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APPROXIMATED EXPRESSIONS FOR THE COHERENT SYNCHROTRON RADIATION EFFECT IN BENDING MAGNETS

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Abstract

In today's X-ray free-electron laser (XFEL), the ultra-bright radiation's strict dependence on emittance has motivated research on advanced electron accelerator techniques. Understanding mechanisms borne in the linac that could jeopardize FEL radiation is important to ensure maximal potential of the machine. One such mechanism, is the Coherent Synchrotron Radiation (CSR) effect. The effect is prominent in latter stages of the linac where the bunch length is short, the peak current is high and the synchrotron radiation emitted in curved sections is temporally coherent. Though, the CSR effect has been comprehensively studied, it still must be quantified for any accelerator system, and requires care in circumventing [1-9].

In this paper, we describe the development of simple and compact analytic expressions for the relative CSR RMS induced energy spread resulting from two typical bending magnet situations in electron particle accelerators. The expressions are compared with the CSR wake field integral expressions derived for a electron bunch with arbitrary linear density, λ_s [10, 11]. Then, the accuracy of each expression is compared against ELEGANT's CSR computational algorithm with the simulation of several idealized examples [15].

INTRODUCTION

The standard derivation of the CSR wakefield begins with the Liénard-Wiechert fields for two electrons traveling along the *same* circular trajectory, ignoring any transverse extent of the beam [10]. The energy transfer due to the electric field of the trailing electron onto the advancing electron is simply given by $dE/d(ct) = e\vec{\beta}\cdot\vec{E}$. The wakefield is inversely proportional to the distance between the two electrons and contains a singularity when the electron separation approaches zero. The singularity is removed by the "normalization process" developed by Saldin *et al*. [10] by subtracting off the contribution of two electrons along a straight-line trajectory. The resulting two electron kernel for the CSR wakefield can then be integrated over any longitudinal bunch distribution to give the collective effect under a wide range of scenarios one might find in a typical accelerator environment. In particular, we concern ourselves with two highly prominent cases found in XFEL applications: First, a linearly chirped electron bunch entering a bending magnet and compressing into the steady-state regime, and second, an electron bunch exiting a bend into the subsequent drift section, both cases of which, are in the ultra-relativistic regime ($\gamma \to \infty$). The complete mathematical expressions for the two situations under the 1-D projected model are as follows [10]:

$$\frac{dE}{dz}\Big|_{\text{bend}} = \frac{4}{R\phi}\left(\lambda(s-S_L) - \lambda(s-4S_L)\right) + \cdots \frac{2}{(3R^2)^{\frac{1}{3}}}\int \frac{d\lambda(s')}{ds'}\left(\frac{1}{(s-s')^{\frac{1}{3}}}\right)ds' \qquad (1)$$

$$\frac{dE}{dz}\Big|_{\text{drift}} = \frac{4}{R}\left(\frac{\lambda(s-S_M)}{\theta_B+2x}\right) + \cdots \frac{4}{R}\left[\int \frac{d\lambda(s\prime)}{ds\prime}\left(\frac{1}{\phi+2x}\right)ds'\right] \quad , \qquad (2)$$

where R is the bending radius, θ_B is the total bend angle, x is the subsequent drift coordinate, $\emptyset$ is the bend angular displacement, λ_s is the normalized longitudinal distribution of the bunch, s and s' are the internalized bunch coordinate of the front and back electron, respectively, and S_L, S_M are the slippage conditions inside the bend and in the subsequent drift, respectively [11]. In in the ultra-relativistic regime, the slippage conditions reduce to:

$$S_L = \mathrm{s} - \mathrm{s}' = \frac{\mathrm{R}\emptyset^3}{24} \qquad (3)$$

$$S_M = \mathrm{s} - \mathrm{s}' = \frac{\mathrm{R}\emptyset^3}{24}\frac{R\emptyset+4x}{R\emptyset+x} \ . \qquad (4)$$

