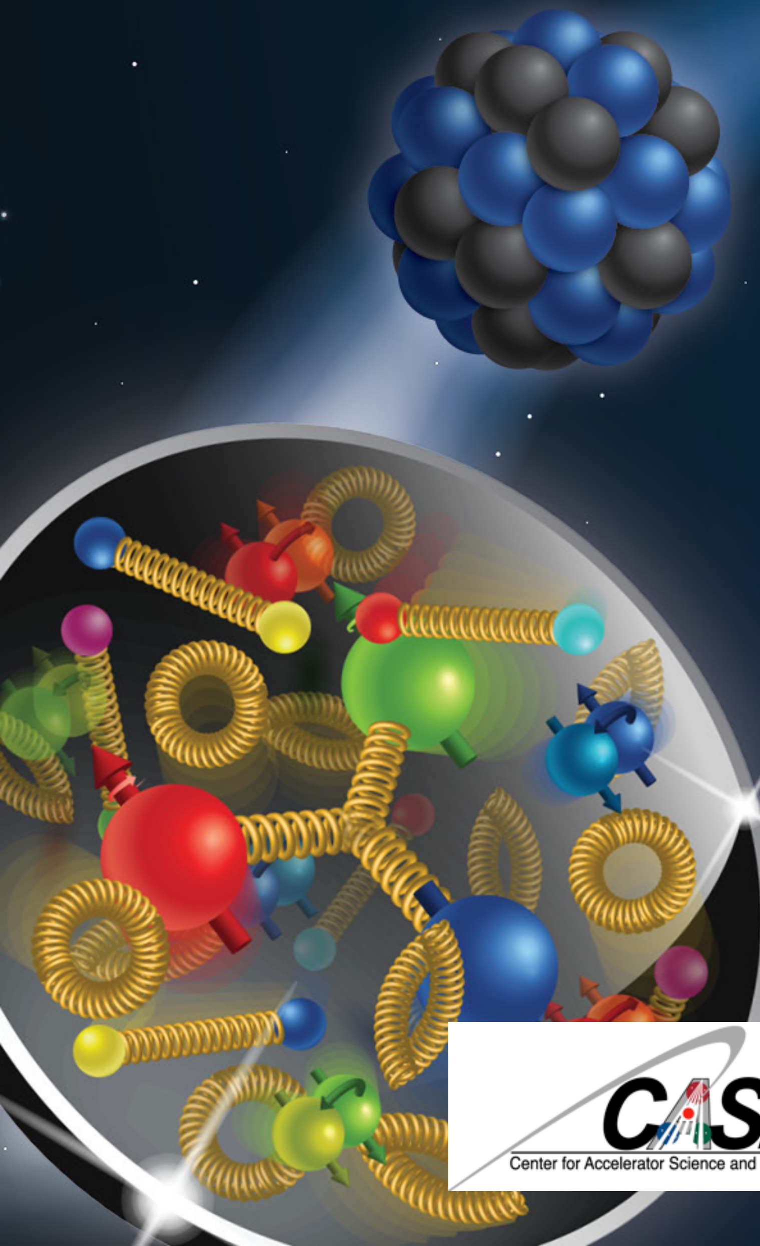

Developing Analytical and Simulation Tools for Coherent Electron Cooling

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Collider-Accelerator Department, BNL

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Tech-X Inc., Boulder, Colorado

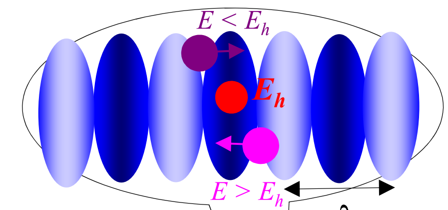
Electron Ion Collider – eRHIC

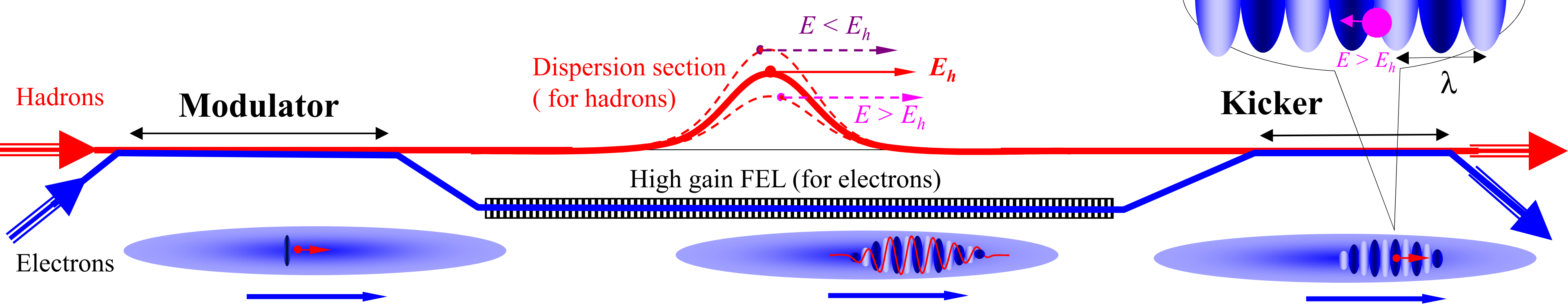
Outline

- Introduction
- Tools predicting electron-ion interactions in a single pass of a CeC system
  - Modulator
  - FEL amplifier
  - Kicker
- Tools to predicting evolution of hadron beam under coherent electron cooling
- Status of the CeC simulations

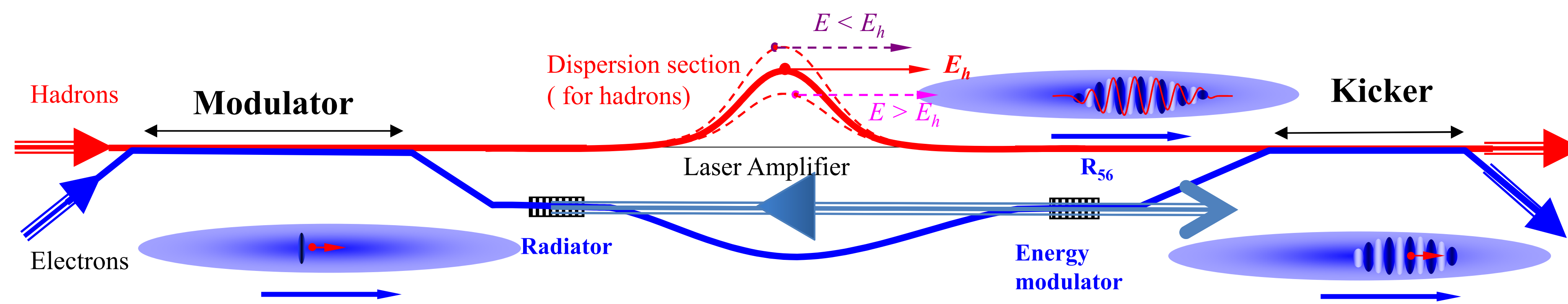
Coherent Electron Cooling Schemes

Classic – FEL amplifier (2006, PRL VL & Y. Derbenev)

