Electrodisintegration of ^{16}O and the Rate Determination of the Radiative Alpha Capture on ^{12}C at Stellar Energies

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$^{12}C/^{16}O$ abundance

- Helium burning stage: $3\alpha \rightarrow {}^{12}C + \gamma$, and $\alpha + {}^{12}C \rightarrow {}^{16}O + \gamma$
- At $T \simeq 2 \cdot 10^8$ K, Gamow window for $\alpha + {}^{12}C$ is around $E_G \simeq 300$ keV and at the moment the rate is know with an uncertainty between 20% and 30%
- Affects evolution of massive stars -> nucleosynthesis of heavier elements
- White dwarfs: ignition of super nova type Ia
- End of stars: ^{16}O rich star - black hole, ^{12}C rich star – neutron star

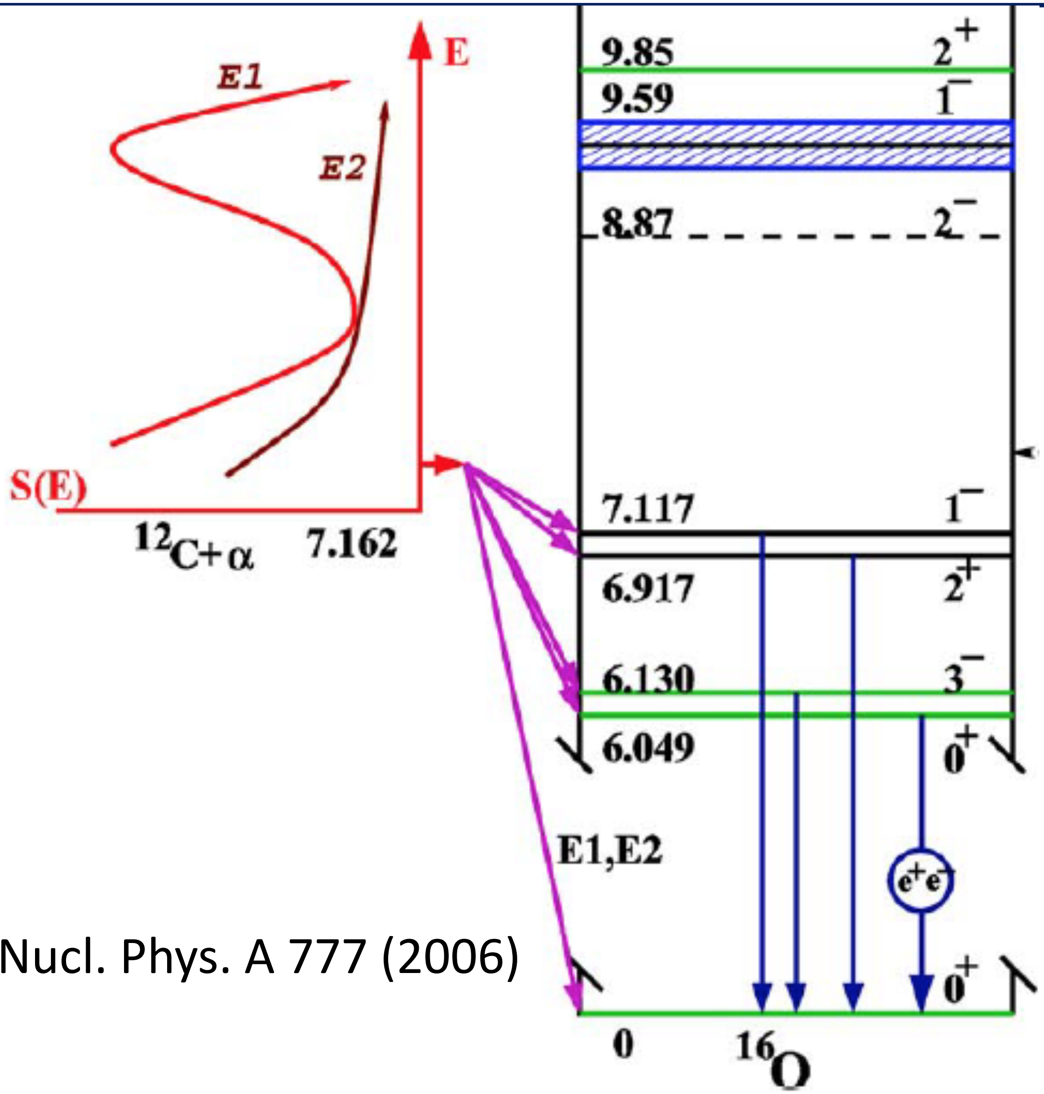
T. A. Weawer and S. E. Woosley, Phys. Rep. 227 (1993) 335.

$\alpha + {}^{12}C \rightarrow {}^{16}O + \gamma$ cross section around E_G

- Large Coulomb barrier → $\sigma \simeq 10^{-5}$ pb (direct measurement is not feasible)
- Around E_G the cross section is dominated by two components:

→ E1 component, $J^\pi = 1^-$: subthreshold state at 7.117 MeV and broad resonance at 9.59 MeV

→ E2 component, $J^\pi = 2^+$: subthreshold state at 6.917 MeV and narrow resonance at 9.85 MeV

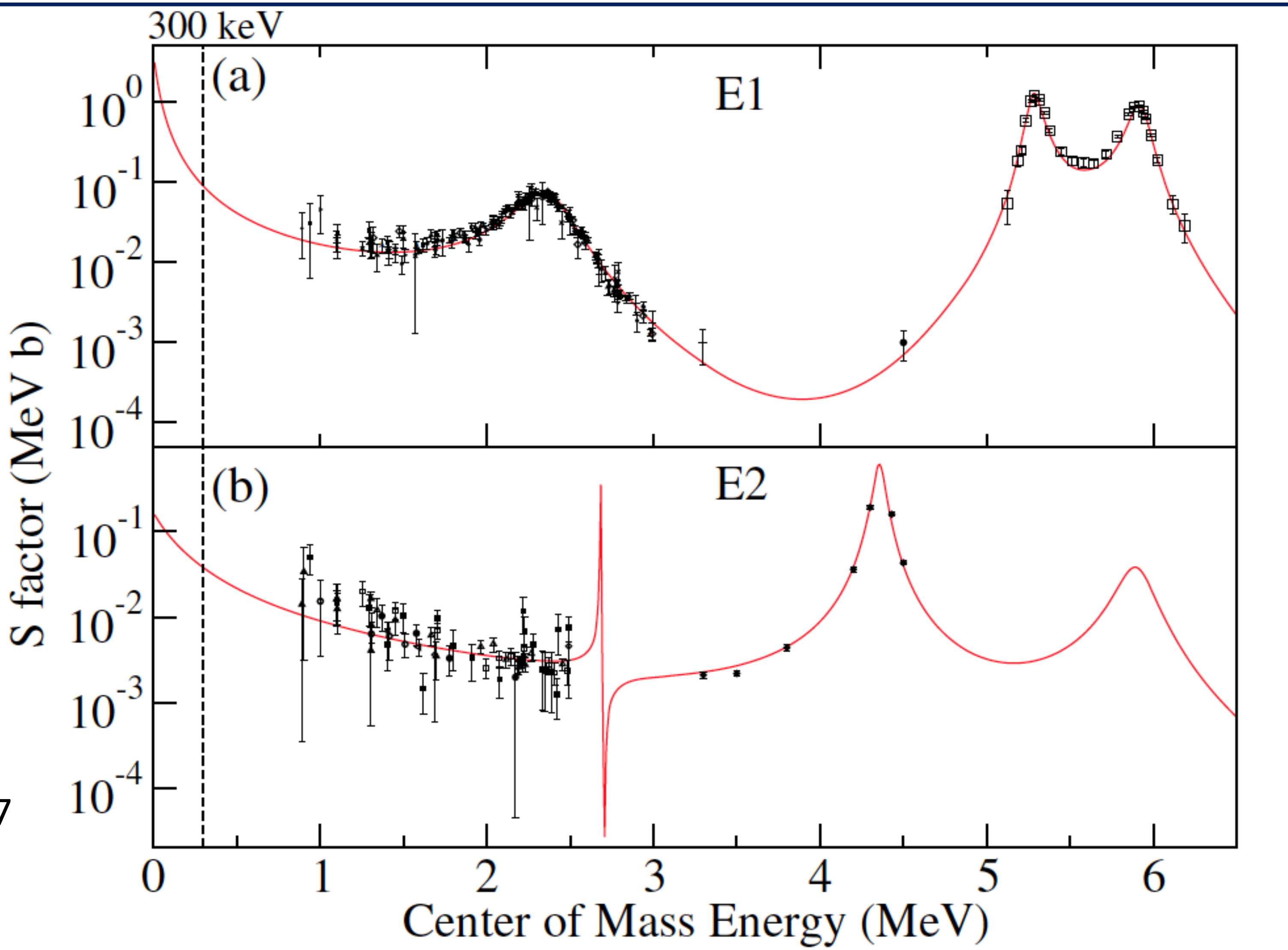


L.R. Buchmann, C.A. Barnes, Nucl. Phys. A 777 (2006)

$\alpha + {}^{12}C \rightarrow {}^{16}O + \gamma$ S-factors

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta}$$

$$\eta = \frac{2\pi Z_1 Z_2 e^2}{h v_{rel}}$$



R. J. deBoer et al., Rev. Mod. Phys. 89, 035007 (2017) and references therein

How to extract $\sigma(E_G)$

- Direct measurements

a) $^{12}C(\alpha, \gamma)^{16}O$; ($\alpha$ beam)
– angular distribution of γ is measured
→ S_{E1} and S_{E2}

b) $\alpha(^{12}C, ^{16}O)\ \gamma$; ($^{12}C$ beam)
– detection of ^{16}O recoils → S_{tot}

- Indirect measurements

a) β decay of ^{16}N: $^{16}O^* \rightarrow \alpha + {}^{12}C$; → S_{E1}

b) inverse reaction:

– photodisintegration of ^{16}O: $^{16}O(\gamma, \alpha)^{12}C$

→ Bubble chamber R. J. Holt et al., (2018), arXiv:1809.10176

→ Time project. chamber M. Gai et al., JINST 5, P12004 (2010)

– **electrodisintegration of ^{16}O: $^{16}O(e, e'\alpha)^{12}C$; THIS TALK**

I. Friščić, W. T. Donnelly and R. G. Milner, Phys. Rev. C 100, (2019) 025804

$$\frac{\sigma(\gamma + {}^{16}O)}{\sigma(\alpha + {}^{12}C)} = \frac{\mu c^2 E_\alpha^{cm}}{E_\gamma^2} \approx 42 \text{ (for } E_\alpha^{cm} = 1 \text{ MeV)}$$

Advantage of $^{16}O(e,e'\alpha)^{12}C$

- inverse reaction: larger cross section than direct reaction
- new generation of e^- energy recovery linear (ERL) accelerators with $I \geq 10$ mA:

  MESA, Univ. of Mainz, Germany, F. Hug et al., Proc. of LINAC'16 28, 313 (2017).

  CBETA, Cornell Univ., USA, D. Trbojevic et al., Proc. of IPAC'17 8, 1285 (2017).
- oxygen cluster gas-jet target with thickness > 10^{18} atoms/cm^2

  MAGIX, Univ. of Mainz, Germany, S. Grieser et al., Nucl. Inst. Meth. Phys. Res. A 906, 120 (2018).