The RMS energy deviation (normalized to the beam energy) that the CSR induces onto the bunch is an important parameter in beam dynamics. Let us now look at the induced CSR energy spread for the specific case of an electron bunch traversing a bending magnet. In this case, when the bunch is fully contained in the magnet, all trailing electron fields will catch-up and interact with advancing electrons (for a given *"s"* coordinate) and the slippage condition effectively approaches infinity, $S_L \to \infty$. Under this condition, the first two terms on the right of eq. 1, the "transient" terms, tend to zero and the lower bound of the integral spans the entire bunch tail domain i.e. the energy transfer is constant along the trajectory dz. In this steady-state regime, the relative CSR RMS energy spread induced on a beam with a Gaussian longitudinal distribution was derived to be [12]:

$$\sigma_{CSR} = 0.22\,\frac{r_e N L_B}{\gamma\rho^{2/3}\sigma_s^{4/3}} \ . \qquad (6)$$

The above expressions can also take into account the evolution of the longitudinal profile of the electron bunch in compression scenarios assuming that the bunch length is short compared to the slippage. We substitute the constant σ_s terms with one that evolves via the R_{56} of the system and energy chirp, h, of the beam to incorporate

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compression [13]. No such calculation, to date, has been published for the relative CSR RMS induced energy spread in the subsequent drift section.

CSR INTEGRAL APPROXIMATIONS

The mobility of the CSR integral expressions, eqs. 1 and 2, are limited when quick estimates are desired (they require a knowledge of the exact longitudinal distribution of the electron beam which demands numerical integration). Deriving analytically accessible expressions, such as eq. 6, in place of the CSR integrals is of great utility for accelerator and FEL applications. In this section, we develop expressions for the relative CSR RMS energy spread induced on a beam in a bend transitioning to steady-state and the subsequent drift in lieu of eqs. 1 and 2. Both derivations take advantage of situational approximations that can be assumed for FEL applications. Both models assume a Gaussian longitudinal distribution.

CSR Energy Spread in Transition to Steady-State for a Bend

The aforementioned σ_{CSR} expression, eq. 6, was derived in the steady-state regime. In actuality, steady-state is not always (sometimes never) achieved in typical FEL bending magnet settings. During bunch compression, for instance, the electron bunch commonly begins out of steady-state ($\sigma_S > S_L$) and compresses into steady-state ($\sigma_S < S_L$) while still in the bend. We would like to have a more comprehensive expression to describe the RMS energy gain/loss of this transitional state to compliment the current σ_{CSR} expression.

To start, we provide a simple conceptual model of the CSR dynamics for this situation. First, we recognize that a key feature of the steady-state regime lies in the integration limits of eq. 1 i.e. the amount of trailing electrons contributing to the wake field. The lower bound is described by the slippage condition in ultra-relativistic limit, $S_L = R\emptyset^3/24$, where we remind the reader that $\emptyset$ is the angular displacement of the observing electron, with internal bunch coordinate s, within the bend. Once the slippage condition exceeds σ_z, the integration bounds virtually encompasses the entire tail-end of the bunch, and the integral is constant; steady-state is achieved. This process is heavily affected if the beam is being compressed and the σ_s is evolving. Second, we incorporate the two transient terms outside of the integral in eq. 1 to properly describe the energy transfer as the steady-state expression neglects them both. The transient terms come about from the entrance wake (case A in [11]) and from the integration-by-part derivation's surface/stationary term of the bend magnet wake (case B in [11]). Both terms will eventually dissipate and slip out of range as the electron bunch traverses the bend. The full description of the transient wakes will take into account the angular dependence, $\emptyset$, of the slippage and compression, but the solution will be non-analytical. For our purposes, we assume the compression of the bunch ($\propto \emptyset$) is negligible relative to the slippage of the wake ($\propto \emptyset^3$) before steady-state is achieved. This will allow us to use the entrance bunch length, σ_{si}, and ignore compression completely to ease the mathematical cost of the RMS calculation. So, our amended model of the σ_{CSR} should reflect this approach to steady-state and take into account the aforementioned assumptions.

