ON APPLICATION OF CHERENKOV RADIATION IN PRESENCE OF DISPERSIVE ANISOTROPIC MATERIALS TO DIAGNOSTICS OF ULTRARELATIVISTIC BEAMS*

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Abstract

Analytical and numerical analyses of Cherenkov radiation in presence of a dispersive, anisotropic metamaterial are presented. A convenient method to determine the Lorenz factor of charged particles, γ, is based on measuring the frequencies of the harmonics generated in a waveguide. In this work, it is shown that this method for determining γ can be performed using certain metamaterials, such as an anisotropic medium with plasmatic dispersion. In particular, it is shown that the medium's parameters can be selected in such a way as to obtain a strong γ-dependence of the mode frequencies in either some predetermined narrow range of γ. As well it is possible to obtain apparent γ-dependence for a wide range including very large values of γ.

INTRODUCTION

Cherenkov radiation is extensively used for the detection of charged particles moving at relativistic speeds [1]. Using modern, artificial metamaterials as Cherenkov radiators can be advantageous over using conventional media [2, 3]. Metamaterials are artificial periodic structures made of small elements that are designed to possess specific electromagnetic properties [4–6]. As long as the periodicity and the size of the elements are smaller than the wavelengths of interest, an artificial structure can be described by a permittivity and permeability, just as in natural materials.

This work mainly focuses on the case of a non-magnetic medium characterized by the following permittivity tensor:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_{||} \end{pmatrix}, \qquad (1)$$

$$\varepsilon_\perp = \varepsilon_{c\perp} - \frac{\omega_{p\perp}^2}{\omega^2 + 2i\omega_{d\perp}\omega}, \quad \varepsilon_{||} = \varepsilon_{c||} - \frac{\omega_{p||}^2}{\omega^2 + 2i\omega_{d||}\omega}, \qquad (2)$$

where $\omega_{p\perp}$ and $\omega_{p||}$ are the plasma frequencies, $\omega_{d\perp}$ and $\omega_{d||}$ are the attenuation parameters, $\varepsilon_{c\perp}$ and $\varepsilon_{c||}$ are some constants. Such a medium can be realized, for instance, with the help of a system of wires with small periods. In this case, plasma frequencies and attenuation parameters are given by the following formulae [4,5,7]:

$$\omega_{p\perp,||} = c\sqrt{2\pi\left[d_{\perp,||}^2 \ln\left(d_{\perp,||}/r_{\perp,||}\right)\right]^{-1}},$$
$$\omega_{d\perp,||} = c^2\left[4\sigma S_{\perp,||} \ln\left(d_{\perp,||}/r_{\perp,||}\right)\right]^{-1}, \qquad (3)$$

where $d_\perp$, $d_{||}$ are periods of systems, and $r_\perp$, $r_{||}$ are radiuses of wires, $S_\perp$, $S_{||}$ are squares of cross sections of wires, σ is a conductivity of wires, and c is speed of light in vacuum (the symbol «$\perp$» are related to wires normal to the z-axis, and the symbol «$||$» are related to wires parallel to the z-axis).

It is assumed that the medium fill a waveguide with a radius a. The main axis of the medium (i.e. z-axis) is coincident with the waveguide axis. A bunch of charge particles moves along the z-axis with a velocity $\vec{V} = c\beta\vec{e}_z$. The bunch dimension is suggested to be essentially less than typical wavelength.

SOME COMMON RESULTS

The components of the electric field behind the charge ($z < Vt$) can be written in the wave area (where the quasi-static field is negligible) using the following approximate form:

$$E_\rho = \frac{4q}{Va}\sum_{m=1}^{\infty} \mathrm{Re}\left\{\frac{i\omega_m J_1(\chi_m\rho/a)\exp(i\omega_m\zeta/V)}{\varepsilon_\perp(\omega_m)\chi_m[J_1(\chi_m)]^2(1+\delta_m+\kappa_m)}\right\},$$
$$E_z = -\frac{4q}{a^2}\sum_{m=1}^{\infty} \mathrm{Re}\left\{\frac{J_0(\chi_m\rho/a)\exp(i\omega_m\zeta/V)}{\varepsilon_{||}(\omega_m)[J_1(\chi_m)]^2(1+\delta_m+\kappa_m)}\right\}, \qquad (4)$$

where $J_n(\xi)$ are Bessel functions, $\zeta = z - Vt$, and

$$\delta_m = \frac{\omega\beta^2}{2\left(\varepsilon_\perp\mu_\perp\beta^2 - 1\right)} \left.\frac{d(\varepsilon_\perp\mu_\perp)}{d\omega}\right|_{\omega=\omega_m},$$
$$\kappa_m = \frac{\omega}{2}\left.\left(\frac{1}{\varepsilon_{||}}\frac{d\varepsilon_{||}}{d\omega} - \frac{1}{\varepsilon_\perp}\frac{d\varepsilon_\perp}{d\omega}\right)\right|_{\omega=\omega_m}. \qquad (5)$$