Luminosity > 10^{35} 1/(cm^2 s)

Systematics from oxygen isotopes

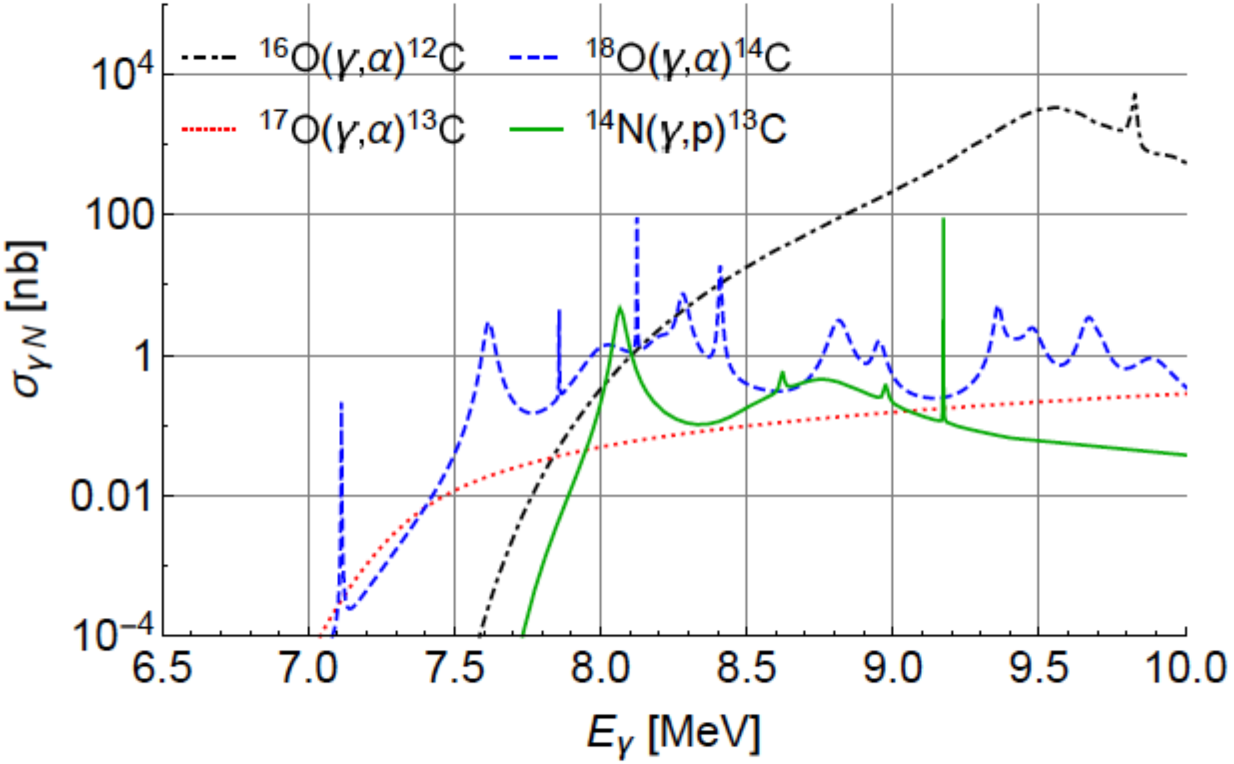
- Oxygen isotope abundance: ^{16}O 99.757%, ^{17}O 0.038% and ^{18}O 0.205%

K. J. R. Rosman, P. D. P. Taylor, Pure Appl. Chem 71 (1999) 1593

$Q(^{16}O \rightarrow \alpha + {}^{12}C) = -7.162$ MeV,
$Q(^{17}O \rightarrow \alpha + {}^{13}C) = -6.357$ MeV
$Q(^{18}O \rightarrow \alpha + {}^{14}C) = -6.228$ MeV

- Photonuclear cross sections: natural abundance of O isotopes + depletion of ^{17}O and ^{18}O by factor 1000, and 5 ppmv for ^{14}N

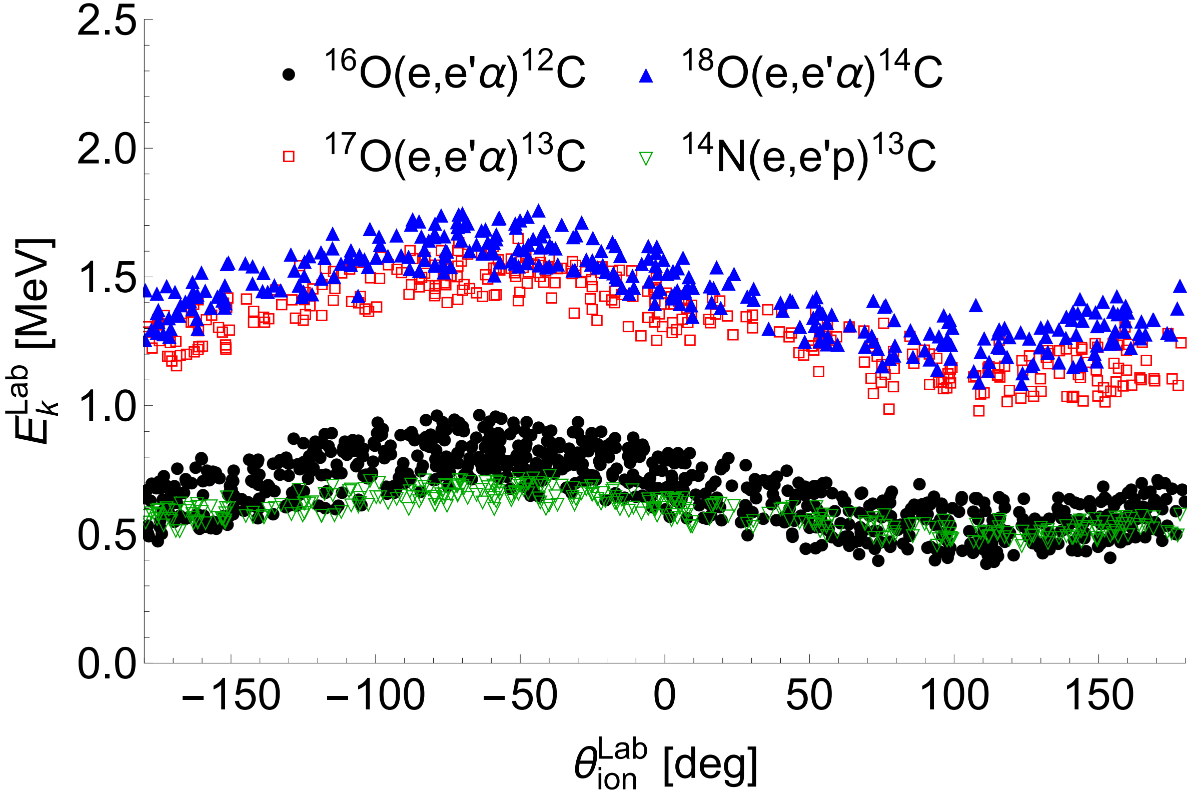
$$E_\gamma = E_\alpha^{cm} + 7.162 \text{ MeV}$$

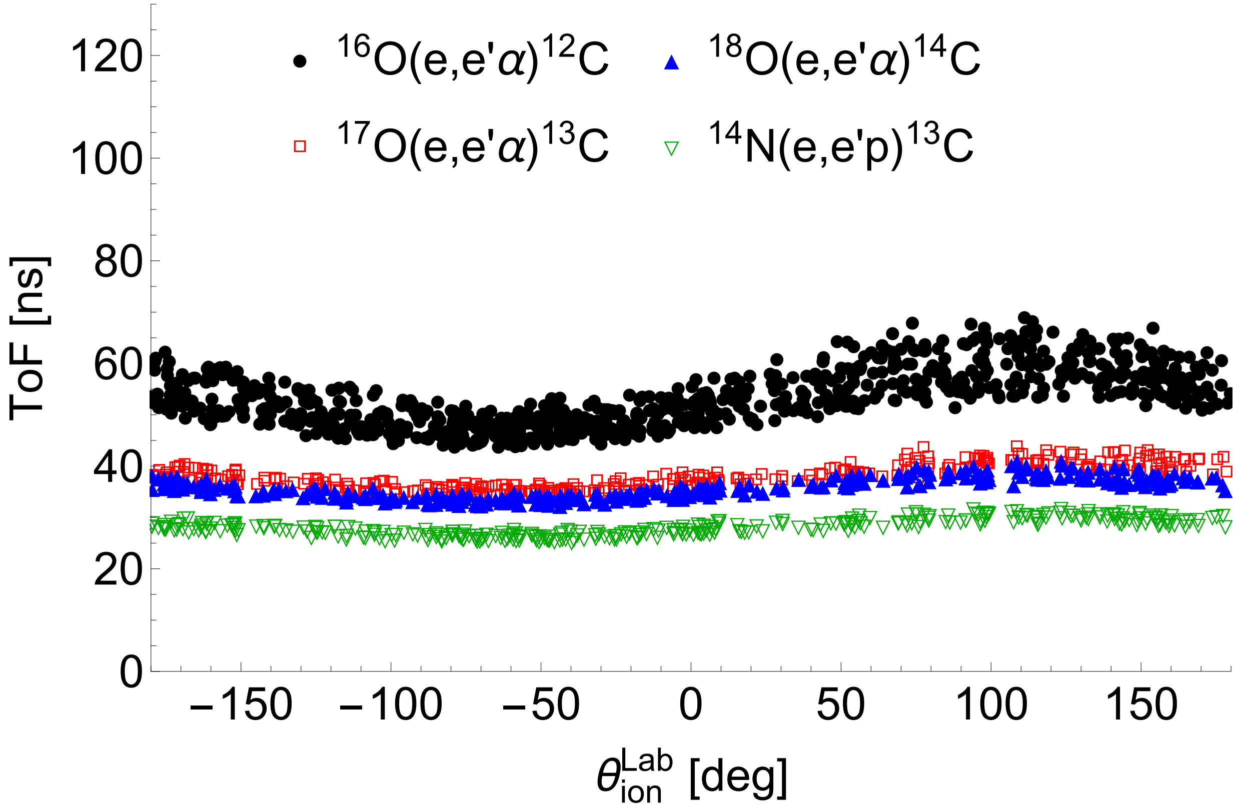


https://wiki.jlab.org/ciswiki/index.php/Simulations_and_Backgrounds#Relevant_Theoretical_Cross_Sections

Systematics from oxygen isotopes: Solution

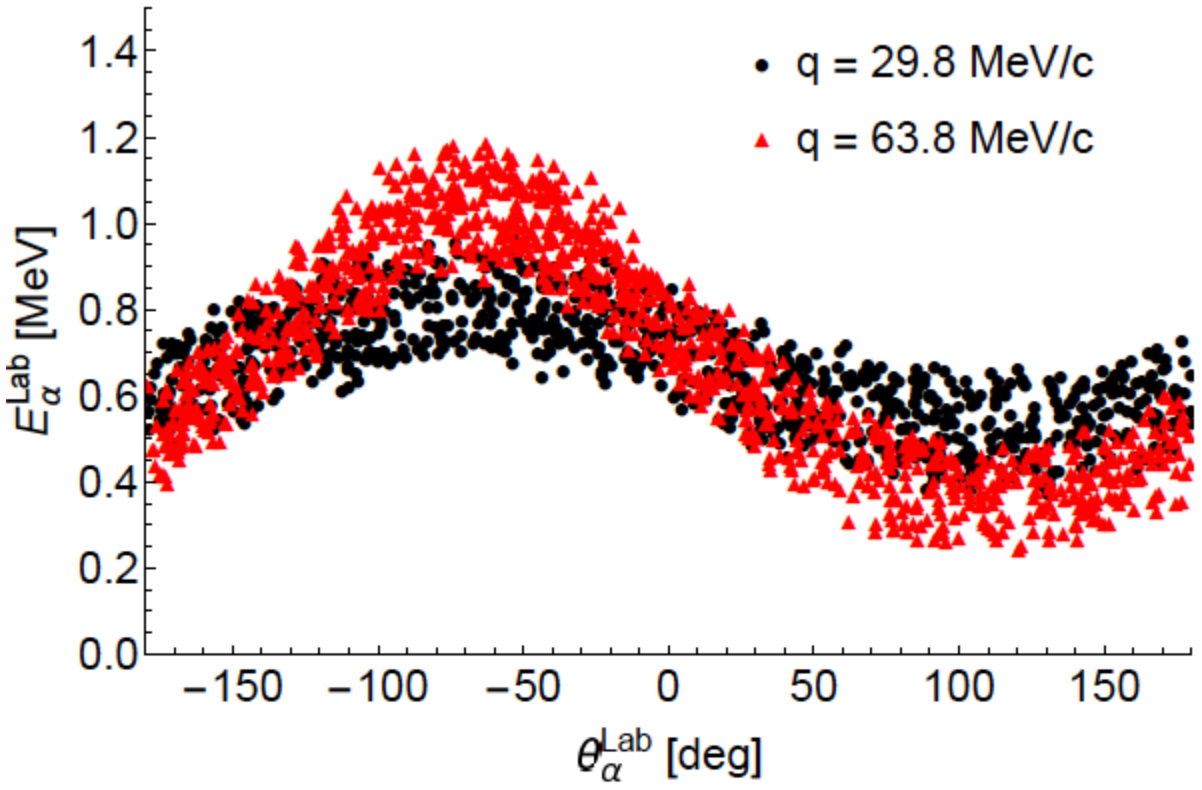
- SRIM simulation: energy loss of α-particles in 2 mm wide oxygen jet, with a density of $6.65 \cdot 10^{-4}$ g/cm³, E_e= 114 MeV, θ_e=15°, 1.0 ≤ E_α^{cm}≤1.1 MeV

