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NRF National Research Foundation
iThemba LABS Laboratory for Accelerator Based Sciences

Design of a spiral inflector at iThemba LABS for injection of the beam into a cyclotron

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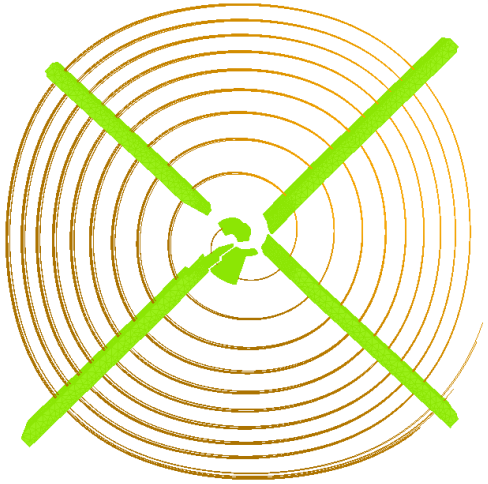
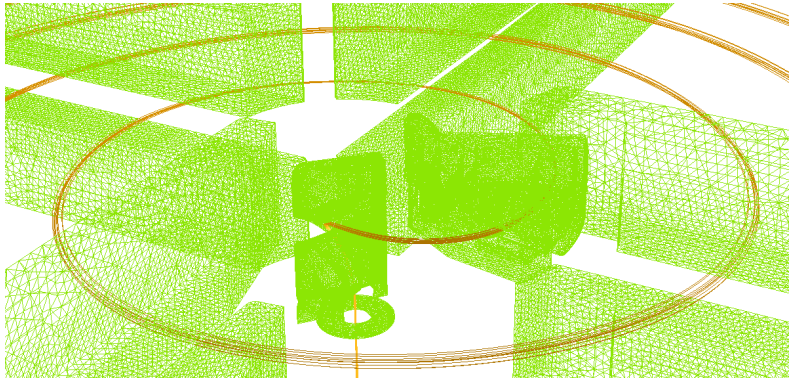
Overview of Talk

- Background on spiral inflectors
- Previous work at iThemba optimising a spiral inflector
- Permanent magnet spiral inflectors
- Optimising: Cyclotron acceptance, injection beam line
- New electrostatic simulation
- Results: Electric vs Magnetic

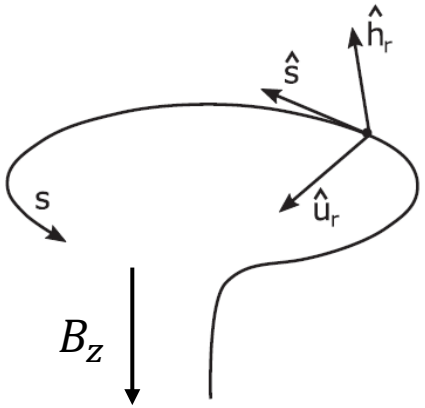
Background – Spiral Inflector



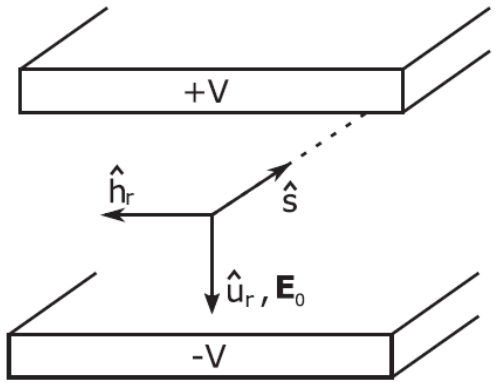
Inflector in central region



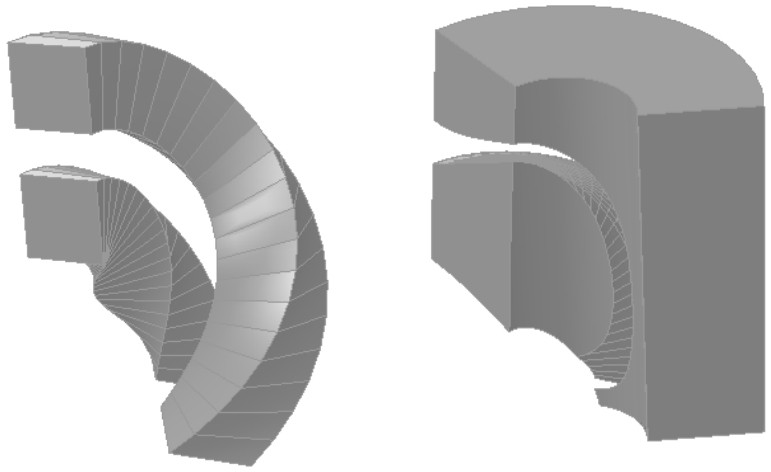
Axial injection



Magnetic field constant
(except in axial hole)



E field between electrodes

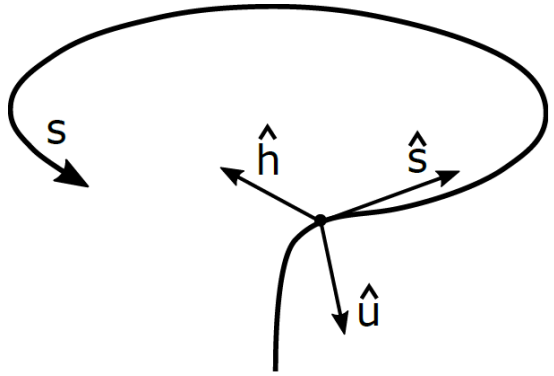


Electrodes

Background – Central path

Low energy non-relativistic

$$\ddot{\boldsymbol{x}} = \frac{q}{m}\boldsymbol{E} + \frac{q}{m}\dot{\boldsymbol{x}} \times \boldsymbol{B}$$



Coordinate system (u, h, s):

$\hat{s}$ - direction of motion

$\hat{h}$ - horizontal

$\hat{u}$ - given by: $\hat{u} = \hat{h} \times \hat{s}$

Criteria for central path:

- $\phi = 0$, so $E_s = 0$
- Analytical solution

"Spiral inflector" has:

1. Constant E_u. This is equivalent to a linear vertical field:

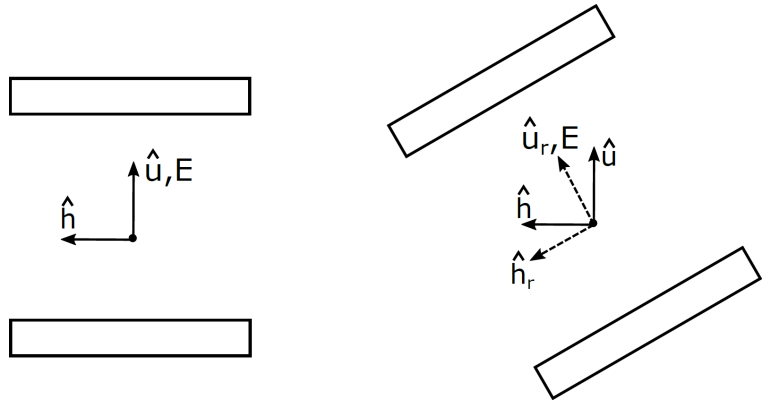
$$E_z = -k\,z$$

2. E_h is selected so it is proportional to the magnetic force. This creates an "effective" magnetic field

For a given magnetic bending radius R_m, the free parameters are:

A – height

k' – tilt parameter:



Properties of Inflector C for the SPC2:

R_m	4.9 cm
A	6.0 cm
k'	0.38

Background - Problems

- Emittance blow-up from coupling (6D unchanged, projected 2D can grow)
- Significant vertical spread

  Central region – no magnetic focusing
- Longitudinal de-bunching

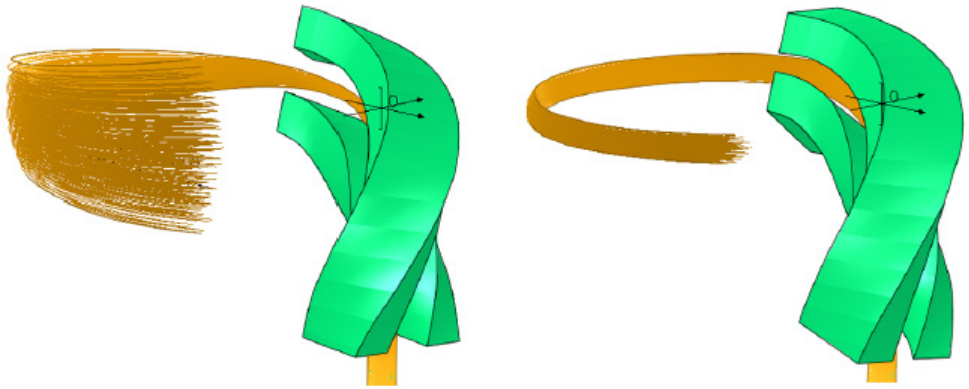
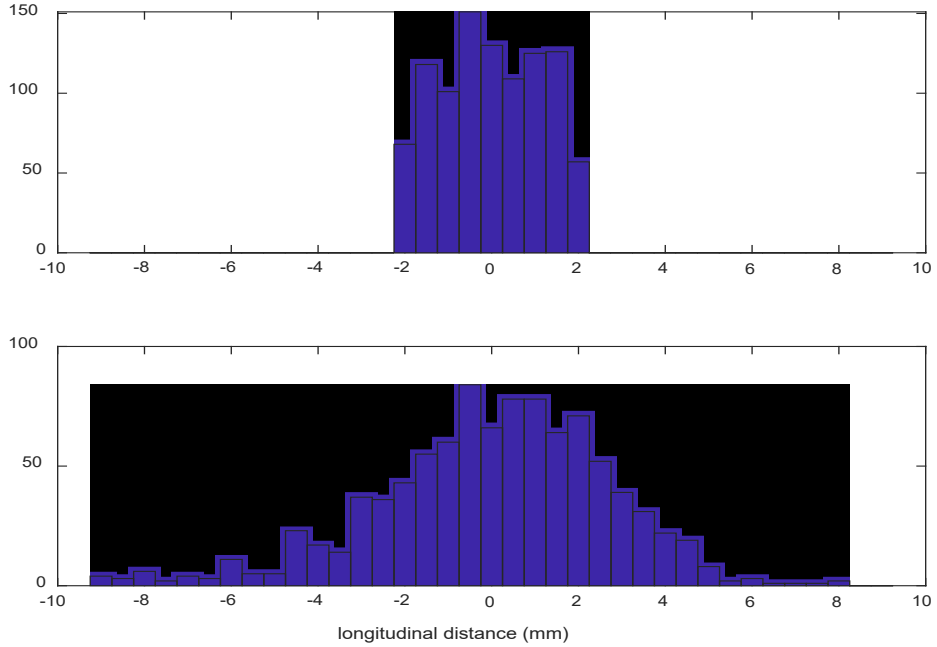


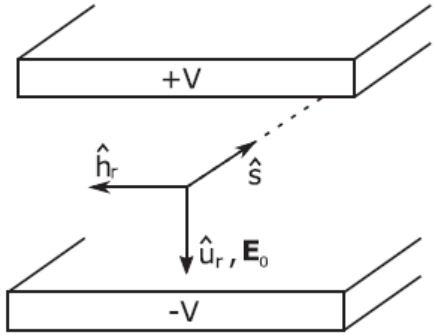
TABLE III. Transfer matrix R of C1 ($|R| = 1.00$).

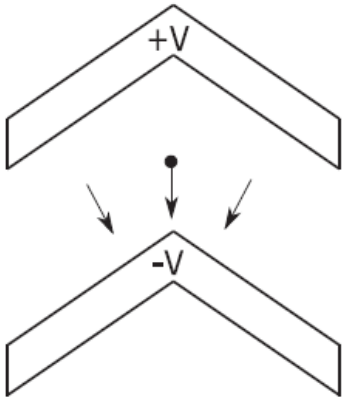
−1.3078	−0.0216	−2.1882	−0.2905	0	−0.2462
−10.951	−0.2732	−21.255	−3.2250	0	−1.9985
0.8273	0.0470	−0.3371	−0.0648	0	−0.0166
−2.3343	0.9307	−3.7095	−1.0723	0	−1.5982
1.4403	0.0834	0.3830	0.1273	1	0.1793
−0.0027	−0.0015	0.0276	0.0031	0	1.0016

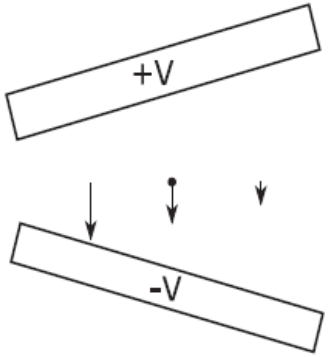
Source [1]

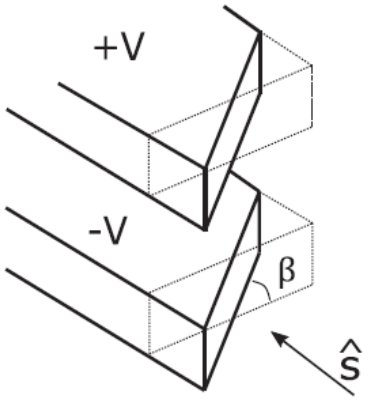


Previous Work – Transverse Gradients

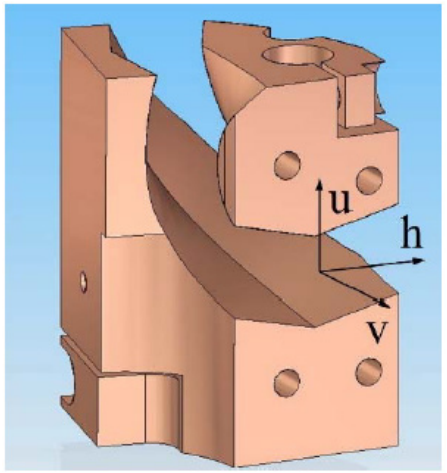






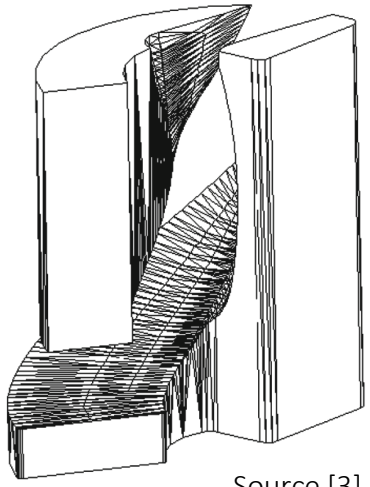


Dubna (1)