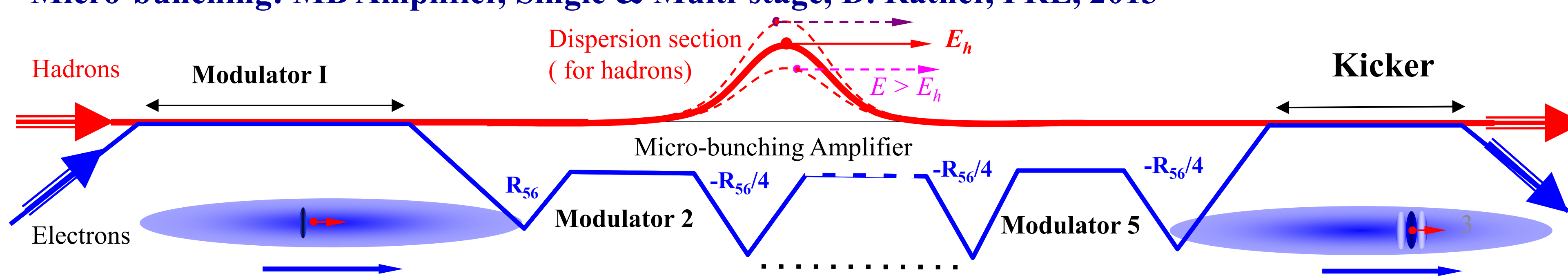




Blended – laser amplifier (2007, VL)



Enhanced bunching: single stage - VL, FEL 2007
Micro-bunching: MB Amplifier, Single & Multi-stage, D. Ratner, PRL, 2013



Classical Coherent electron Cooling scheme

At a half of plasma oscillation

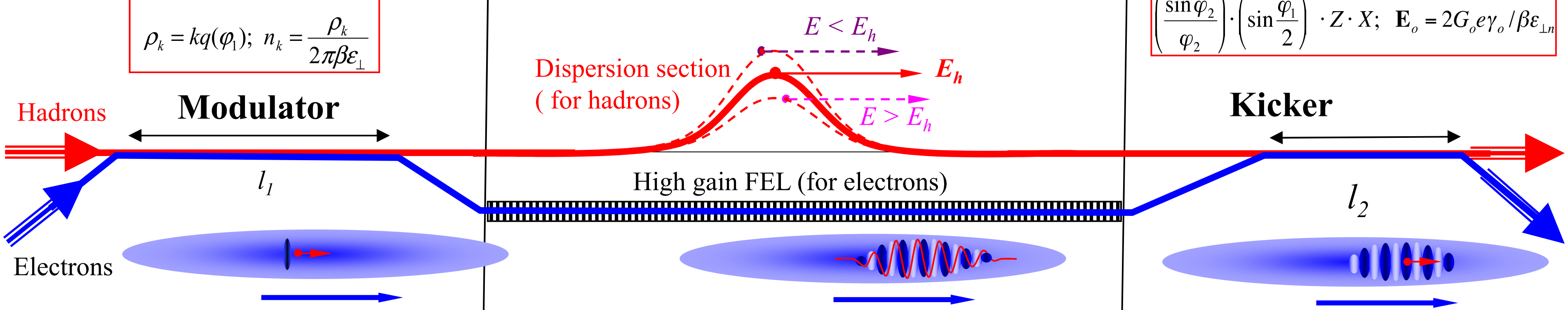
$$q_{\lambda_{FEL}} \approx \int_0^{\lambda_{FEL}} \rho(z)\cos(k_{FEL} z)\,dz$$

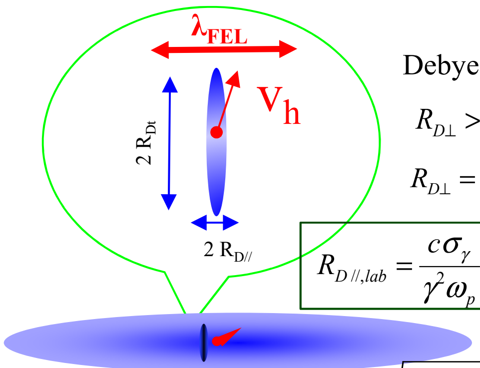
$$\rho_k = kq(\varphi_1);\; n_k = \frac{\rho_k}{2\pi\beta\varepsilon_\perp}$$

Dispersion

$$c\Delta t = -D_{zh}\cdot\frac{\gamma-\gamma_o}{\gamma_o};\; D_{free} = \frac{L}{\gamma^2};\; D_{chicane} = l_{chicane}\cdot\theta^2 \ldots\ldots$$

$$\Delta E_h = -e\cdot \mathbf{E}_o\cdot l_2\cdot \sin\left(k_{FEL} D\frac{E-E_o}{E_o}\right)\cdot \left(\frac{\sin\varphi_2}{\varphi_2}\right)\cdot\left(\sin\frac{\varphi_1}{2}\right)^2\cdot Z\cdot X;\;\; \mathbf{E}_o = 2G_o e\gamma_o/\beta\varepsilon_{\perp n}$$





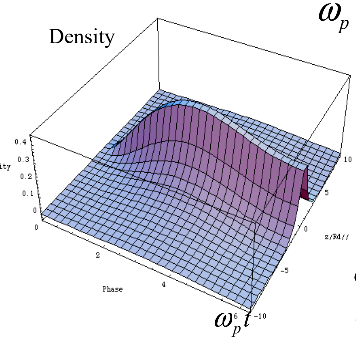
Debye radii

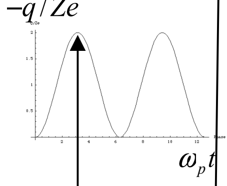
$$R_{D\perp} >> R_{D//}$$

$$R_{D\perp} = \frac{c\gamma\sigma_{\theta e}}{\omega_p}$$

$$R_{D//,lab} = \frac{c\sigma_\gamma}{\gamma^2\omega_p} << \lambda_{FEL}$$

$$\omega_p = \sqrt{4\pi n_e e^2/\gamma_o m_e}$$



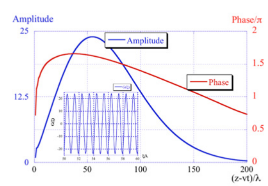


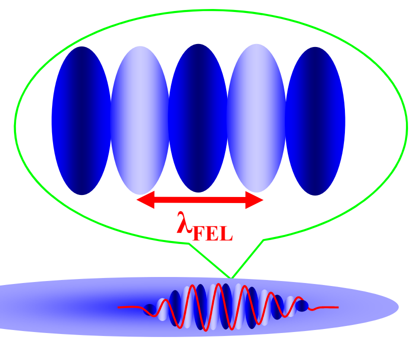
$$q = -Ze\cdot(1-\cos\varphi_1)$$

$$\varphi_1 = \omega_p l_1/c\gamma$$

$$q_{peak} = -2Ze$$

Amplifier of the e-beam modulation in an FEL with gain G_{FEL}~10^2





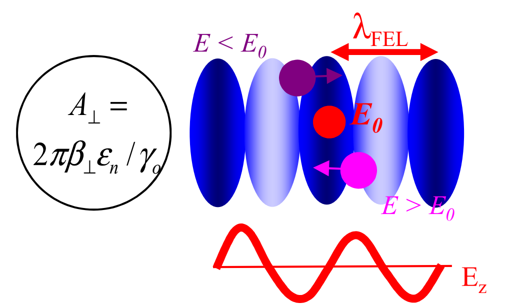
$$\lambda_{fel} = \lambda_w\left(1+\langle\vec{a}_w^2\rangle\right)/2\gamma_o^2$$