The values χ_m are roots of the function $J_0(\chi)$, and the harmonic frequencies ω_m are determined by the equation

$$\omega^2\varepsilon_{||}(\omega)\left[1 - \beta^{-2}\varepsilon_\perp^{-1}(\omega)\right] = c^2 a^{-2}\chi_m^2. \qquad (6)$$

In the case of non-dispersive medium, it can be shown that the dependence of ω_m on the Lorenz-factor $\gamma = \left(1-\beta^2\right)^{-1/2}$ is minor when $\gamma >> 1$, which is unfavourable for determining the energy of the ultrarelativistic particles. However, this disadvantage can be partially overcome by using certain anisotropic, dispersive media.

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In the paper [3] where we considered the case $\varepsilon_\perp = \varepsilon_{c\perp} = \text{const}$, we showed that under the condition $\varepsilon_\perp = 1$, $\omega_m(\gamma)$ is sensitive to values of γ below some maximum value $\gamma_{m\lim}$ (if $\gamma > \gamma_{m\lim}$, the wave harmonics is not excited). The value of $\gamma_{m\lim}$ increases with decreasing $\varepsilon_\perp$. It is interesting that it is possible to obtain an almost linear dependence on γ for $\omega_1(\gamma)$ [3]. For certain values of $\varepsilon_\perp < 1$, the first mode is generated for an arbitrary magnitude of γ, and the dependence of ω_1 on γ is readily apparent for $\gamma \le 10^4$ [3].

However, the case $\varepsilon_\perp = \text{const} < 1$ can be realized only approximately. Therefore, we consider the case when both $\omega_{p\perp} \ne 0$ and $\omega_{p\|} \ne 0$, i.e. both elements of the permittivity tensor in Eq. (1) depend on frequency. The frequencies of the harmonics are given by the following expression:

$$\omega^2_{m1,2} = \left[2\left(1-\beta^2\varepsilon_{c\perp}\right)\varepsilon_{c\|}\right]^{-1}\left[\Omega^2 \pm \sqrt{\Omega^4 + 4\beta^2\varepsilon_{c\|}\left(1-\beta^2\varepsilon_{c\perp}\right)\omega^2_{p\perp}\left(\omega^2_{p\|}+c^2\chi^2_m a^{-2}\right)}\right], \quad (7)$$

where $\Omega^2 = \omega^2_{p\|} - \beta^2\left(\omega^2_{p\|}\varepsilon_{c\perp} + \omega^2_{p\perp}\varepsilon_{c\|} + \varepsilon_{c\perp}c^2\chi^2_m a^{-2}\right)$ (the parameters $\omega_{d\perp}$, $\omega_{d\|}$ are assumed to be negligible).

Note that under the condition $\beta^2\varepsilon_{c\perp} < 1$, a single series of travelling harmonics with frequencies of ω_{m1} is generated (the ω_{m2} frequencies are imaginary). Under the condition $\beta^2\varepsilon_{c\perp} > 1$, two series of travelling modes with frequencies ω_{m1} and ω_{m2} are excited. An analysis shows that the frequency ω_{m1} is more convenient for measuring the Lorenz factor γ.

PERSPECTIVE APPLICATIONS TO BEAMS DIAGNOSTICS

By varying the medium parameters and the waveguide radius, different γ-dependencies for the harmonic's frequency, $\nu_m(\gamma)$, can be achieved. If the parameters a, $\varepsilon_{c\perp}$, $\varepsilon_{c\|}$, $\omega_{p\|} = 2\pi\nu_{p\|}$ are assumed to be fixed, a maximum γ-dependency of $\nu_m(\gamma)$ for the case when $\gamma >> 1$ can be achieved by selecting the value of $\omega_{p\perp} = 2\pi\nu_{p\perp}$ (which can be changed by varying the distance between wires and the wire thickness). Figure 1 shows an example of the frequencies' ($\nu_m = \omega_{m1}/2\pi$) dependence on γ, and figure 2 shows some typical spectra and wave fields. It can be shown that the γ-dependence of $\nu_m(\gamma)$ is enough to determine γ for $\gamma \le 10^4$.