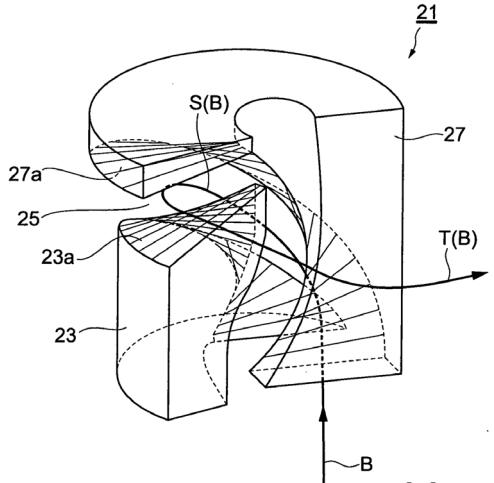
Source [2]

Dubna (2)



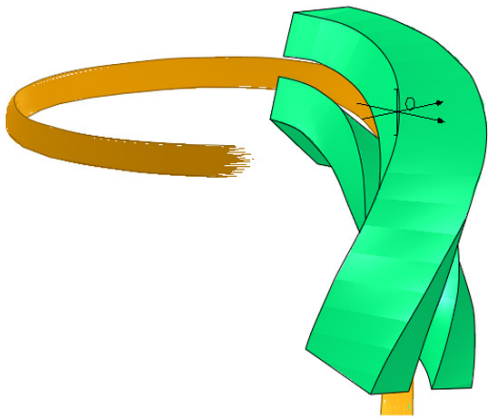
Source [3]

Sumitomo



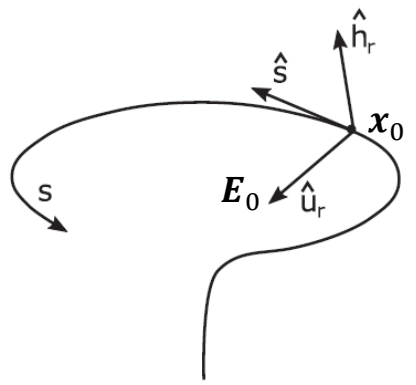
Source [4]

iThemba (C2)



Previous Work – Electric potential

Rotated (u_r, h_r, s) coordinate frame aligned with main electric field: $\boldsymbol{E}_0 = E_0 \hat{u}_r$



$$\boldsymbol{x}(u_r, h_r, s) = \boldsymbol{x}_0(s) + u_r \hat{u}_r(s) + h_r \hat{h}_r(s)$$

Rotation of coordinate frame - $\boldsymbol{\kappa}$

$$\hat{u}_r' = \kappa_s \hat{h}_r - \kappa_h \hat{s}$$
$$\hat{h}_r' = \kappa_u \hat{s} - \kappa_s \hat{u}_r$$
$$\hat{s}_r' = \kappa_h \hat{u}_r - \kappa_u \hat{h}_r$$

Coordinates form a non-orthogonal system, with base and dual base vectors:

$$\boldsymbol{b}_1 = \frac{\partial \boldsymbol{x}}{\partial u_r} = \hat{u}_r \qquad \boldsymbol{b}^1 = \hat{u}_r + \frac{h_r \kappa_s}{1 - u_r \kappa_h + h_r \kappa_u}\,\hat{s}$$
$$\boldsymbol{b}_2 = \frac{\partial \boldsymbol{x}}{\partial h_r} = \hat{h}_r \qquad \boldsymbol{b}^2 = \hat{h}_r - \frac{u_r \kappa_s}{1 - u_r \kappa_h + h_r \kappa_u}\,\hat{s}$$
$$\boldsymbol{b}_3 = \frac{\partial \boldsymbol{x}}{\partial h_r} = \hat{s}_r(1 - u_r \kappa_h + h_r \kappa_u) + \kappa_s(u_r \hat{h}_r - h_r \hat{u}_r) \qquad \boldsymbol{b}^3 = \frac{1}{1 - u_r \kappa_h + h_r \kappa_u}\,\hat{s}$$

Laplace:

$$\nabla^2 \phi = \frac{1}{\sqrt{g}} \sum_i \frac{\partial}{\partial u^i} \left(\sqrt{g} \sum_j \frac{\partial \phi}{\partial u^j} g^{ij} \right) = 0$$

$g_{ij} = \boldsymbol{b}_i \cdot \boldsymbol{b}_j$ is the metric matrix

$g = \det g_{ij}$

Expand ϕ to second order in terms of u_r, h_r:

$$\frac{\partial^2 \phi}{\partial u_r^2} + \frac{\partial^2 \phi}{\partial h_r^2} + \hat{s}' \cdot \boldsymbol{E}_0 = 0$$

This gives $\boldsymbol{E}$ to first order, i.e. linear optics

Previous Work – Quadrupole parameters

At each point s, the solution is:

$$\phi = -u_r E_0 - Q_1 E_0 \frac{u_r^2 - h_r^2}{2} - Q_2 E_0 h_r u_r - \frac{u_r^2}{2} \hat{s}' \cdot \boldsymbol{E}_0$$

Quadrupole terms $Q_1(s)$ and $Q_2(s)$ are functions that the inflector designer can select. $Q_1 = Q_2 = 0$ is a traditional inflector.

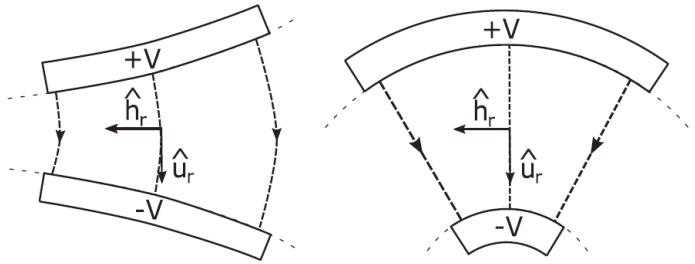
Electric fields:

$$\boldsymbol{E} = -\nabla\phi = -\sum_i \frac{\partial \phi}{\partial u^i} \boldsymbol{b}^i$$

