Longitudinal Phase Space Dynamics in ERLs

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Outline

- General features of longitudinal matches
- FEL match
- NP match
- Cooling match
- Conclusions

Top-Level Features of ERL Architectures

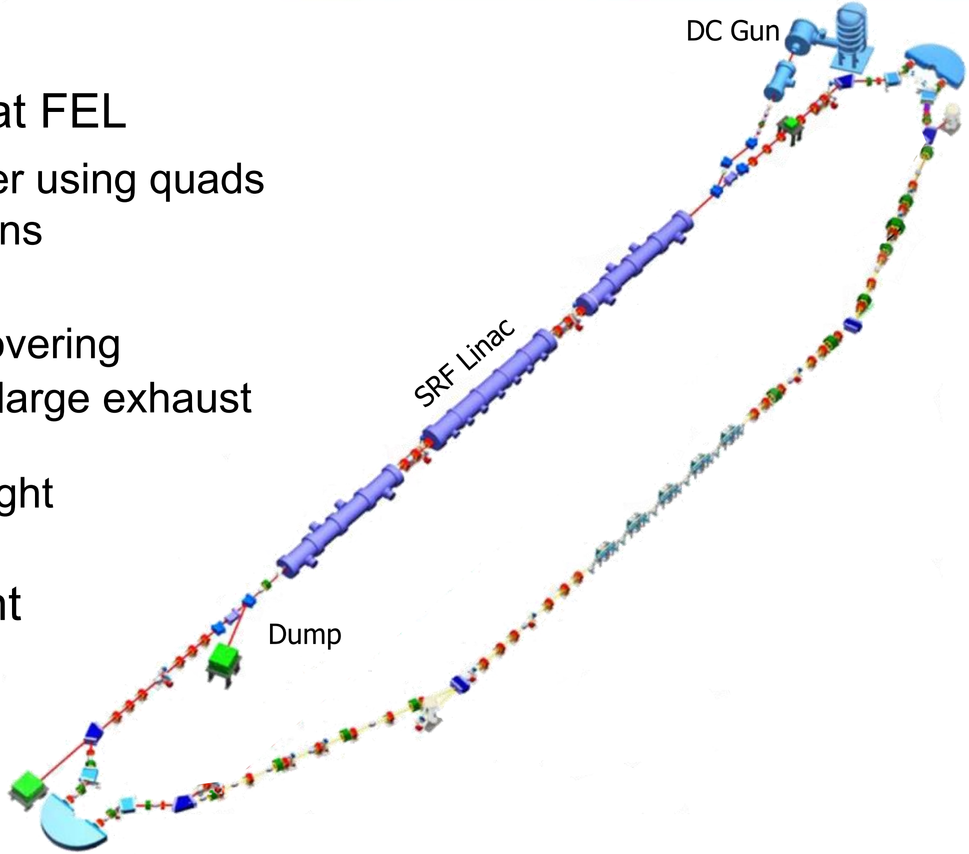
- Motivation for ERL architecture: *save money on* ***RF drive*** *while* ***delivering bright high power beams***
  - ideal ERLs have $P_{beam} >> P_{RF}$; involve very high CW beam powers
  - beam quality preservation and control of paramount importance
- ERLs are basically just ***time-of-flight spectrometers***
  - they exist to create specific conspiracies between time and energy
  - longitudinal motion (“longitudinal match”) must be carefully controlled
- ERLs are ***non-equilibrium systems***
  - ERLs look like rings, but behave like injector chains
    - high power beam $\Rightarrow$ “injection efficiency” (99.999+%) critical
  - beam and lattice are *different*, and beam is not Gaussian
    - beam and lattice may evolve independently, be mismatched
  - Beam degrades at the target and then gets anti-damped.

FEL Longitudinal Matching Scenario

Requirements on phase space:

- high peak current (short bunch) at FEL
  - bunch length compression at wiggler using quads and sextupoles to adjust compactions
- “small” energy spread at dump
  - energy compress while energy recovering
  - “short” RF wavelength/long bunch, large exhaust $\delta p/p$ (~10%)

  $\Rightarrow$get slope, curvature, *and* torsion right (quads, sextupoles, octupoles)
- Note that this is a parallel-to-point longitudinal match



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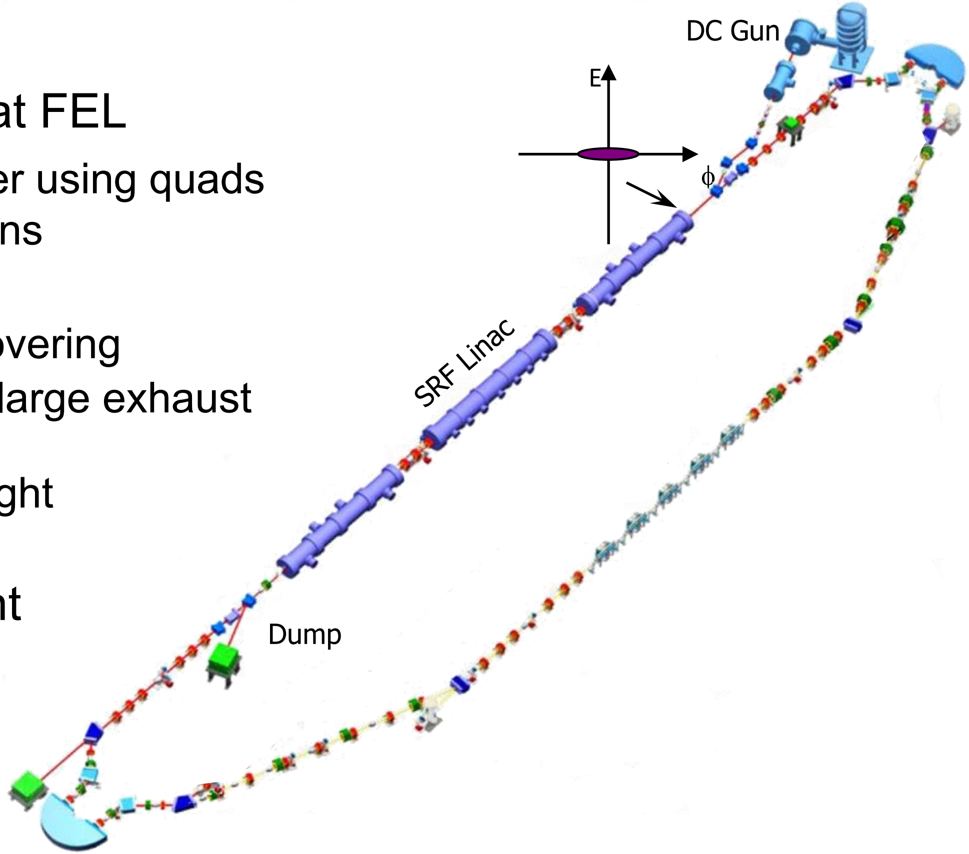

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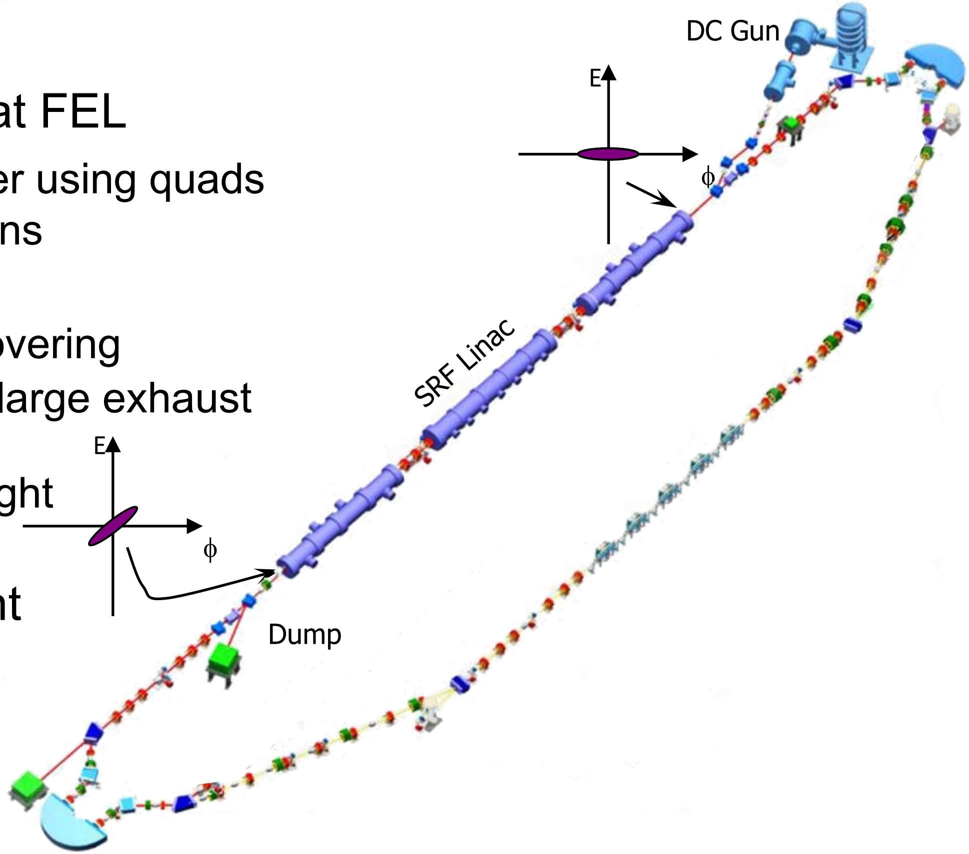

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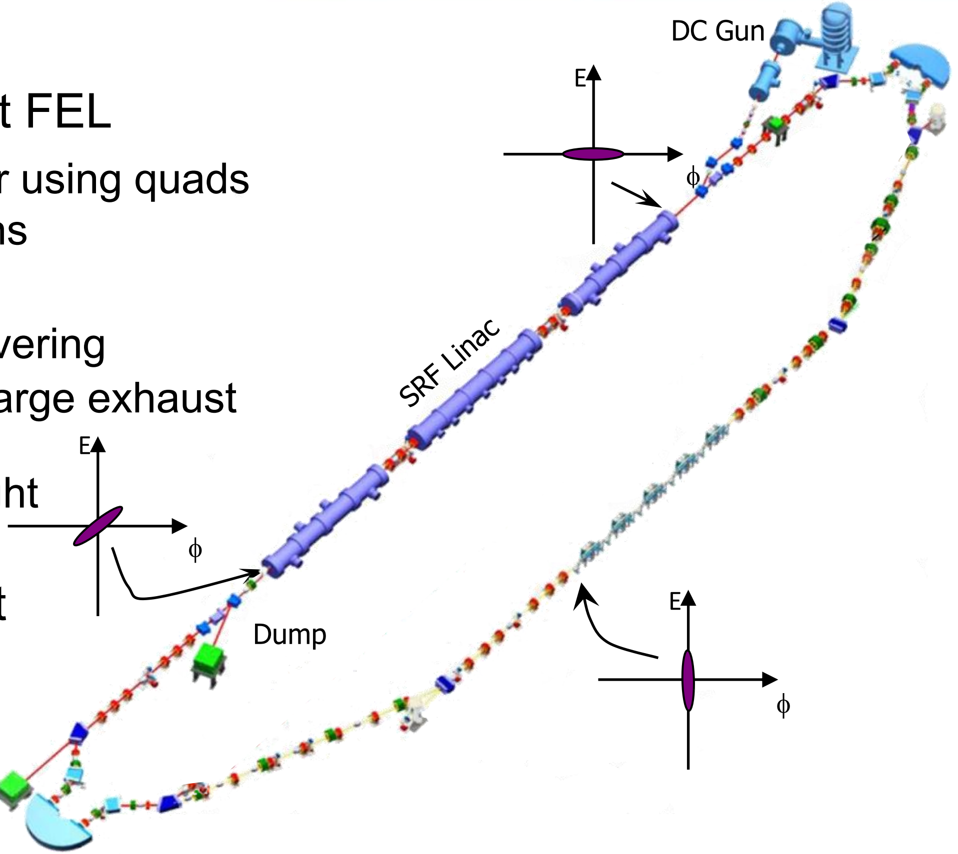

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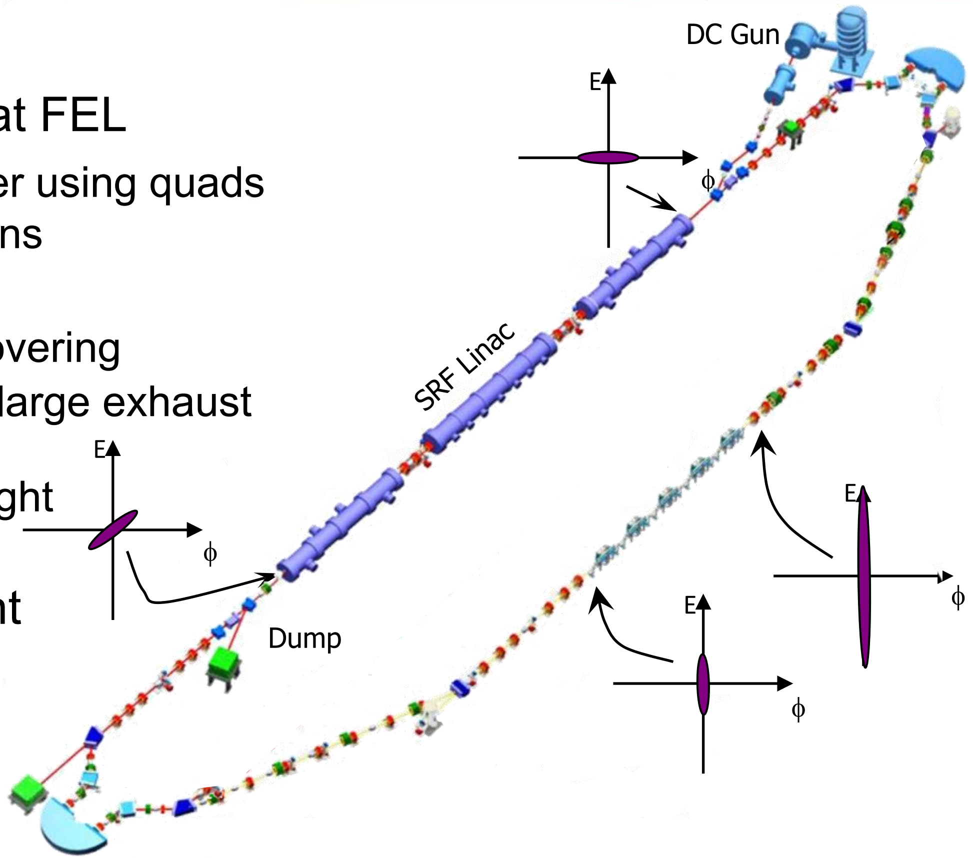