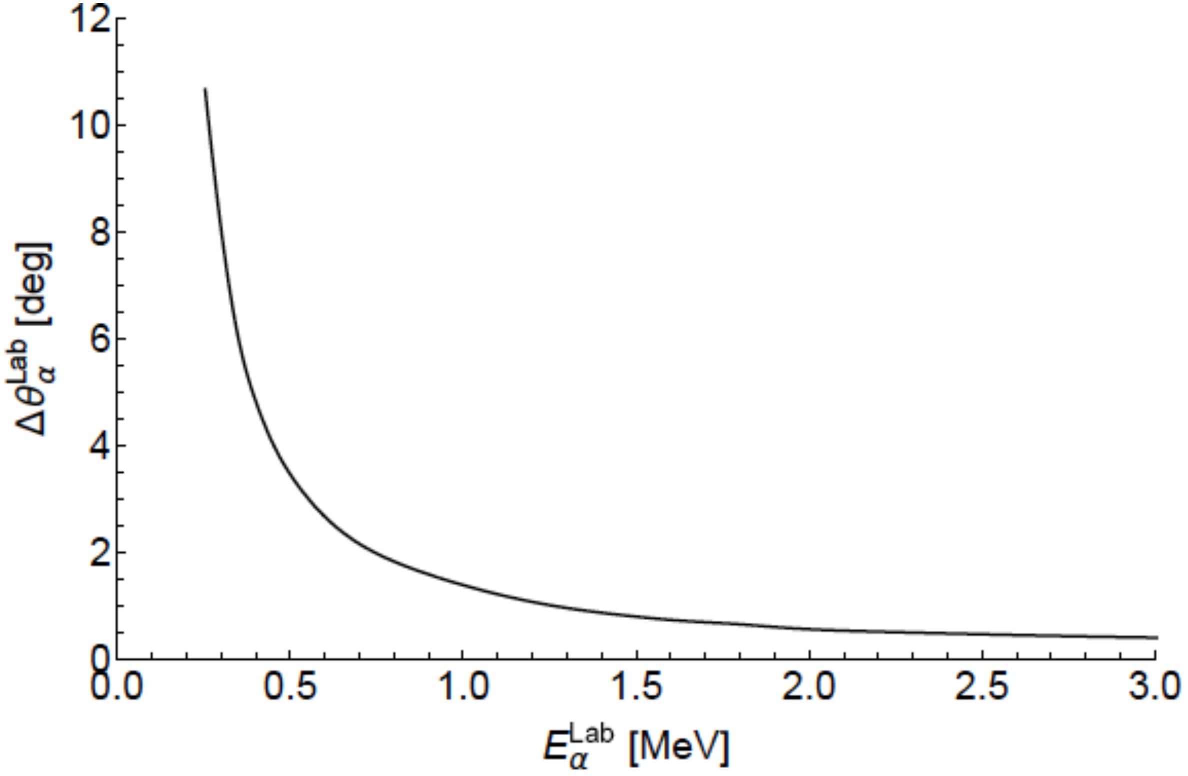




Virtual photon advantage

- SRIM simulation: angular spread of α-particles in 2 mm wide oxygen jet, with a density of $6.65 \cdot 10^{-4}$ g/cm^3, E_e= 114 MeV, θ_e=15° and 35°, $1.0 \le E_\alpha^{cm} \le 1.1$ MeV

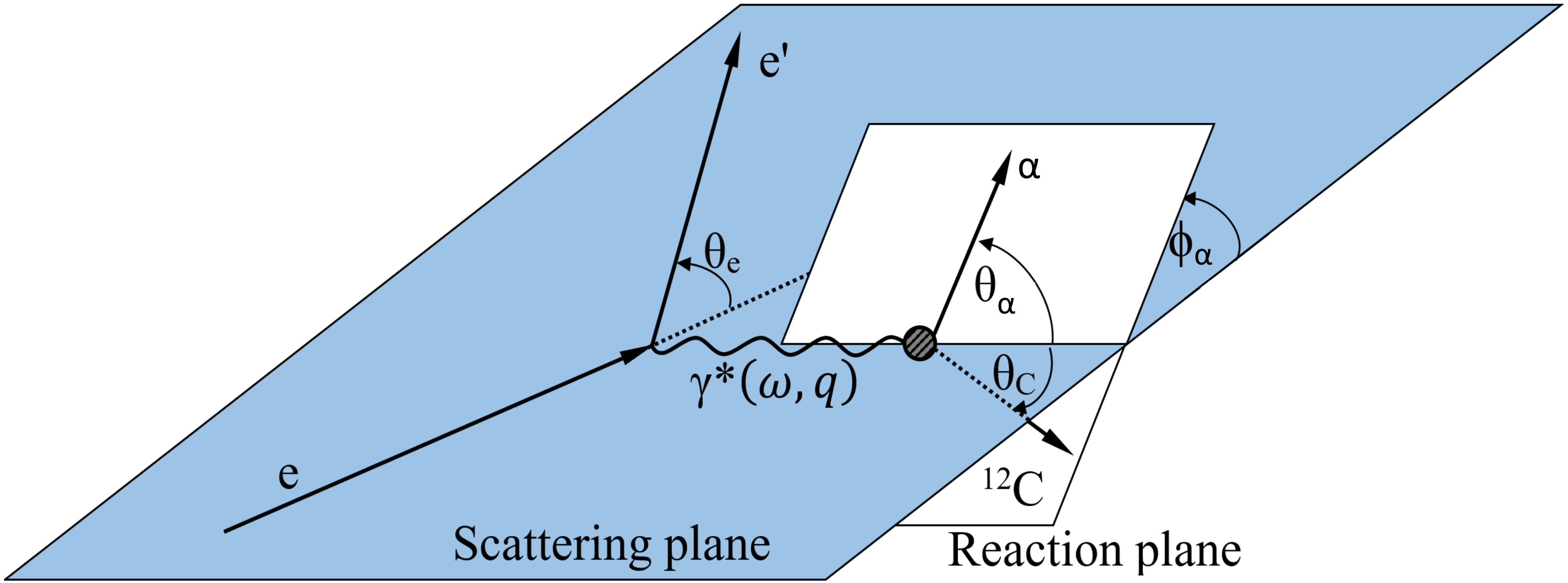




Detection of α-particles and other ions

- Requirements:
  - measure the total energy of the α-particles to about ∼10%
  - distinguish between protons, α-particles and C-isotopes
  - measure the position to ∼ mm and the timing to a few ns
  - ion detection system have to be blind to scattered e^- and photons
- Options:
  - Silicon detectors → high position resolution, needs to be cooled to min. radiation damage
  - Micro-channel-plate electron (MCP) detector → good timing resolution
  - Parallel-plate avalanche counter (PPAC) → good timing resolution and position resolution
  - Time Projection Chamber → reconstruction of the ion's trajectory

Kinematics: $^{16}O(e,e'\alpha)^{12}C$



The cross section formulas

- Electrodisintegration of ^{16}O:

$$\frac{d\sigma}{dE'_e d\Omega_e d\Omega_\alpha^{cm}} = \frac{M_\alpha M_{12C}}{8\pi^3 W}\frac{p_\alpha^{cm}}{(\hbar c)^3}\sigma_{Mott}(\tilde{v}_L R_L + \tilde{v}_T \boldsymbol{R_T} + \tilde{v}_{LT} R_{LT} + \tilde{v}_{TT} R_{TT})$$

$$W = \sqrt{(M_{16O} + \omega)^2 - q^2} \qquad \boldsymbol{E_\alpha^{cm} = W - W_{th}}$$

A. S. Raskin and T. W. Donnelly, Ann. of Phys. 191 (1989)

- Direct reaction ^{12}C(α, γ)^{16}O :

$$\left.\frac{d\sigma}{d\Omega_\gamma^{cm}}\right|_{(\alpha,\gamma)} = \frac{M_\alpha M_{12C}}{2\pi W}\frac{p_\alpha^{cm}}{\hbar c}\frac{\alpha}{E_\gamma}\boldsymbol{R_T}$$

$$W = \sqrt{M_\alpha^2 + M_{12C}^2 + 2M_{12C}E_\alpha^{lab}} \qquad \boldsymbol{E_\alpha^{cm} = \frac{M_{12C}}{M_{12C}+M_\alpha}E_\alpha^{lab}}$$

Response functions for $J^\pi = 0^+$ nuclei

$$
\begin{aligned}
R_L = & P_0(\cos\theta_\alpha)\left(|t_{C0}|^2 + |t_{C1}|^2 + |t_{C2}|^2\right) \\
& + P_1(\cos\theta_\alpha)\left(2\sqrt{3}|t_{C0}||t_{C1}|\cos(\delta_{C1} - \delta_{C0}) + 4\sqrt{\frac{3}{5}}|t_{C1}||t_{C2}|\cos(\delta_{C2} - \delta_{C1})\right) \\
& + P_2(\cos\theta_\alpha)\left(2|t_{C1}|^2 + \frac{10}{7}|t_{C2}|^2 + 2\sqrt{5}|t_{C0}||t_{C2}|\cos(\delta_{C2} - \delta_{C0})\right) \\
& + P_3(\cos\theta_\alpha)\left(6\sqrt{\frac{3}{5}}|t_{C1}||t_{C2}|\cos(\delta_{C2} - \delta_{C1})\right) \\
& + P_4(\cos\theta_\alpha)\left(\frac{18}{7}|t_{C2}|^2\right)
\end{aligned}
$$

$$
\begin{aligned}
R_T = & P_0(\cos\theta_\alpha)\left(|t_{E1}|^2 + |t_{E2}|^2\right) \\
& + P_1(\cos\theta_\alpha)\left(\frac{6}{\sqrt{5}}|t_{E1}||t_{E2}|\cos(\delta_{E2} - \delta_{E1})\right) \\
& + P_2(\cos\theta_\alpha)\left(-|t_{E1}|^2 + \frac{5}{7}|t_{E2}|^2\right) \\
& + P_3(\cos\theta_\alpha)\left(-\frac{6}{\sqrt{5}}|t_{E1}||t_{E2}|\cos(\delta_{E2} - \delta_{E1})\right) \\
& + P_4(\cos\theta_\alpha)\left(-\frac{12}{7}|t_{E2}|^2\right)
\end{aligned}
$$

$$
R_{TT} = -R_T\cos(2\phi_\alpha)
$$

Matrix elements and coefficients

- Multipole matrix elements (q_0 = 1.2 fm^{-1}):

$$t_{EJ} = \frac{\omega}{q}\left(\frac{q}{q_0}\right)^J a'_{EJ}\left[1+\left(\frac{q}{q_0}\right)^2 b'_{EJ}(q)\right]e^{-\left(\frac{q}{q_0}\right)^2} \qquad t_{CJ} = \left(\frac{q}{q_0}\right)^J a'_{CJ}\left[1+\left(\frac{q}{q_0}\right)^2 b'_{CJ}(q)\right]e^{-\left(\frac{q}{q_0}\right)^2}$$

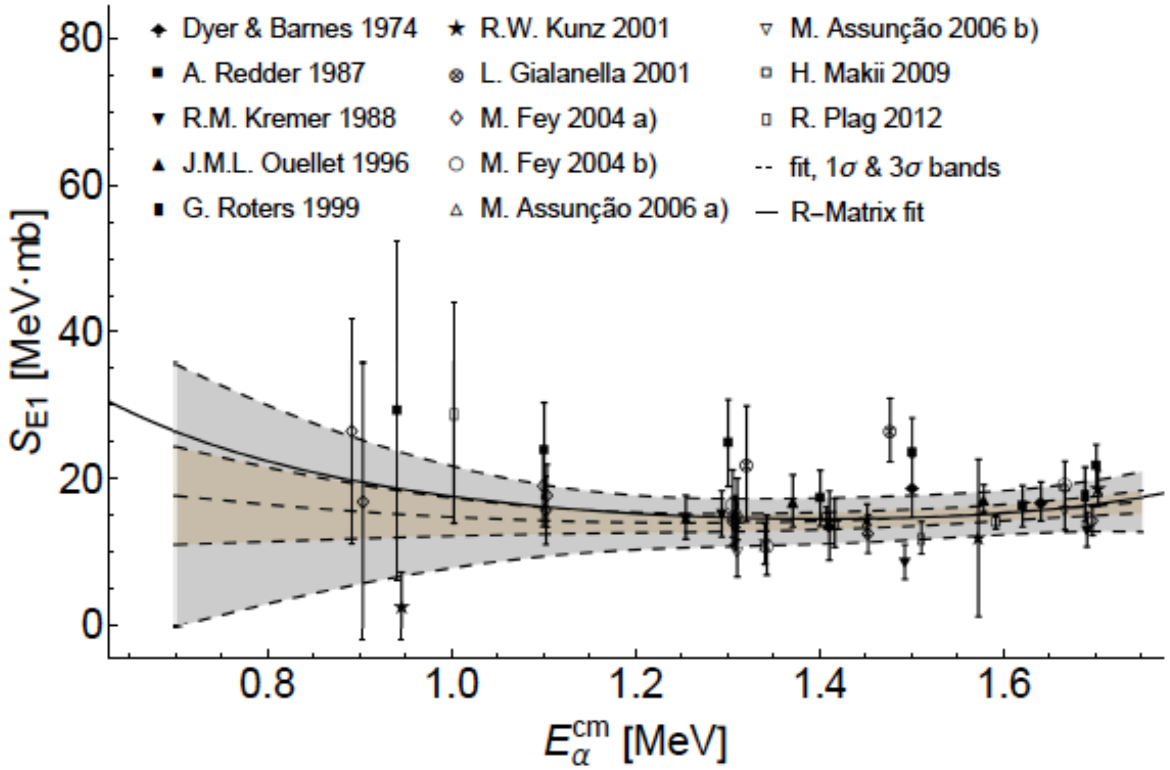
(t_{C0} leading dependence cannot occur due to orthogonality of initial and final state)