$$E_{u_r} = E_0 + u_r(Q_1 E_0 + \hat{s}' \cdot \boldsymbol{E}_0) + h_r Q_2 E_0$$

$$E_{h_r} = u_r Q_2 E_0 - h_r Q_1 E_0$$

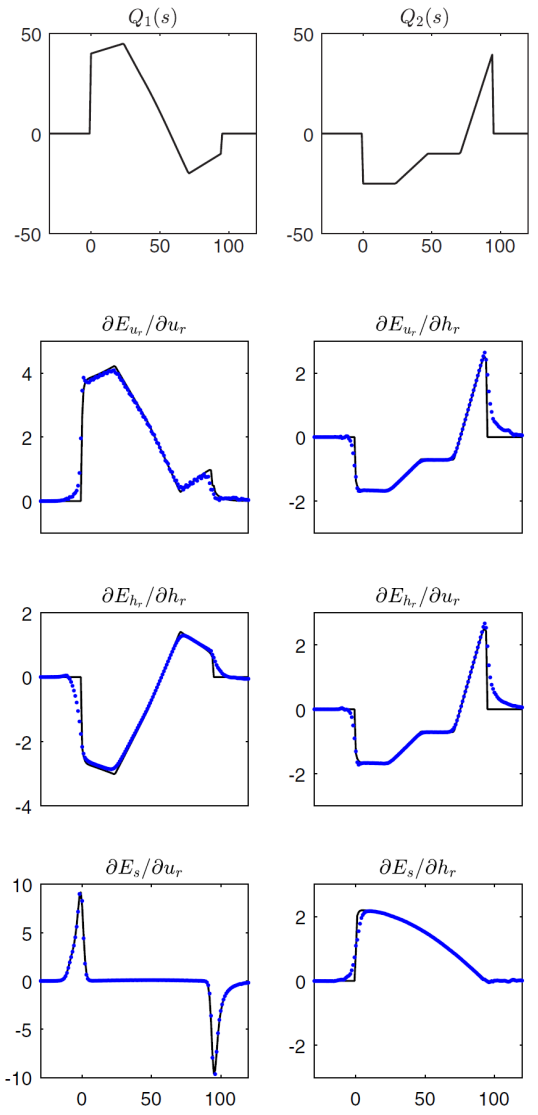
$$E_s = u_r E_0' - h_r \kappa_s E_0$$



Q_2 Q_1

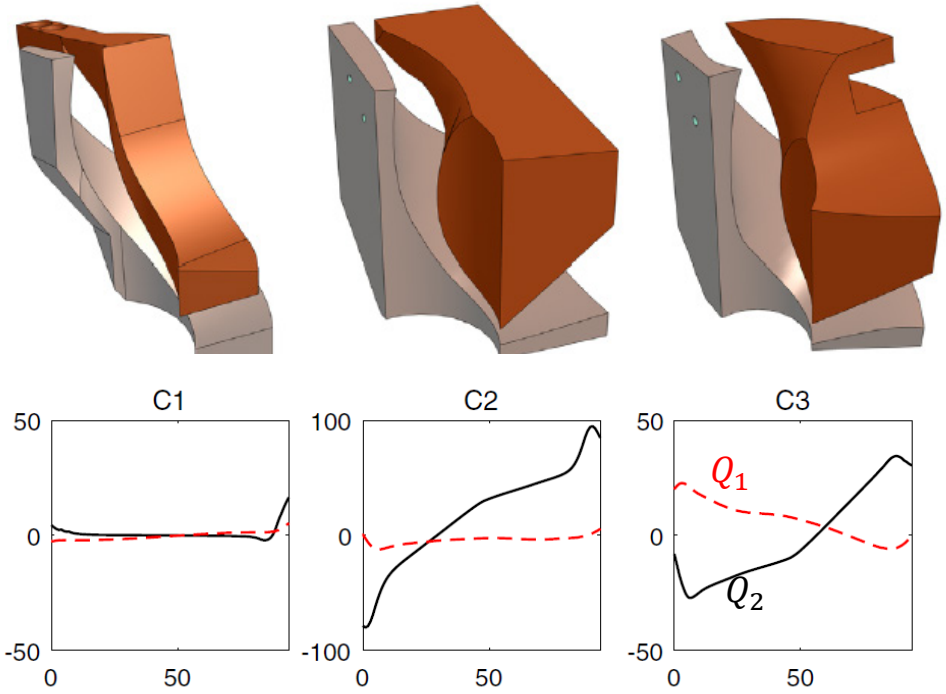
Numerical verification:

- electrodes lie on equipotential surfaces
- TOSCA and theoretical field gradients agree



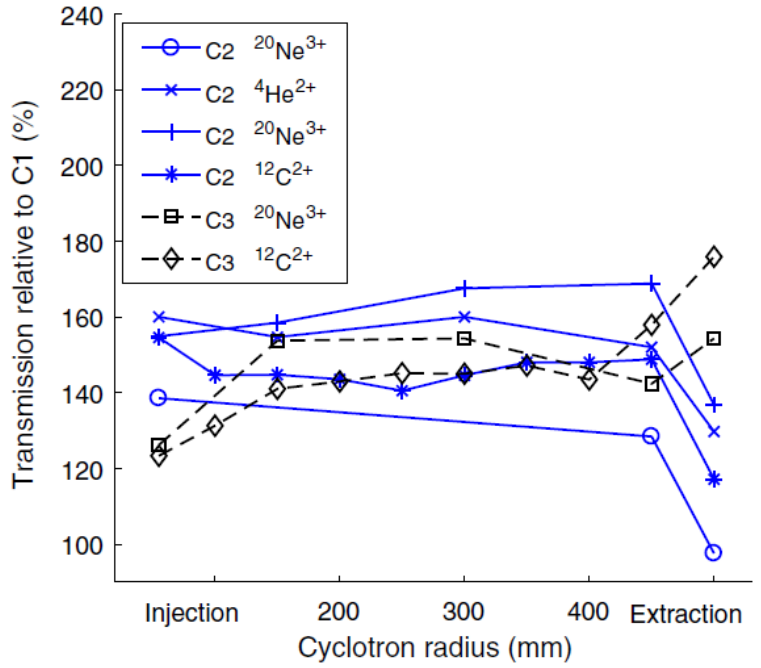
Source [1]

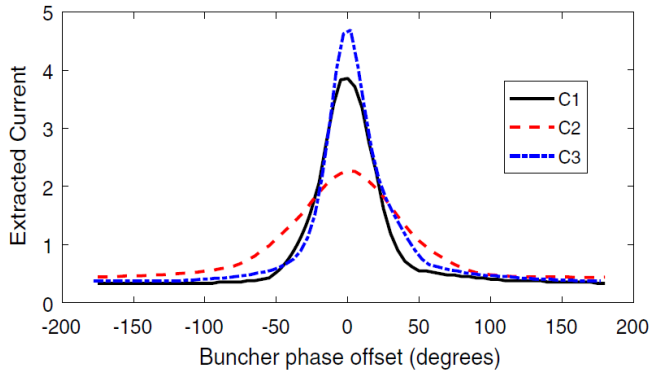
Previous Work – Experimental Results



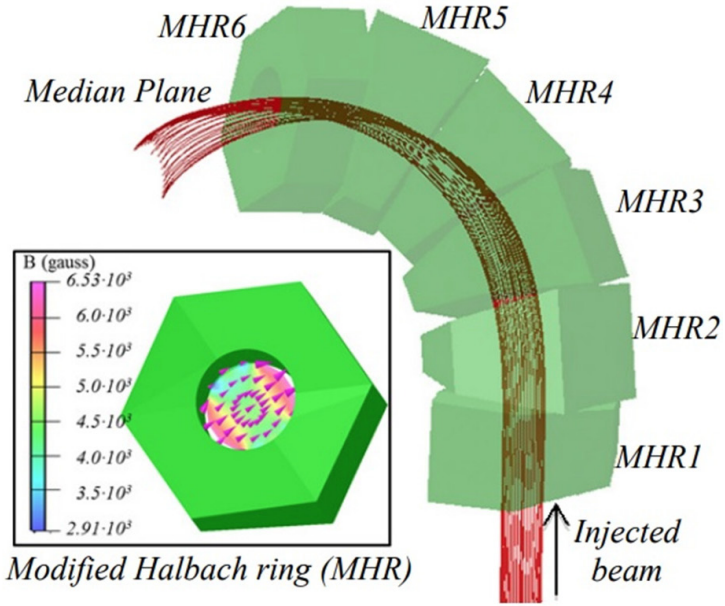
	C1	C2	C3
Vertical emittance	145	105	120
Vertical half width (mm)	8.9	2.9	7.0
Horizontal emittance	145	130	120
Horizontal half width (mm)	3.0	1.5	2.0
Longitudinal half width (mm)	5.5	12.9	3.1
rf phase spread (degrees)	±39	±90	±22

Source [1]





Permanent Magnet Spiral Inflectors



Source [5]