To begin, we introduce a normalization factor, A_{bend}, in the σ_{CSR} expression to model the beam transitioning into the steady state regime:

$$\begin{cases} A_{\text{bend}} = \dfrac{\sigma_{ss}}{\sigma_{si}(1+hR_{56})}, & \sigma_s > \sigma_{ss} \\ A_{\text{bend}} = 1, & \sigma_s \le \sigma_{ss} \end{cases}$$

where σ_{ss} is steady-state bunch length defined by the slippage condition, σ_{si} is the bunch length at the entrance of the bend, h is the longitudinal energy chirp, and R_{56} is the momentum compaction of the bend. Now, we can modify the expression for the relative CSR RMS energy spread with the normalization factor for a bunch is being compressed into the steady-state regime:

$$\sigma_{CSR} = \frac{\sigma_{ss}}{\sigma_{si}(1+hR_{56})}\left(0.22\frac{Nr_eL_B}{\gamma\rho^{\frac{2}{3}}\left(\sigma_{si}(1+hR_{56})\right)^{\frac{4}{3}}}\right)$$
$$= 0.22\frac{Nr_eL_B\sigma_{ss}}{\gamma\rho^{2/3}\left(\sigma_{si}(1+hR_{56})\right)^{7/3}} . \quad (7)$$

The above expression is the relative CSR RMS energy spread for a non-steady-state electron bunch undergoing linear compression of its RMS bunch length. It should be noted that the normalization factor, A_{bend}, assumes a linearity towards the bunch approaching steady-state when, in actuality, the correct behaviour is a dictated by the third power of $z^3/24R^2$ taken from the slippage condition. This assumption is only valid for small z excursions which is nearly always satisfied in the short bending magnets of FELs.

Next, we incorporate the transient wake terms [10, 11]:

$$\frac{dE}{dz}\Big|_{\text{transient}} = \frac{4}{R\phi}\left(\lambda(s-S_L) - \lambda(s-4S_L)\right),$$

under the assumption that there is no compression of the bunch length and that the wake is only valid for small angles, $\emptyset$. This will permit a small angle expansion of $\emptyset$, and cause the nested RMS integration over the path length ($Rd\emptyset$) and internal bunch coordinate, s, to be analytical. Computing the RMS energy spread gives:

$$\sigma_{\text{transient}} = 2.9169\frac{Nr_eL_B\theta_B{}^2}{\gamma\sigma_{si}{}^2} . \quad (8)$$

Therefore, the cumulative CSR RMS energy spread induced onto the beam while in transition to steady-state in a bend is simply the sum of eqs. 7 and 8:

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$$\sigma_{\text{transient}} = \frac{N r_e L_B {\theta_B}^2}{\gamma}\left(2.9\frac{1}{{\sigma_{si}}^2} + \left(7.7\times 10^{-7}\frac{L_B {\theta_B}^2}{\left(\sigma_{si}(1+hR_{56})\right)^7}\right)^{1/3}\right), \quad (9)$$

where the unit of the energy spread is in percent(s). The above expression is only valid when the bunch length has not reached the steady-state condition as dictated in the A_{bend} boundary conditions i.e. the σ_{CSR} steady-state expression (eq. 6) can be used in post.

CSR Energy Spread for a Drift

The CSR RMS energy spread induced on the beam due to the subsequent drift section is typically more detrimental than that produced solely in the bend. The reason is that the CSR wake in the drift has much more distant over which sustained energy transfer to the beam can occur. The difficulty in solving the drift space CSR integral (eq. 2) is in its requiring of numerical methods for even simple bunch distributions e.g. a Gaussian distribution. Again the utility of a simple expression to describe the CSR RMS energy spread in a drift is desired.

To begin, we conceptually map a set of simple assumptions that will ease the mathematical complexity of the CSR integral. First, we recognize that the wake in the drift section (with drift coordinate x) after a bend is essentially the CSR wake from the bend co-propagating with the beam. We can assume the beam reached steady-state in the bend prior to entering the drift so that transient effects are not present in the co-propagating CSR wake. In essence, if the bunch has achieved steady-state the slippage condition, eq. 4, will drop its $\emptyset$ dependence and only depend on x. The x-dependence of the slippage will effectively contain the entire tail-end of the bunch for any x value and we can equivalently allow the integration lower bound to negative infinity. Second, while the CSR wake propagates further into the drift, its power decreases as radiating electrons leave the bend, which will amount to a x^{-1} decay of the wake. Integrating over the drift distance x, the energy transfer should tend as the natural log and will contain a singularity at $x = 0$. To circumvent the singularity, we restrict the integration limits over drift intervals where the entire bunch has transitioned out of the bend and completely into the drift section ($\sigma_s < x \leq d$, where d is the entire drift distance).