$$\vec{a}_w = e\vec{A}_w/mc^2$$

$$L_G = L_{Go}(1+\Lambda)$$

$$G_{FEL} = e^{L_{FEL}/L_G}$$

$$A_\perp = 2\pi\beta_\perp\varepsilon_n/\gamma_o$$



$$k_{FEL} = 2\pi/\lambda_{FEL};\; k_{cm} = k_{FEL}/2\gamma_o$$

$$n_{amp} = G_o\cdot n_k\cos(k_{cm} z)$$

$$\Delta\varphi = 4\pi en \Rightarrow \varphi = -\varphi_0\cdot\cos(k_{cm} z)$$

$$\vec{\mathbf{E}} = -\vec{\nabla}\varphi = -\hat{z}\mathbf{E}_o\cdot X\sin(k_{cm} z)$$

$$\mathbf{E}_o = 2G_o\gamma_o\frac{e}{\beta\varepsilon_{\perp n}}$$

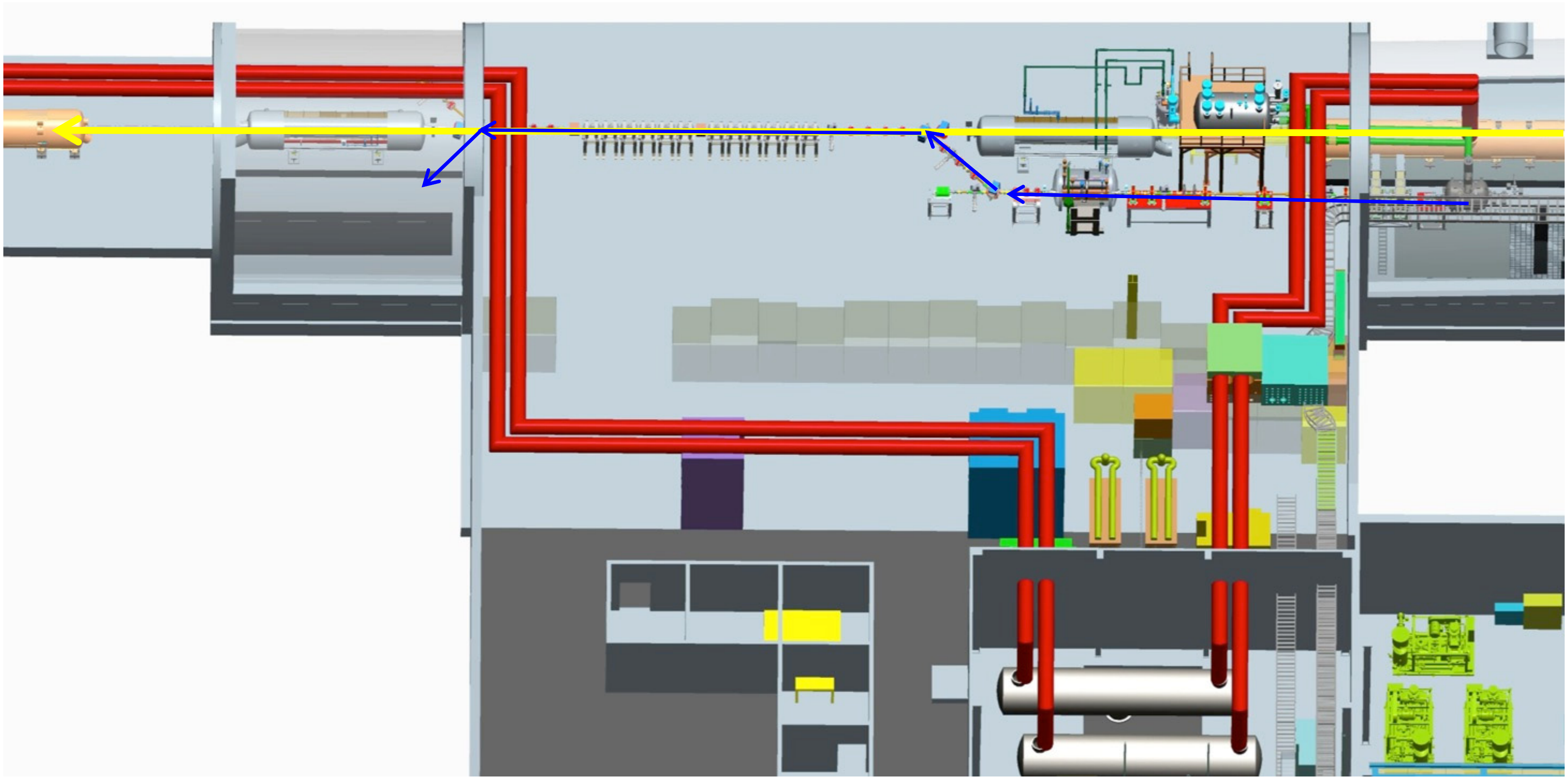
$$X = q/e \cong Z(1-\cos\varphi_1) \sim Z$$

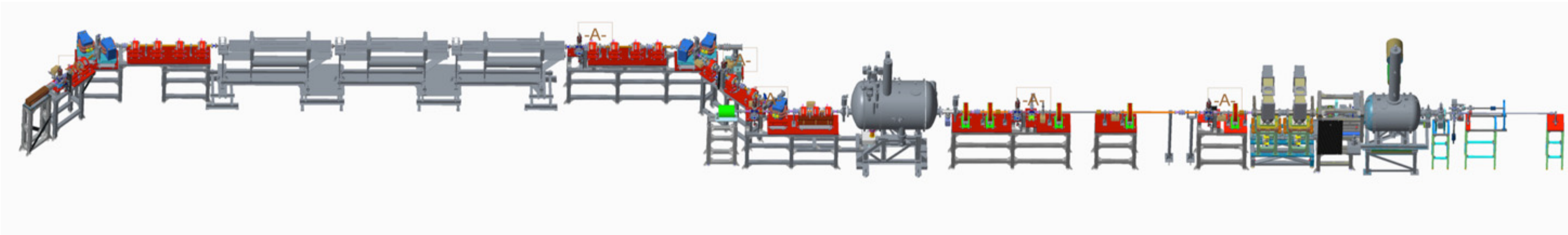
PRL 102, 114801 (2009) PHYSICAL REVIEW LETTERS week ending 20 MARCH 2009

Coherent Electron Cooling

Vladimir N. Litvinenko[1,*] and Yaroslav S. Derbenev[2]