Other goal is obtaining strong dependence $\nu_m(\gamma)$ for some narrow range of values of γ. This can be achieved, for example, by carefully selecting the magnitudes of $\varepsilon_{c\perp}$. Note that we can vary this parameter because it is determined by the permittivity and the thickness of the base plates of the metamaterials. Figure 3 shows some examples of the γ-dependence of $\nu_m(\gamma)$ for different magnitudes of $\varepsilon_c = \varepsilon_{c\perp} = \varepsilon_{c\|}$. One can see that the range of strong γ-dependence for ν_m drifts towards lower frequencies when ε_c increases.

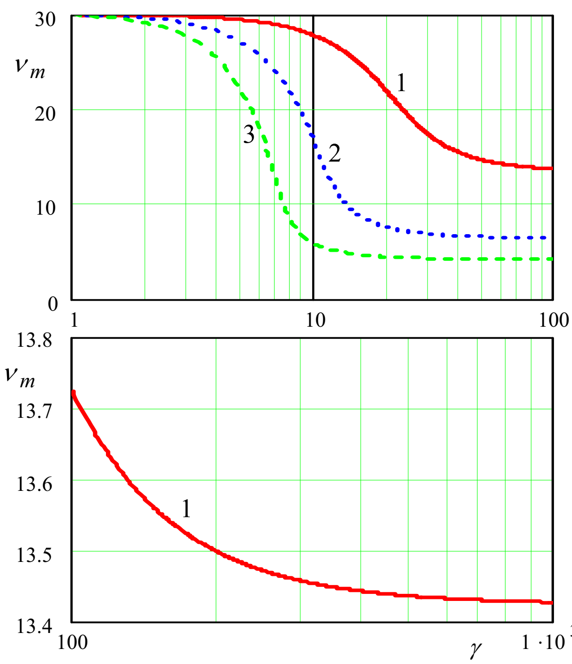

Figure 1: Dependence of the harmonics' frequencies ν_m (GHz) on γ: $a = 10\,\text{cm}$, $\nu_{p\|} = 30\,\text{GHz}$, $\nu_{p\perp} = 0.574\,\text{GHz}$, $\varepsilon_{c\|} = \varepsilon_{c\perp} = 1$; the harmonics' numbers are shown near the curves.

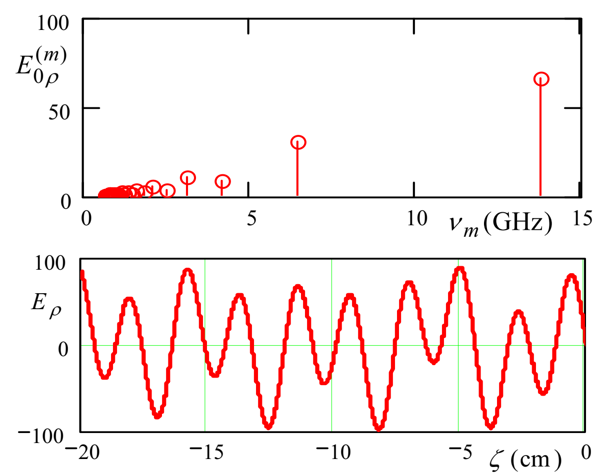

Figure 2: Amplitudes of harmonics of E_ρ and the wave field behind the charge (V/m): $q = -1\,pC$, $\gamma = 100$, $\rho = 5\,cm$, and other parameters are the same as in Fig.1.