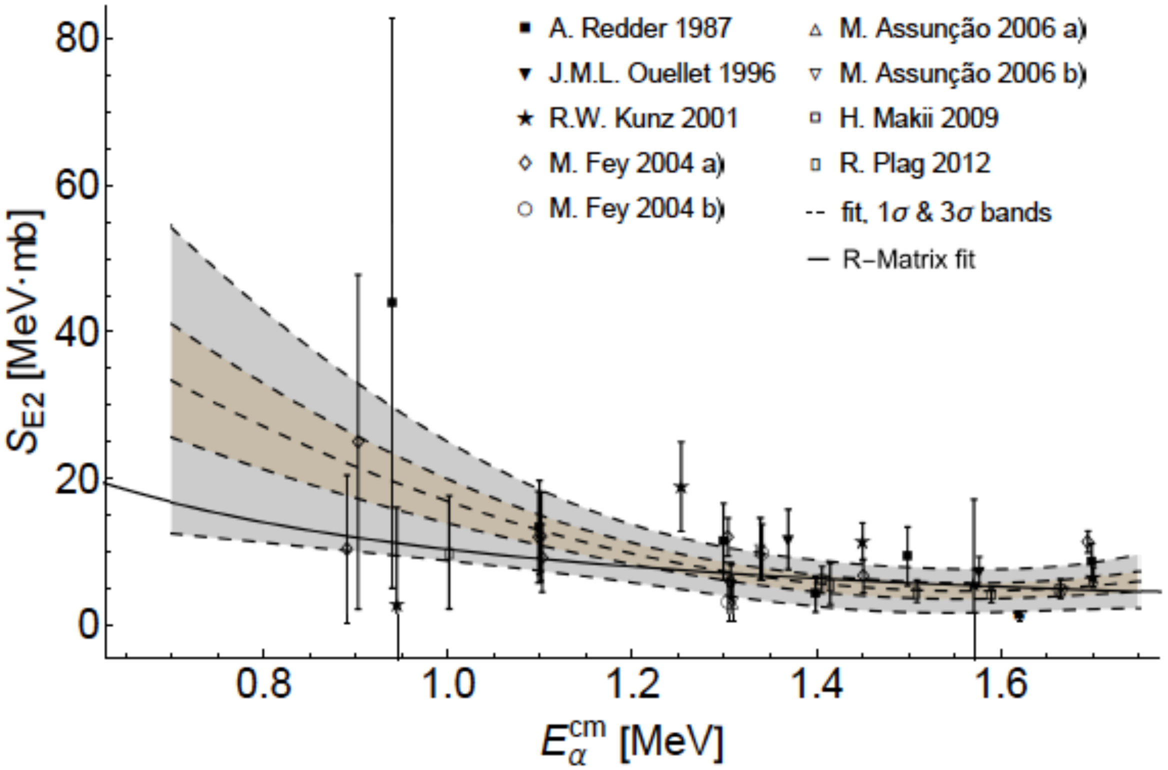
- Long wavelength limit (q→0) and continuity equation:

$$t_{EJ} \to -\sqrt{\frac{J+1}{J}}\left(\frac{\omega}{q}\right) t_{CJ} \qquad a'_{EJ} = -\sqrt{\frac{J+1}{J}}\, a'_{CJ}$$

S-factor modeling

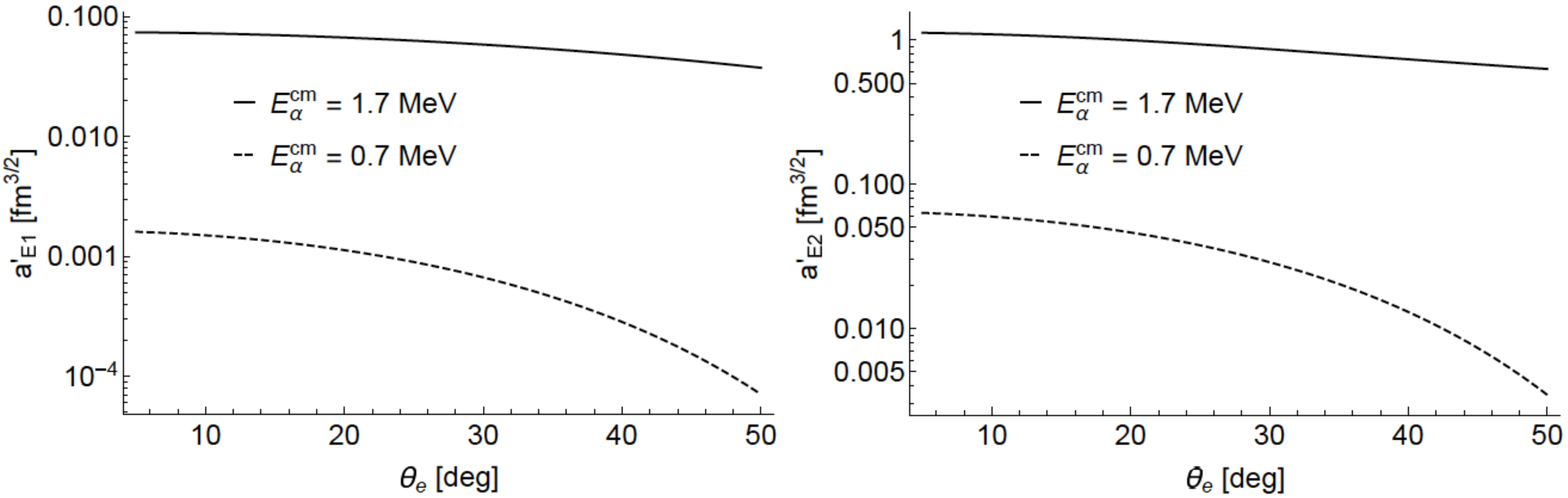
- Second order polynomial fit to data $E_\alpha^{cm} < 1.7$ MeV





Leading order coefficients

$$a'_{EJ} = \left(\frac{q_0}{\omega}\right)^J \sqrt{\frac{\hbar c \cdot p_\alpha^{cm} \cdot W}{2\alpha \cdot \omega \cdot M_\alpha M_{^{12}C}} \frac{S_{EJ}(E_\alpha^{cm}) \cdot e^{-2\pi\eta(E_\alpha^{cm})}}{E_\alpha^{cm}}} \; ; \quad J = 1, 2.$$



Next-to-leading order coefficients

- No knowledge about next to leading order coefficients $b'_{EJ,CJ}$ with $J = 1, 2$

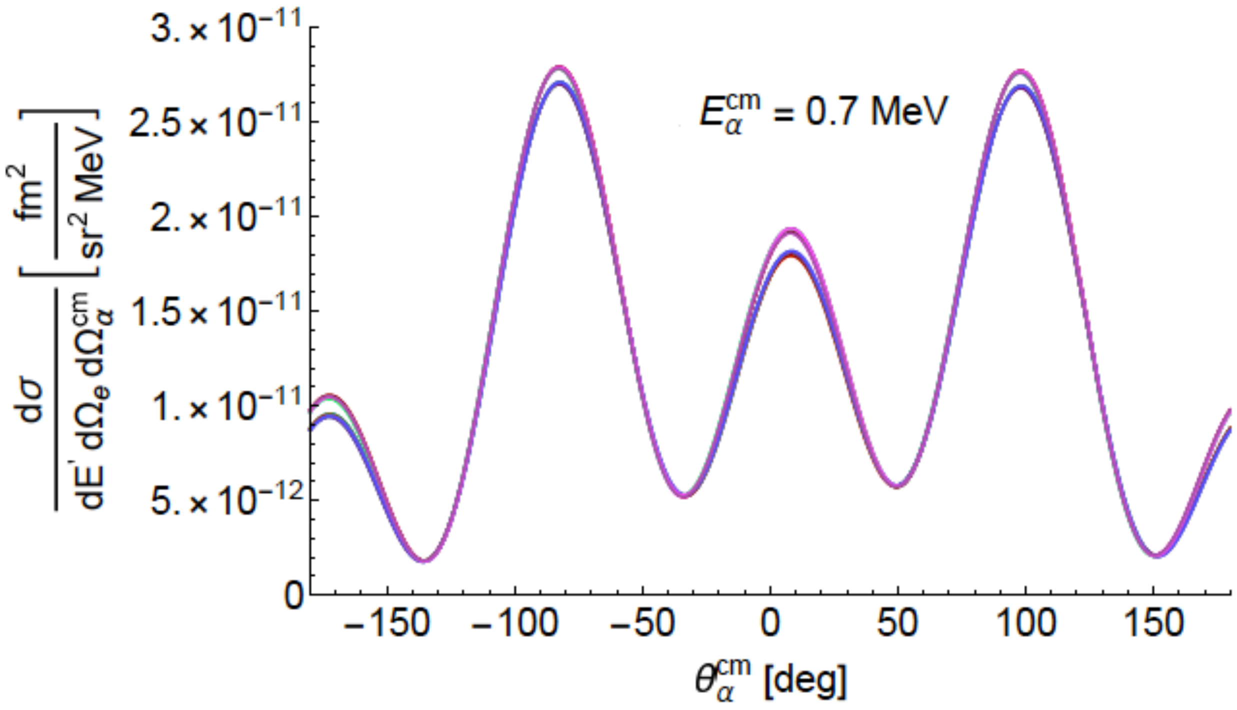
  → Assuming $b'_{EJ,CJ} \approx 1$ and "+" sign
- No knowledge about C0 multipole and $b'_{C0} \cdot a'_{C0}$

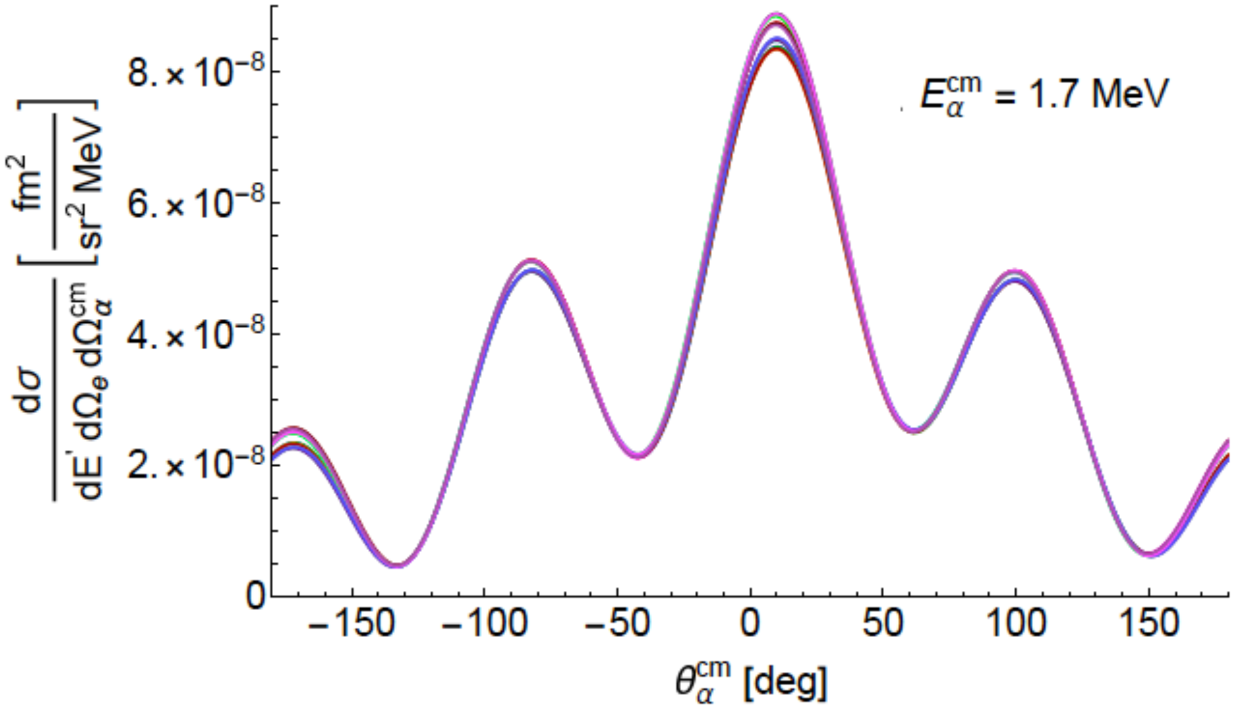
  → Assuming $b'_{C0} \approx 1$ and "+" sign, **Case A** $a'_{C0} = a'_{E2}$ and **Case B** $a'_{C0} = 0.5a'_{E2}$
- For E_α^{cm}< 1.7 MeV only Coulomb phase contributes:

$$\delta_{Cl} - \delta_{C0} = \delta_{El} - \delta_{E0} = \sum_{n=1}^{l} \arctan \frac{\eta}{l}$$

Differential cross section: $^{16}O(e,e'\alpha)^{12}C$

- E_e= 114 MeV, θ_e=15°
- Minor differences corresponding to the local maxima from $+b'_{C2}$ and $-b'_{C2}$