With these simplifications in place, we begin to calculate the CSR integral from a slightly different approach than previously published [10]. Standard derivation of eq. 2 is obtained via integration by parts that incorporates the derivative of the longitudinal bunch distribution. Instead, we opt to not employ the integration by parts routine and simply integrate the two-electron kernel with the longitudinal bunch distribution:

$$\frac{dE}{dz}\Big|_{\text{drift}} = \int_{-\infty}^{s}\frac{-32e^2(R\emptyset+x)^2}{\emptyset^2(R\emptyset+2x)^4}\lambda(s')ds' ,$$

where the two-particle wake is taken from Stupakov and Emma in the ultra-relativistic regime [11]. From here, we are able to integrate over the bend angle using $ds' = \frac{-R\phi^2}{8}\frac{(R\phi+2x)^2}{(R\phi+x)^2}d\phi$ derived from the slippage condition:

$$\frac{dE}{dz}\Big|_{\text{drift}} = \int_{-\infty}^{0}\frac{-32e^2(R\emptyset+x)^2}{\emptyset^2(R\emptyset+2x)^4} \times\lambda(s-S_M)\left(\frac{-R\phi^2}{8}\frac{(R\phi+2x)^2}{(R\phi+x)^2}d\phi\right).$$

This integral is non-analytical. To make the integral solvable for the RMS energy spread we employ one last simplification to the bunch length and expand it about small bunch coordinates s. For any given order, we have:

$$\frac{dE}{dz}\Big|_{\text{drift}} = \int_{-\infty}^{0}\frac{4e^2R}{(R\emptyset+2x)^2}\frac{N}{\sqrt{2\pi}\sigma_z}e^{-\frac{s^2}{2{\sigma_s}^2}} \times\left(1+\frac{sR\phi^3}{6{\sigma_s}^2}-\frac{sR^2\phi^3}{8x{\sigma_s}^2}+\ldots\right)d\phi.$$

Taking only the zeroth order term, we can solve for the total energy transfer by integrating over the drift distance (under our second assumption's bounds $[\sigma_s, d]$), and solve for the for the relative CSR RMS energy spread analytically given by:

$$\sigma_{CSR} = \frac{N r_e}{\gamma\sqrt{\pi}\sigma_s}\sqrt{\left(\frac{2}{\sqrt{3}}-1\right)}\log\left(\frac{24d}{R{\theta_B}^3}\right). \quad (10)$$

This expression is valid for short bunches, as it will justify the steady-state condition in the bend ($\sigma_S < S_L$) and the expansion about small bunch positions, s.

SIMULATION STUDIES

The 6-D particle tracking code, ELEGANT, has been used extensively in the simulation of charged particle beams for FEL applications [14]. ELEGANT also includes a computationally low cost evaluation of the CSR energy kicks a charged particle beam will experience while traversing a bend magnet and coasting into a subsequent drift section based on the projected longitudinal model [15]. This makes ELEGANT an ideal candidate for testing the accuracy of our newly formed CSR RMS expressions.

The simulation study will consist of benchmark tests for both relative CSR RMS energy spread expressions, eqs. 9 and 10. For this, we simulate a 100-pC Gaussian electron bunch ($\beta_{x,y} = 100\,m$, $\gamma\epsilon_{x,y} = 0.5\,\mu m$, $E_0 = 1.6\,GeV$) to inject into an idealized bend-drift ($L_B = 0.55\,m$) system while scanning through prominent parameters: initial bunch length σ_{si}, bend angle θ_B, and drift distance d.

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CSR Energy Spread in Transition to Steady-State for a Bend

To test eq. 9, we impose a linear energy chirp along the beam's longitudinal dimension and inject the beam into a typical second-stage 4-bend chicane bunch compressor ($R_{56} \approx 37\ mm$). The third bend is where the majority of the compression cycle occurs, and typically, where the beam will compress into the steady-state regime (compresses from $\sim 100\ \mu m$ to $\sim 10\ \mu m$) making it an ideal location to test our first expression.

