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THE DESIGN OF A SUPERCONDUCTING DIPOLE MAGNET BASED ON TILTED SOLENOIDS

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Abstract

As a core component of proton therapy equipment, the gantry can project the proton beam onto a tumor from different angles. The weight of the gantry with normal conducting magnets (mainly normal dipole magnets and quadrupole magnets) is usually more than 150 tons, which demands high requirements for the design, processing and fabrication. For the realization of light-weight gantry, this article puts forward a design of Canted-Cosine-Theta (CCT) superconducting magnet (to be used on superconducting gantry). Since the superconducting CCT magnet can produce higher magnetic field compared with normal magnets, for the proton beam with the same magnetic stiffness, the deflection radius of the magnet can be significantly reduced, thus reducing the radius and volume of the gantry.

In this article, the finite element analysis software and Biot-Savart principle were adopted to establish the method of magnetic field calculation for CCT superconducting magnet, and MATLAB was used to simulation and validation of particle path, which finally realize the design of CCT superconducting magnet applied on gantry.

INTRODUCTION

Table 1: Physical Requirements for the Deflecting CCT Magnet

Parameter	Value
Maximum proton energy	280 MeV
Deflecting Angle	58.5°
Deflecting Radius	0.9 m
Magnetic field intensity	2.8 T
Effective length	918.9 mm
Good field radius	30 mm
Magnetic field homogeneity	1×10^{-3}

CCT superconducting magnet has the following characteristics: high magnetic field strength, flexible multi-stage field combination, high uniformity of integral field and transverse field; and when it comes to structure, it has the characteristics of light weight, simple winding process, large range of good field (two-thirds of conductor coil radius), etc [1]. Both LBNL (USA) and CERN (Europe) have developed superconducting CCT dipole prototypes (linear CCT magnets) [2]. In order to optimize the rotating gantry, this paper specially studies the deflection CCT superconducting magnet used in the gantry, and determines its feasibility. According to beam optics requirements of the gantry, the physical requirements for the deflecting CCT superconducting magnet are shown in Table 1.

THEORETICAL MODEL OF DEFLECTING CCT SUPERCONDUCTING MAGNET

Deflecting CCT superconducting magnet trajectory equation can be established in two ways: one is in the cylindrical coordinates and another is in the toroidal coordinates [3-5].

The trajectory equation of the linear CCT superconducting magnet in the cartesian coordinates is as shown in Eq. (1), where the origin of coordinates is at the center of the cylindrical surface:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = \sum\left[\dfrac{r}{n\tan\alpha}\sin(n\theta) + \dfrac{\omega}{2\pi}\theta\right] \end{cases} \tag{1}$$

where: r is the radius of the cylindrical surface where the trajectory of the single-layer coil is located, unit mm; α is the tilt angle between single layered coil and horizontal plane, unit rad; θ is the azimutrical angle, the changing angle of trajectory in the cylindrical coordinate system, unit rad, with changing range of $[-N\pi, N\pi]$, N is the number of coil turns; ω is the distance between turns of single-layer coil, unit mm; n is the magnetic pole series (n = 1 for dipole, n = 2 for quadrupole, and so on).

It can be understood that the deflecting magnet, according to the definition, is like the linear magnet bends along the center trajectory with a radius of R, which is the deflecting radius of the deflecting magnet. Therefore, according to Eq. (1), the trajectory equation of deflecting CCT superconducting magnet can be shown as following in Eq. (2):

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$$\begin{cases} x = (R + r\cos\theta)\cos\varphi \\ y = (R + r\cos\theta)\sin\varphi \\ z = r\sin\theta \\ \varphi = \sum\left[\dfrac{r}{n\tan\alpha}\sin(n\theta) + \dfrac{\omega}{2\pi}\theta\right] \times \dfrac{1}{R} \end{cases} \quad (2)$$

