STABILITY CHANGE OF FOURTH-ORDER RESONANCE WITH APPLICATION TO MULTI-TURN EXTRACTION SCHEMES

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Abstract

Recently, a novel multi-turn extraction scheme was proposed, based on particle trapping inside stable resonances. Numerical simulations and experimental tests confirmed the feasibility of such a scheme for low order resonances. While the 3rd order resonance is generically unstable and those higher than 4th order are generically stable, the 4th order resonance can be either stable or unstable depending on the details of the system under consideration. By means of the normal form approach a general formula to control the stability of the 4th order resonance is derived. Numerical simulations confirm the analytical results and show that by crossing the unstable 4th order resonance the region around the centre of phase space is depleted and particles are trapped only in the four stable islands. This indicates that a four-turn extraction could be envisaged based on this technique.

INTRODUCTION

In recent years, a novel type of extraction based on particle trapping inside stable islands of the horizontal phase space was proposed [1]. The beam manipulation is intrinsically linked with non-linear beam dynamics. The beam is swept through a stable non-linear resonance and whenever the crossing is adiabatic, some particles can be trapped inside the stable islands and then transported towards higher amplitudes. At this stage the extraction proper is performed.

Such an extraction mode is primarily aimed at replacing the current Continuous Transfer extraction [2] from the CERN Proton Synchrotron (PS) to the Super Proton Synchrotron (SPS) as an improved extraction mode, in the sense of losses (that are highly reduced, if not completely suppressed) and of injection matching. Parenthetically, a long measurement campaign was performed [3] to assess the performance of the proposed method, and a project setup for the implementation of such an extraction in the PS machine [4].

Further studies, however, showed that the method has a much broader scope, being applicable to resonances other that the $1/4$, which is the one selected for the CERN-specific application [5]. More than this, the approach can be time-reversed and used for a novel injection type [6].

In the case of a stable resonance of order n, $n+1$ beamlets are generated, n corresponding to the islands and one to the beam core part remaining after the trapping process. The beamlets at the end of the trapping process form two disconnected structures in phase space: a ribbon closing up after n turns around the machine circumference and the part, one-turn long, at the centre of phase space.

At the level of the extraction system proper, a set of kickers should generate a fast bump constant over the $n+1$ extraction turns, noting that the kickers' strength should be increased for ejecting the last turn. This might be a limiting factor, imposing higher demands on the strength requirements. In this respect, an unstable resonance, for which there is basically no beam left at the centre of phase space, might be advantageous.

In Ref. [5] the case of the third-order resonance was studied. However, one could also consider the fourth-order resonance and make it unstable so to generate a four-turn extraction scheme. This can be done by using the tool of Normal Forms. The application of such a tool to the non-linear betatronic motion was proposed in [7], while a review can be found in [8]. This approach can be used to change the stability type of a resonance [9]. The theory and the simple model used for this study will be presented together with some results of numerical simulations.

THEORY AND MODEL

To simulate the adiabatic capture process the composition of two symplectic maps is used:

$$\begin{pmatrix} x \\ p_x \end{pmatrix}_{n+1} = R(\psi_2)\cdot\begin{pmatrix} x \\ p_x + k_{3,2}x^3 \end{pmatrix} \circ \\ R(\psi_1)\cdot\begin{pmatrix} x \\ p_x + x^2 + k_{3,1}x^3 \end{pmatrix}_n , \qquad (1)$$

where (x, p_x) are phase space coordinates, $R(\varphi)$ is a rotation matrix of an angle φ, $k_{3,1}$ and $k_{3,1}$ are the strengths of two octupolar elements. Without any loss of generality the sextupolar strength can be set equal to one. During the iteration of the map the value of ψ_1 is kept constant letting ψ_2 varying with n so to set $\omega = 2\pi\nu = \psi_1 + \psi_2$ to the actual value of the linear frequency of the system.

Via Normal Forms theory applied to polynomial maps [8] it is known that the $1/4$ resonance could be either stable on unstable, depending on the value of the first amplitude detuning term, as the Hamiltonian can be written

$$H(\theta,\rho) = \epsilon\,\rho + \Omega_2\,\rho^2 + A\,\rho^2\,\cos 4\theta + \mathcal{O}(\rho^3), \qquad (2)$$

where (θ, ρ) stand for the angle-action variable and Ω_2 is the first amplitude-dependent detuning term.

A general recurrence to evaluate Ω_2 in the case of the composition of M polynomial maps of the form

$$\begin{pmatrix} x \\ p_x \end{pmatrix}_{n+1} = R(\psi_j) \cdot \begin{pmatrix} x \\ p_x + k_{2,j}x^2 + k_{3,j}x^3 \end{pmatrix}_n , \quad (3)$$

is described in Ref. [9]. This case represents a realistic model of a machine, where M sextupoles and octupoles are installed. Of course, for the case of M maps, $\omega = \sum_{i=0}^{M} \psi_i$ is the global tune.