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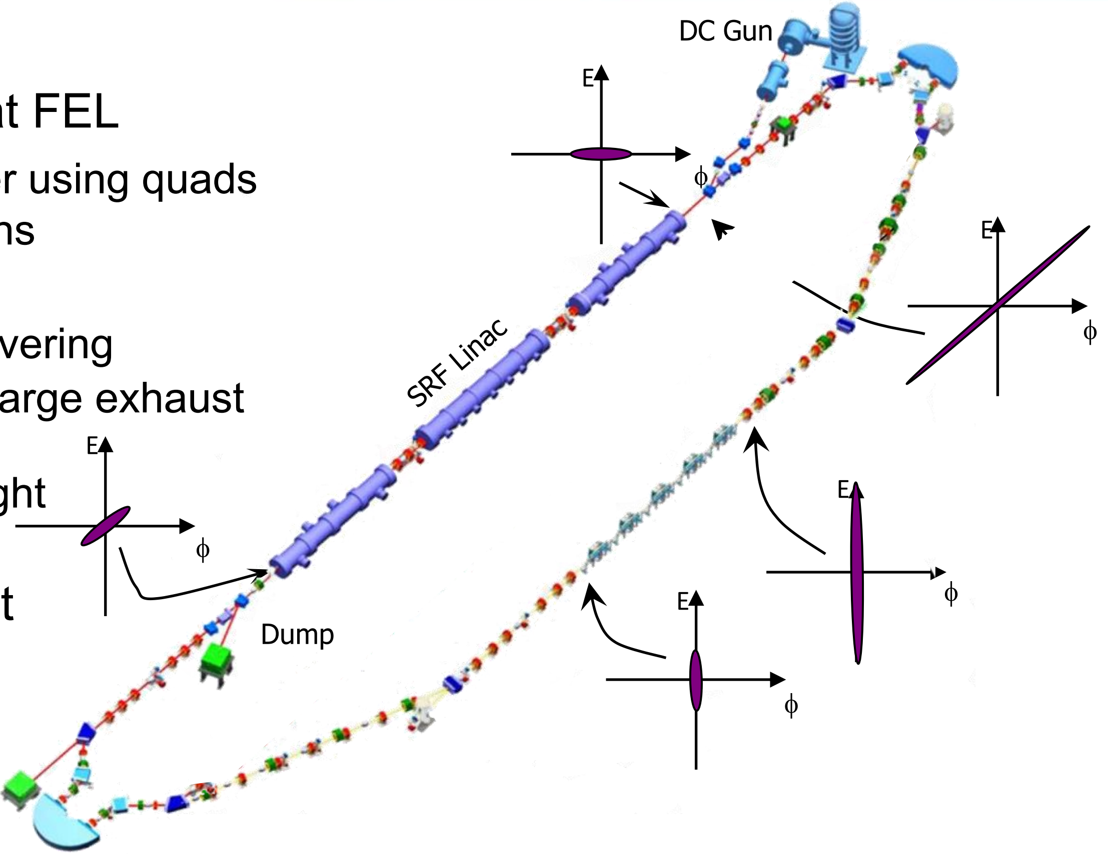

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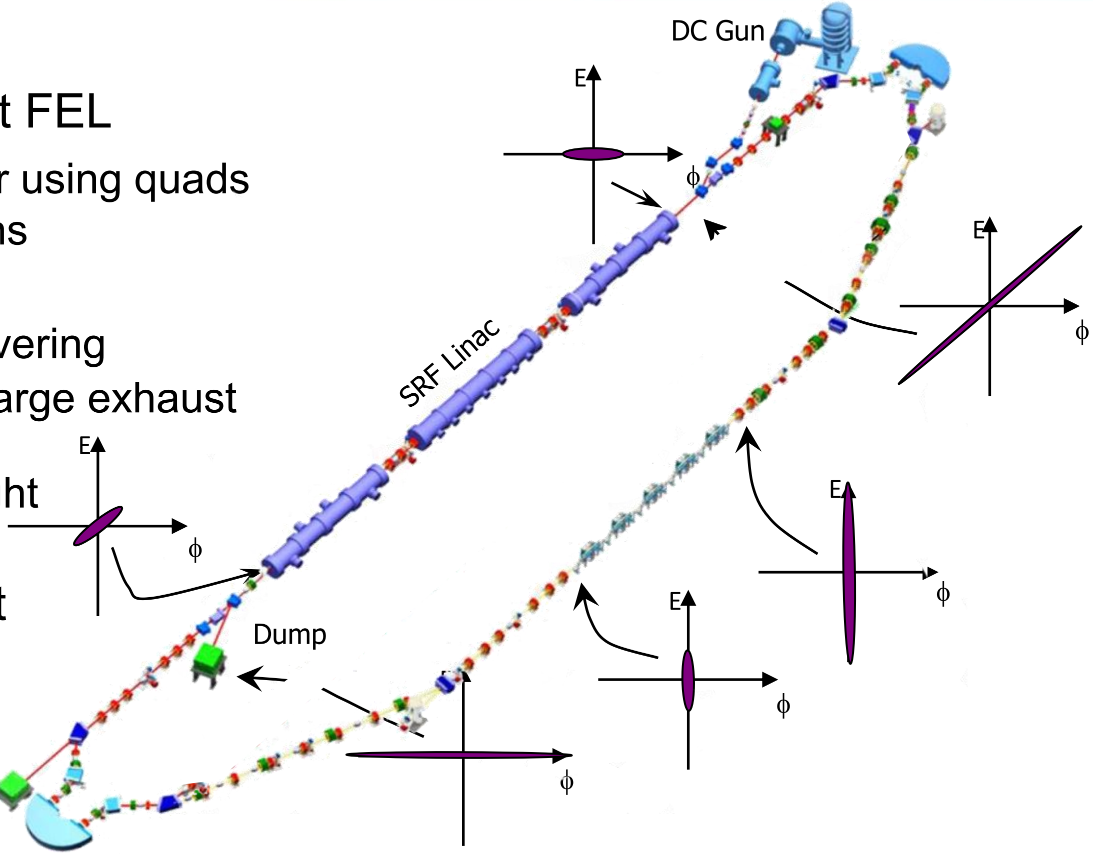
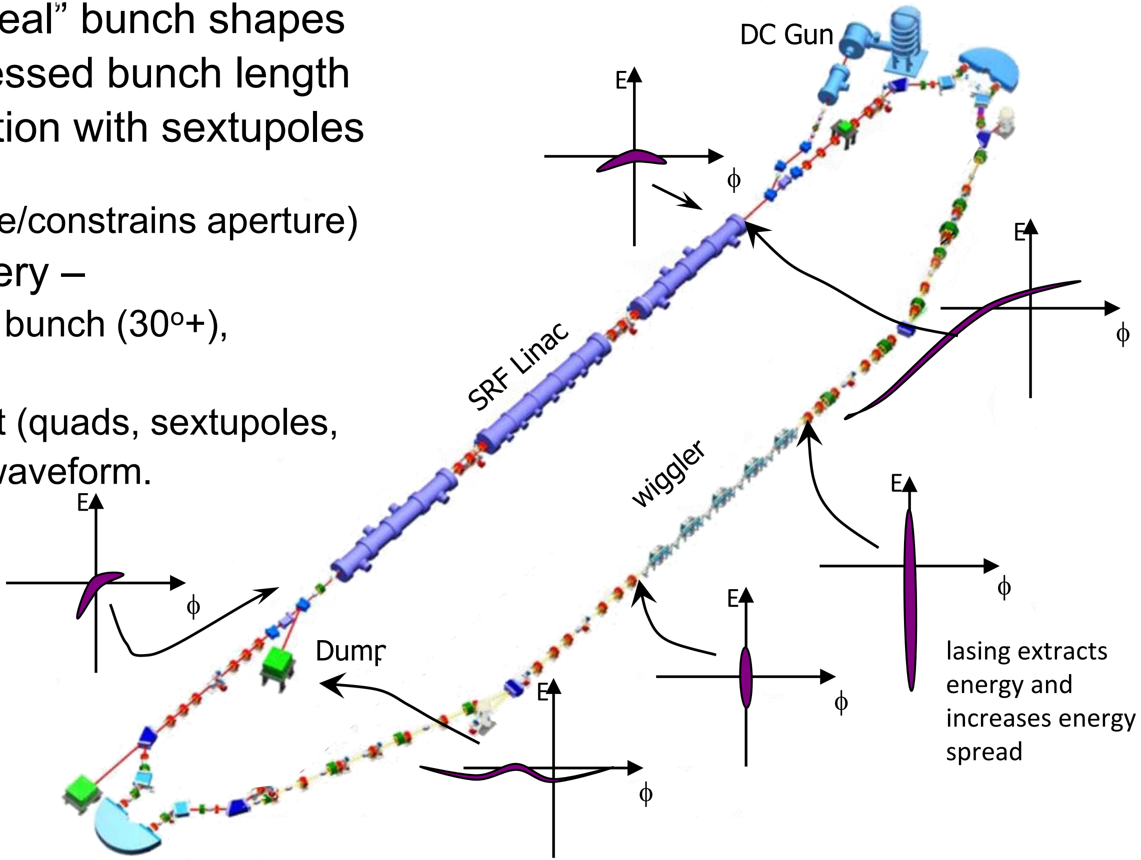


Nonlinear FEL Longitudinal Matching

As shown earlier – but with typical "real" bunch shapes

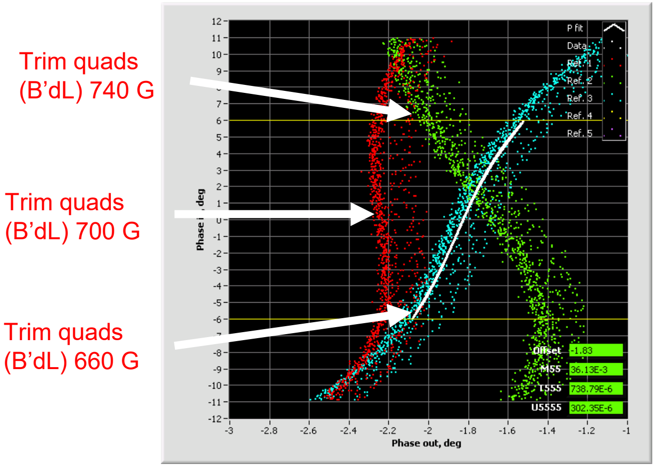
- RF curvature can degrade compressed bunch length
- Set nonlinear momentum compaction with sextupoles to compensate & linearize bunch
  - Avoids use of harmonic RF (expensive/constrains aperture)
- Energy compression during recovery –
  - "short" RF wavelength/ultimately long bunch (30°+), large exhaust $\delta p/p$ (~10-15%)

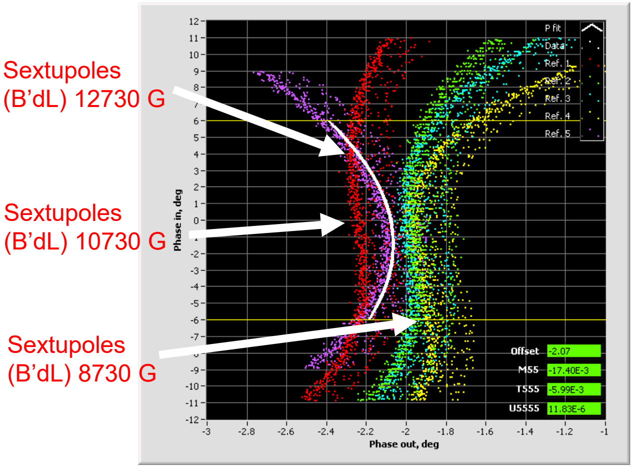
  $\Rightarrow$get slope, curvature, *and* torsion right (quads, sextupoles, octupoles…) to match bunch to RF waveform.