Coherent electron Cooling (CeC) Demonstration Experiment

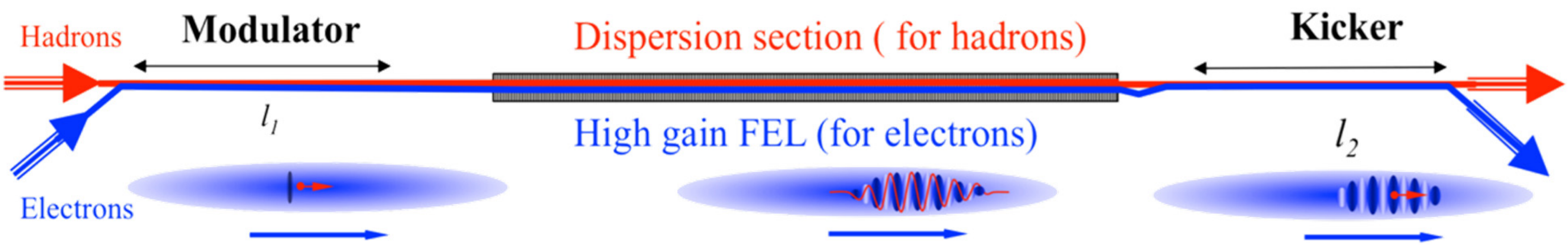




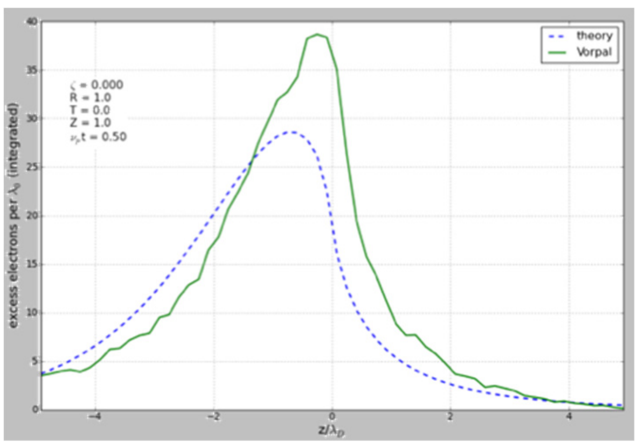
BROOKHAVEN NATIONAL LABORATORY

Stony Brook University

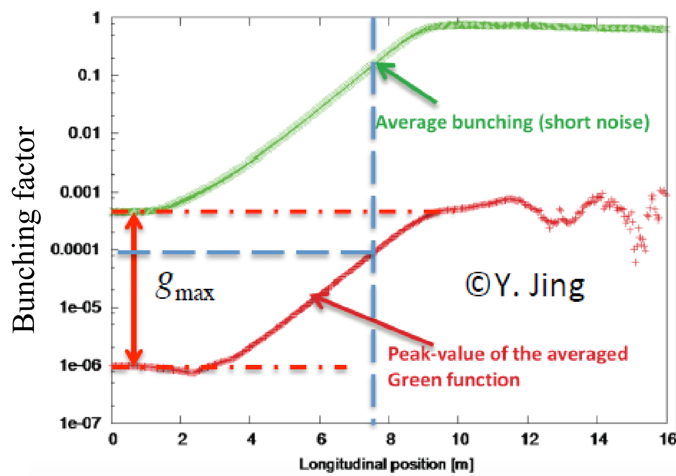
Our Proof-of-Principle is an economic version of CeC, where electrons and hadrons are co-propagating along the entire CeC system



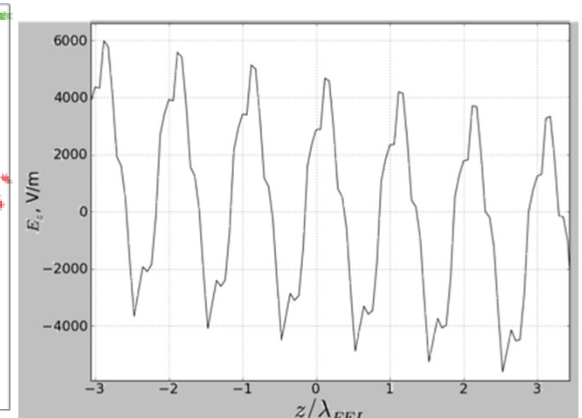
Param.' s from 40 GeV proof-of-principle exp. at BNL



VORPAL 3D δf PIC computation of e-density perturbation near Au^{+79} ion (green) vs. idealized theory (blue). On Cray XE6 cluster at NERSC.



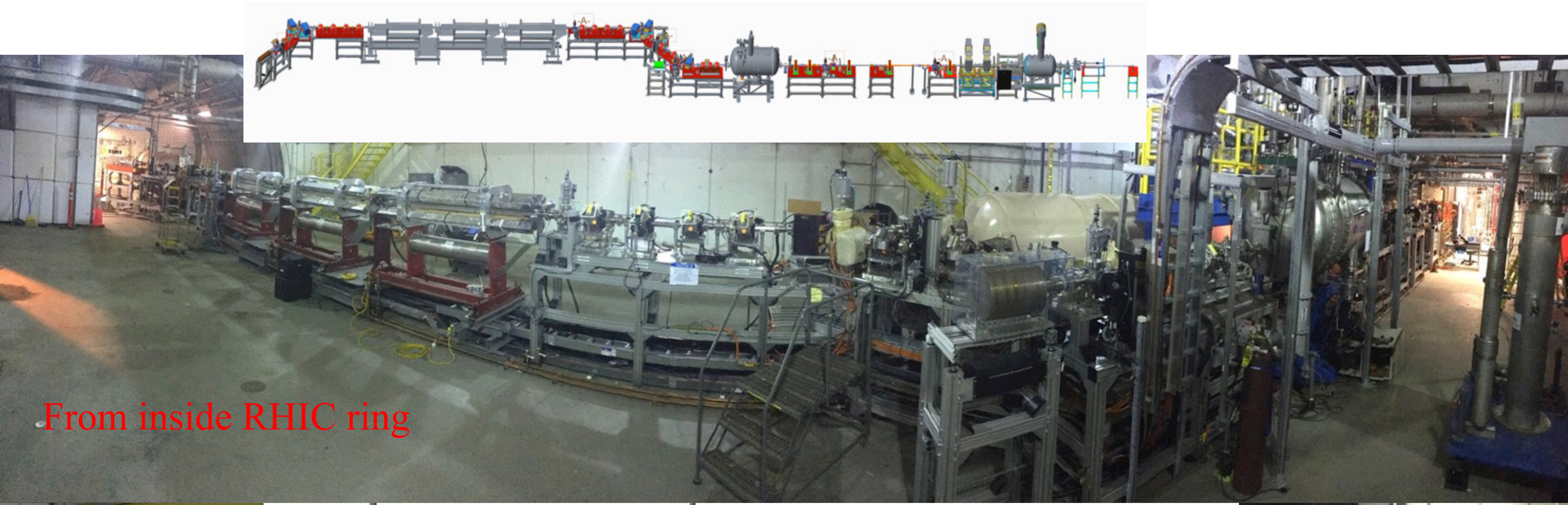
GENESIS parallel computation of electron beam bunching in free electron laser (FEL) shows amplification of modulator signal.