The bending angle and incoming bunch length at the third bend are systematically and independently varied while never allowing the beam to achieve steady-state. Figure 1 plots the bunch length's approach to the steady-state condition for the angle and bunch length scans, respectively.

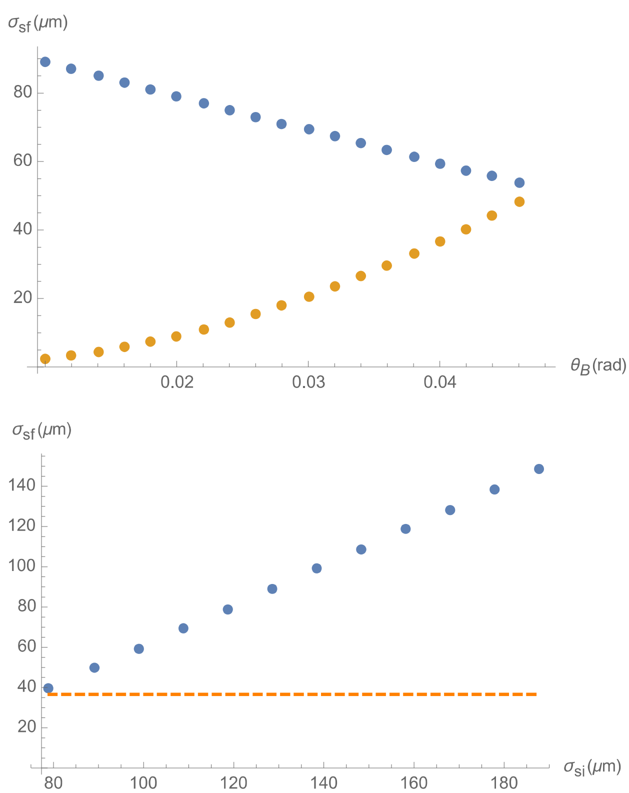

Figure 1: Top: The exiting bunch length of the beam after compression (blue marker) and steady-state bunch length (orange marker) for various bending angles. Bottom: The exiting bunch length of the beam (blue marker) and the steady-state condition (orange dashed line) for various initial bunch lengths.

The relative CSR RMS energy spread is measured at the exit of the bend and compared with that of equations 1 and 9. The results are compared in Figure 2.

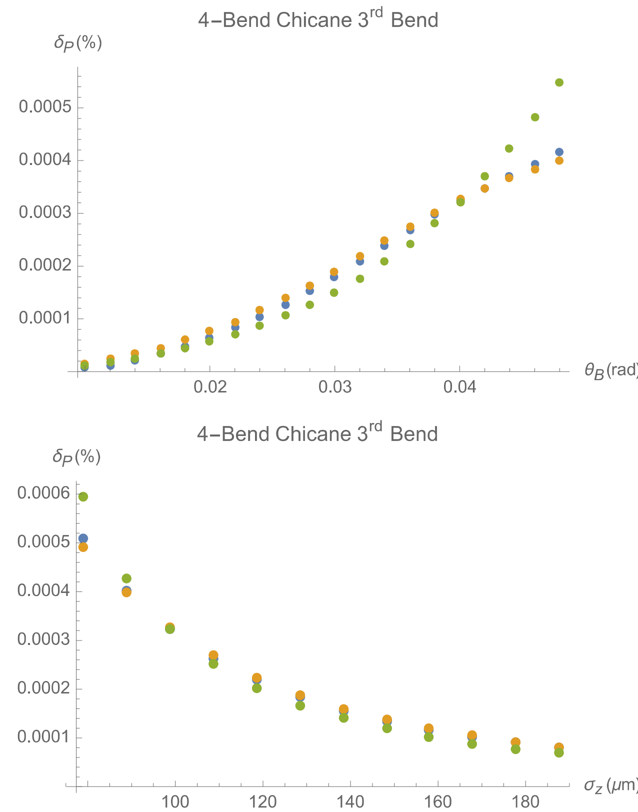

Figure 2: Top: The CSR RMS energy spread calculated by ELEGANT (blue marker), full CSR integral in eq. 1 (orange marker), and the newly derived approximate expression in eq. 9 (green marker) for various bending angle values. Bottom: The CSR RMS energy spread calculated by ELEGANT (blue marker), full CSR integral in eq. 1 (orange marker), and the newly derived approximate expression in eq. 9 (green marker) for various initial bunch lengths. In both plots, the ELEGANT and CSR integral markers mostly overlap.