JLab FEL bunch compression and diagnostics

- The Jlab FEL operated with a bunch compression ratio of 17–25 using nonlinear compression – compensating for LINAC RF curvature (up to 2nd order during acceleration and 3rd order during recovery).
- The RF curvature compensation is made with multipoles installed in dispersive locations of 180° Bates bend with separate function magnets - no harmonic RF
- Operationally longitudinal match relies on:
  a. Longitudinal transfer function measurements R_{55}, T_{555}, U_{5555}
  b. Bunch length measurements at full compression (Martin-Puplett Interferometer)
  c. Energy spread measurements in injector and exit of the LINAC





Connecting R_{56} & T_{566} to M_{55}

- R_{56} and T_{566} are validated via longitudinal transfer function measurements.
- Arrival phase is measured with a pillbox cavity + heterodyne receiver.
- Phase of the injector is modulated relative to the LINAC phase
- Essential ~ 15 % energy acceptance and ~ 30 % phase acceptance

Connecting R_{56} & T_{566} to M_{55}

$$\varphi_W = \left(1 + R_{56}^C \cdot R_{65}^L\right)\varphi_0 + \left[R_{56}^C \cdot T_{655}^L + \left(R_{65}^L\right)^2 \cdot T_{566}^C\right]\varphi_0^2$$

taking second order transport matrix elements

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taking second order transport matrix elements

directly measured

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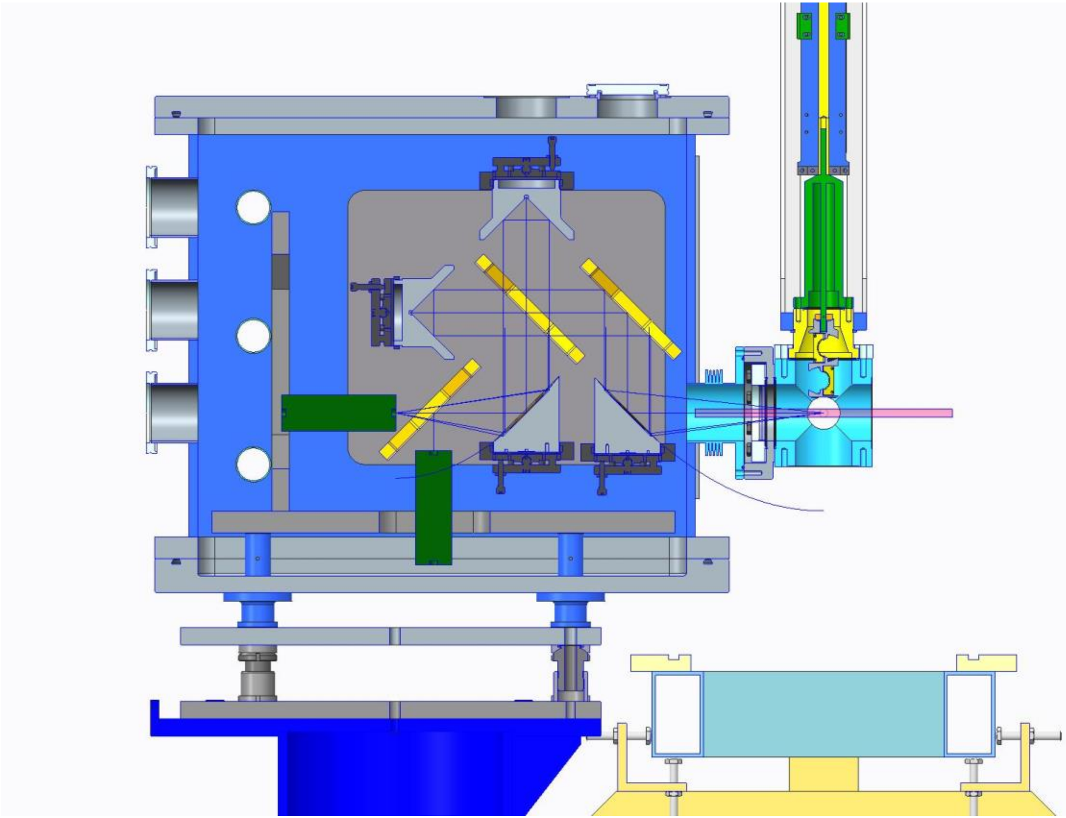
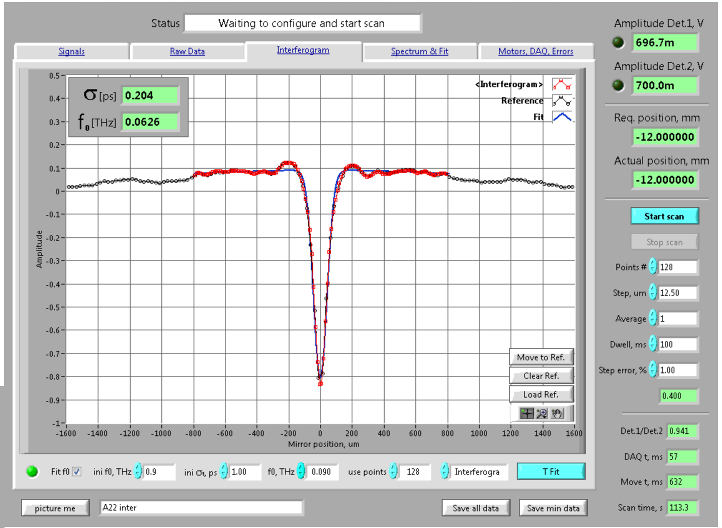
ELBE – Martin-Puplett Interferometer

- Interferometer for ELBE – proper Martin-Puplett interferometer
- Wire-grid polarizers scaled down by factor 2 to allow shorter than 50 fs measurements
- Built with vacuum chamber to reduce air absorption

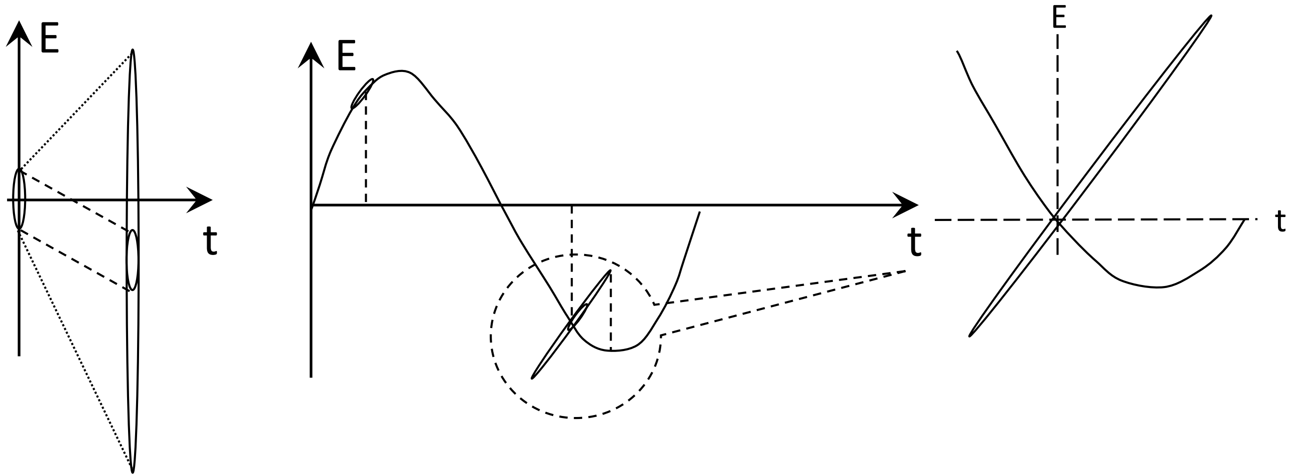


MPI @U37 - Data Evaluation

- At IR/UV Upgrade interferometer data evaluation – bunch length extraction was made in frequency domain, NLSF + Gaussian beam assumption
- With ELBE MPI data frequency domain fit is often difficult
- Changed to data evaluation with time domain NLSF
- Always used all data points for fit
- Much more robust



Energy Compression During Recovery



- Beam central energy drops, relative beam energy spread grows
- Recirculator and beam central energies must match to maximize acceptance
- Beam rotated, curved, torqued to match shape of RF waveform
- Maximum energy can't exceed peak *deceleration* available from linac
  - Corollary: entire bunch must precede trough of RF waveform

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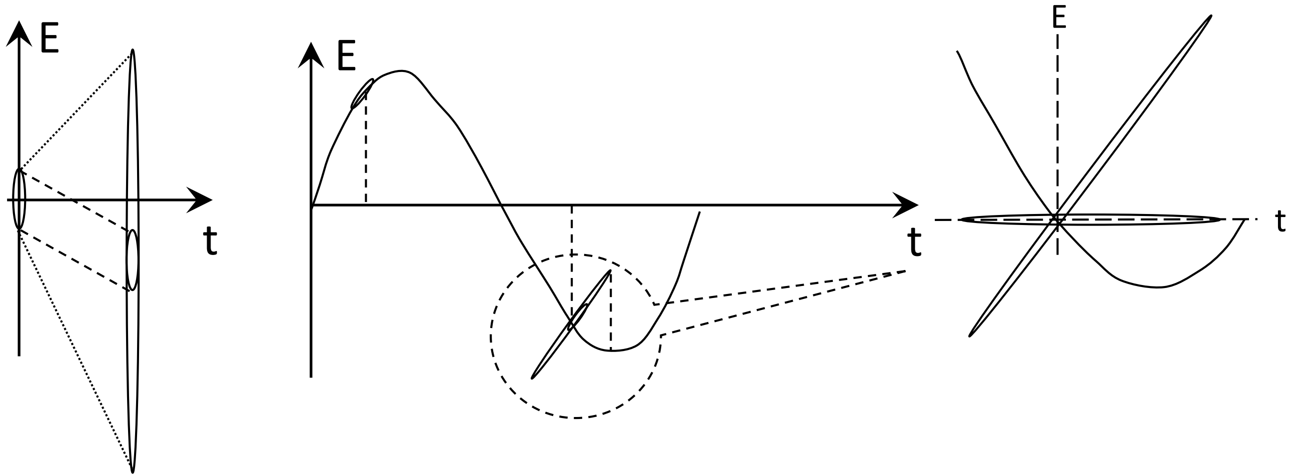

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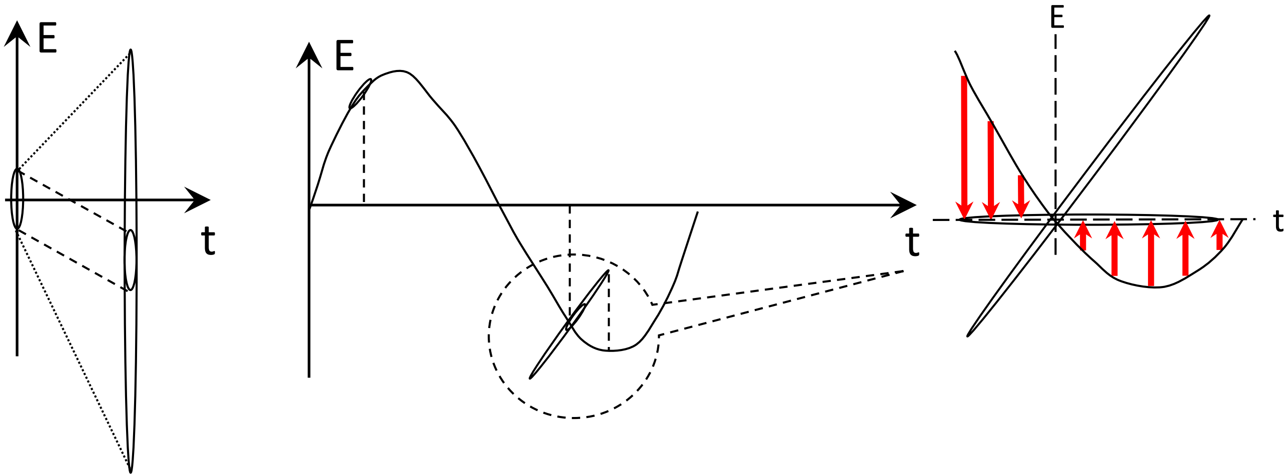

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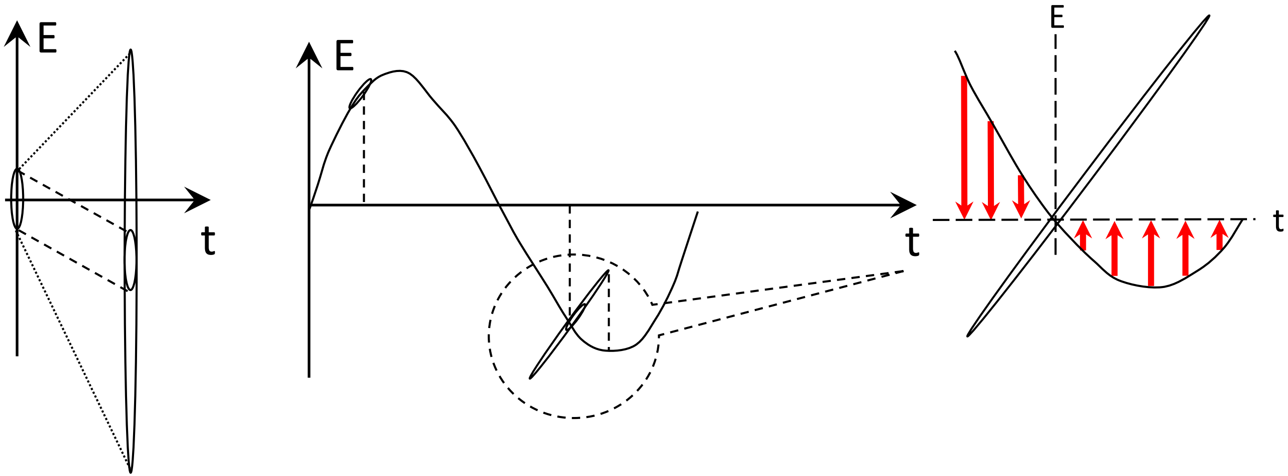