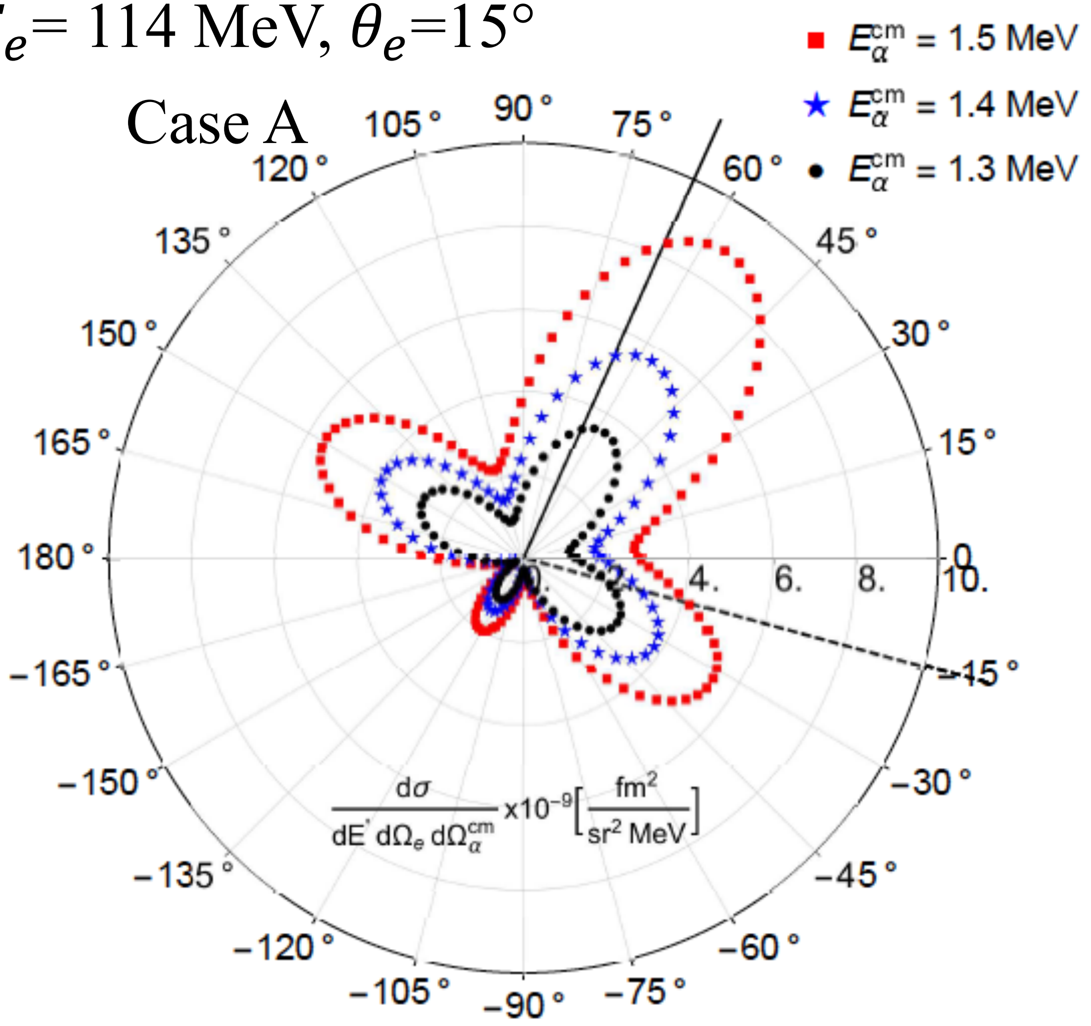




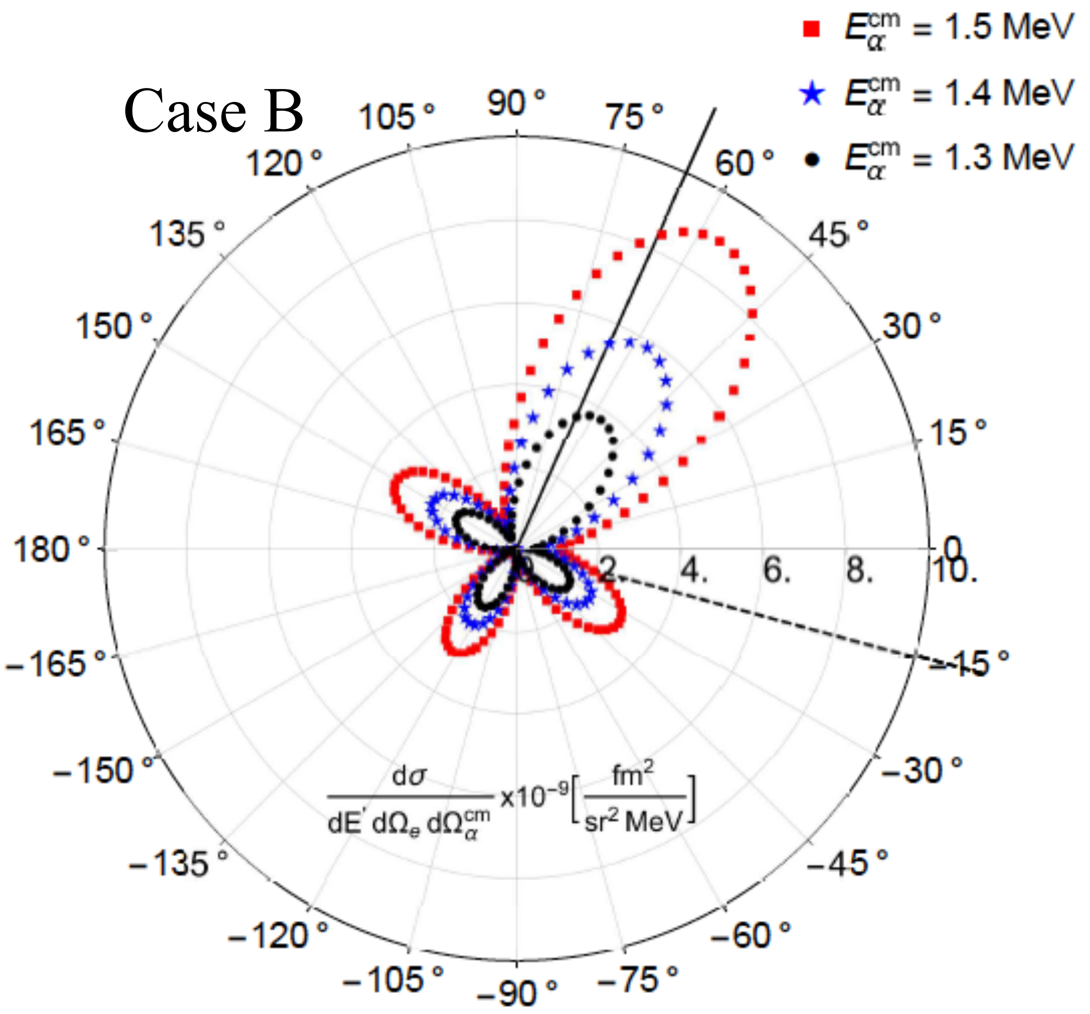
Differential cross section: $^{16}O(e,e'\alpha)^{12}C$

- E_e= 114 MeV, θ_e=15°

Case A



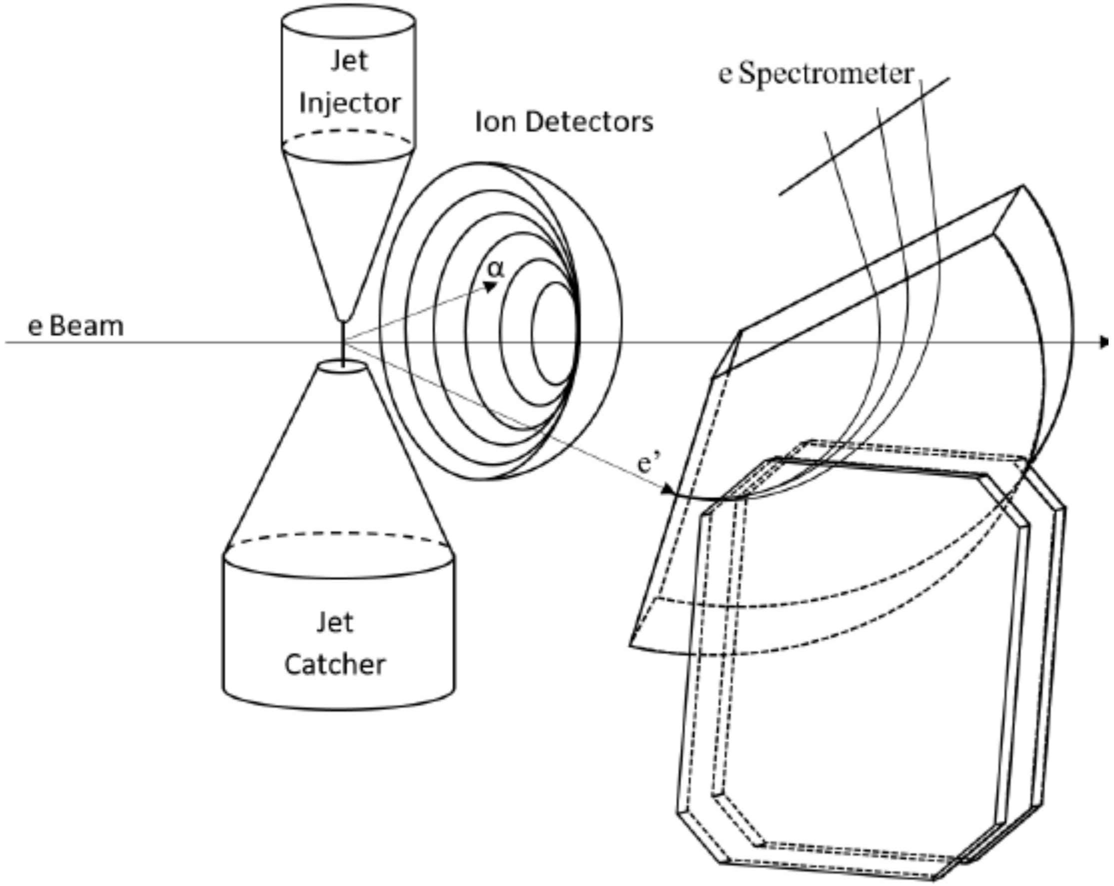
Case B

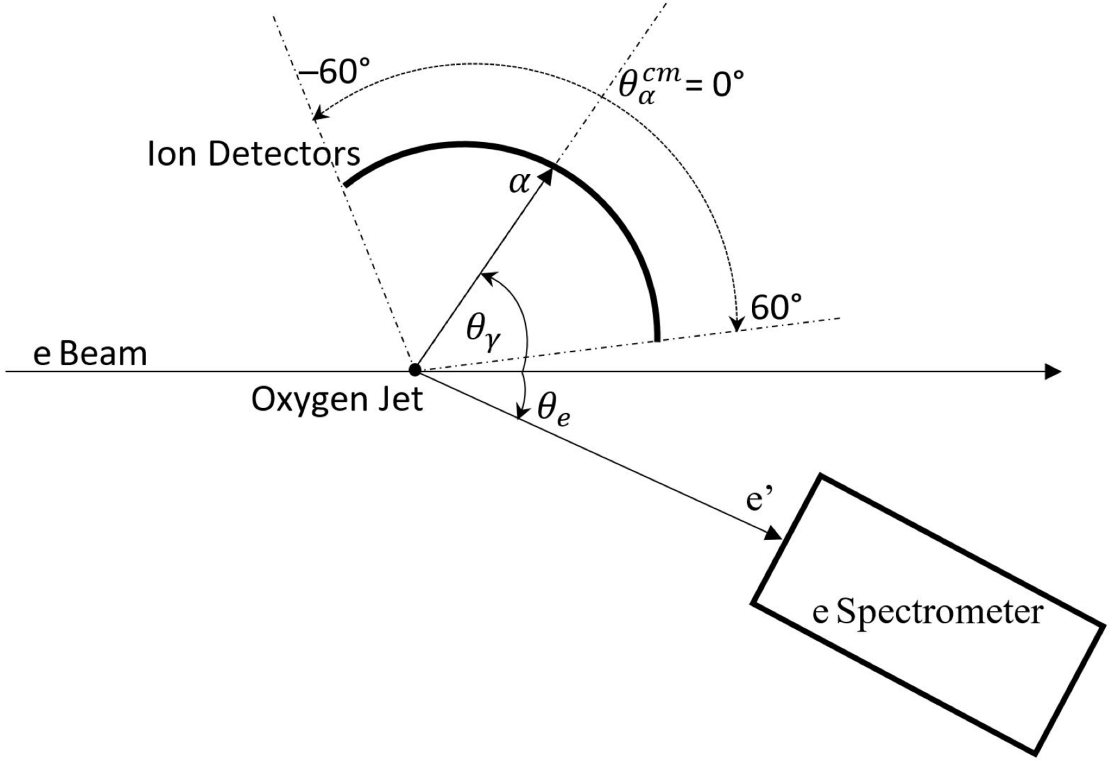


Previous experiments and proposals

- G. De Meyer et al., Phys. Lett. B 513 (2001): α-knockout experiment at NIKHEF; 3μA e^- beam at 639 MeV and 615 MeV; E_α: from 20 to 35 MeV; target: CO_2 at 1.6 bar and 300 K → $1.46 \cdot 10^{34}$ ($cm^{-2}s^{-1}$)
- E. Tsentalovich et al., (2000), MIT-Bates PAC proposal 00-01: 100 mA e^- beam at 400 MeV in a storage ring, cluster jet target $2 \cdot 10^{16}$ at/cm^2 → 10^{34} ($cm^{-2}s^{-1}$); Blast detector for the scattered e^- and silicon detectors for α
- T. W. Donnelly, "Electron scattering and the nuclear many-body problem", in The Nuclear Many-Body Problem 2001, edited by W. Nazarewicz and D. Vretenar (Springer Netherlands, Dordrecht, 2002) pp. 19
- S. Lunkenheimer, "Studies of the nucleosynthesis $^{12}C(\alpha,\gamma)^{16}O$ in inverse kinematics for the MAGIX experiment at MESA", (2017), 650. WE-Heraeus-Seminar.