In the toroidal coordinates (η,ζ,ϕ), the parametric equation of deflecting CCT superconducting magnet is as shown in Eq. (3):

$$\begin{cases} \eta = \eta_0 \\ \zeta = \zeta \\ \phi(\zeta) = \sum\left[\dfrac{\cot\alpha}{n\sinh\eta_0}\sin(n\zeta) + \dfrac{\phi_0}{2\pi}\zeta\right] \end{cases} \quad (3)$$

where: α is the tilt angle between single layer coil and horizontal plane, unit rad; ζ is the torus angle, unit rad, with a changing range of $[-N\pi, N\pi]$, N is the number of coil turns; ϕ_0 is the angle between turns of single-layer coil, unit rad; n is the magnetic pole series (n = 1 for dipole, n = 2 for quadrupole, and so on).

Linear CCT is periodic in the axial direction, and the turns spacing ω of any two adjacent points is constant. The deflecting CCT has symmetry in the bending direction, and the angular distance ϕ_0 between any two adjacent points is constant. Considering that the trajectory equation established by Eq. (2) is not conducive to the inter-turn analysis, Eq. (3) is used for the analysis of deflecting CCT magnet.

STRUCTURAL MODEL OF DEFLECTING CCT SUPERCONDUCTING MAGNET

Solution of the Angular Distance ϕ_0

The distance between adjacent wiring turns in the deflecting CCT magnet and the tangent value of the tilt angle of the deflected CCT magnet are values that vary with the torus angle ζ, which can be obtained by establishing a local orthogonal coordinate system on the trajectory path. The solution is as follows:

$$\delta(\zeta) = \frac{a\phi_0}{\cosh\eta_0 - \cos\zeta}(\sinh^{-2}\eta_0 + \phi'(\zeta))^{-1/2} \quad (4)$$

$$\tan(\alpha) = \frac{1}{\sinh\eta_0\phi'(\zeta)} \quad (5)$$

where: a is the pole value of bipolar coordinates, unit mm. $\delta(\zeta)$ is the turn-to-turn distance, unit mm. Using Eq. (4) and Eq. (5), we can get:

$$\delta(\zeta) = a\phi_0\sin\alpha\frac{\sinh\eta_0}{\cosh\eta_0 - \cos\zeta} \quad (6)$$

The analysis shows that the turn-to-turn distance is the smallest at the inner side of the deflect CCT coil, where $\zeta = \pi$. Outside of the coil, where $\zeta = 0$, the turn–to-turn distance is the largest. The extreme value of turn-to-turn distance is shown as follows:

$$\begin{cases} \min(\delta)\big|_{\zeta=\pi} = a\phi_0\sin\alpha\tanh(\eta_0/2) \\ \max(\delta)\big|_{\zeta=0} = a\phi_0\sin\alpha\coth(\eta_0/2) \end{cases} \quad (7)$$

Let δ_0 be the minimum rib thickness of skeleton wire slot, and wd be the width of skeleton superconducting wire slot. According to the minimum turn-to-turn distance in Eq. (7), the angular distance ϕ_0 can be obtained as follows:

$$\phi_0 = \frac{\delta_0 + \mathrm{wd}}{a\sin\alpha\tanh(\eta_0/2)} \quad (8)$$

In the formula, the pole coordinates a and η_0 can be determined by the deflecting radius R and the cylinder radius r:

$$\begin{cases} a = \sqrt{R^2 - r^2} \\ \eta_0 = \dfrac{1}{2}\ln\dfrac{R+a}{R-a} \end{cases} \quad (9)$$

Model Analysis

According to the physical requirements in Table 1 and the above solution process, the structural parameters of the deflecting CCT superconducting magnets are determined as shown in Table 2.