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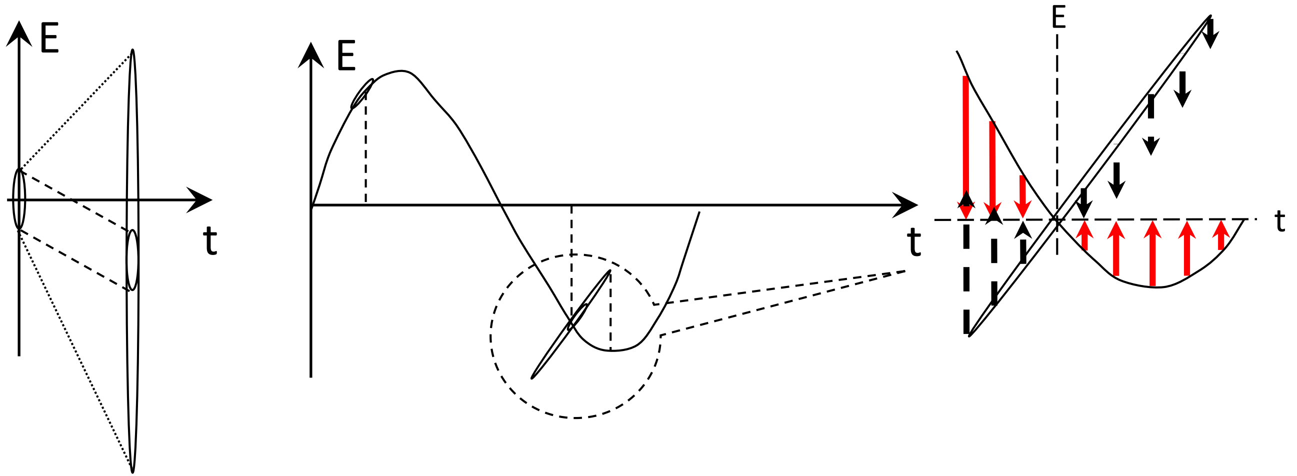

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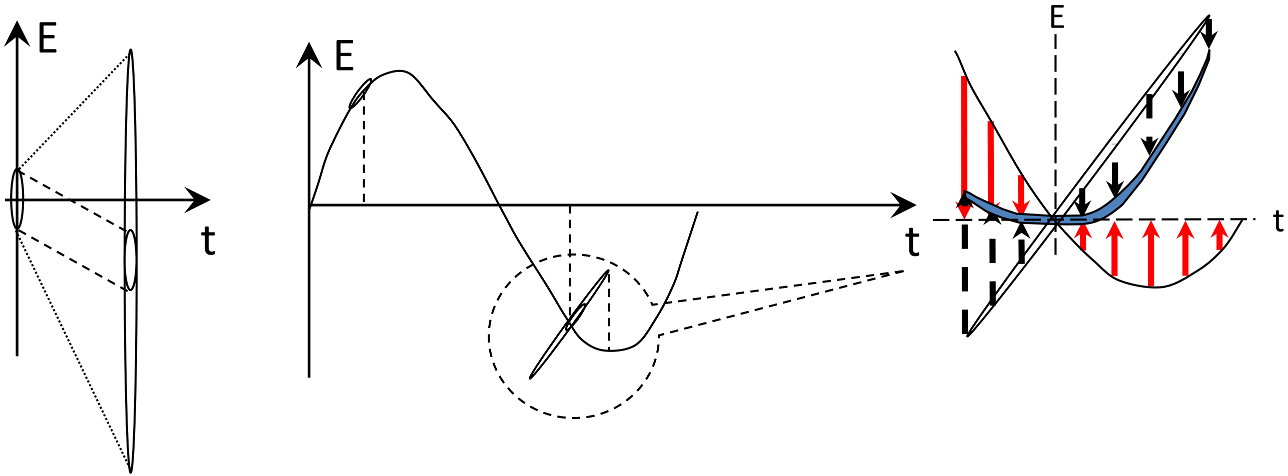

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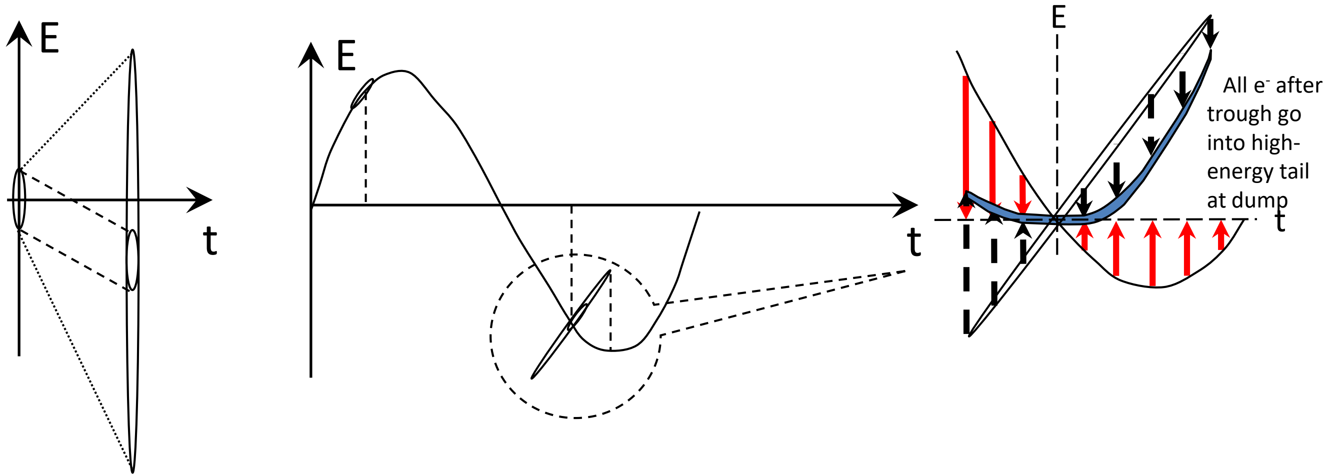

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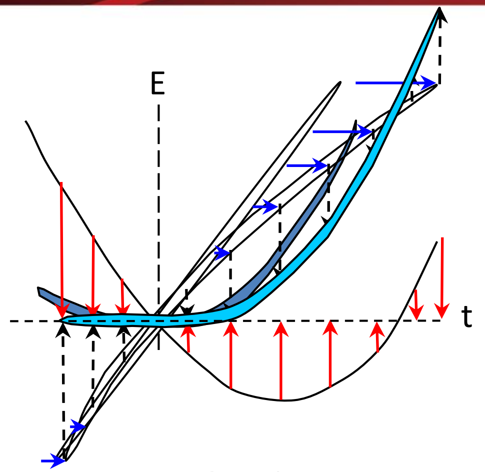
Higher Order Corrections

- Without nonlinear corrections, phase space becomes distorted during deceleration
- Curvature, torsion,… can be compensated by nonlinear adjustments
  - differentially move phase space regions to match gradient required for energy compression
- Required phase bite is $\cos^{-1}(1-\Delta E_{FEL}/E_{LINAC})$; at modest energy this is

  >25° at RF fundamental for 10%

  >30° for 15%

  - typically need 3rd order corrections (octupoles)
  - also need a few extra degrees for tails, phase errors & drifts, irreproducible & varying path lengths, etc, so that system operates reliably
- In this context, **harmonic RF very hard to use…**



$$M_{56} = -\frac{\lambda_{RF}}{2\pi}\left(\frac{E_0}{E_{linac}}\right)\frac{1}{\sin\phi_0}$$

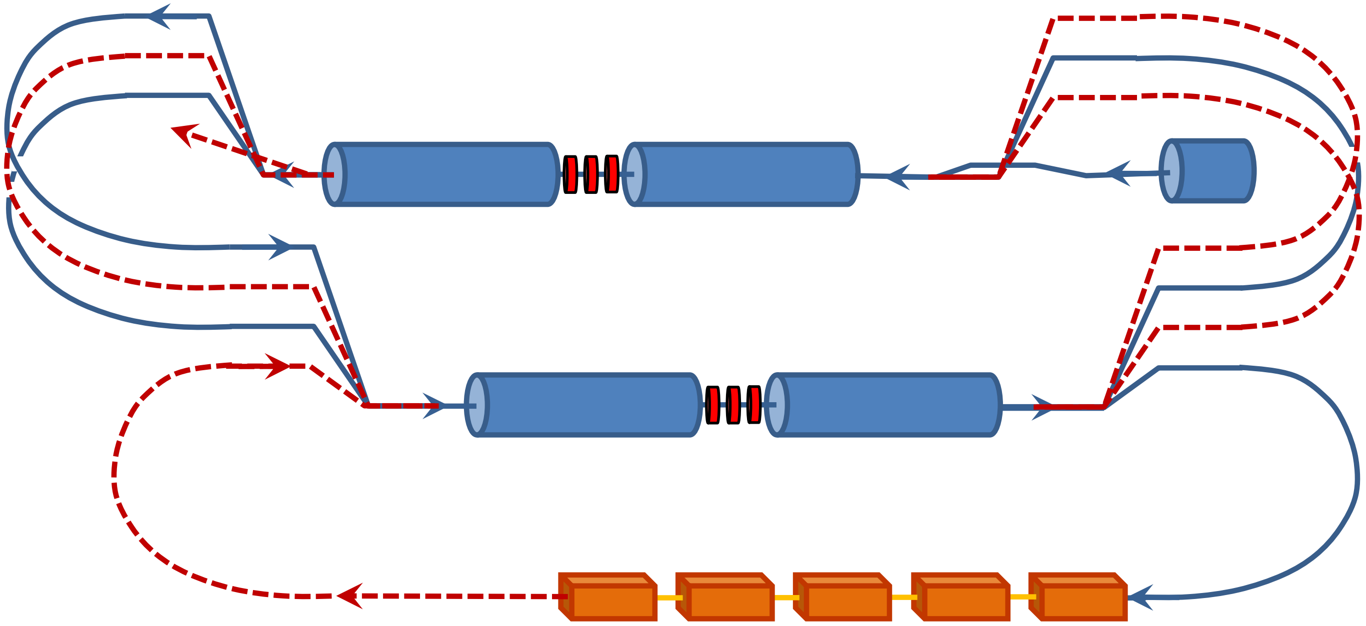
$$T_{566} = -\frac{1}{2}\left(\frac{2\pi}{\lambda_{RF}}\right)(M_{56})^2\frac{\cos\phi_0}{\sin\phi_0}$$

$$W_{5666} = -\left[\frac{1}{6}+\frac{1}{2}\frac{\cos^2\phi_0}{\sin^2\phi_0}\right]\left(\frac{2\pi}{\lambda_{RF}}\right)^2(M_{56})^3$$

$$U_{56666} \propto \left(\frac{2\pi}{\lambda_{RF}}\right)^3(M_{56})^4\text{, etc.}$$

Machine Topology for Multipass Machine

- At high energy, a telescopic match is better suited to the requirement for small energy spread but high peak current.
- User separated function arcs to accelerate in two pass up and two passes down.



Longitudinal Matching Solution: Delivery to FEL

- Inject long, low momentum spread bunch
- Initially accelerate on rising portion of RF waveform
- Perform mild compression, full RF curvature compensation at mid-energy

  NOTE*: intermediate stages of compression* **must** *be performed mid-way through acceleration cycle when using multistage compression in an ERL (if transport is common to both accelerated and recovered beams)*

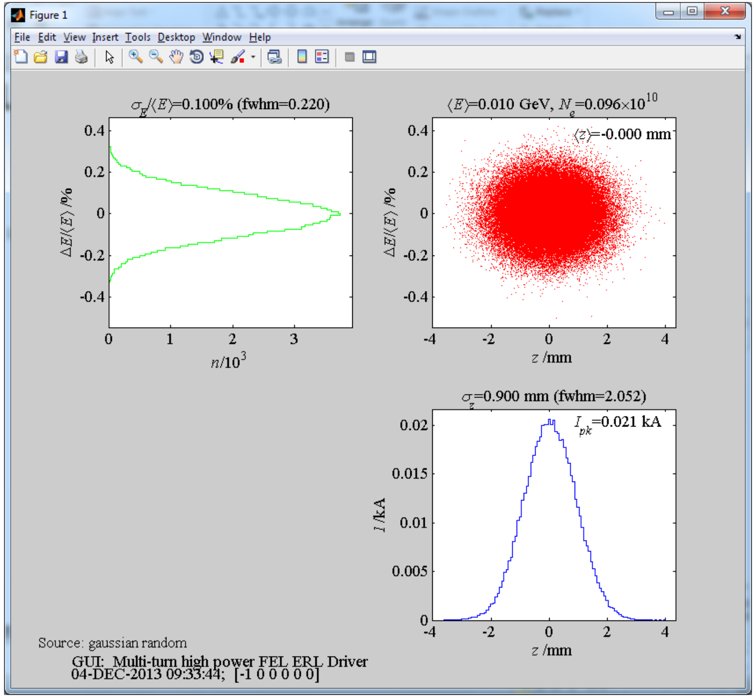
- Accelerate to full energy on falling portion of RF waveform to de-chirp beam to produce small final momentum spread
- Compress bunch length during transport to FEL

Longitudinal Matching Solution: Energy Recovery

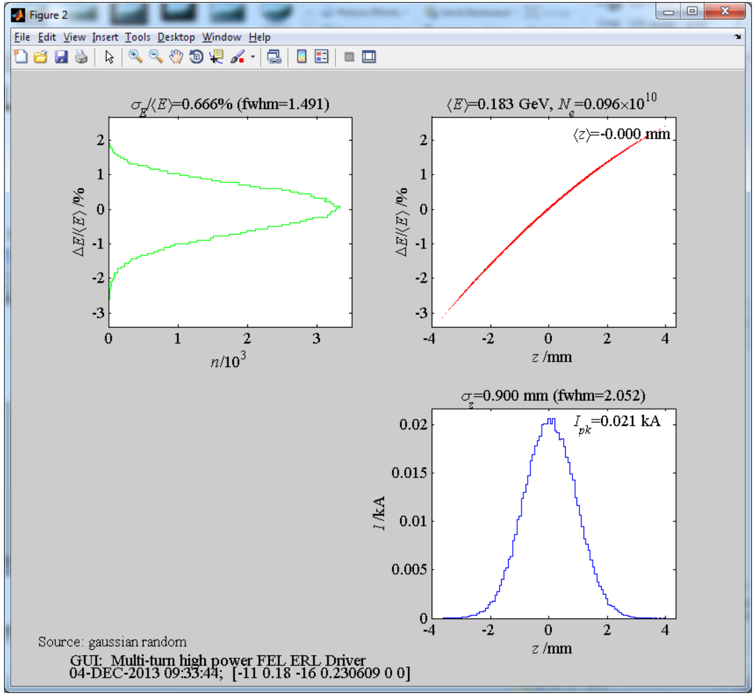
- Complete energy recovery while lasing
  - $E_{dump} < E_{injection}$
- Multistage nonlinear energy compression during energy recovery
  - Curvature/torsion compensation
  - provides small $\delta p/p$ at dump
  - Keeps E_{dump} constant as FEL turns off/on
  - Defines RF drive requirements (which are modest)