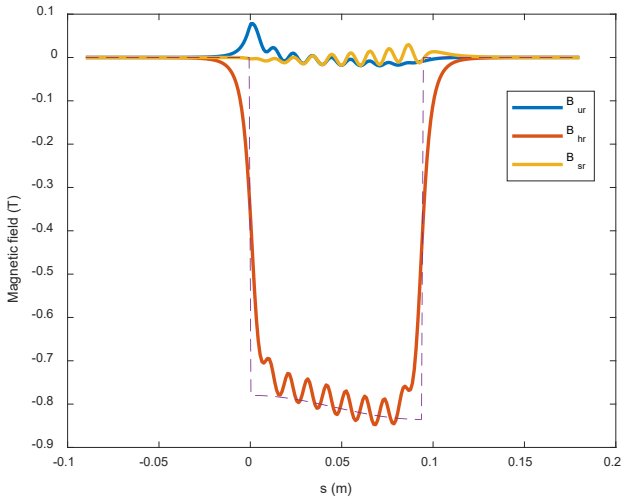
REVIEW OF HIGH POWER CYCLOTRONS AND THEIR APPLICATIONS

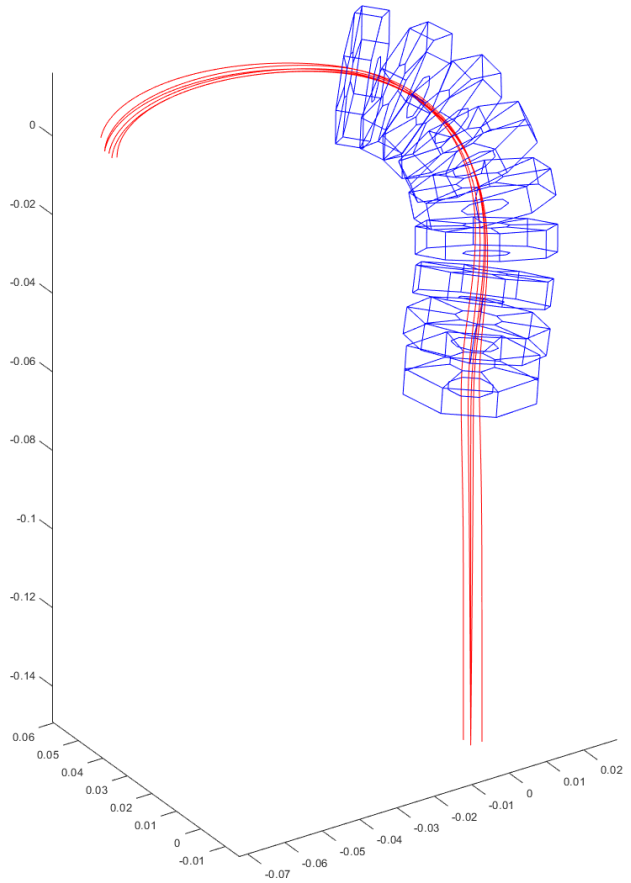
L. Calabretta, D. Rifuggiato, Istituto Nazionale di Fisica Nucleare- LNS, Catania, Italy
M. Maggiore, Istituto Nazionale di Fisica Nucleare- LNL, Legnaro, Italy

$$\boldsymbol{F}_M = \boldsymbol{F}_E$$

$$\boldsymbol{E} = E_0 \hat{u}_r$$

$$\boldsymbol{B} = -\frac{E_0}{v}\hat{h}_r$$





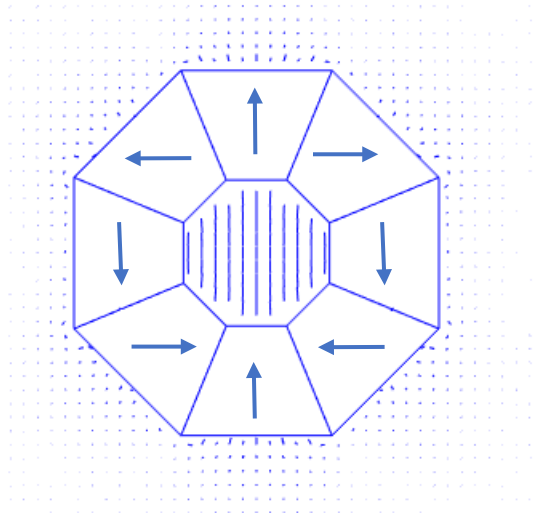
Modified Halbach rings (1)

Inflector field: potential similar to electric case

$$B = -\nabla\psi$$
$$\nabla^2\psi = 0$$

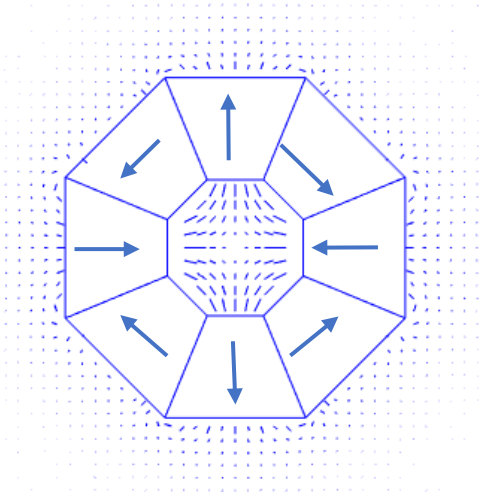
$$\psi = h_r B_0 - Q_1 B_0 \frac{u_r^2 - h_r^2}{2} - Q_2 B_0 u_r h_r - \frac{u_r^2}{2}\,\hat{s}' \cdot \boldsymbol{B}_0$$

B_0



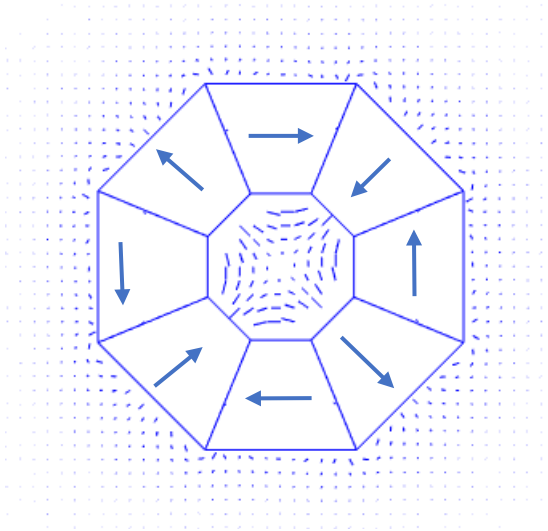
$$M_r = M_0 \sin\theta$$
$$M_\theta = -M_0 \cos\theta$$

Q_1 type



$$M_r = -M_0 \cos 2\theta$$
$$M_\theta = -M_0 \sin 2\theta$$

Q_2 type



$$M_r = -M_0 \sin 2\theta$$
$$M_\theta = M_0 \cos 2\theta$$

Q_1, Q_2 (field gradients) depend on ring size, spacing etc, so rather work with K_1, K_2:

$$M_r = M_0(\sin\theta - K_1 \cos 2\theta - K_2 \sin 2\theta)$$
$$M_\theta = M_0(-\cos\theta - K_1 \sin 2\theta + K_2 \cos 2\theta)$$

Optimisation space: $K_1(n), K_2(n)$ where $n = 1,..,$ number of rings

Modified Halbach rings (2)

Numerical verification of:

$$\psi = h_r B_0 - Q_1 B_0 \frac{u_r^2 - h_r^2}{2} - Q_2 B_0 u_r h_r - \frac{u_r^2}{2}\, \hat{s}' \cdot \boldsymbol{B}_0$$