Schematic layout of the proposed experiment





Parameters for the rate calculation

Oxygen Target	Thickness	5×10^{18} atoms/cm^2
	Density	6.65×10^{-4} g/cm^3
Electron Beam	Current	40 mA
	Energies	78, 114, 150 MeV
Electron arm	In-plane acceptance	$\pm2.08^\circ$
	Out-of-plane acceptance	$\pm4.16^\circ$
	Solid angle acceptance	10.5 msr
α-particle arm	In-plane acceptance	60°
	Out-of-plane acceptance	360°
	Solid angle acceptance	3.14 sr
Luminosity		1.25×10^{36} cm^{-2}s^{-1}
Integrated Luminosity (100 days)		1.08×10^7 pb^{-1}
Central electron scattering angles		15°, 25°, 35°
E_α^{cm}-range of interest		$0.7 \leq E_\alpha^{cm} \leq 1.7$ MeV

Number of events after 100 days

- Events were sorted in:

→ four 1.91 MeV wide q-bins

→ ten 100 keV wide E_α^{cm}-bins

→ six 10° wide θ_α^{cm}-bins

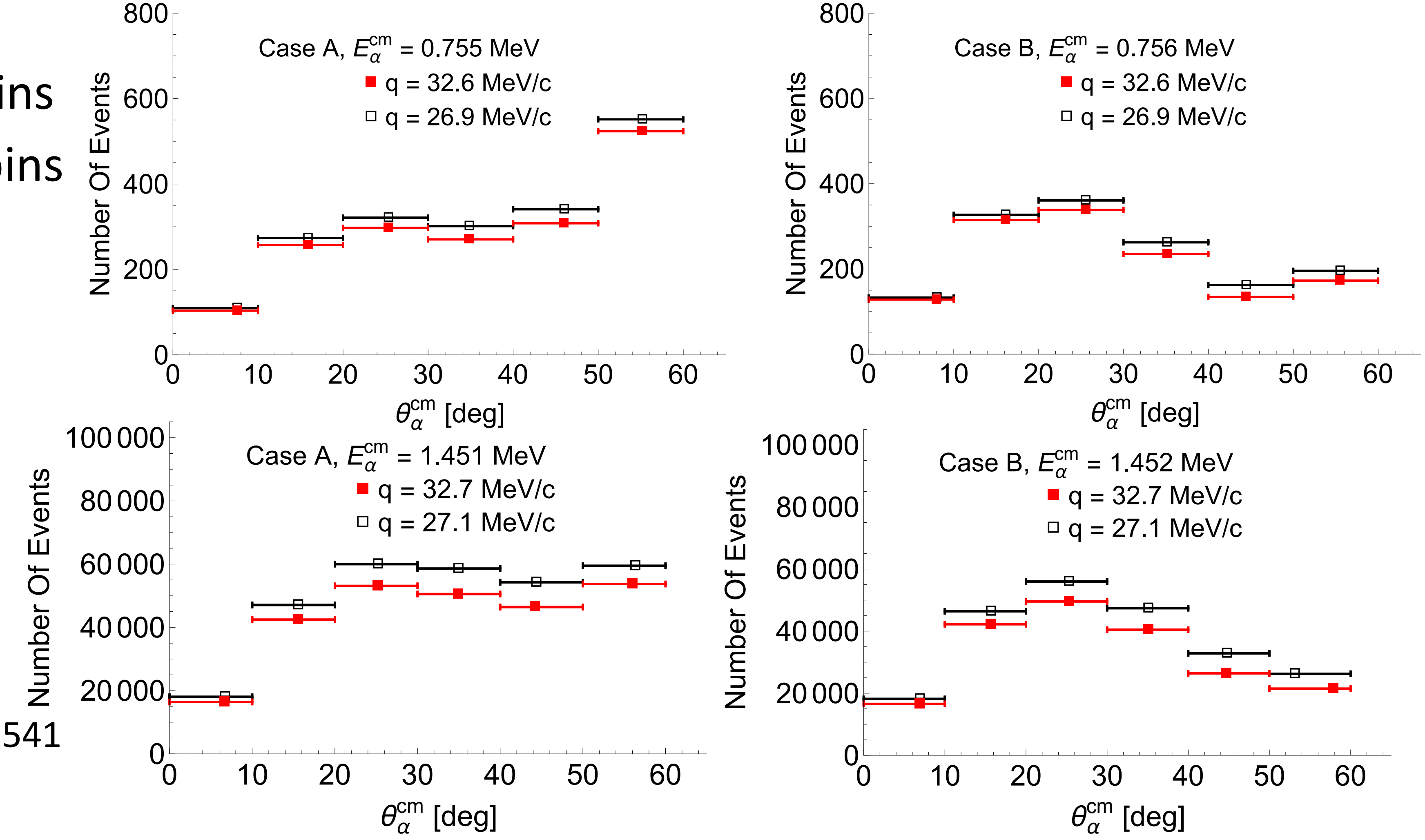
- E_e= 114 MeV, θ_e=15°,

Case A and Case B

- **Now we can compute statistical uncertainties**

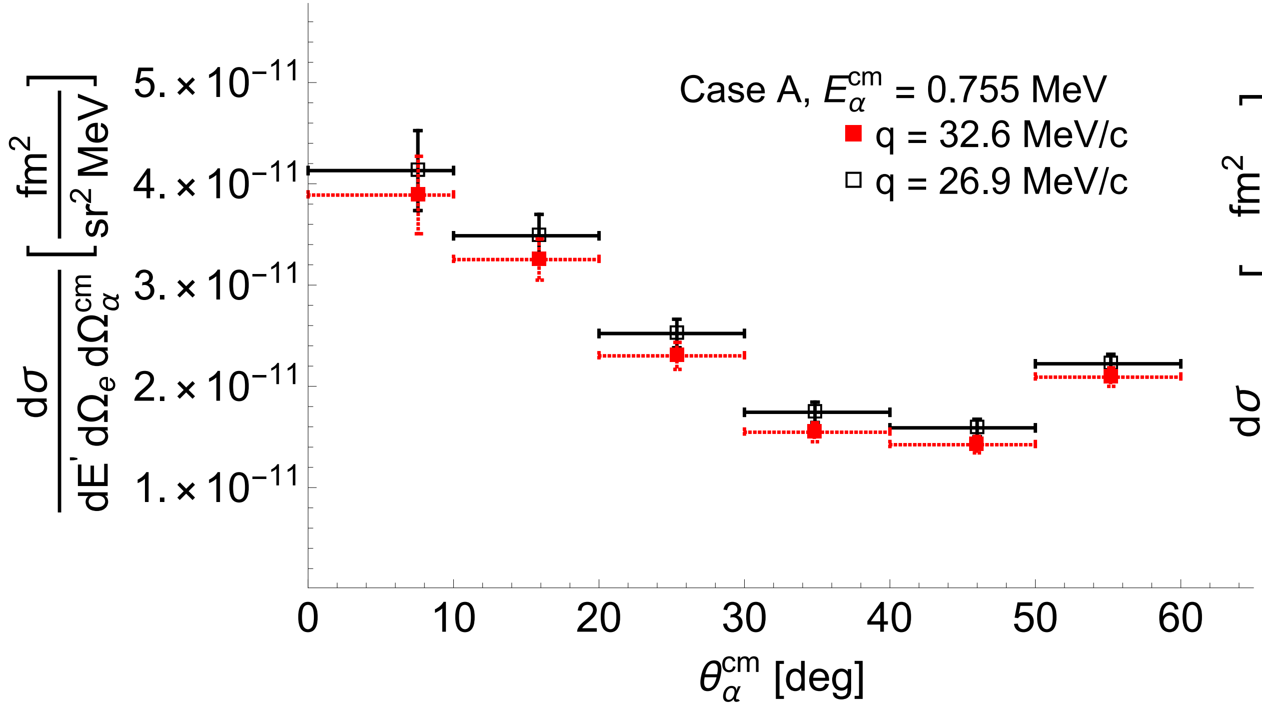
- Horizontal placement of data points according to:

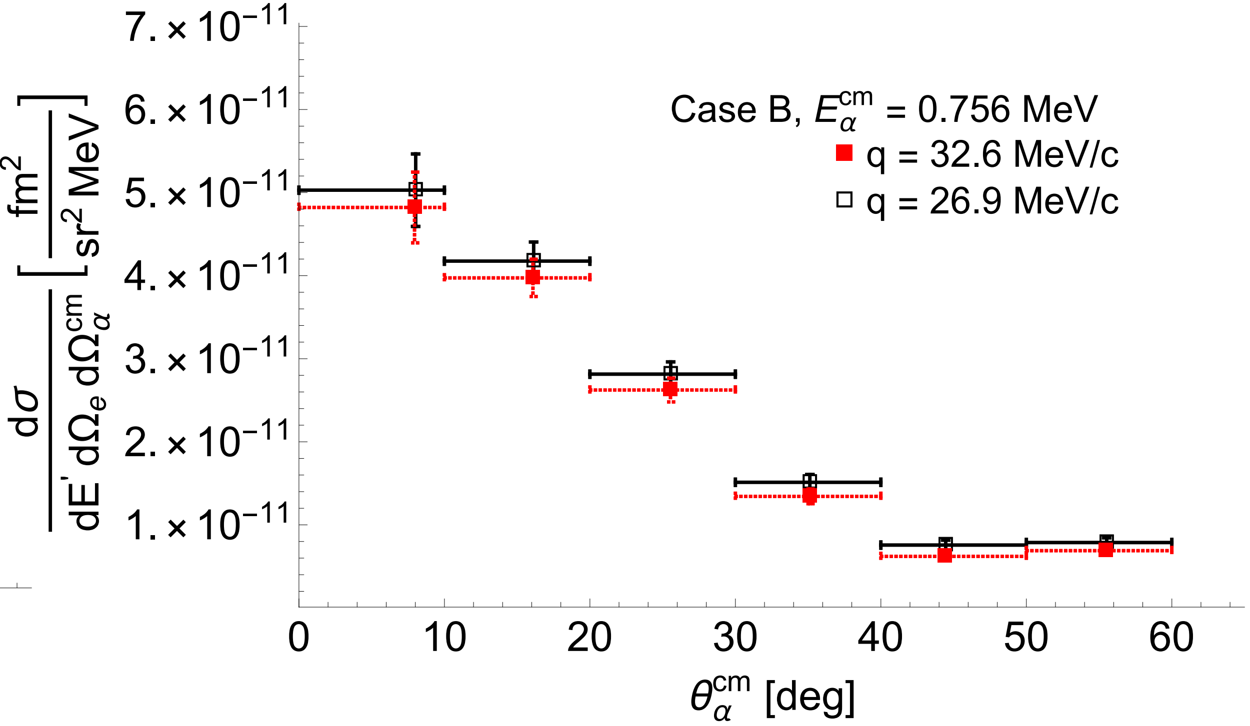
G. D. Lafferty and T. R. Wyatt, Nucl. Instrum. Methods Phys. Res. A 355, 541 (1995).