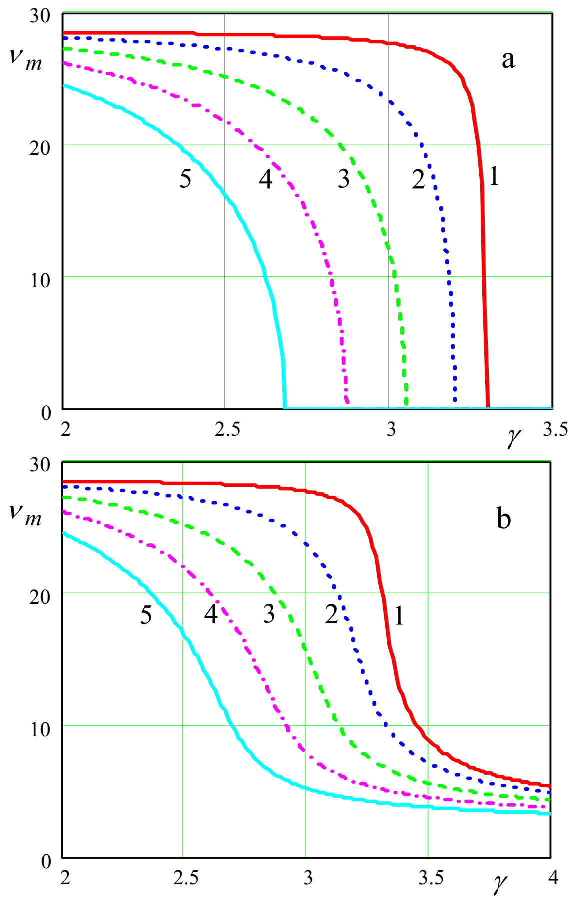

Figure 3: Dependence of harmonics' frequencies on γ: $a = 10\,\text{cm}$, $\nu_{p\|} = 30\,\text{GHz}$, $\varepsilon_{c\|} = \varepsilon_{c\perp} = 1.1$, $\nu_{p\perp} = 0$ (a) and $\nu_{p\perp} = 1\,\text{GHz}$ (b); the harmonics' numbers are shown near the curves.

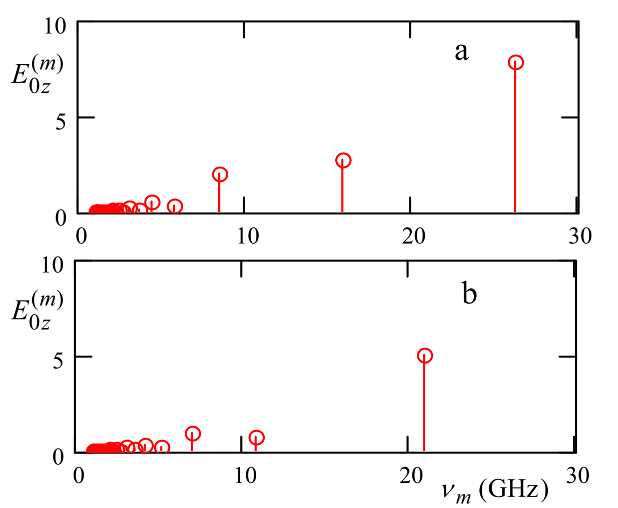

Figure 4: Amplitudes of harmonics of component E_z (V/m): $\gamma = 3.2$ (a), $\gamma = 3.3$ (b), $\nu_{p\perp} = 1\,\text{GHz}$, $|q| = 1\,pC$, and other parameters are the same as in Fig.3.

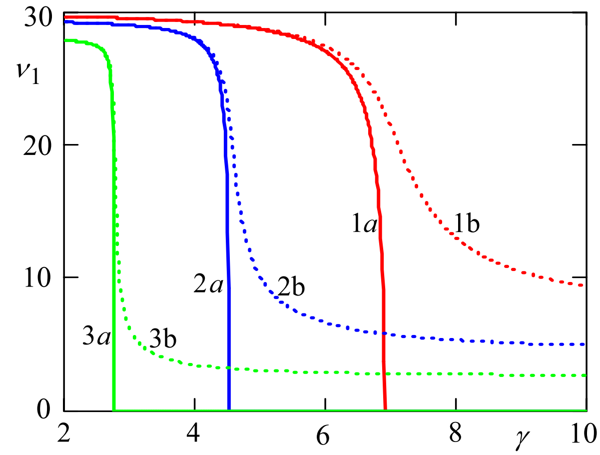

Figure 5: Dependence of the 1st harmonic frequency ν_1 (GHz) on γ: $a = 10\,\text{cm}$; $\nu_{p\|} = 30\,\text{GHz}$; $\varepsilon_{c\|} = \varepsilon_{c\perp} = 1.02\,(1a, 1b),\ 1.05\,(2a, 2b),\ 1.15\,(3a, 3b)$; $\nu_{p\perp} = 0\,(1a, 2a, 3a)$, $\nu_{p\perp} = 1\,\text{GHz}\,(1b, 2b, 3b)$.

Figure 4 shows the spectra for two close magnitudes of γ. One can see that they are essentially distinguishable.

Figure 5 illustrates the dependence of $\nu_1(\gamma)$ for different values of $\varepsilon_c = \varepsilon_{c\|} = \varepsilon_{c\perp}$ and $\nu_{p\perp}$. One can see that the parameter ε_c has an influence on the smoothness of this dependence. If $\nu_{p\perp} = 0$, the real frequency $\nu_1(\gamma)$ is equal to 0 for $\gamma > \gamma_{1\lim}$. This fact allows one to design a "reverse" threshold Cherenkov detector. Such a detector would register all particles with $\gamma < \gamma_{1\lim}$. Increasing $\omega_{p\perp}$ leads to smoother functions for $\nu_m(\gamma)$. It is important to note that relatively small variations in the parameter ε_c leads to an essential shift of the range of the strong dependence $\nu_m(\gamma)$. These facts may be useful for designing Cherenkov detectors for different purposes.

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