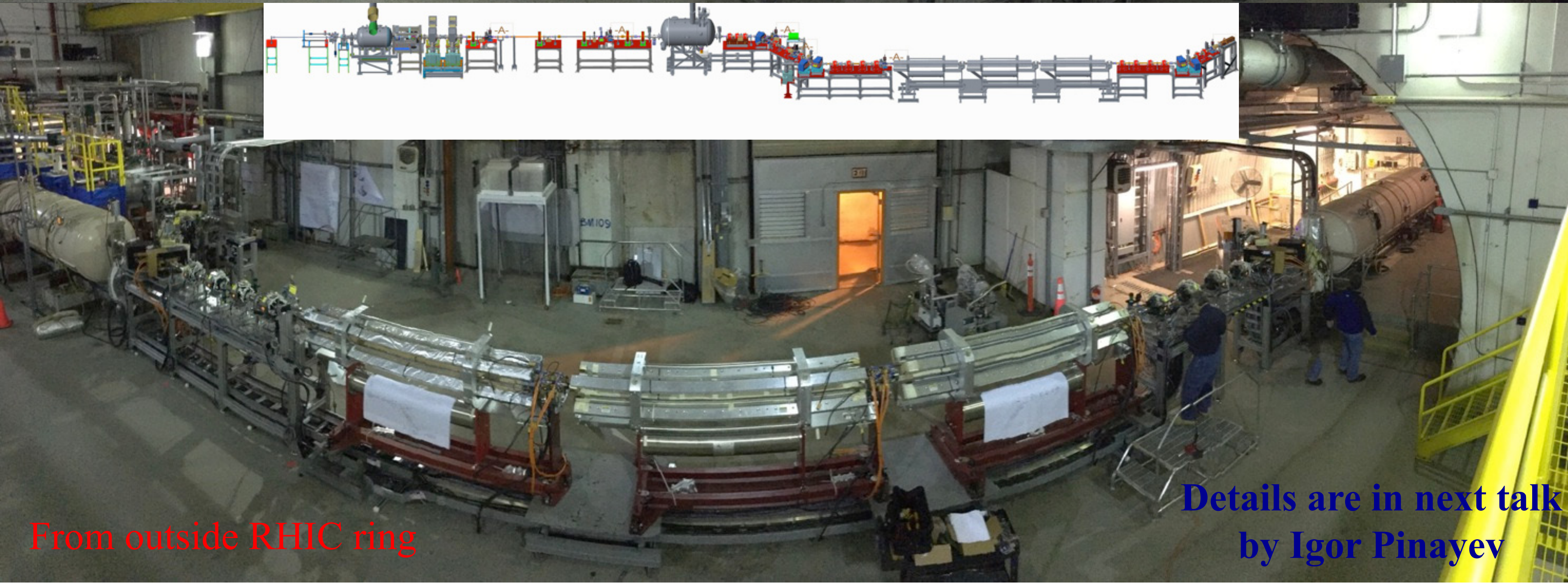


VORPAL prediction of the coherent kicker electric field E_k due to e-density perturbation from modulator, amplified in the FEL.

Simulations by Tech-X and Y. Jing

Panoramic views





Sub-set of analytical tools

Debye shielding in an uniform electron plasma with anisotropic velocity distribution:

World's first analytical *t*-dependent solution

G. Wang, 2007

The system can be described by linearized Vlasov-Maxwell equations

$$\frac{\partial}{\partial t} f_1(\vec{x},\vec{v},t) + \vec{v}\cdot\frac{\partial}{\partial \vec{x}} f_1(\vec{x},\vec{v},t) - \frac{e\vec{E}}{m_e}\frac{\partial}{\partial \vec{v}} f_0(\vec{v}) = 0$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_o}\left\{Ze\delta(\vec{x}) - e\tilde{n}_1(\vec{x},t)\right\}$$

$$\tilde{n}_1(\vec{x},t) = \int f_1(\vec{x},\vec{v},t)\, d^3v.$$

$$f_0(\vec{v}) = \frac{1}{\pi^2\sigma_x\sigma_y\sigma_z}\left(1 + \frac{v_x^2}{\sigma_x^2} + \frac{v_y^2}{\sigma_y^2} + \frac{v_z^2}{\sigma_z^2}\right)^{-2}$$

In 3-D Fourier domain, the equations reduces to a non-homogeneous 2nd ODE

$$\frac{d^2}{dt^2}\tilde{H}_1(\vec{k},t) + \omega_p^2\tilde{H}_1(\vec{k},t) = \omega_p^2 Z_i e^{-\lambda(\vec{k})t}$$

$$\tilde{H}_1(\vec{k},t) = \tilde{n}_1(\vec{k},t)e^{-\lambda(\vec{k})t}$$

The solution for zero initial density and velocity modulation in Fourier domain can be found

$$\dot{\tilde{n}}_1(\vec{k},t) = Z_i\omega_p \sin(\omega_p t)e^{\lambda(\vec{k})t}$$

$$\lambda(\vec{k}) = i\vec{k}\vec{v}_o - \sqrt{k_x^2\sigma_x^2 + k_y^2\sigma_y^2 + k_z^2\sigma_z^2}$$

By inverse Fourier transformation, we obtain the density modulation in space domain

$$\tilde{n}_1(\vec{x},t) = \frac{Z_i}{\pi^2 a_x a_y a_z}\int_0^{\omega_p t}\frac{\tau\sin\tau\cdot d\tau}{\left[\tau^2 + \left(\frac{x}{a_x} + \frac{v_{0,x}}{\sigma_x}\tau\right)^2 + \left(\frac{y}{a_y} + \frac{v_{0,y}}{\sigma_y}\tau\right)^2 + \left(\frac{z}{a_z} + \frac{v_{0,z}}{\sigma_z}\tau\right)^2\right]^2}$$

$$a_{x,y,z} = \sigma_{x,y,z}/\omega_p$$

Analytical Tools for the Modulation Process I

- Cold uniform electron beam (VL)
  - Density modulation: $q = -Ze\cdot(1-\cos\varphi_1)$ $\quad \varphi_1 = \omega_p l_1 / c\gamma_0$
  - Energy modulation ($\varphi_1 << 1$): $\left\langle \frac{\delta E}{E} \right\rangle \cong -2Z\frac{r_e}{a^2}\cdot\frac{L_{pol}}{\gamma}\cdot\left(\frac{z}{|z|}-\frac{z}{\sqrt{a^2/\gamma^2+z^2}}\right)$

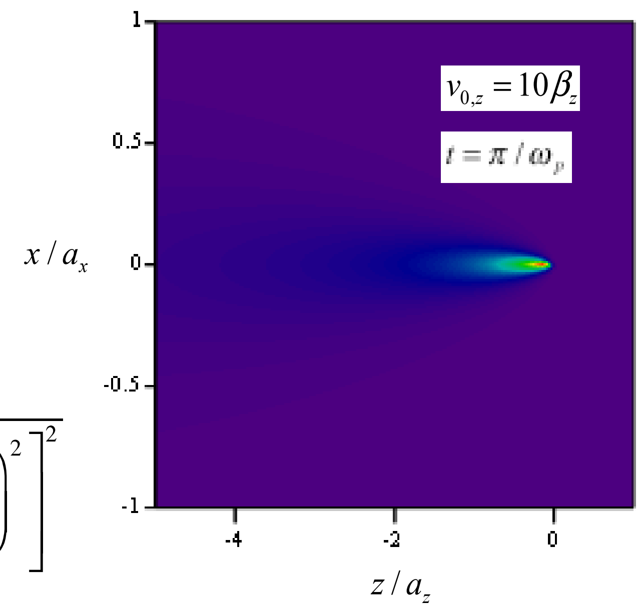
- Warm uniform electron beam with k-2 anisotropic velocity distribution

$$f_0(\vec{v}) = \frac{1}{\pi^2\sigma_x\sigma_y\sigma_z}\left(1+\frac{v_x^2}{\sigma_x^2}+\frac{v_y^2}{\sigma_y^2}+\frac{v_z^2}{\sigma_z^2}\right)^{-2}$$