Equation 9 shows strong agreement in both scans, but tends to stray when steady-state is being approached. This is seen in the large bend angles of Figure 2 (top) and the small bunch lengths of the Figure 2 (bottom). These are the regions where the beam is closest to steady-state and are corroborated in Figure 1.

CSR Energy Spread for a Drift

To test eq. 10, we inject the mono-energetic ($\sigma_\delta = 0$) Gaussian beam into a 0.55 m, 0.04 radian bending magnet and allow the beam to coast into a drift section. We systematically scan the incoming bunch length, drift length, and bend angle while measuring the relative CSR RMS energy spread at the end of the drift section. The results are displayed in Figure 3.

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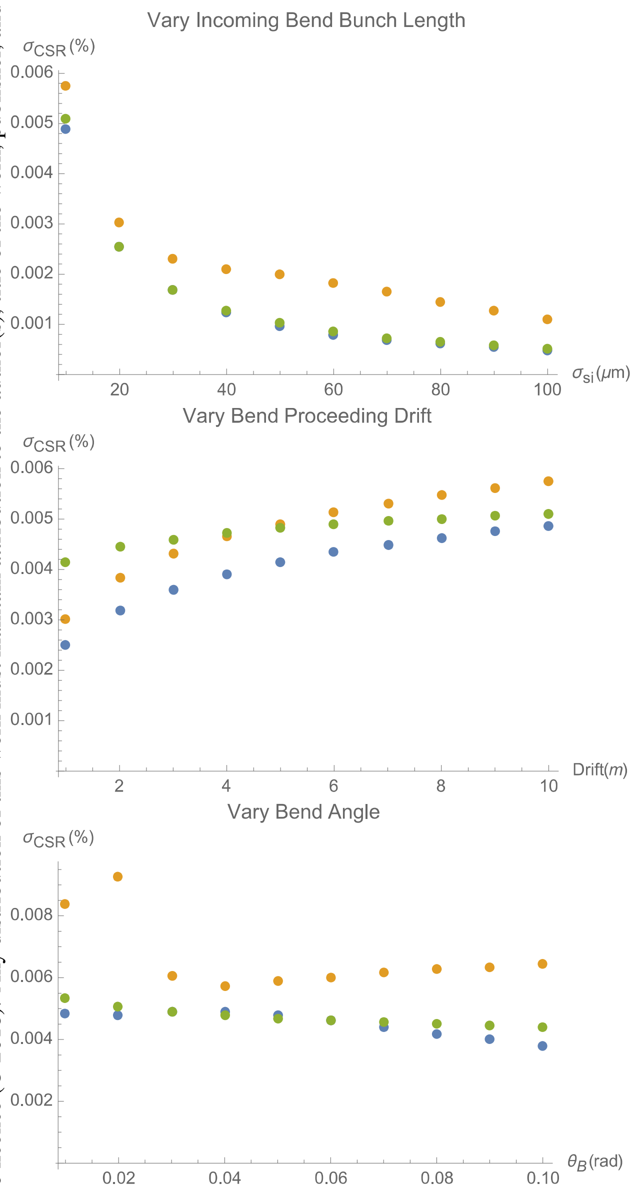

Figure 3: The relative CSR RMS energy spread measured at the end of the bend-drift system. ELEGANT (blue marker), the full CSR integral in eq. 2 (orange marker), and eq. 10 (green marker) are plotted together while varying the incoming bunch length (top), the subsequent drift length (middle), and bend angle values (bottom). In the top plot, the ELEGANT and the relative CSR RMS energy spread expression markers mostly overlap.

CONCLUSION

We have demonstrated, through simple mathematical approximations, that the relative CSR RMS energy spread for a beam transitioning into steady-state and a beam propagating in a post-bend drift section can be simply modelled. The expressions serve well for performing quick estimates of the CSR effect that is commonly found in FEL bending magnets, and in particular, bunch compressor chicanes. The utility of such expressions can be used to highlight, and thereby circumvent, features of the CSR effect that may lead to potential degradation of the beam phase space quality.

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