Differential cross section after 100 days

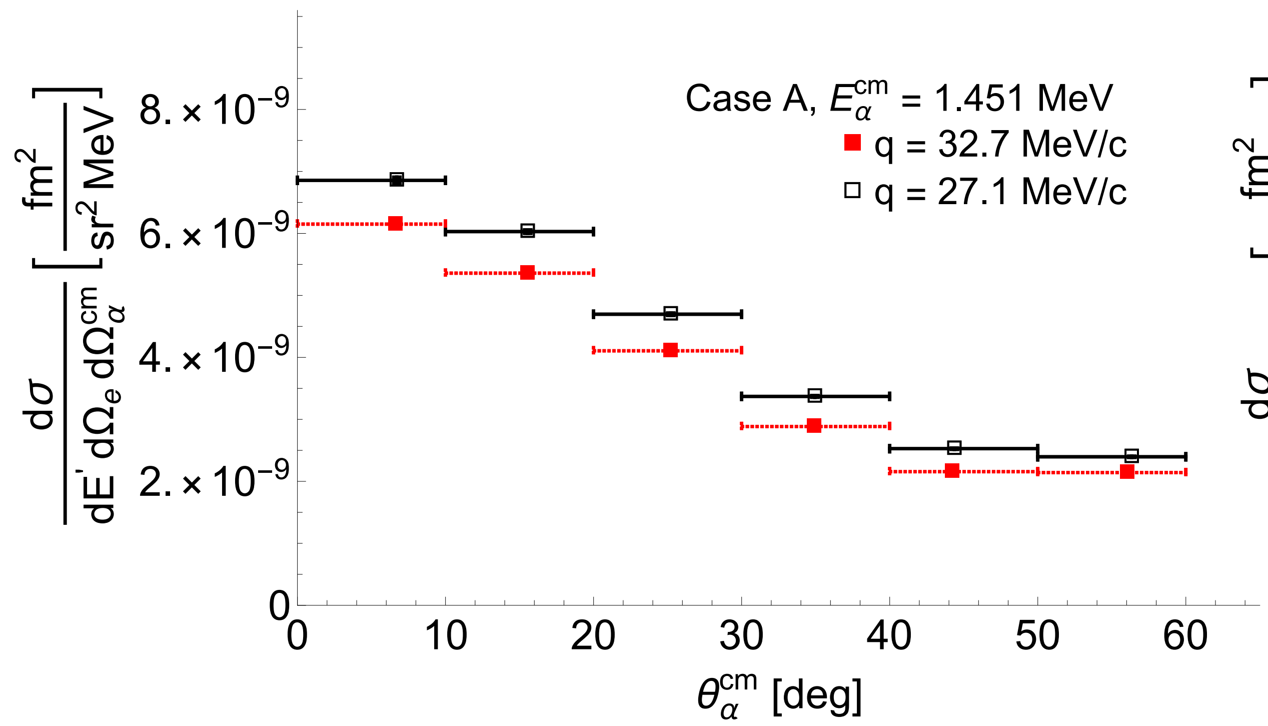
- E_e= 114 MeV, θ_e=15°, Case A and Case B

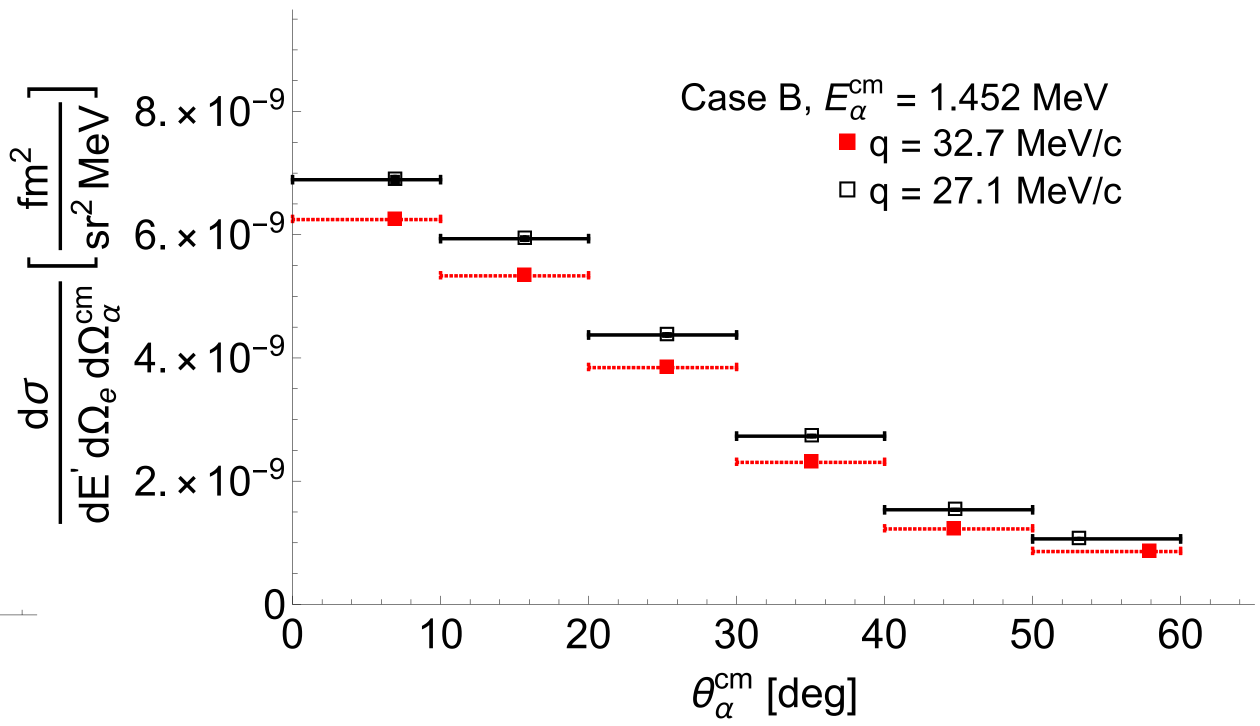




Differential cross section after 100 days

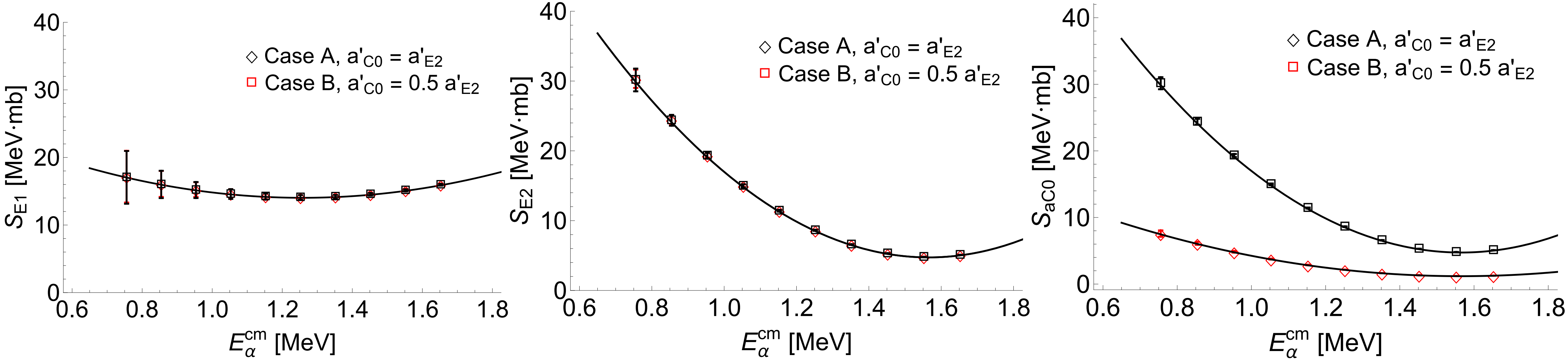
- E_e= 114 MeV, θ_e=15°, Case A and Case B





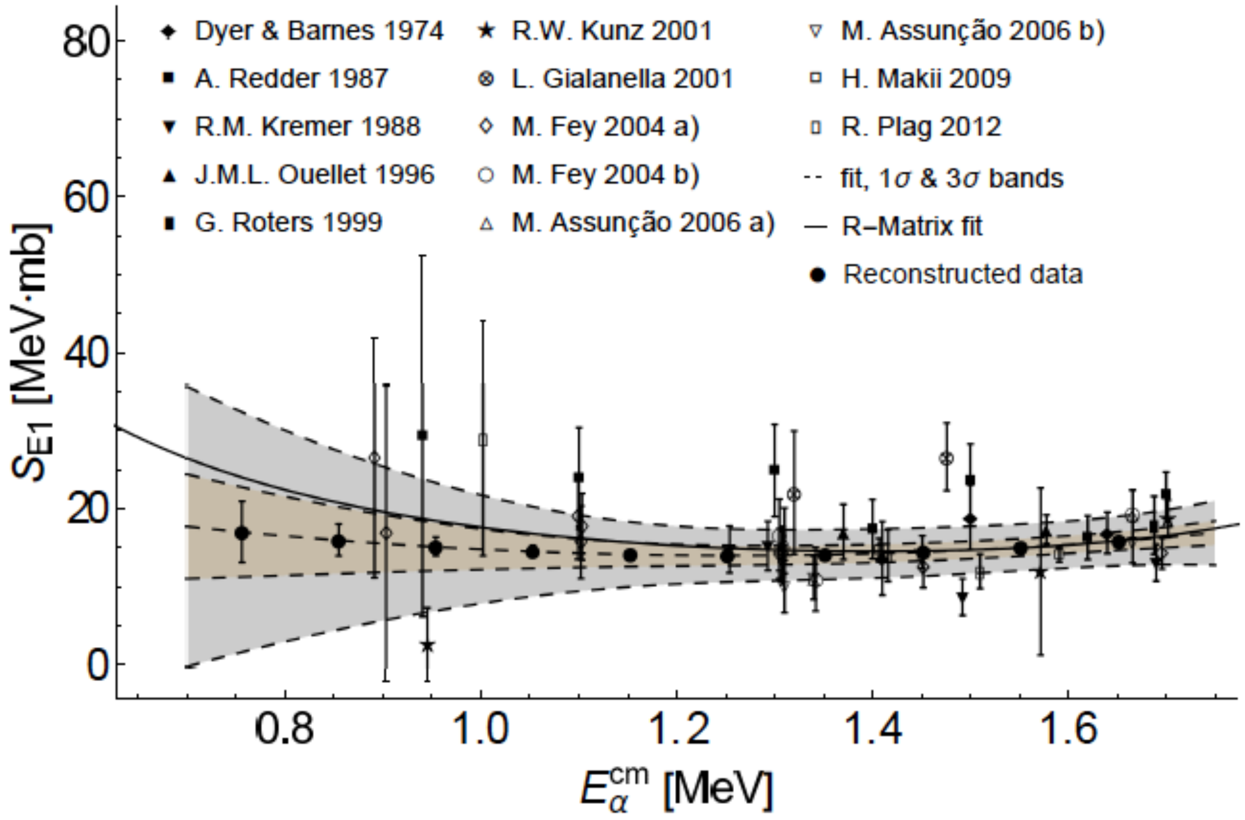
S-factors with projected statistical uncertainties

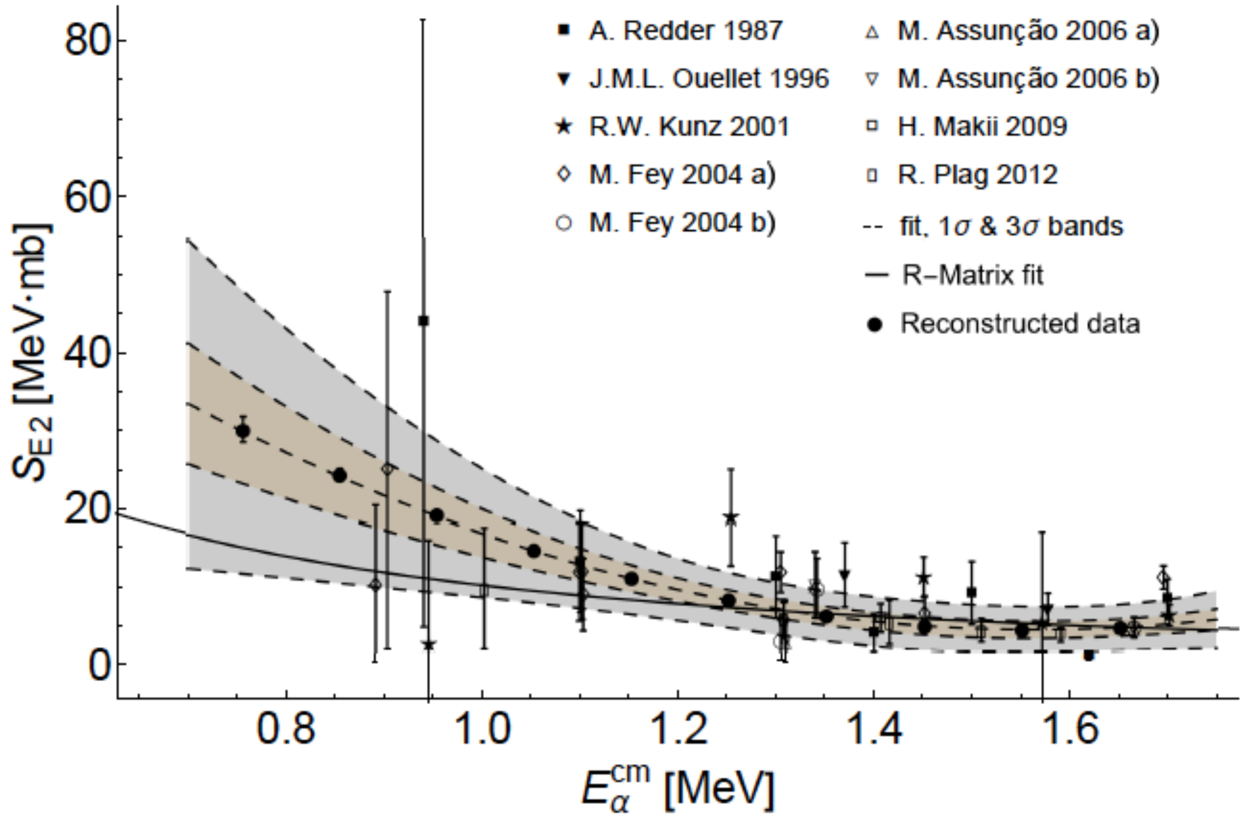
- E_e= 114 MeV, θ_e=15°, Case A and Case B
- Three fitting parameters a'_{E1}, a'_{E2} and a'_{C0} -> S_{E1}, S_{E2} and S_{aC0} non-astrophysical factor