Longitudinal Phase Space

Injected

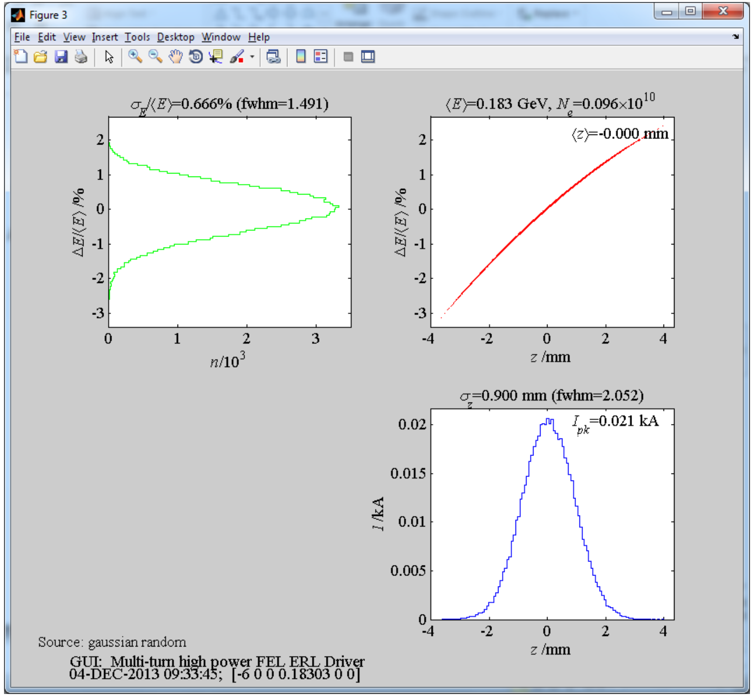


Linac 1: Exit Pass

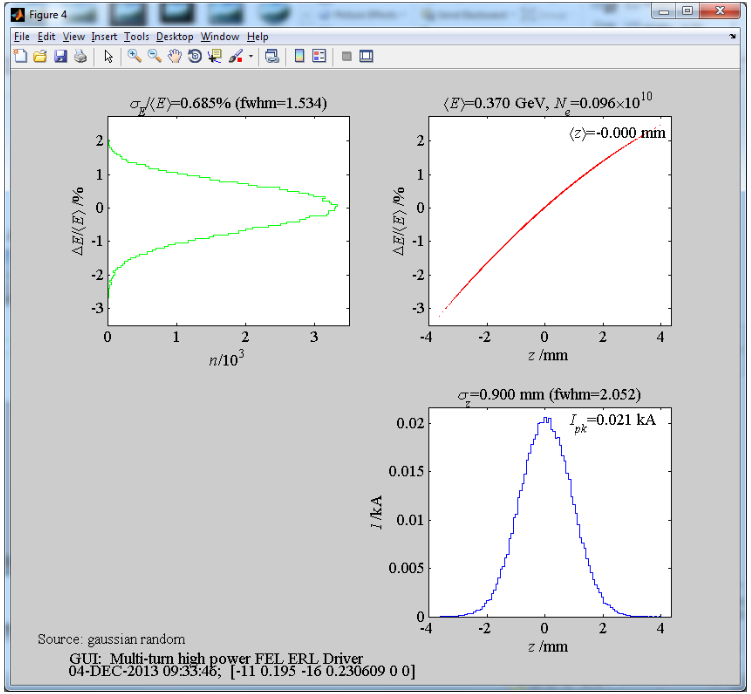


Longitudinal Phase Space

Arc 1: Exit



Linac 2: Exit Pass 1



Longitudinal Phase Space

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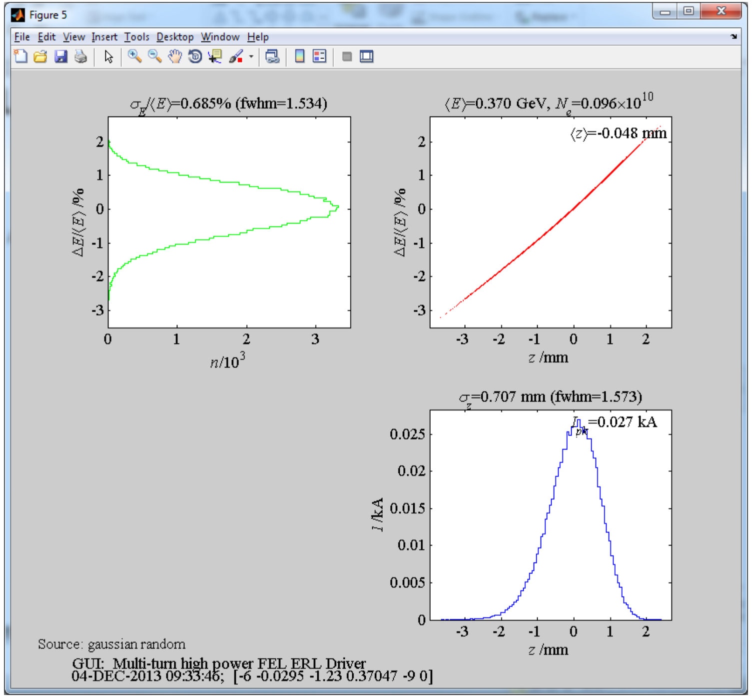
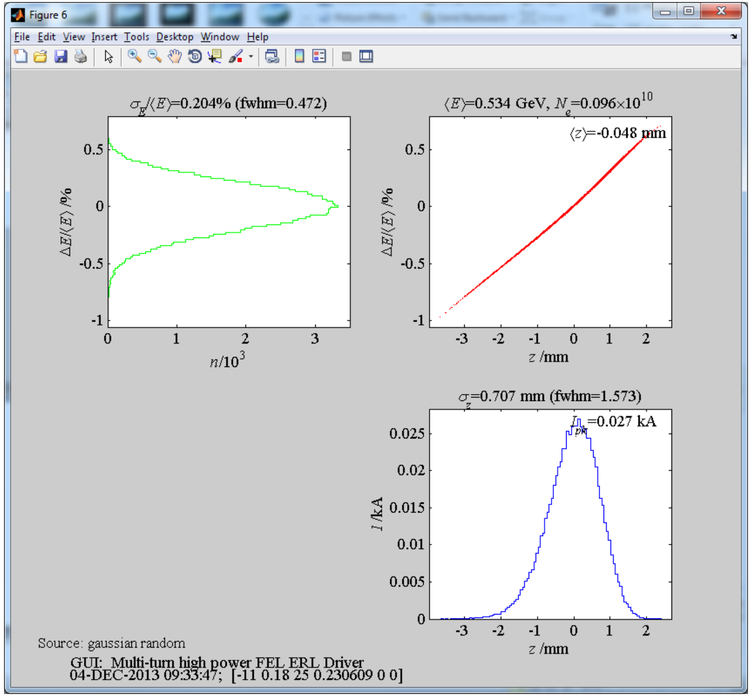
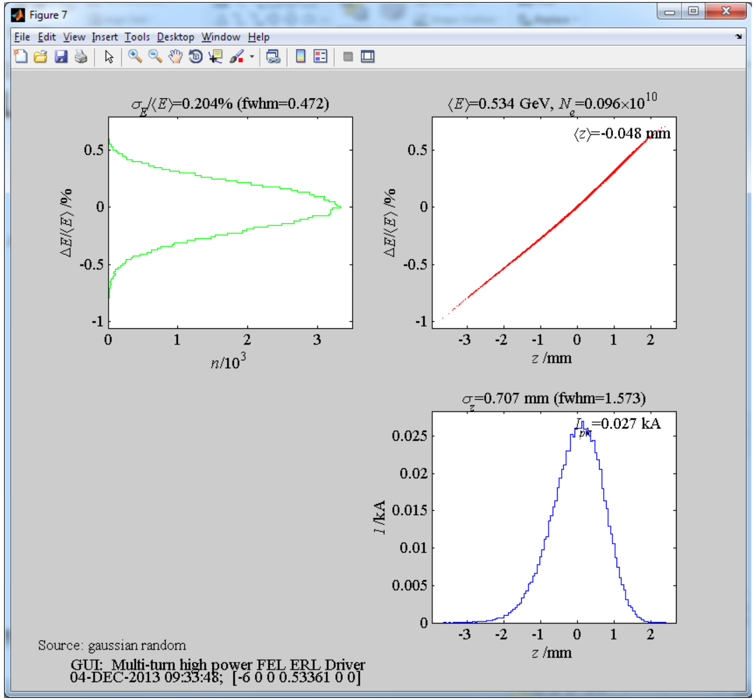


