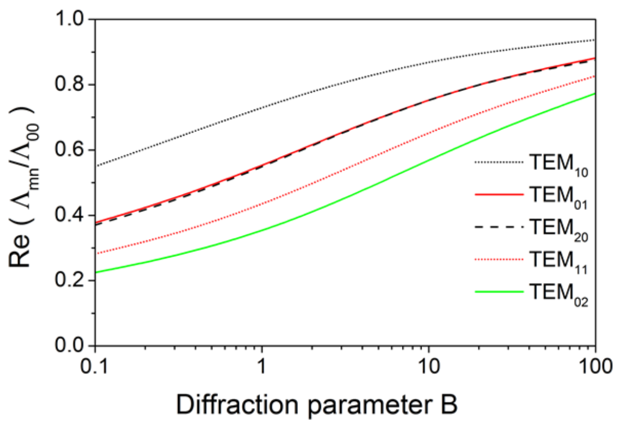
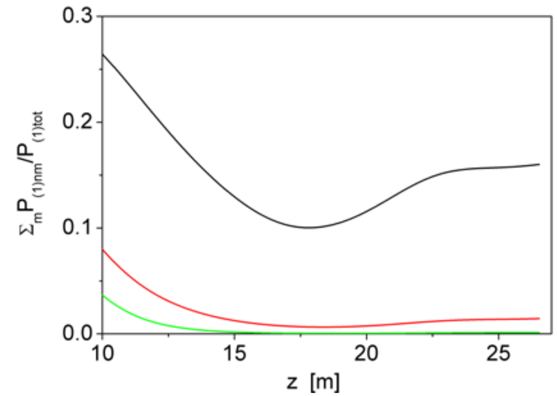
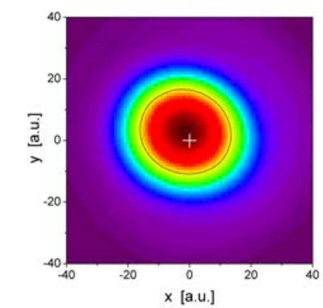






FLASH: FAST simulations



Parameter space of FLASH:

Large values of diffraction parameter (B = 10 - 25) and "cold" electron beam.

Mode degeneration effect is strong (gain of TEM$_{10}$ mode is 0.8 – 0.83 of the fundamental TEM$_{00}$).

Contribution of the first azimuthal mode to the total power is 10 to 15% ($\zeta \sim 0.8$).

Result: unstable shape and pointing of the photon pulse.

E.A. Schneidmiller, M.V. Yurkov, J. Mod. Optics (2015), DOI: 10.1080/09500340.2015.1066456.

Is it possible to generate fully coherent radiation in seeded and self-seeded hard x-ray FELs?

- The answer is "NO". By the time there are no schemes providing generation of fully coherent radiation in the hard x-ray regime.
- HGHG seedeng schemes:

  E.L. Saldin, E.A. Schneidmiller and M.V. Yurkov, Study of a noise degradation of amplification process in a multistage HGHG FEL, Opt. Commun. 202 (2002) 169-187:

  "... Our studies have shown that the frequency multiplication process produces noise degradation proportional at least to the square of the frequency multiplication factor. This prevents operation of HGHG FEL at a very short wavelength range."
- Self-seeded FEL:
  - Growth of the noise within amplification bandwidth of the second stage.
  - Strong fluctuations of the input seed signal of the second stage. In fact, its power fluctuates according to negative exponential distribution.
  - One more concern refers to the problem of bad transverse coherence of the radiation after the first stage when operating in the hard x-ray regime.

Summary

- Application of similarity techniques allows to describe coherence phenomena in SASE FEL in an elegant way on both, qualitative and quantitative level. Main parameters of the problem are FEL parameter ρ for temporal coherence, and diffraction parameter *B* for transverse coherence.
- Inverse value of the FEL parameter ρ gives the scale of the coherence length in the units of the radiation wavelength. Diffraction parameter is the ratio of the electron beam size to the area of diffraction expansion of the radiation on a scale of the field gain length, and is the figure of merit for the strength of diffraction effects.
- Application of similarity techniques to the results of numerical simulations allows to derive simple physical dependencies. For instance, all characteristics of optimized x-ray FEL with small energy spread depend on the only parameter, $2\pi\varepsilon/\lambda$, the ratio of the geometrical emittance to the radiation wavelength.
- It is predicted that X-ray FELs operating at short wavelengths will demonstrate degradation of the transverse coherence, slice field patterns will significantly deviate from gaussian, and the pointing stability will degrade with the increase of the parameter $2\pi\varepsilon/\lambda$ > 1.