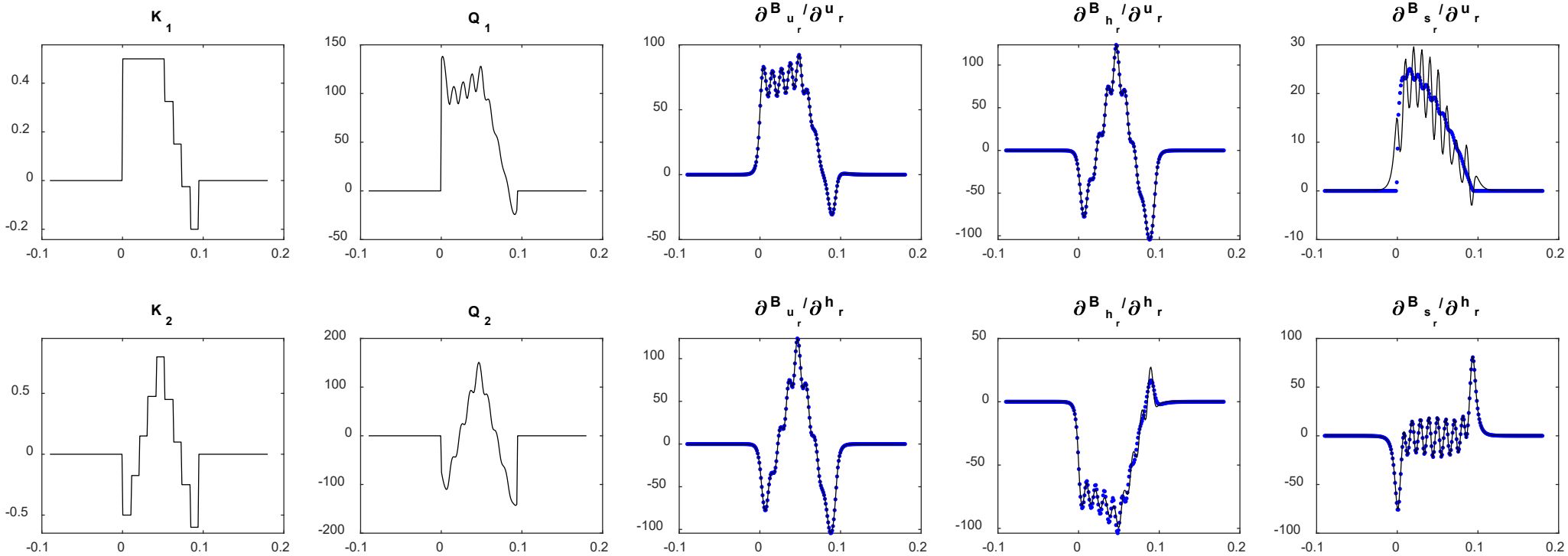
$$B_{u_r} = u_r(Q_1 B_0 + \hat{s}' \cdot \boldsymbol{B}_0) + h_r Q_2 B_0$$

$$B_{h_r} = -B_0 + u_r Q_2 B_0 - h_r Q_1 B_0$$

$$B_s = -h_r B_0' + u_r \kappa_s B_0$$

Actual quadrupoles $Q \approx 215\ K$.

- Q is a "smoothed" version of K
- When $K = 0, Q \neq 0$ since Q definition was made for electrical inflector



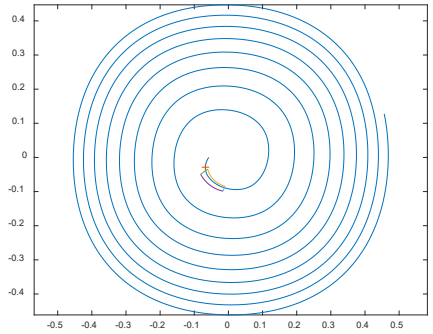
Cyclotron model – SPC2

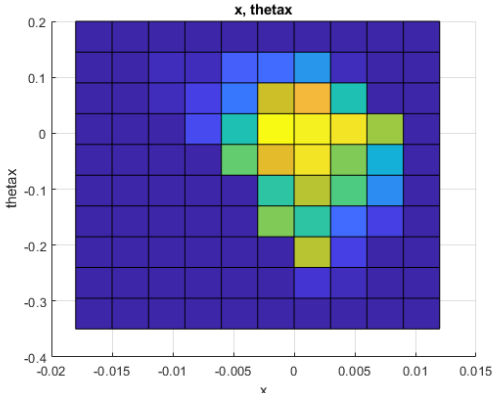
Previous optimization: $\min f$ (emittance and spread)
No real model of the cyclotron acceptance

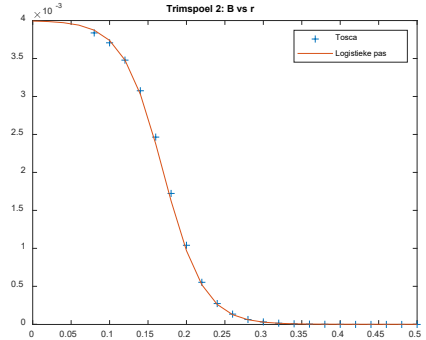
Simple model of solid pole cyclotron

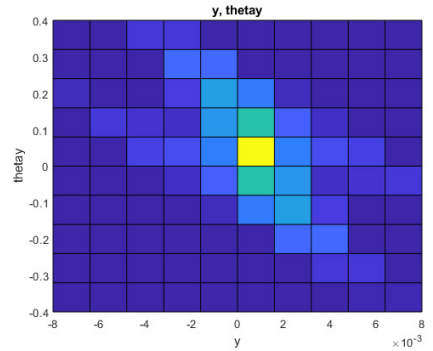
- Main B field is an offset sinusoid
- Flutter disappears in central region
- Trim coils for isochronism (from TOSCA)
- E fields are oscillating DC fields (numerical solutions of Dee-Dummy Dee field)
- Central path obtained by backtracking along the accelerated equilibrium orbit at extraction
- Losses at slits in central region and at large vertical excursions

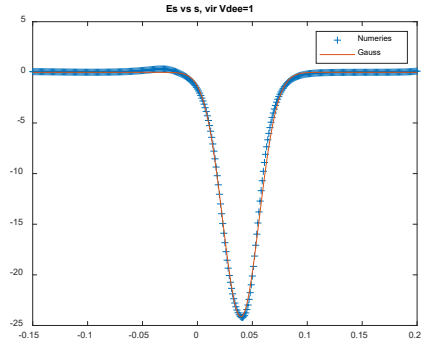
Injection point: 3 cm in front of first acceleration gap
Extraction point: location of electrostatic extraction channel
Acceptance: ellipse
within set distance from sampled points in acceptance