Linac 1: Exit Pass 2

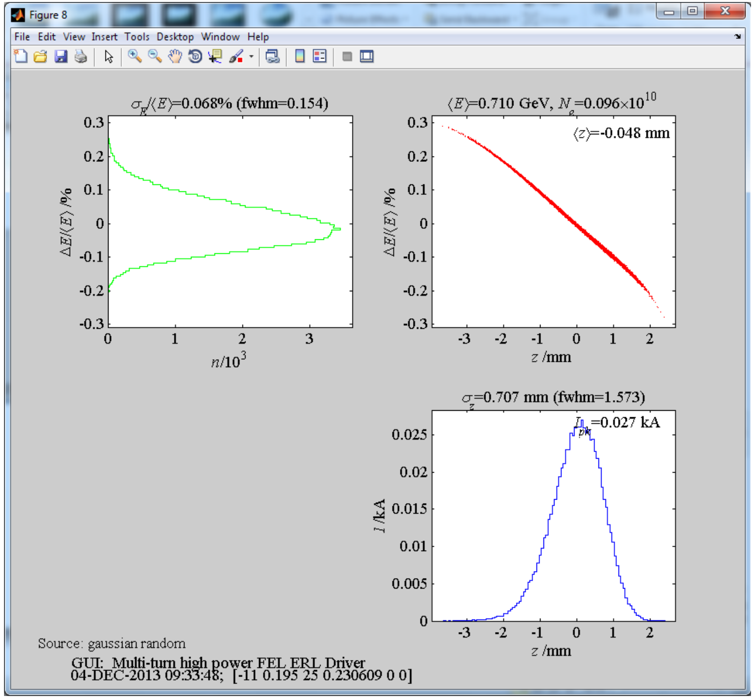


Longitudinal Phase Space

Arc 3: Exit

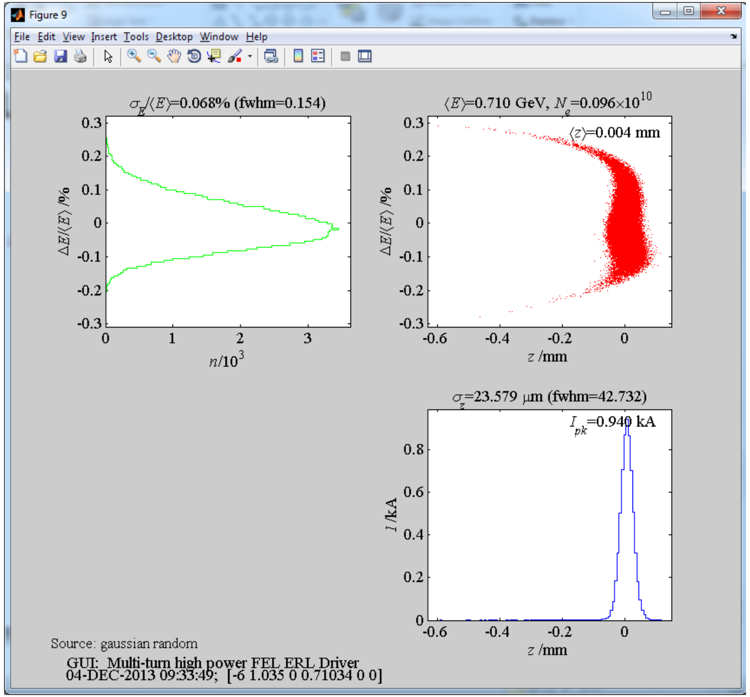


Linac 2: Exit Pass 2

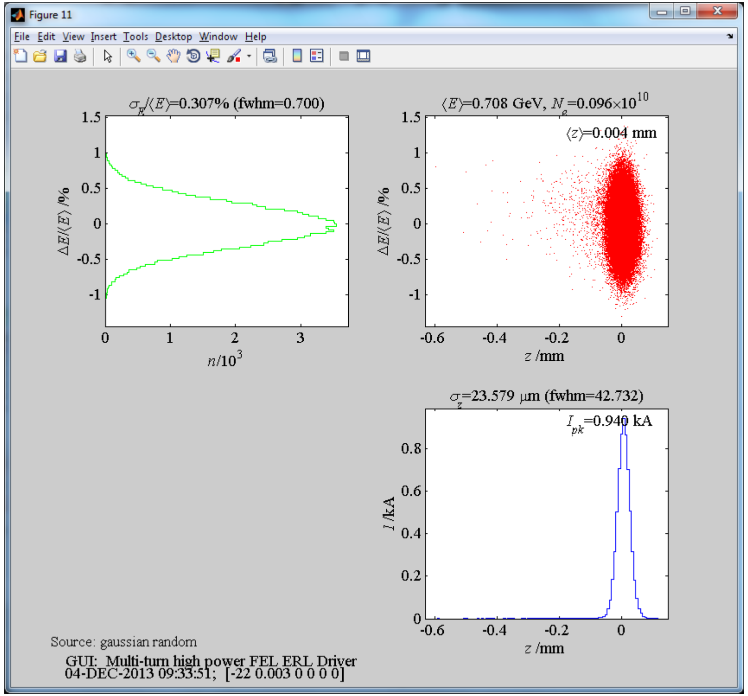


Longitudinal Phase Space

Arc 4: Exit

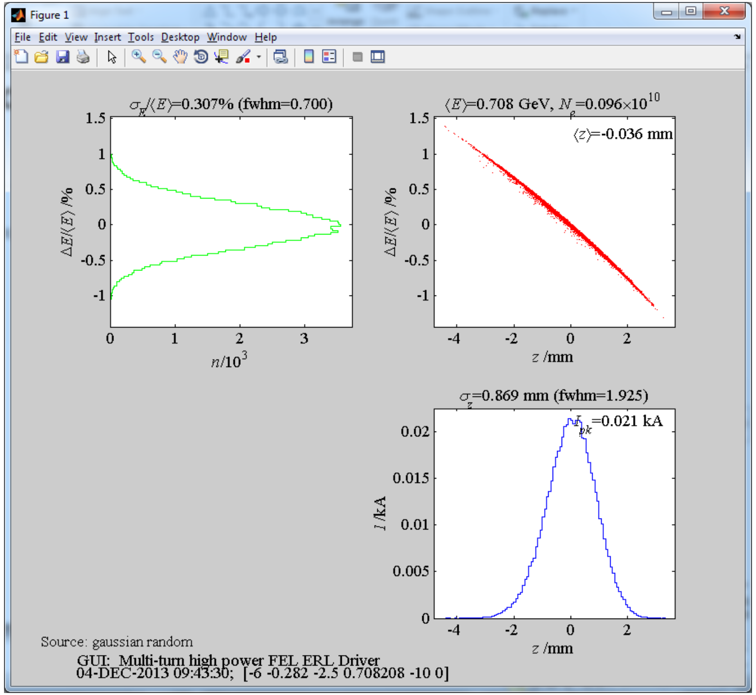


After FEL

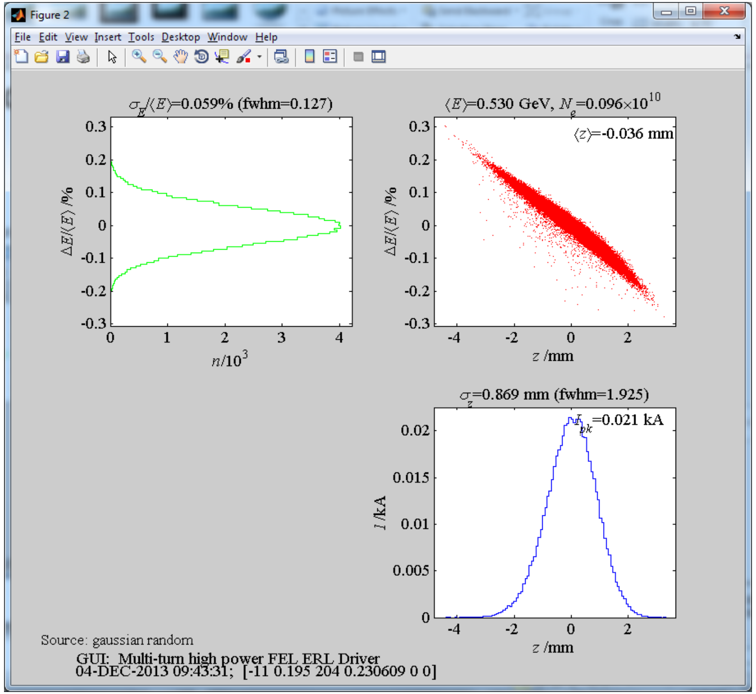


Longitudinal Phase Space

Arc 5: Exit

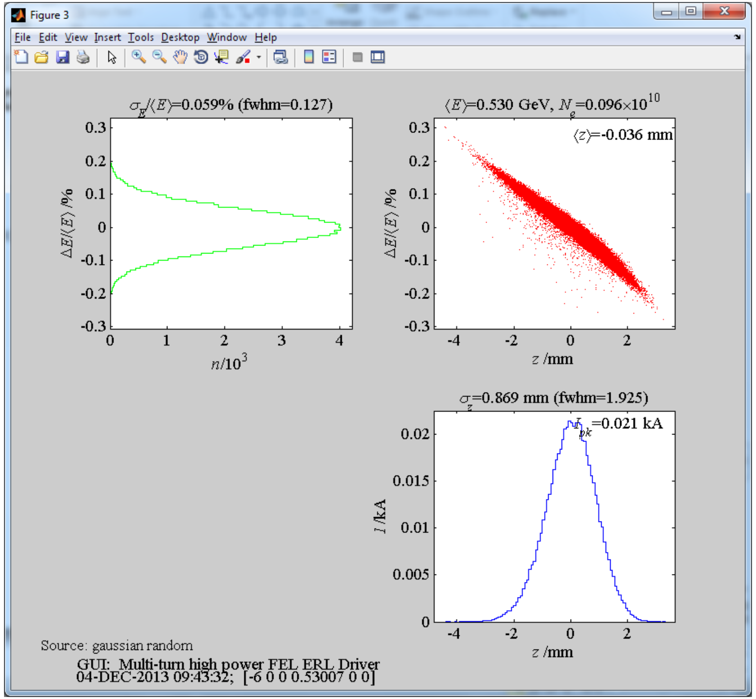


Linac 2: Exit Pass 3

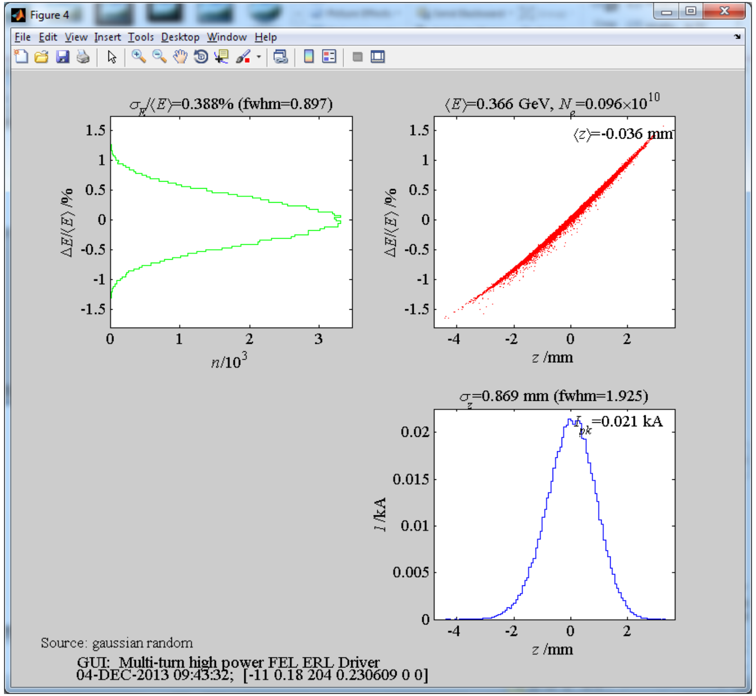


Longitudinal Phase Space

Arc 6: Exit

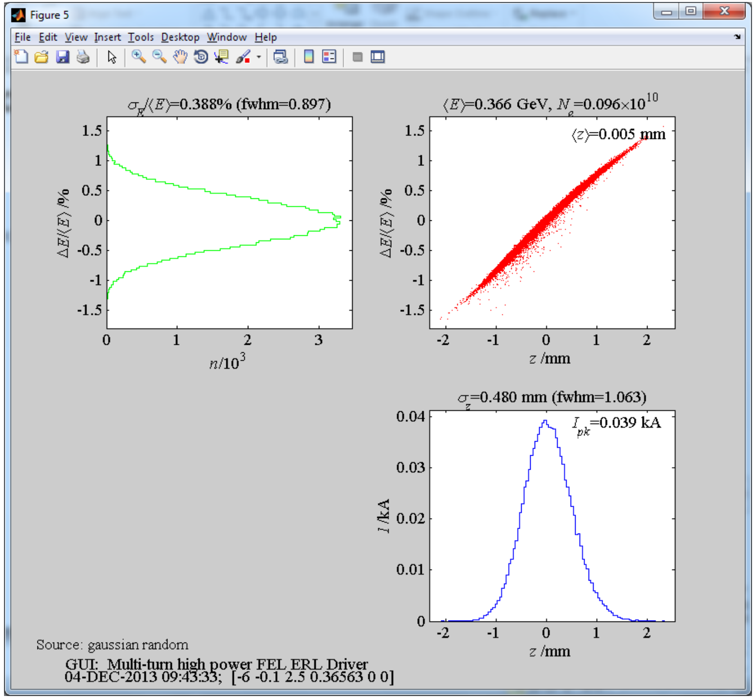


Linac 1: Exit Pass 3

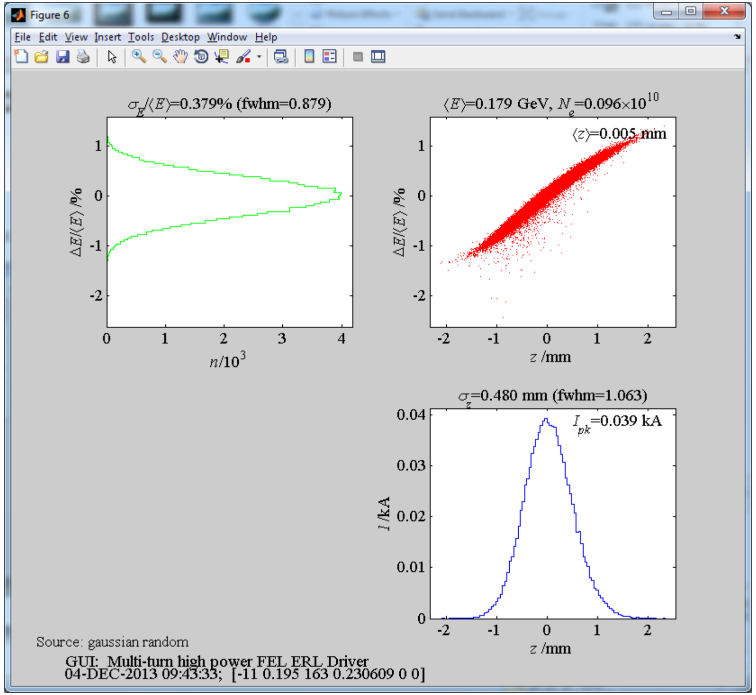


Longitudinal Phase Space

Arc 7: Exit

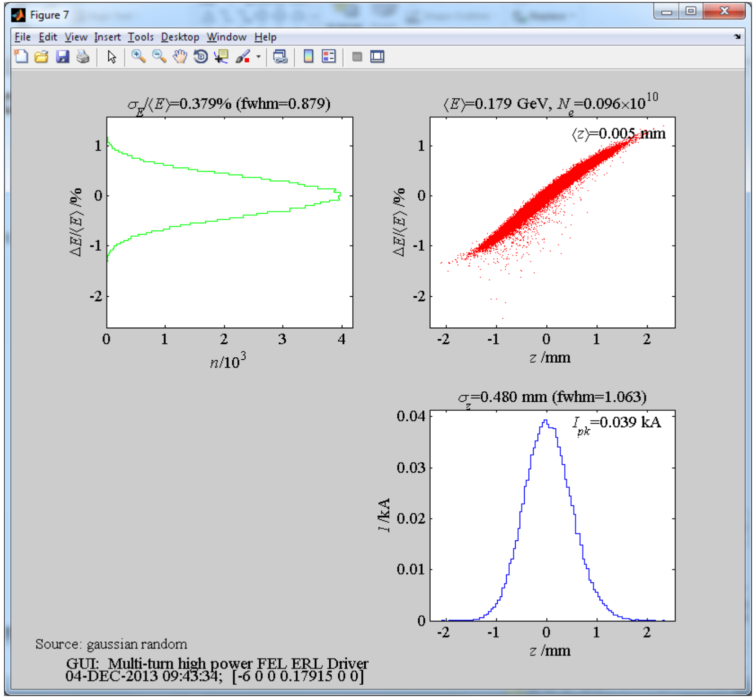


Linac 2: Exit Pass 4

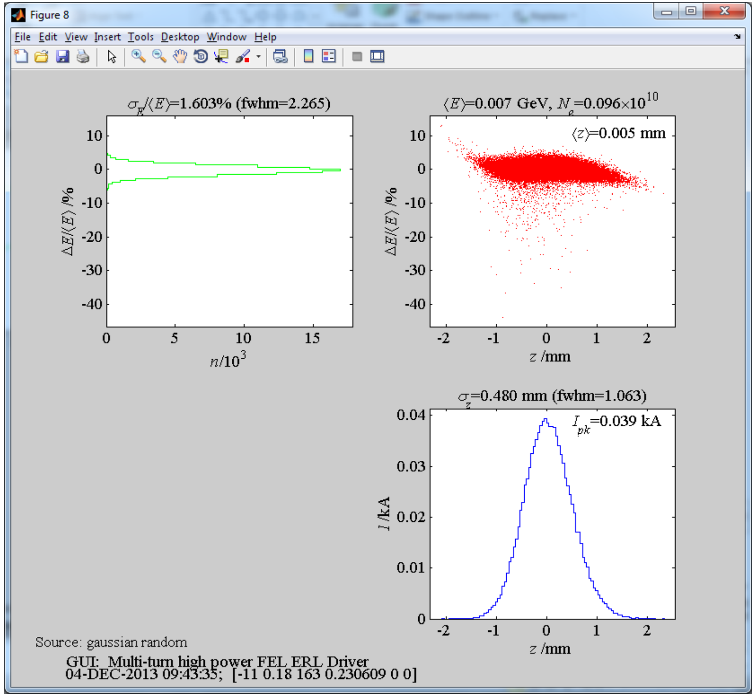


Longitudinal Phase Space

Arc 8: Exit

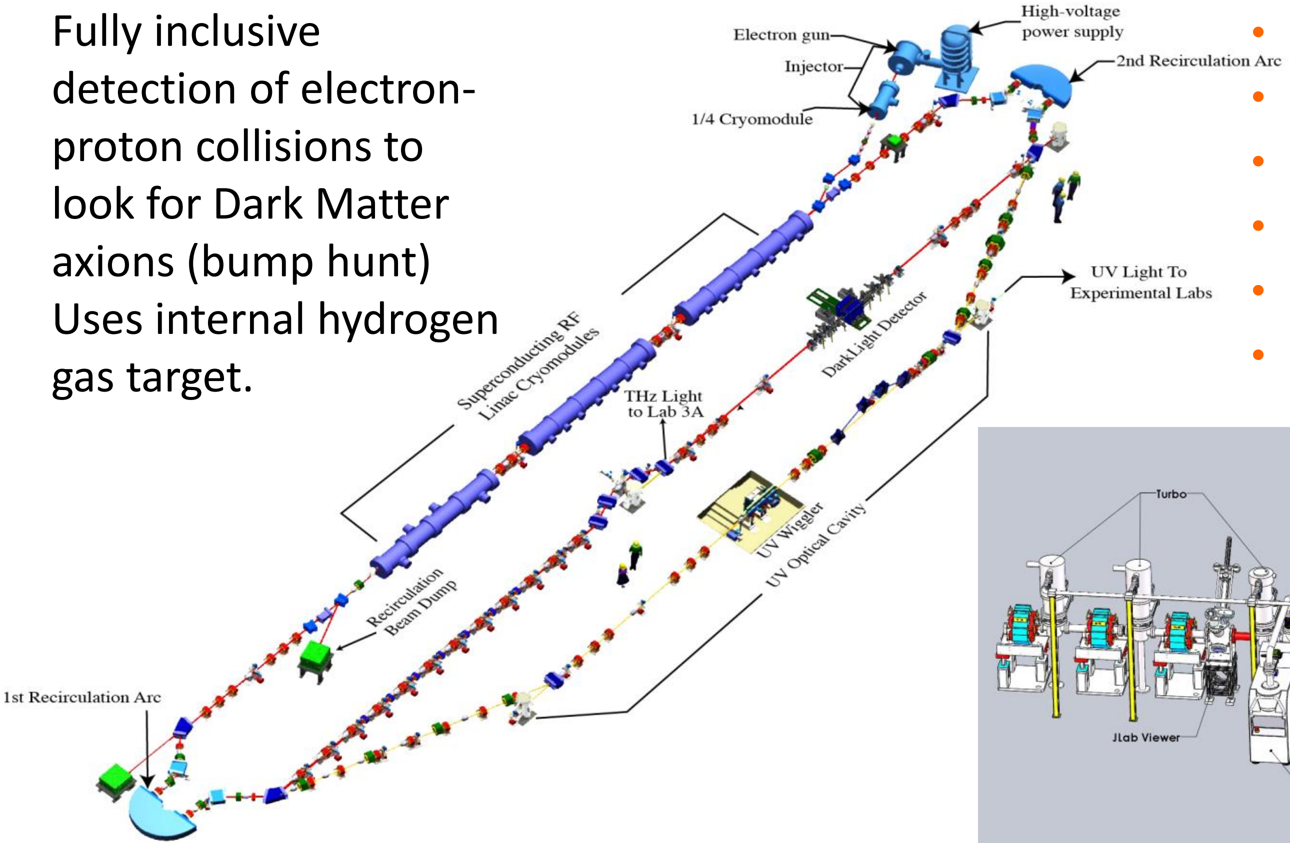


Linac 1: Exit Pass 4

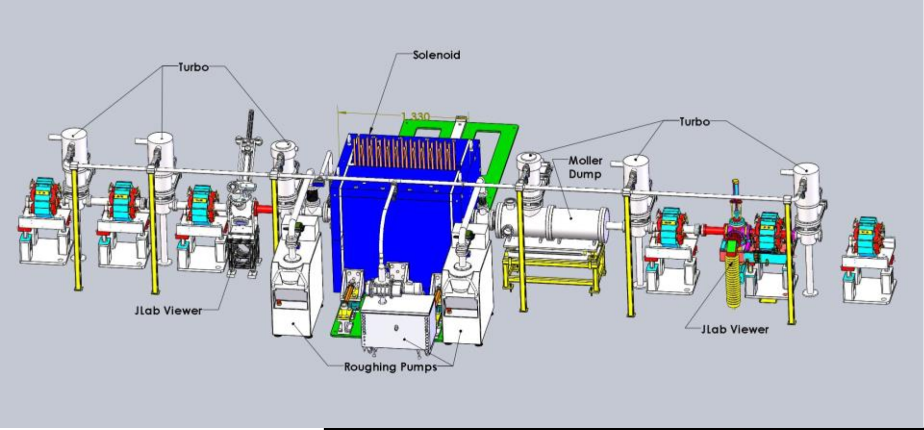


DarkLight Experiment

Fully inclusive detection of electron-proton collisions to look for Dark Matter axions (bump hunt)
Uses internal hydrogen gas target.

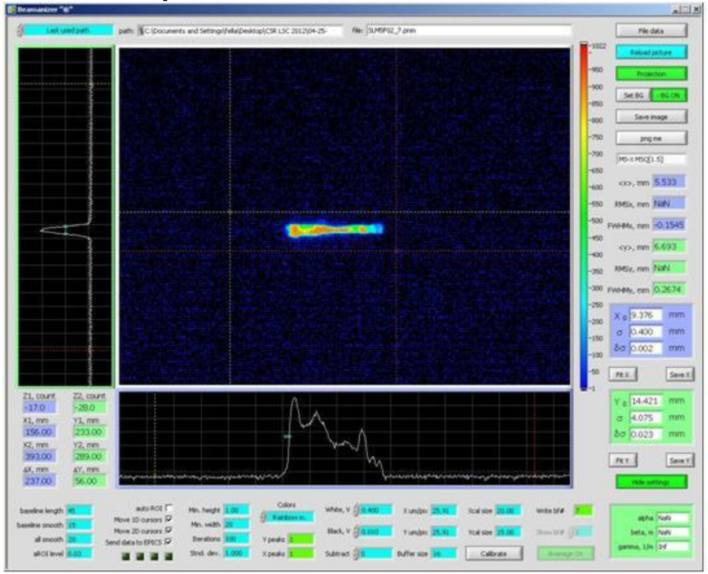


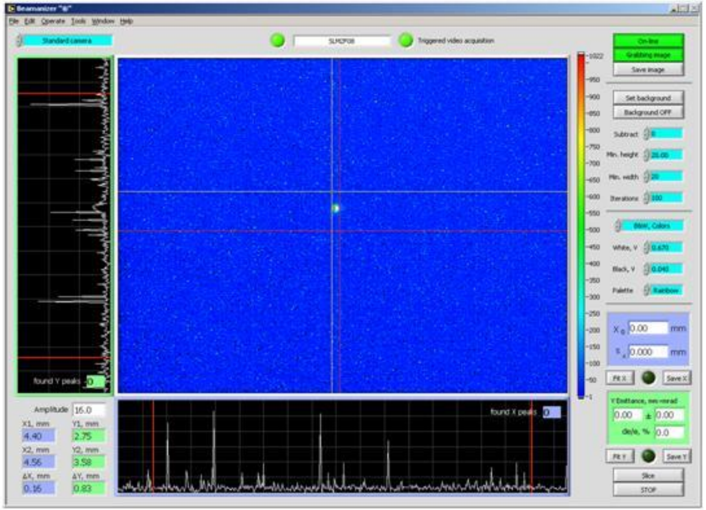
- Energy 100 MeV
- Current 5 mA
- Bunch length 3 psec
- Energy spread <0.1%
- Emittance <15 mm-mrad.
- Solenoid field 0.5 T



DarkLight Longitudinal Match

- Bunch is no longer compressed at the target.