G. Wang and M. Blaskiewicz, Phys Rev E 78, 026413 (2008)

  - Density modulation:

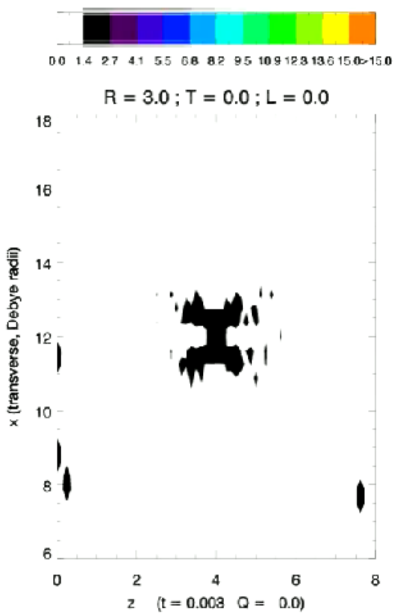
$$\tilde{n}_1(\vec{x},t) = \frac{Z_i}{\pi^2 a_x a_y a_z}\int_0^{\omega_p t}\frac{\tau\sin\tau\cdot d\tau}{\left[\tau^2+\left(\frac{x}{a_x}+\frac{v_{0,x}}{\sigma_x}\tau\right)^2+\left(\frac{y}{a_y}+\frac{v_{0,y}}{\sigma_y}\tau\right)^2+\left(\frac{z}{a_z}+\frac{v_{0,z}}{\sigma_z}\tau\right)^2\right]^2}$$

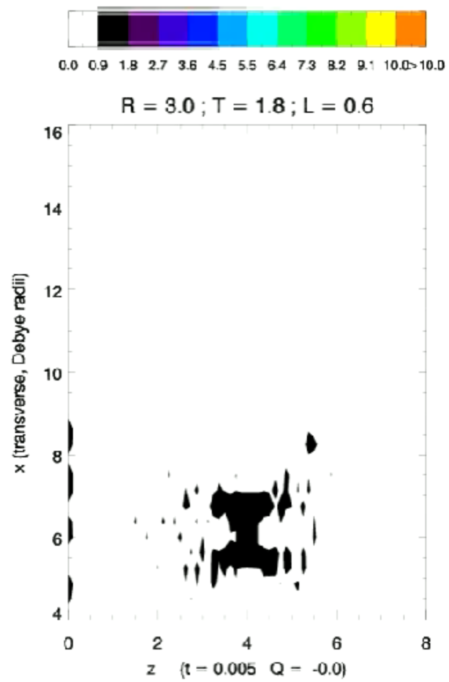


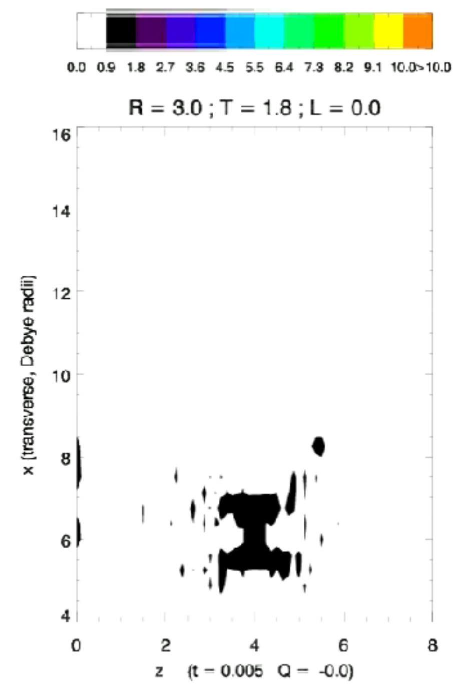
Simulation Tools for the Modulation Process I

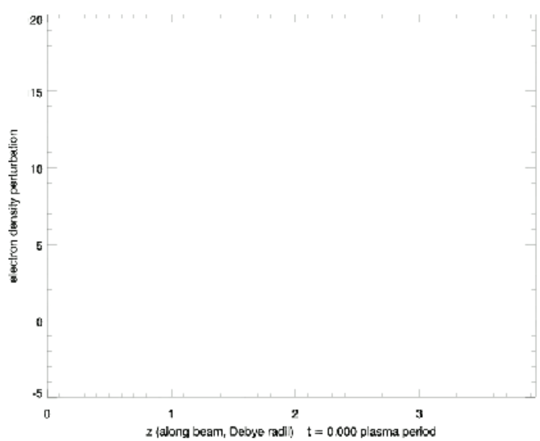
- Simulations based on δf approach (©Tech-X)

$$R = \frac{\sigma_{v_\perp}}{\sigma_{v_z}};\ T = \frac{V_{hx}}{\sigma_{v_z}};\ L = \frac{V_{hz}}{\sigma_{v_z}};$$







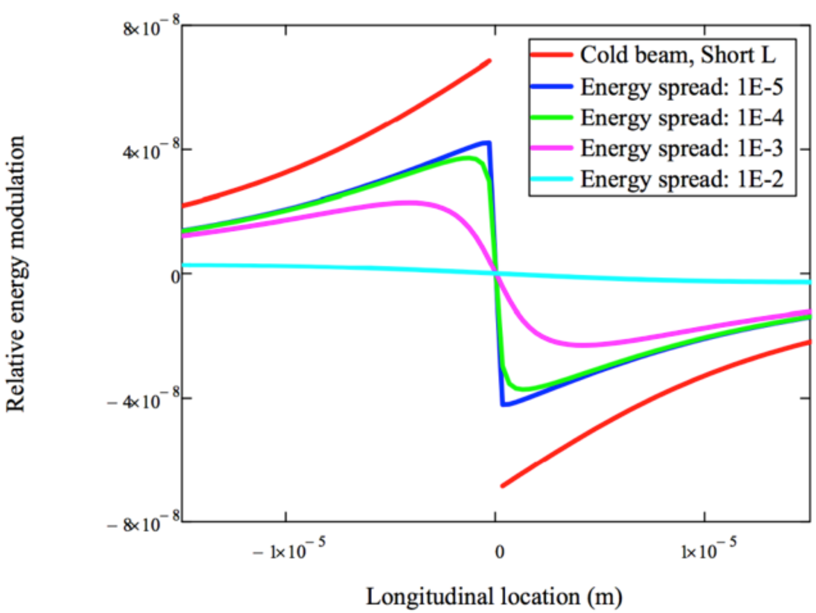


Analytical Tools for the Modulation Process II

– Energy modulation: $$\boxed{\left\langle \frac{\delta E}{E_0} \right\rangle = \frac{\langle v_z \rangle}{c} = -\frac{1}{e n_0 \pi a^2 c} I_d\left(\gamma_0 z_l, \frac{L_{\text{mod}}}{\beta_0 \gamma_0 c}\right)}$$