Table 2: Parameters of Deflecting CCT Superconducting Magnets

Parameter	**Value**
Pore size at room temperature	40 mm
Number of coil layers	2 layer
Deflecting Radius	900 mm
Coil tilt Angle	30°
Number of coil turns	120 t
Inner coil radius	56 mm
Outer coil radius	66 mm
Minimum interturn rib thickness	0.4 mm
Maximum interturn rib thickness	3.9/4 mm
Inter-turn angular distance	0.0085 rad
Current of a single superconducting wire	< 1000 A

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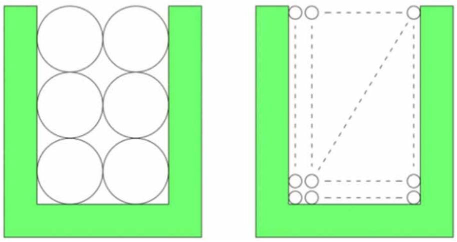
Figure 1: Schematic diagram of multi-step linear approximation.

The Opera-3D finite element analysis software and MATLAB mathematical tool were used to analyze and calculate the magnetic field simulation and optimization analysis of the deflecting CCT magnet, and the results was ensured by comparison and verification. In Opera-3D finite element simulation analysis, 8-node blocks in the software were used for splicing modelling, and the tangent direction of the node blocks was the same as that of the coil track, so as to ensure that the conductor height direction was always perpendicular to the surface of the cylindrical skeleton. When using Biot-Savart magnetic field integral law for analysis in MATLAB, in order to ensure the consistency between the analyzed conductor and the actual conductor and improve the accuracy of calculation, the conductor section is divided by multi-step linear approximation method, as shown in Fig. 1.

According to the parameter equation of Eq. (3) and the magnet parameters of Table 2, when n = 1, the deflecting dipole magnet model was established by node modelling in Opera-3D, as shown in Fig. 2. The center trajectory model is established in MATLAB, as shown in Fig. 3.


Figure 2: Opera-3D CCT model.

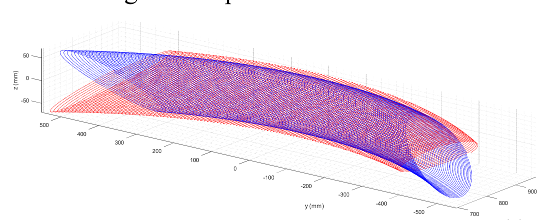
Figure 3: Center trajectory model.

Magnetic Field Optimization

According to the above analysis, when n = 1, the Biot-Savart law is used in MATLAB to calculate the transverse magnetic field and the magnetic field in the direction of the trajectory respectively, and the results are shown in Figs. 4 and 5, in which, By_inner and By_outer represent the contribution of the inner and outer coils to the magnetic field respectively. Through the analysis, it can be clearly seen that the transverse magnetic field uniformity is poor and there is a negative gradient, which is caused by the higher order components, especially the higher order components of the first and second order.

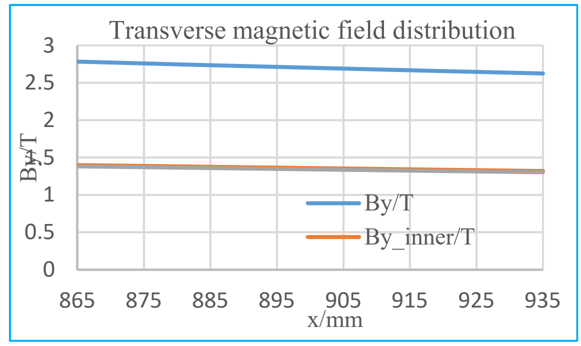

Figure 4: Transverse magnetic field by distribution (x = -35 ~ 35, y = 0, z = 0).