- Energy spread is very small, thus insensitive to multipoles.
- Desire fo low repetition rate to take advantage of time-of-flight in particle identification.





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- Bunch is no longer compressed at the target.
- Energy spread is very small, thus insensitive to multipoles.
- Desire fo low repetition rate to take advantage of time-of-flight in particle identification.
- Can switch between quite different operational modes with only minor parametric changes
  - e.g. cross-phasing of linac cavities/modules: change single phase setpoint and go from short bunch to small dp/p
- Can use short bunch setup to optimize longitudinal transfer map.
- Can use two-pass setup to have very small energy spread.
- Note: Thompson backscattering requires high charge with moderately short bunches. This setup could be very advantageous for that application as well.

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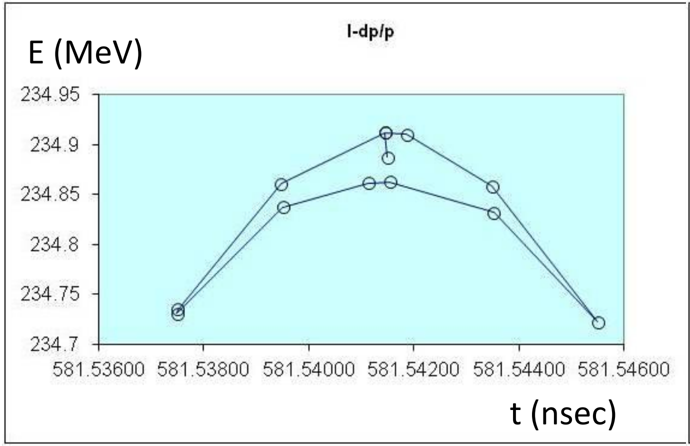

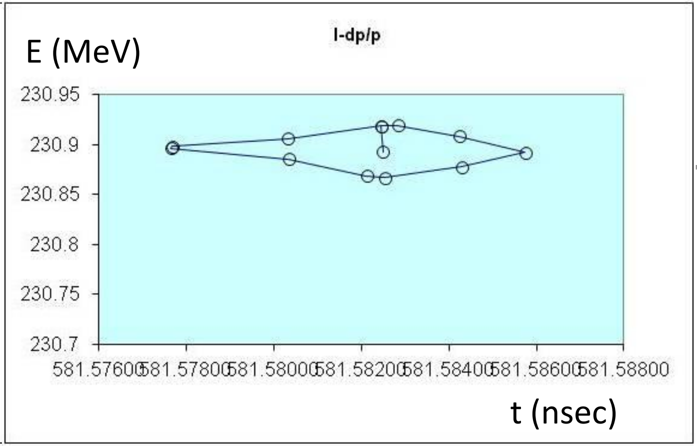

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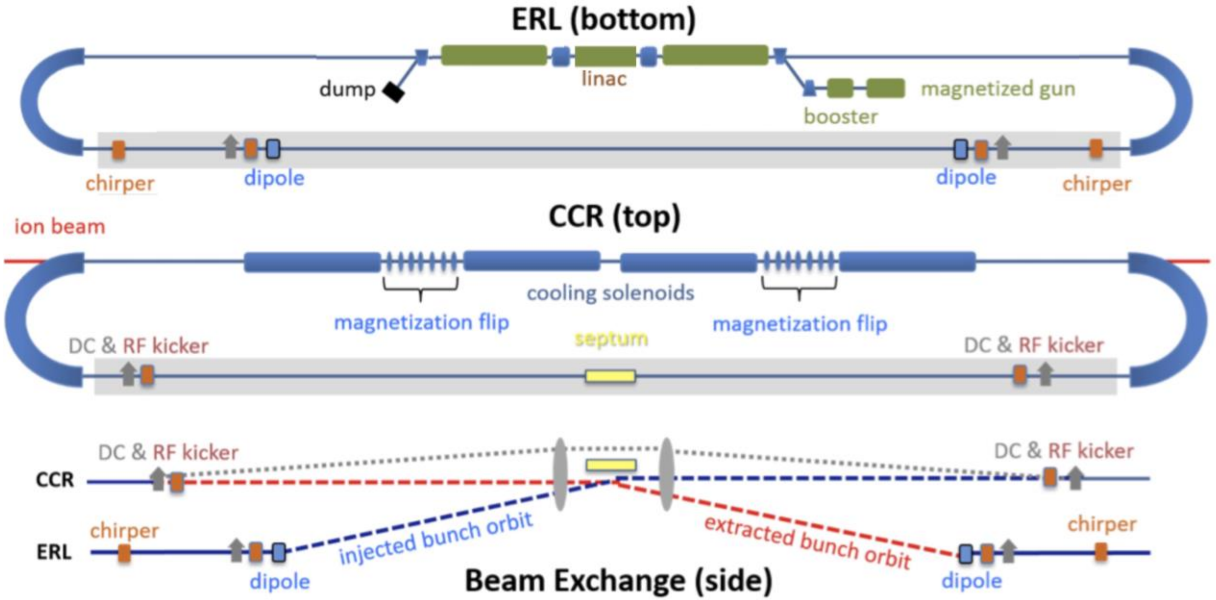
EIC Cooler ERL and CCR

- Need very long, small energy spread bunch with very high charge.
- Magnetized electron beam for higher cooling efficiency
- Repetition rate of bunches is 476.3 MHz.
- Assume high charge, low rep-rate injector (w/ harmonic linearizer acceleration)