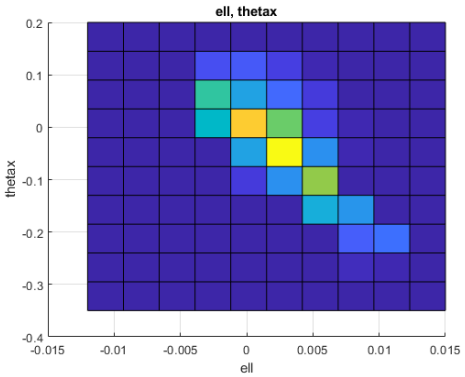




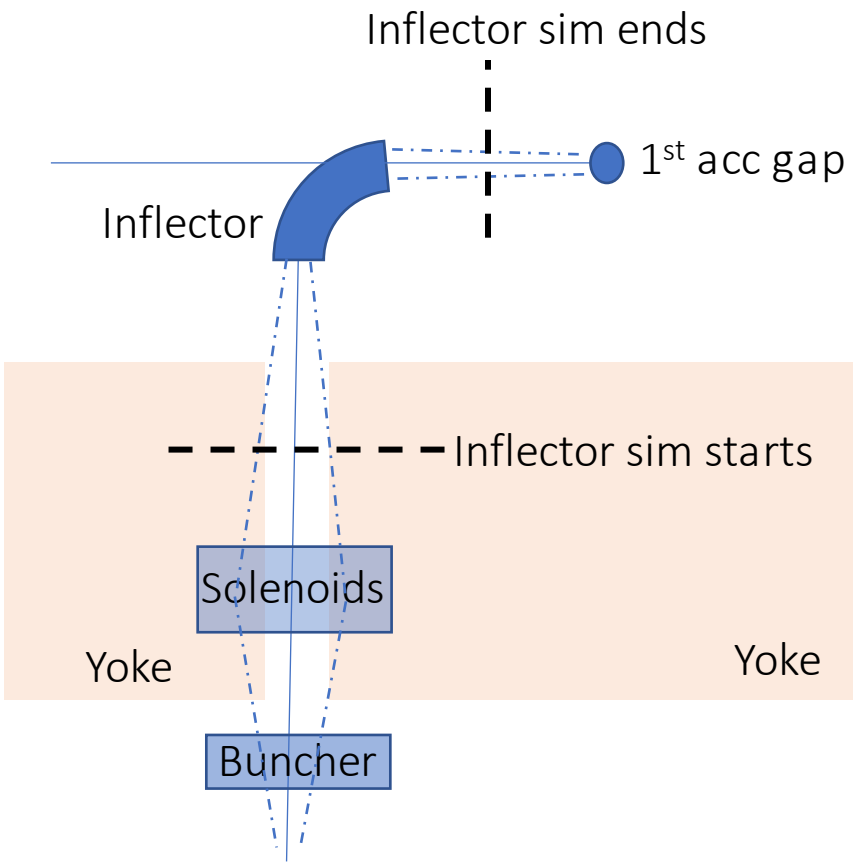








Injection beam line



Inflector simulation:

- starts in region where main magnetic field is zero, based on TOSCA simulation of field in axial hole
- ends in region where inflector field, and acceleration field, are zero

Beam focused at inflector entrance

Buncher

- first harmonic
- focusses at first acceleration gap
- Includes optical length of inflector, R_{56}

Transmission through cyclotron:

1. Pick random particle in DC beam, upstream of buncher
2. Propagate to $\boldsymbol{X}_{in}$ at inflector start (not linear)
3. $\boldsymbol{X}_{out} = M\boldsymbol{X}_{in}$
4. Check if it is in cyclotron acceptance
5. Repeat for ~ 10^5 particles

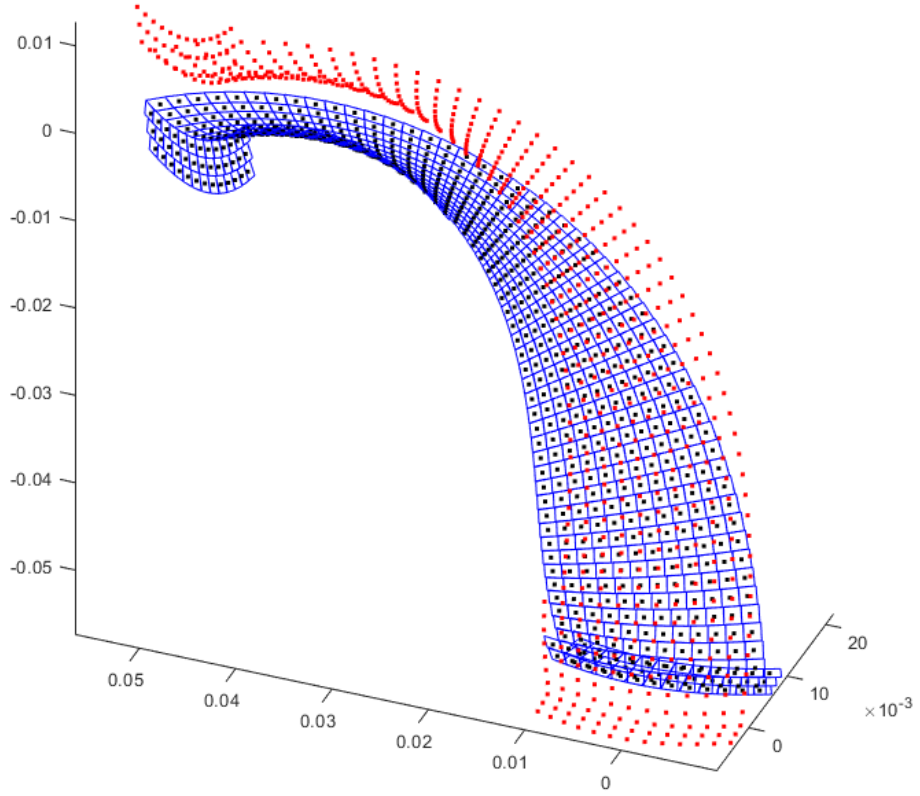
Electrostatic solver (1)

During optimization: need to evaluate lots of inflectors

TOSCA takes too long, needs to change geometry and mesh every time
- ~ 1 hour to compute accurate inflector transfer matrix

Previous solution: pre-calculate 7 TOSCA fields and linearly extrapolate
- ~ 5 seconds to compute approximate inflector transfer matrix

New method: compute the surface charge density on the electrodes by minimizing the potential energy
- Only mesh the surface, not the volume
- Compute electric field at any position during run-time
- The field is “smooth”, so integration runs faster
- ~ 3 seconds to compute accurate inflector transfer matrix



Square meshing of negative electrode

Electrostatic solver (2)

Suppose that the surface is divided into approximate squares of side length L_i and charge q_i spread out uniformly over the square

$$U = \frac{1}{2}\boldsymbol{q}\cdot\boldsymbol{V}_q + \boldsymbol{q}\cdot\boldsymbol{V}_E$$

Write the voltage $\boldsymbol{V}_q = D\boldsymbol{q}$

$D_{ij} = \dfrac{f_{ij}}{4\,\pi\epsilon_0|\boldsymbol{r}_i - \boldsymbol{r}_j|}$ f_{ij} is a geometric factor, set it to 1

Special case $i = j$, self-energy of a square:

$$D_{ii} = \frac{2}{4\,\pi\epsilon_0 q_i^2}\int\int \frac{1}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|}dq_1 dq_2 = 2\frac{1.4865}{4\pi\epsilon_0 L_i}$$

Apply a voltage V^n to electrode $\boldsymbol{n}$ by fixing its total charge:

$$\boldsymbol{q}\cdot\boldsymbol{c}^n = Q^n$$

where

$c_i^n = \begin{cases} 1 & \text{if node } i \text{ is part of electrode } \boldsymbol{n} \\ 0 & \text{otherwise} \end{cases}$

Optimise, using Lagrange multipliers:

$$U = \frac{1}{2}\boldsymbol{q}\cdot D\boldsymbol{q} + \boldsymbol{q}\cdot\boldsymbol{V}_E + \lambda_1\boldsymbol{q}\cdot\boldsymbol{c}^1 + \cdots + \lambda_N\boldsymbol{q}\cdot\boldsymbol{c}^N$$

$$D\boldsymbol{q} + \boldsymbol{V}_E + (\lambda_1\boldsymbol{c}^1 + \cdots + \lambda_N\boldsymbol{c}^N) = 0$$

But $D\boldsymbol{q} + \boldsymbol{V}_E$ is the total voltage, so $\lambda_n = -V^n$ and:

$$\lambda_1\boldsymbol{c}^1 + \cdots + \lambda_N\boldsymbol{c}^N = -\boldsymbol{V}$$