S-factors with projected statistical uncertainties

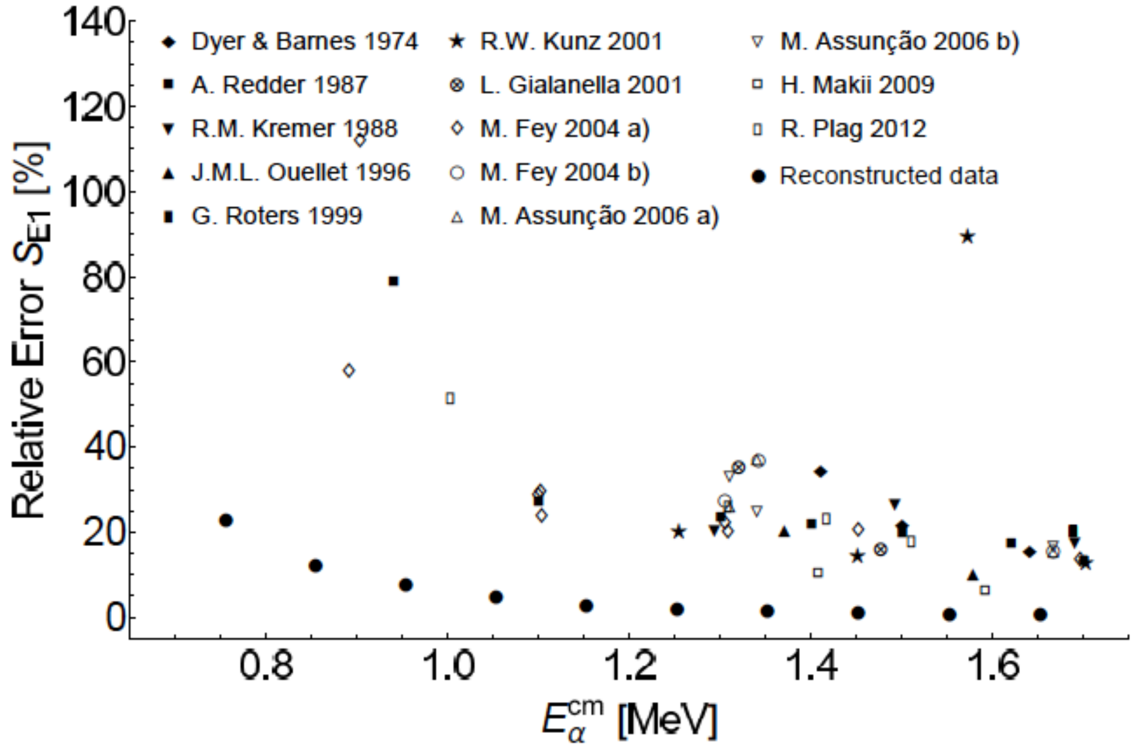
- E_e= 114 MeV, θ_e=15°, Case A

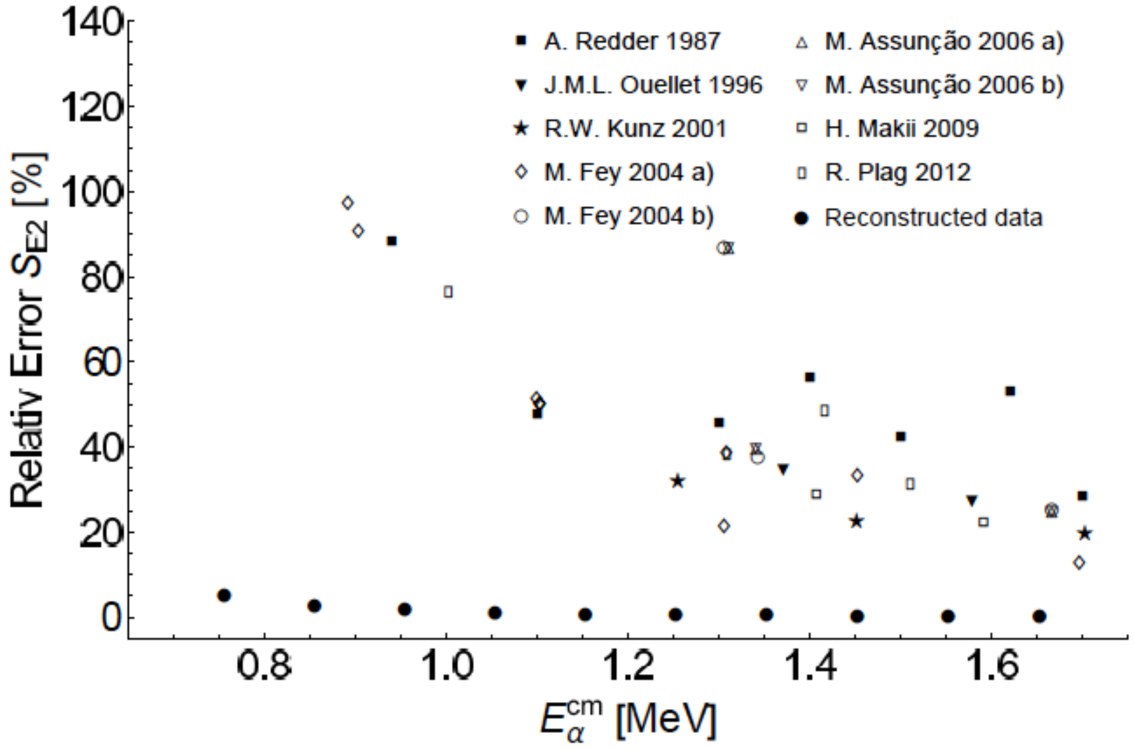




S-factors with projected statistical uncertainties

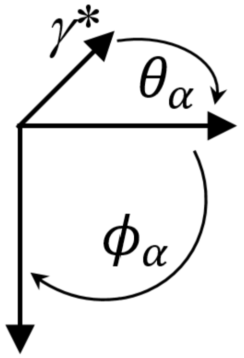
- E_e= 114 MeV, θ_e=15°, Case A
- Compared to most accurate measurements, statistical uncertainties of S_{E1} and S_{E2} are improved at least by factors 5.6 and 23.9, respectively

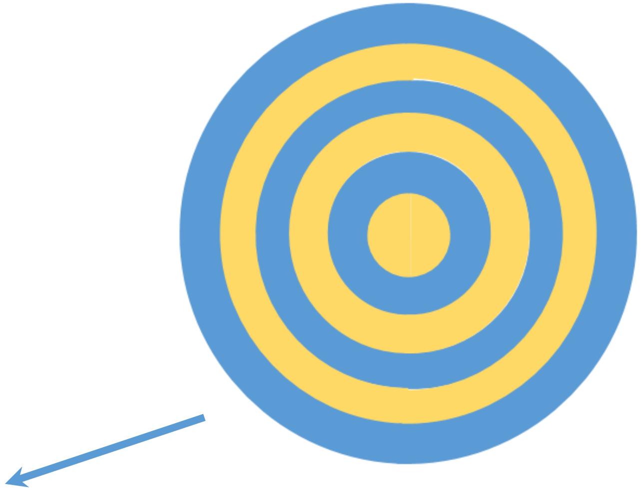




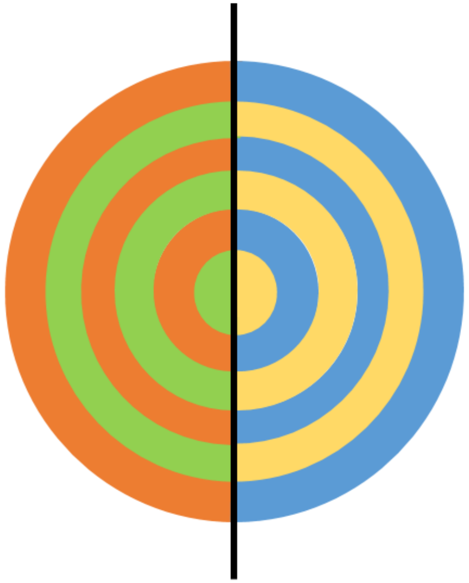
Outlook

- Shorter run at higher E_α^{cm} to test the particle identification and all assumptions

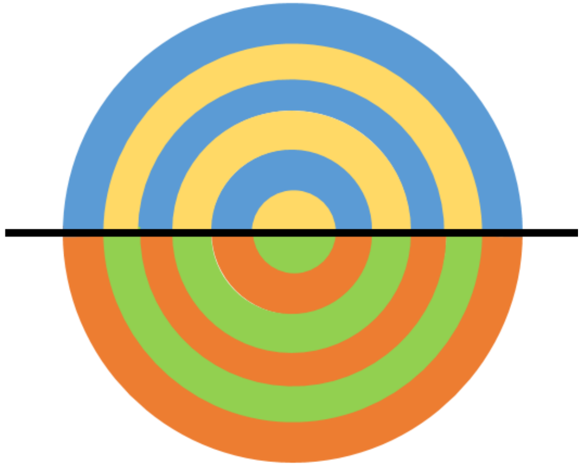




- Only R_T and R_L contribute to the rate
- Rosenbluth separation of R_T and R_L to extract the S-factors and the phases separately



- $\sigma_{Right} - \sigma_{Left} \rightarrow 2R_{TL}$
- $\sigma_{Right} + \sigma_{Left} \rightarrow 2(R_T + R_L)$
- Form an asymmetry



- $\sigma_{Up} = \sigma_{Down}$

→ test of efficiency and systematics

Conclusion

- Using a simple model, possibilities of new ERL accelerators and the gas-jet target, we studied the rate of $^{16}O(e,e'\alpha)^{12}C$ reaction in range 0.7 < E_{α}^{cm}< 1.7 MeV and showed how to determine $^{12}C(\alpha, \gamma)^{16}O$ reaction rate with unprecedented statistical precision

- At E_e= 114 MeV and electron spectrometer with 10% E_e' acceptance the full range 0. < E_{α}^{cm}< 10.2 MeV is accessible

- For more details: **I. Friščić, W. T. Donnelly and R. G. Milner, Phys. Rev. C 100, (2019) 025804; or arXiv:1904.05819**

Work supported by the DOE Office of Nuclear Physics under grant No. DE-FG02-94ER40818.

Backup

The cross section formulas

A. S. Raskin and T. W. Donnelly, Ann. of Phys. 191 (1989)

$$\frac{d\sigma}{dE'_e d\Omega_e d\Omega_\alpha^{cm}} = \frac{M_\alpha M_{12C}}{8\pi^3 W}\frac{p_\alpha^{cm}}{(\hbar c)^3}\sigma_{Mott}(\tilde{v}_L R_L + \tilde{v}_T \boldsymbol{R_T} + \tilde{v}_{LT} R_{LT} + \tilde{v}_{TT} R_{TT})$$

$$W = \sqrt{(M_{16O}+\omega)^2 - q^2} \qquad \boldsymbol{E_\alpha^{cm} = W - W_{th}}$$

$$\left.\frac{d\sigma}{d\Omega_\alpha^{cm}}\right|_{(\gamma,\alpha)} = \frac{M_\alpha M_{12C}}{4\pi W}\frac{p_\alpha^{cm}}{\hbar c}\frac{\alpha}{E_\gamma}\boldsymbol{R_T}$$

$$W = \sqrt{M_{16O}(M_{16O} + 2E_\gamma)}$$

$$\boldsymbol{E_\alpha^{cm} = E_\gamma - 7.162\ \mathrm{MeV}}$$

$$\left.\frac{d\sigma}{d\Omega_\gamma^{cm}}\right|_{(\alpha,\gamma)} = \frac{M_\alpha M_{12C}}{2\pi W}\frac{p_\alpha^{cm}}{\hbar c}\frac{\alpha}{E_\gamma}\boldsymbol{R_T}$$

$$W = \sqrt{M_\alpha^2 + M_{12C}^2 + 2M_{12C}E_\alpha^{lab}}$$

$$\boldsymbol{E_\alpha^{cm} = \frac{M_{12C}}{M_{12C} + M_\alpha}E_\alpha^{lab}}$$

Differential Cross Section: $^{16}O(e,e'\alpha)^{12}C$

A. S. Raskin and T. W. Donnelly, Ann. of Phys. 191 (1989)

$$\frac{d\sigma}{dE'_e d\Omega_e d\Omega_\alpha^{cm}} = \frac{M_\alpha M_{12C}}{8\pi^3 W}\frac{p_\alpha^{cm}}{(\hbar c)^3}\sigma_{Mott}(\tilde{v}_L R_L + \tilde{v}_T R_T + \tilde{v}_{LT} R_{LT} + \tilde{v}_{TT} R_{TT})$$

$$\rho \equiv |Q^2/q^2| = 1 - (\omega/q)^2$$

Total invariant mass:

$$W = \sqrt{(M_{^{16}O} + \omega)^2 - q^2}$$

$$E_\alpha^{cm} = W - W_{th}$$

$$v_L = \rho^2 \qquad \tilde{v}_L = (W/M_{^{16}O})^2 v_L$$

$$v_T = \frac{1}{2}\rho + \tan^2\theta_e/2 \qquad \tilde{v}_T = v_T$$

$$v_{TL} = -\frac{1}{\sqrt{2}}\rho\sqrt{\rho + \tan^2\theta_e/2} \qquad \tilde{v}_{TL} = (W/M_{^{16}O}) v_T$$

$$v_{TT} = -\frac{1}{2}\rho \qquad \tilde{v}_{TT} = v_{TT}$$