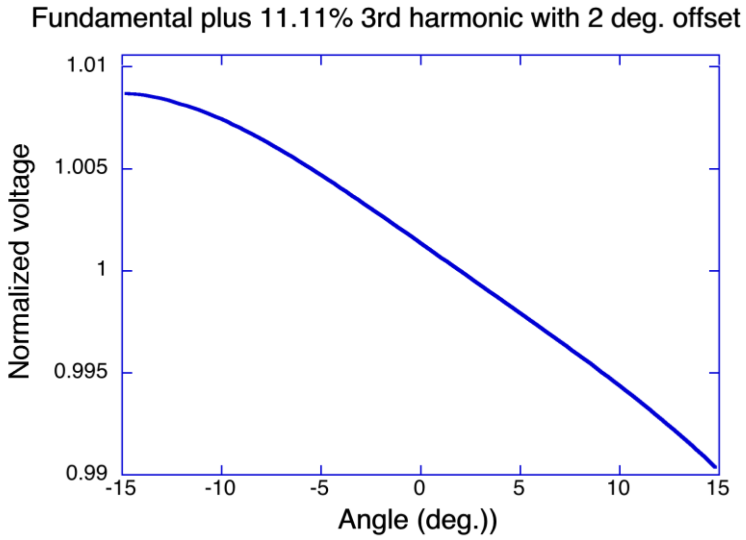


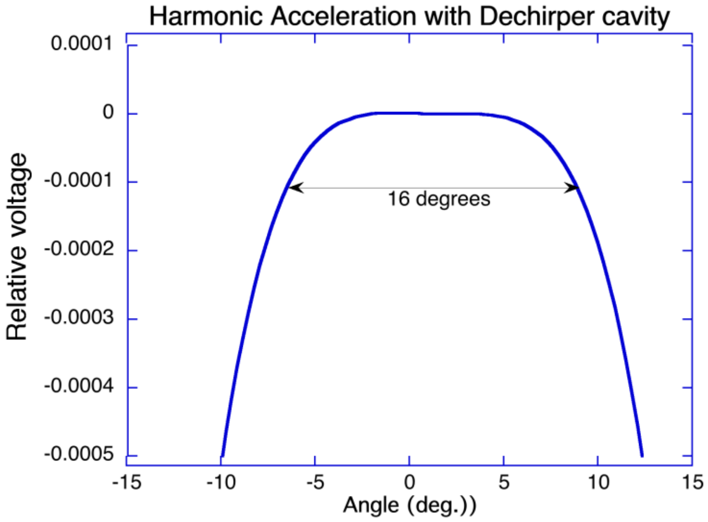
JLEIC BBU Cooler Specifications

Energy	20–110 MeV
Charge	3.2 nC
CCR pulse frequency	476.3 MHz
Gun frequency	43.3 MHz
rms Energy spread (uncorr.)	3×10^{-4}
Energy spread (p-p corr.)	$< 6 \times 10^{-4}$
Bunch length (tophat)	3 cm (17°)
Thermal (Larmor) emittance	<19 mm-mrad
Cathode spot radius	3.1 mm
Cathode field	0.05 T
Normalized hor. drift emittance	36 mm-mrad
Solenoid field	1 T
Electron beta in cooler	37.6 cm
Solenoid length	4x15 m
Bunch shape	beer can

Voltage with 3rd Harmonic and phase and amplitude offsets

- If we want to accelerate a very long bunch and then stretch it out even more we can use 3rd harmonic cavities in the linac.





Before going into the CCR, take out the slope using a 952.6 MHz de-chirper.
We can also put in a quartic correction if necessary by changing the amplitude

Conclusions

- ERL architecture is determined by the longitudinal design.
- Transverse design follows the longitudinal settings.
- For FELs one wants a high peak current:
  - For small long wavelengths a parallel to point focus is optimal
  - For short wavelengths a telescopic focus is better.
- Nuclear Physics applications do not need high peak current but need small relative energy spread.
  - Can use either lattice of harmonic RF to get a good energy spread
  - Low charge, high repetition rate is a better match to these applications.
- Electron Cooling applications need extremely long bunches and extremely small energy spread.
  - Harmonic RF is almost required for such bunches.
  - Microbunching and CSR are now the big challenges.

Compaction Management

LINAC	Energy Gain (MV)	Phase (Deg)	ARC	M_{56} (m)	T_{566} (m)	W_{5666} (m)
1	0.1800	-16	1	0	0	0
2	0.1950	-16	2	-0.0295	-1.23	-9
3	0.1800	+25	3	0	0	0
4	0.1950	+25	4	+1.035	0	0
5	0.1950	204	5	-0.282	-2.5	-10
6	0.1800	204	6	0	0	0
7	0.1950	163	7	-0.1	+2.5	0
8	0.1800	163	8	0	0	0

ERLs are time-of-flight spectrometers

- Exist solely to create conspiracies between phase and energy
  - no closed orbit; may not be betatron stable/have "matched" beam envelopes $\Rightarrow$ beam and lattice are different (mismatch often advantageous)
  - longitudinal match constrains system architecture
- need full suite of *longitudinal* diagnostics for both machine (lattice) and beam
  - phase transfer function system (M_{55}, T_{555})
  - bunch length monitoring/noninvasive energy spread
  - tomography to capture/correct nonlinear phase space distortion
- spectrometer-grade components
  - perturbations at high energy anti-damp during recovery
- aberration management critical: nonlinear modeling/diagnosis/control needed

Features of ERL Architectures (cont.)

- no equilibrium ⇒ stability a challenge
  - CEBAF parity-quality beam provides benchmark
- high beam power, absence of equilibrium ⇒ CW is a game-changer
  - beam loss monitoring/suppression
- beam quality generation and preservation:
  - beam quality declines from cathode onward* ; “best” injected beam not necessarily “best” delivered beam

*unless emittance compensation implemented; can be applied at high energy for, e.g. CSR management

  - beam degrades at full energy ⇒ anti-damping makes things worse during recovery

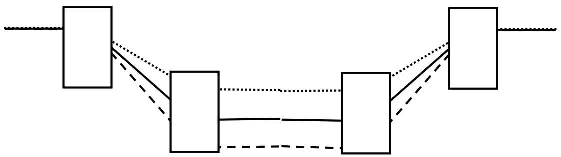
- Recirculator/ERL ⇒ ***multiple beams/common transport*** (at least in linac!)
  - creates challenges for monitoring & control

180° Bates bend

Bates band - design by Sargent/Flanz from MIT (combined function magnets)

J. B. Flanz and C. P. Sargent, "Operation of an Isochronous Beam Recirculation System," *Nucl. Instrum. and Methods* **A241** (1985) 325–333

D. Douglas
separated sextupoles
and added quads



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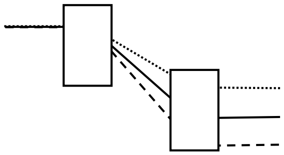

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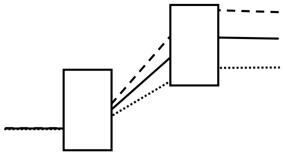

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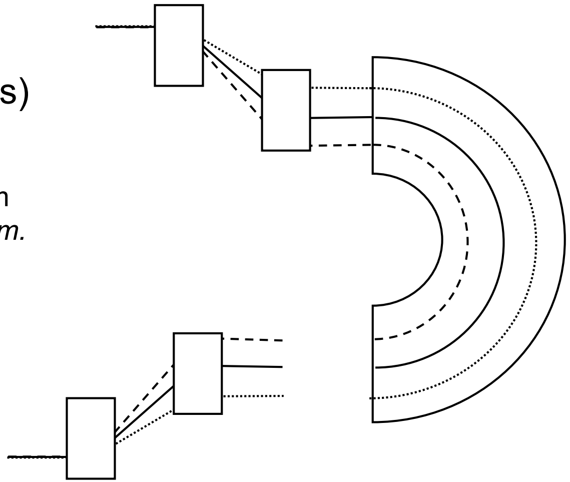

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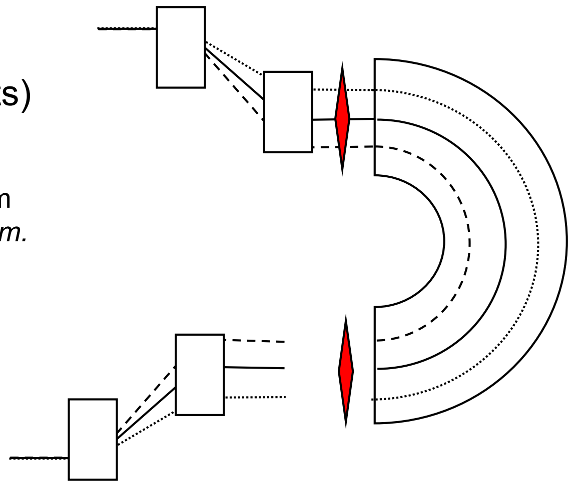

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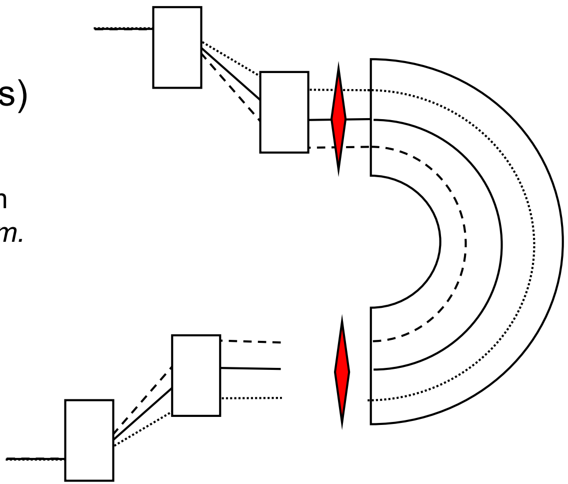
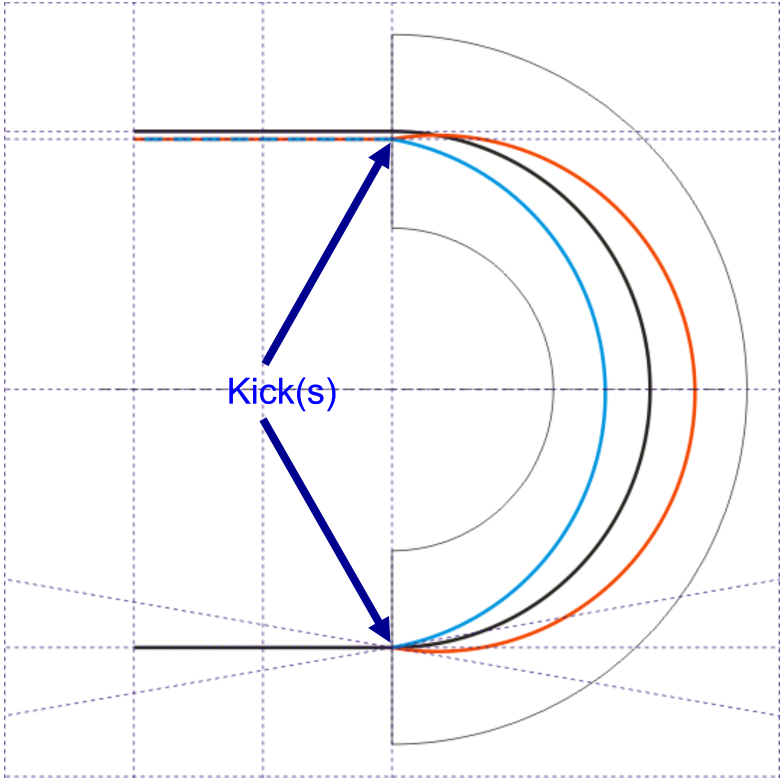


Courtesy of D. Douglas

- Really robust
- Really easy to operate (if it is instrumented)
- Really simple (if you think about it the right way)
- Good acceptance (>10% energy, 30-40 deg phase)
- Symmetry – aberrations corrections
- Match in/out with chromatically balanced telescopes

180° Bates bend (1)

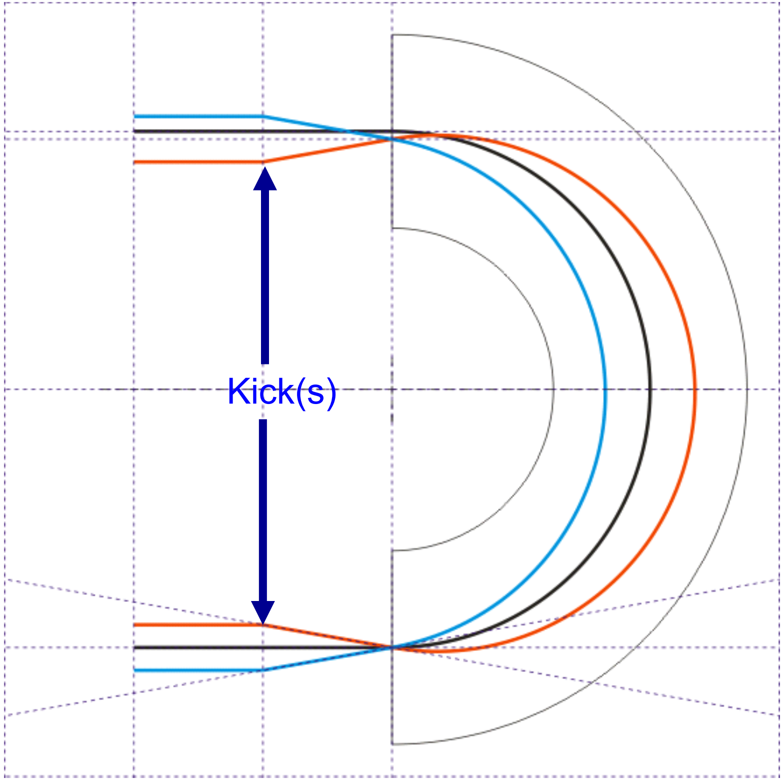


Path length change with kick;

$$\delta L = 2\rho\,\delta x'$$

Used to adjust the path length i.e. phase of the energy recovered beam

180° Bates bend (2)



Path length change with kick;

$$\delta L = 2\rho\, \delta x'$$

Kick by quadrupole;

$$\delta x'(x) = A \cdot x$$

Kick by sextupole;

$$\delta x'(x) = B \cdot x^2$$

Due to dispersion created by first two dipoles;

$$E \propto x$$