In general, solve:

$$\boldsymbol{q} = \mathrm{D}^{-1}(\mathbf{V} - \mathbf{V}_\mathrm{E})$$

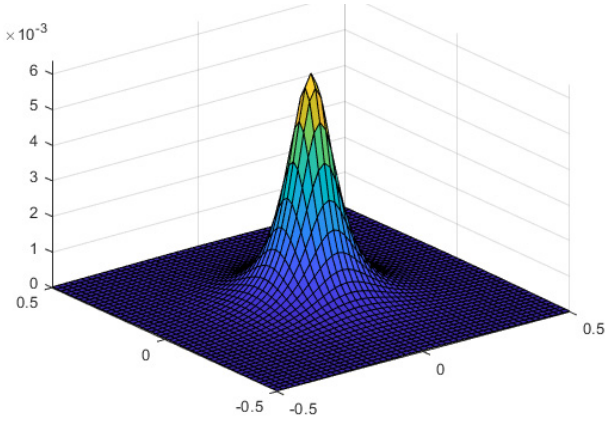
$$\boldsymbol{E}(\boldsymbol{x}) = \frac{1}{4\,\pi\epsilon_0}\sum_i q_i\frac{\boldsymbol{x} - \boldsymbol{r}_i}{|\boldsymbol{x} - \boldsymbol{r}_i|^3}$$ Caveat: $\boldsymbol{x}$ not close to $\boldsymbol{r}_i$

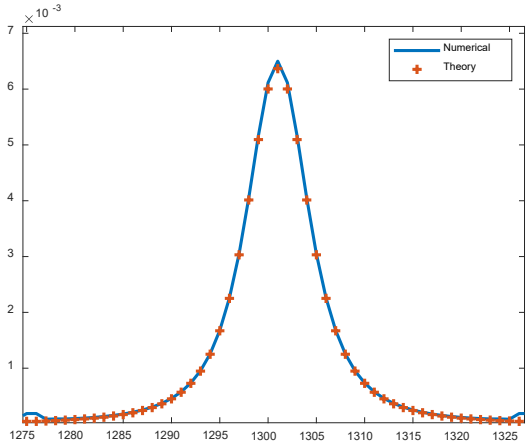
Electrostatic solver (3)

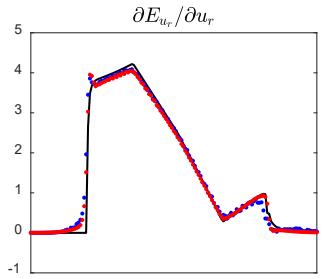
Tests:

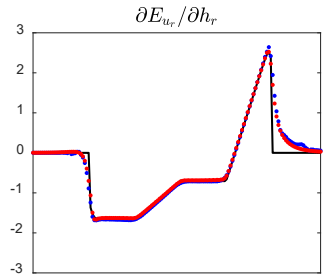
- Parallel plate capacitor, E field in middle, error 0.5%
- Surface charge induced in an earthed plate by a point charge
- Inflector: compare to TOSCA and analytic expression

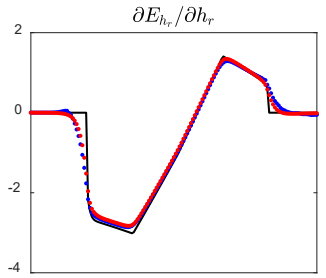
Induced charge

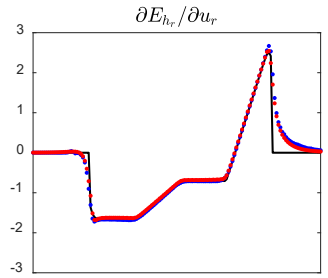


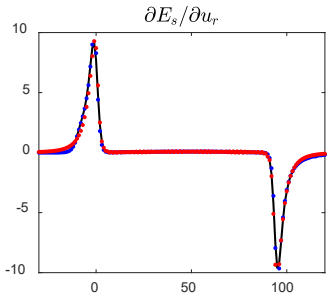


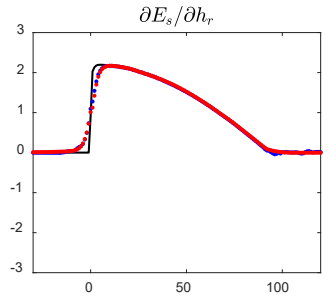












Solid – theory
Blue – TOSCA
Red – this method

Results of optimization

- Optimisation space:
  - Electric: discretize $Q_1(s), Q_2(s)$ into Q_{1i}, Q_{2i} where $i = 1,..,3$
  - Magnetic: K_{1i}, K_{2i} where $i = 1$ to the number of rings
- Cost function: -Transmission through cyclotron
- Method: Random sampling, followed by steepest descent

Transmission Table	Electric Inflector	Magnetic Inflector
Neutral: $Q, K = 0$	26%	33%
Optimised	43%	45%
Relative improvement	65%	36%

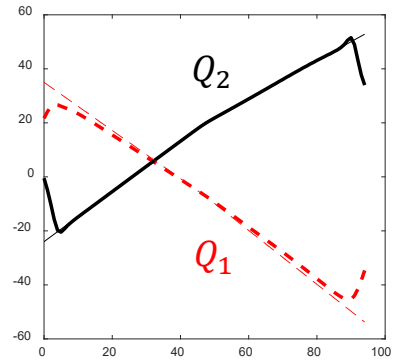
To note:

- Significant improvement
- Similar performance for magnetic and electric
- The electric improvement of 65%, similar to C3 experimental value

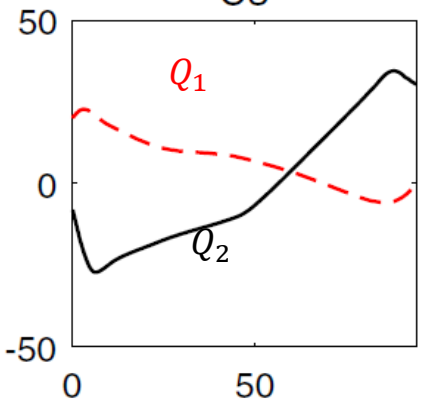
Actual transmission in SPC2 ~5%

- Extraction channel not included in simulation yet, factor ~3 loss
- Emittance of injected beam uncertain

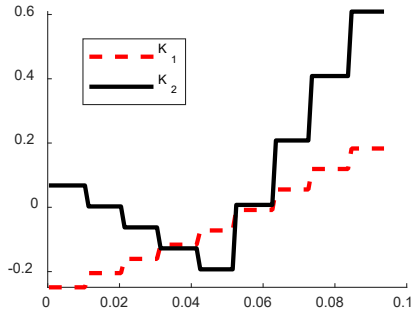
Electric - Similar to C3 design:



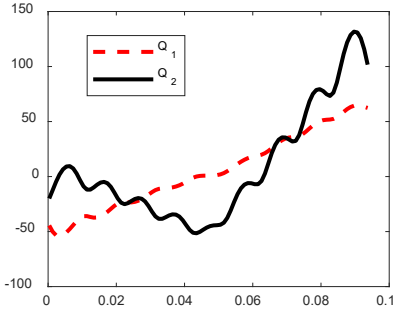
C3



Optimal K



Actual Quadrupoles



Conclusion

- Optimisation of inflectors improves performance substantially
- Similar performance from electric and magnetic spiral inflectors

Important:

- Space charge not accounted for (!) so this is strictly speaking only applicable to low intensity machines. But inflector optimization will most probably also be important for high intensity machines.

Thank you!

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