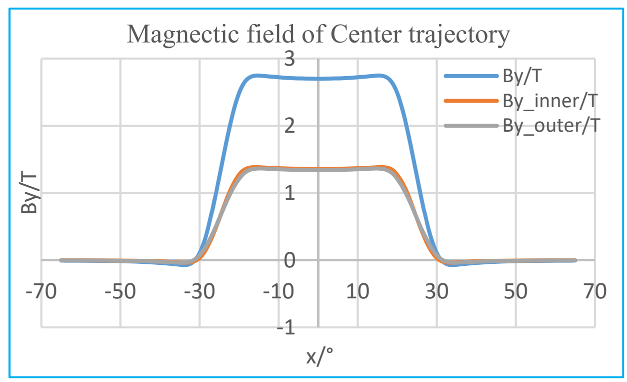

Figure 5: By distribution on the particle central trajectory(x = 0, y = 0, z = -65 ~ 65°).

In order to improve the uniformity of the magnetic field and reduce the harmonic wavelength, paths such as quadrupole (n = 2), six-level field (n = 3) and eight-level field (n = 4) can be added to the pure bipolar CCT path (n = 1), and the corresponding adjustment coefficients (k_1, k_2, k_3, k_4 and etc.) can be introduced for harmonic adjustment. The $\phi(\zeta)$ in Eq. (3) can be changed as shown in Eq. (10). In this way, the periodicity of each order component can be guaranteed, and the turn-to-turn angular distance can be equal.

$$\phi(\zeta)=\sum\left[k_n\frac{\cot\alpha}{n\sinh\eta_0}\sin(n\zeta)\right]+\frac{\phi_0}{2\pi}\zeta \qquad (10)$$

where k_n is the adjustment coefficient (n = 1 is the first-order adjustment coefficient, n = 2 is the second-order adjustment coefficient, n = 3 is the third-order adjustment coefficient, etc.); therefore, when n = 1 and $k_1 = 1$, k_n / n (when $n \geq 2$) represents the proportion of each higher-order magnetic field in the main magnetic field. After several iterations, each higher-order harmonic component is shown in Table 3. The magnetic field distribution on the circumference of the cross section of the magnet and the magnetic field distribution inside the circumference are shown in Figs. 6 and 7 respectively.

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Table 3: Harmonic Components

Item	Parameter	Value
Adjustment coefficient	K1	1
	K2	0.0548
	K3	0.003
Harmonic component	1	1.92E-04
	2	1.74E-04
	3	7.36E-05
	4	4.72E-06
	5	2.06E-07
	6	7.40E-09

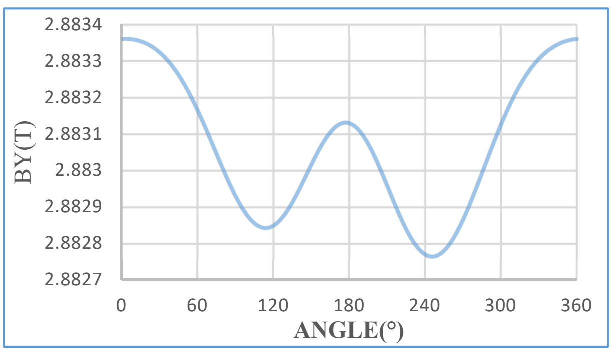

Figure 6: Magnetic field distribution on the cross-section circumference(r = 35mm).

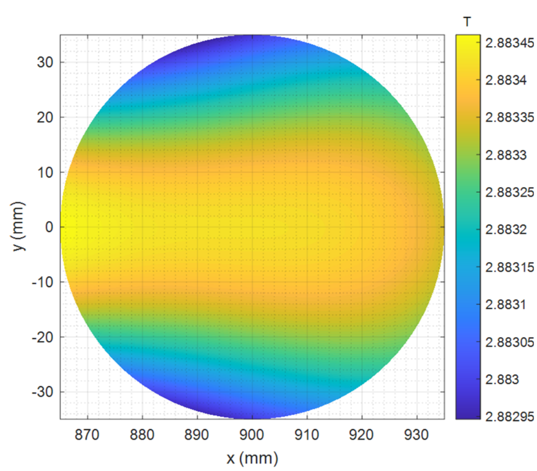

Figure 7: Magnetic field distribution in the circumferential section (r ≤ 35mm).