$$I_d(z,t) = -\frac{Z_i e \omega_p^2}{\pi}\int_0^t d\tau \left(z+v_{0,z}\tau\right)\left\{\frac{a_z \sin(\omega_p \tau)}{\left[\bar{\beta}^2\tau^2+\left(z+v_{0,z}\tau\right)^2\right]\left[1+\bar{\beta}^2\tau^2+\left(z+v_{0,z}\tau\right)^2/a^2\right]} - \cos(\omega_p\tau)\left[\frac{\arctan\left(\left|z+v_{0,z}\tau\right|/\left(\bar{\beta}\tau\right)\right)}{\left|z+v_{0,z}\tau\right|} - \frac{\arctan\left(\sqrt{\left(z+v_{0,z}\tau\right)^2+a^2}/\left(\bar{\beta}\tau\right)\right)}{\sqrt{\left(z+v_{0,z}\tau\right)^2+a^2}}\right]\right\}$$

where $I_d(z,t)$ is an 1-D integral with finite integration limits



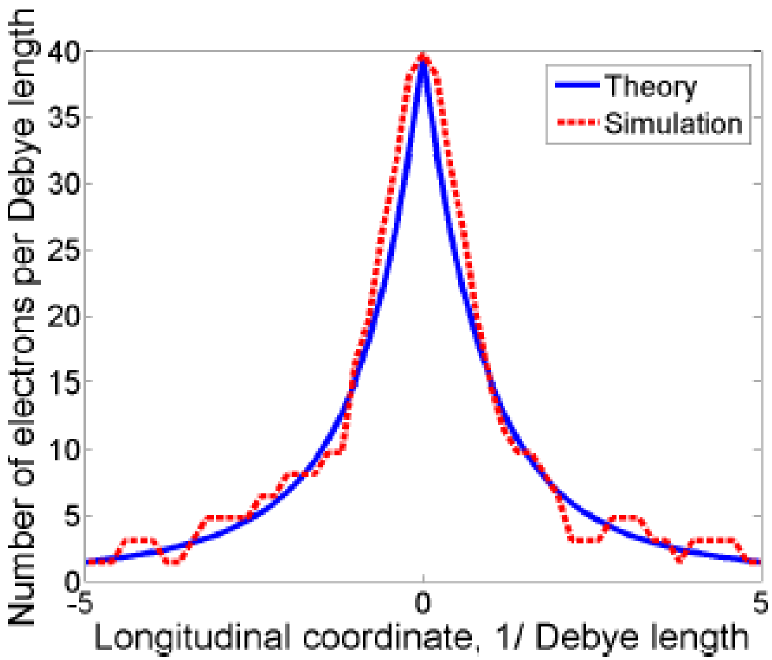
It reduces to the previously derived cold beam result at the corresponding limits:

$$\bar{\beta} = 0 \quad v_{0,z} = 0 \quad L_{\text{mod}} \ll \beta_0 \gamma_0 c / \omega_p$$

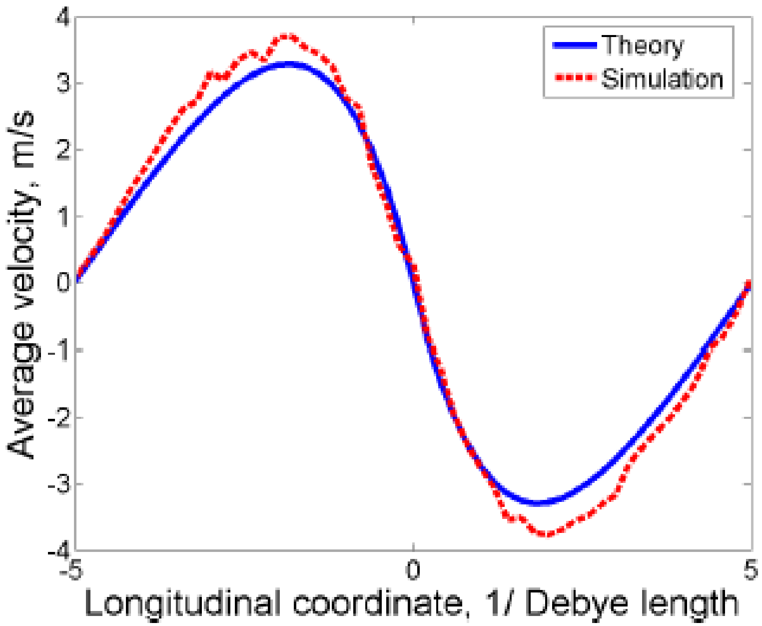
$$\left\langle \frac{\delta E}{E} \right\rangle \approx -2 Z_i \frac{r_e}{a^2} \frac{L_{\text{mod}}}{\gamma} \cdot \left[\frac{z_l}{|z_l|} - \frac{z_l}{\sqrt{z_l^2 + a^2/\gamma^2}}\right].$$

Simulation Tools for the Modulation Process II

- Simulations based on perturbative trajectory approach (© J. Ma, with code SPACE)
  - Benchmarked with theory for uniform warm beam



(a) Longitudinal density

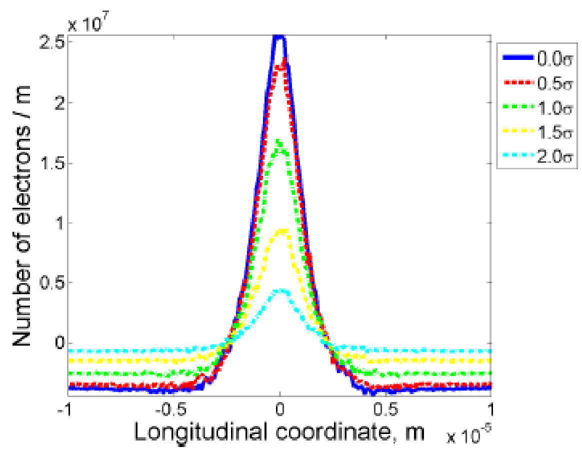


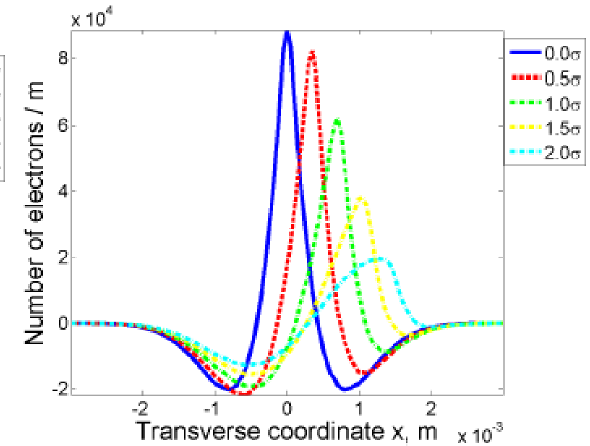
(b) Longitudinal velocity

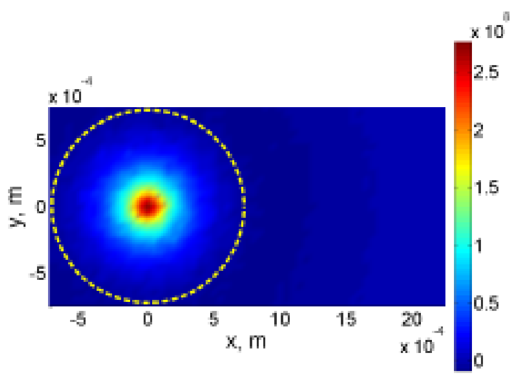
Simulation Tools for the Modulation Process III