For the model used in the numerical simulations presented in this paper the values $\psi_1/2\pi = 0.62$, $k_{3,1} = 1$, and $k_{3,1} = -1/3$ were used. The variable $k_{3,1}$ was used to set $\Omega_2 = 0$ during the crossing of the $1/4$ resonance, which corresponds to changing its stability type.

A typical sequence of phase space topology obtained during the resonance crossing process is shown in Fig. 1. By changing the tune the separatrices related to the hyper-

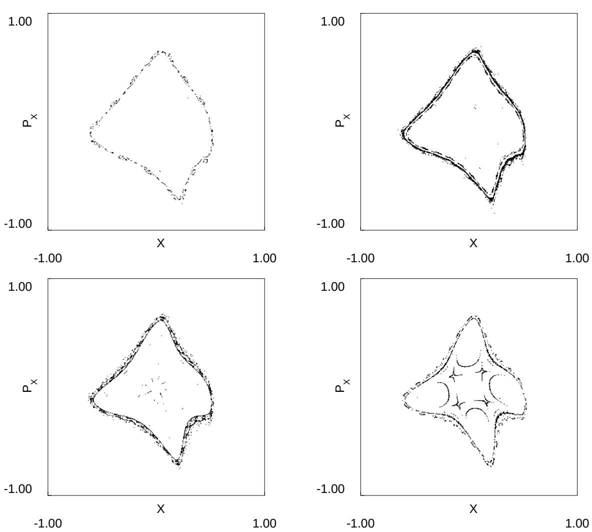

Figure 1: Phase portrait of the dynamical system Eq. (1) as the linear tune ν is changed and approaching the resonant value $\nu = 1/4$ from above.

bolic fixed points shrink towards the origin, thus making it unstable.

NUMERICAL SIMULATIONS

The numerical simulations have been performed using the model of Eq. (1) and considering a Gaussian distribution of particles in the horizontal phase space space (x, p_x)

$$\rho(x, p_x) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+p_x^2}{2\sigma^2}} . \quad (4)$$

The tune $\nu(n)$ is changed with time as a polynomial function f_k of order k, where the key parameters are ν_s, ν_r, and ν_f, the starting, resonant, and final tune values, respectively. n_1, and N represent the turn at which the resonance is crossed and the total number turns involved in the capture process. The order of the polynomial can change before and after n_1 with the constraint that $\nu'(n_1) = 0$ so to achieve a smooth transition at exact resonance crossing. Henceforth, we refer to the $i-j$ curve if we have a polynomial curve of ith order before the resonance crossing and a jth polynomial curve after the resonance crossing.

Typically, 2×10^6 particles were tracked for about 5×10^4 iterations of the map, with $\nu_s = 1.253$, $\nu_r = 1.25$, and $\nu_e = 1.248$, while the resonance crossing occurs for $n_1 = 3.5 \times 10^4$. An example of splitting process is shown in Fig. 2 The corresponding tune variation is reported in

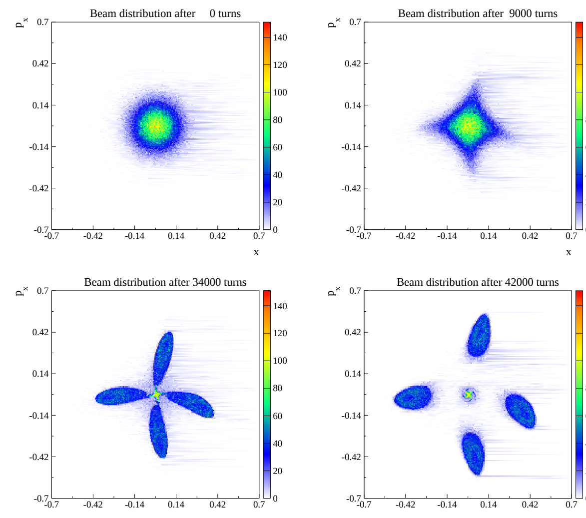

Figure 2: Evolution of the initial distribution during the resonance crossing process.

Fig. 3. While the separatrices collapse towards the origin,

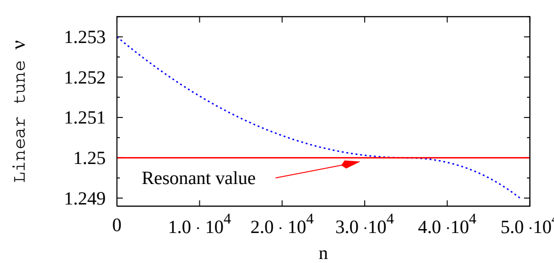

Figure 3: Tune variation used for the splitting presented in Fig. 2. The tune curve is of type $2-3$.

the particles are pushed towards the islands and trapped inside.