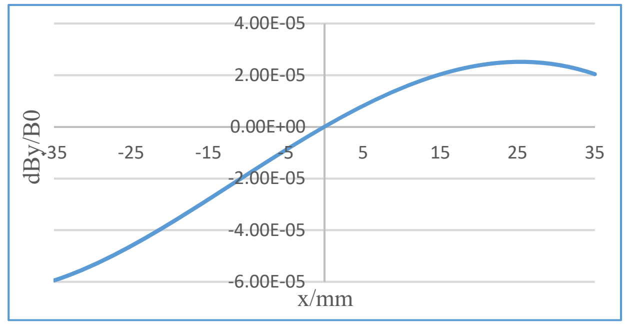

Figure 8: Transverse field uniformity(r = ±35 mm, y = 0, z = 0).

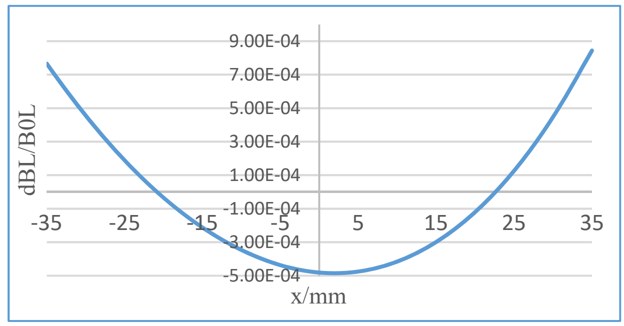

Figure 9: Integral field uniformity (r ≤ ±35 mm, y = 0).

After reducing the harmonics, the transverse field uniformity and the integral field uniformity of the magnet are improved, as shown in Figs. 8 and 9, which is in the range of ±1E-3 and meet the design requirements. The polar plane deflection angle is 4 degrees by calculation.

CONCLUSION

In this paper, the path equations of CCT magnets are established in cylindrical coordinate system and toroidal coordinate system, and the key parameters in the model are analyzed and calculated based on the circular coordinate system.

By setting the adjustment coefficient of high order harmonics, revising the parametric equation and iterative calculating, the high order harmonics component was reduced and the uniformity of magnetic field was improved after several iterations.

The analysis model is established by means of node modelling and coordinate transformation using Opera-3d finite element analysis software, and the magnetic field analysis is carried out. At the same time, the center trajectory path is established using MATLAB, and the multi-step linear approximation method is adopted to analyze the magnetic field by using Biot-Savart integral law. Comparing the two results and selecting the optimized design of CCT magnet that meets the physical requirements.

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REFERENCES

[1] Y. Liang, "R&D of the Canted-Cosine-2eta Superconducting Magnets", Beijing, China: University of Chinese Academy of Sciences, 2018.

[2] A. Hafalia *et al.*, "The structural design for a 'Canted Cosine–Theta' superconducting dipole coil and magnet structure-CCT1", *IEEE Trans. Appl. Supercond.*, vol. 24, no. 3, p. 4001904, Jun. 2014. doi:10.1109/TASC.2013.2284429

[3] L. Brouwer, S. Caspi, and S. Prestemon, "Structural analysis of an 18 T hybrid Canted–Cosine–Theta superconducting dipole", *IEEE Trans. Appl. Supercond.*, vol. 25, no. 3, p. 4000404, Jun. 2015. doi:10.1109/TASC.2014.2361027

[4] G. Montenero *et al.*, "Mechanical structure for the PSI canted-cosine-theta (CCT) magnet program", *IEEE Trans. Appl. Supercond.*, vol. 28, no. 3, p. 4002805, Apr. 2018. doi:10.1109/TASC.2017.2787596

[5] L. N. Brouwer, "Canted-cosine-theta superconducting accelerator magnets for high energy physics and ion beam cancer therapy", Ph.D. thesis, University of California, Berkeley, CA, USA, 2015.

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