In Figs. 4, 5 the results concerning the fraction of trapped particles in the resonance island and the relative emittances as a function of the σ of the initial distribution are shown. For the particles left close to the origin of the phase space the so-called halo parameter h [11] defined as

$$h = \frac{\langle x^4 \rangle}{\langle x^2 \rangle^2} - 2 \quad (5)$$

is shown in Fig. 4. The bigger is the σ the greater is the fraction of beam trapped into the resonance islands. This

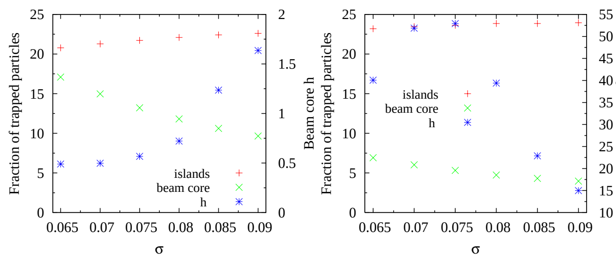

Figure 4: Fraction of trapped particles as a function of the initial sigma. Two types of tune curves are used: $1-1$ (left) and $2-3$ (right). The value of the factor h for the beam core is also given.

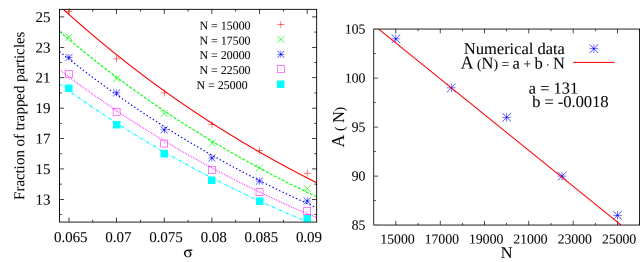

Figure 6: Fit results of the fraction of trapped particles as a function of the total number of turns N with an exponential function (left). The amplitude of such an exponential is fitted as a linear function of N (right).

trend is valid for every type of tune curve. On the other hand it is also clearly seen that the higher the order of the polynomial the smaller is the fraction of beam left in the centre. For the curve $1-1$, ≈ 10 % of the beam is left in the core distribution. The best result is obtained for the curve $2-3$ with a fraction of ≈ 4 % of the whole particles distribution. It is worth mentioning that almost no particle loss is observed ($\approx 0.01\%$ of the total number of particles).

For the numerical simulations of the adiabatic splitting

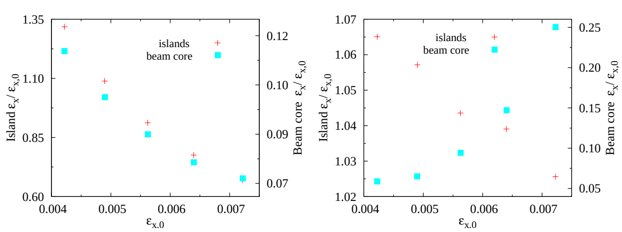

Figure 5: Relative islands emittance as a function of the initial emittance. Two types of tune curves are used: $1-1$ (left) and $2-3$ (right).

the condition $\Omega_2 = 0$ was set only for the resonant value of the tune. Comparisons with simulations performed with $\Omega_2 = 0$ throughout the whole process showed a decrease of the fraction of particle left in the centre to ≈ 0.5 %.

A set of numerical simulations was made to assess the dependence of the fraction of trapped particles on the total turn number N over which the tune variation $\Delta\nu = \nu_f - \nu_i$ is performed and the σ of the initial distribution. A tune curve of type $1-1$ was used. The dependence on σ is perfectly fitted with an exponential function, while that on N is a linear function and the combined result gives $N_{\text{Core}}/N_{\text{part}} = A\, e^{-\tau\sigma}$ with $A(N) = a + b\,N$. The numerical results together with the fitted curves are shown in Fig. 6. The excellent agreement between data and fit function is clearly visible. From the numerical simulations it emerges clearly that the parameter τ does not depend on N and its value is $\tau \approx 22.3$. Still, it could be a function of the non-linear parameters of the system.

CONCLUSIONS

In this paper the use of Normal Forms for the change of the stability type of a non-linear resonance was presented. Applied to a simple model describing the horizontal betatronic motion it can be successfully used to generate a scheme for a four-turn extraction. A parametric study of the beam parameters after splitting was also discussed, showing that the process can be accurately controlled.

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