– Simulation results for a continuous focusing channel (Beam is matched and transverse beam size does not vary.) (© J. Ma, with code SPACE)

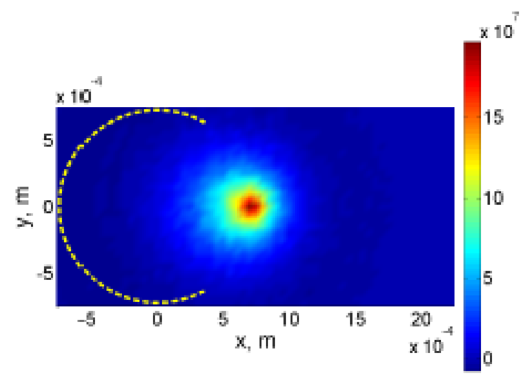
- Modulation is less effective for an off-centered ion. For an ion sitting at 1σ away from transverse electron beam center, the longitudinal density modulation reduces by ~40%.
- The transverse density modulation profile induced by an off-centered ion is significantly different from that induced by an ion at beam center



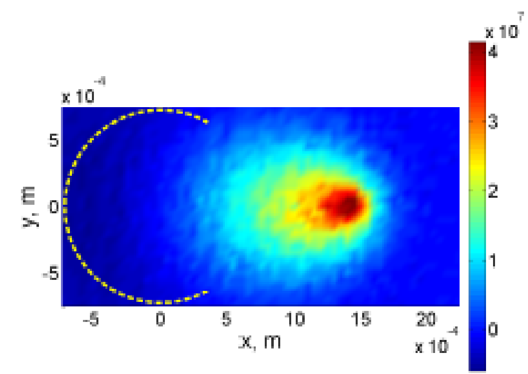




(a) Ion at center

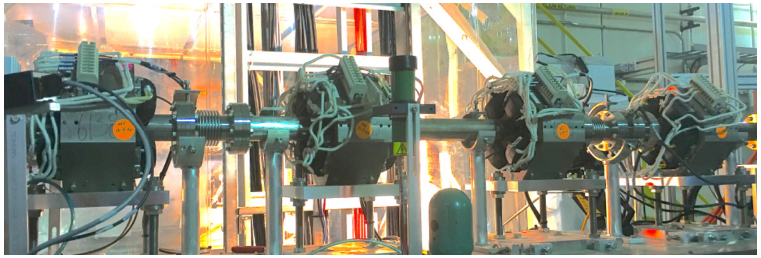


(b) Ion at $x = 1\sigma$



(c) Ion at $x = 2\sigma$

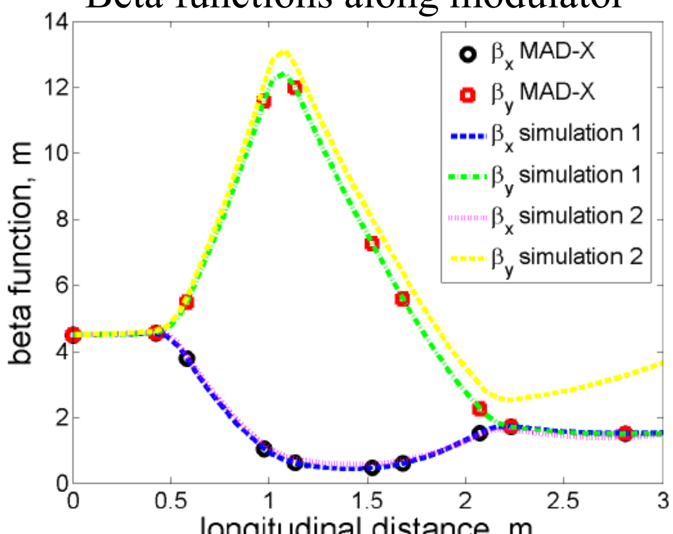
Simulation Tools for the Modulation Process III



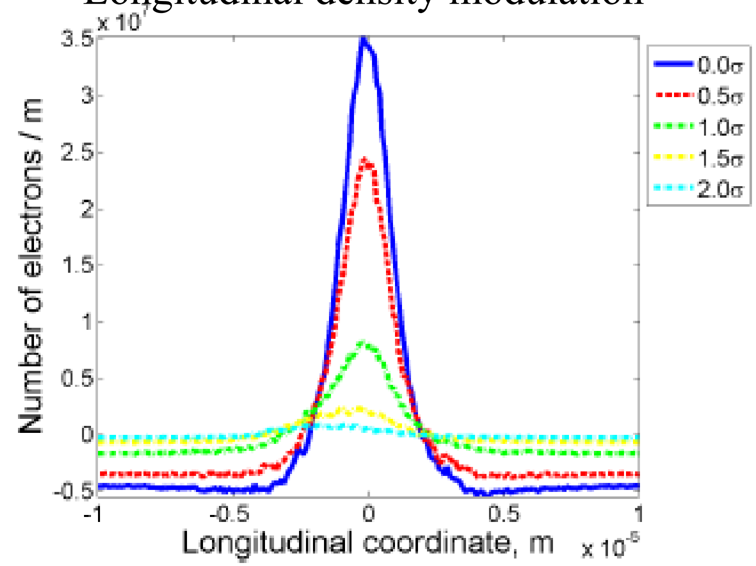
(© J. Ma, with code SPACE)

– Simulation results for a CeC quadrupole beamline with transverse beam size varying along the modulator
  - Beta functions extracted from SPACE agree with those from MAD-X calculations, when space charge is turned off in SPACE simulation.
  - When space charge is turned on, the vertical lattice function at the end of modulator deviates significantly from that calculated by MAD-X.
  - The efficiency of longitudinal density modulation depends strongly on the quadrupole settings, which has to be taken into account in optimizing the system.

Beta functions along modulator



Longitudinal density modulation



Analytical tools for FEL amplifier I

- The 1D FEL amplifier model with collinear radiation field and κ-1 (Lorentzian) energy distribution has been applied to study how the wave-packet forms inside the undulator.

$$\tilde{j}_1(z) = -\left(\frac{\theta_s}{2\varepsilon_0 c}\right)^{-1}\left[A_1\lambda_1 e^{\lambda_1 \hat{z}} + A_2\lambda_2 e^{\lambda_2 \hat{z}} + A_3\lambda_3 e^{\lambda_3 \hat{z}}\right]$$

$$\lambda^3 + 2\left(\hat{q} + i\hat{C}\right)\lambda^2 + \left[\hat{\Lambda}_p^2 + \left(\hat{q} + i\hat{C}\right)^2\right]\lambda - i = 0$$

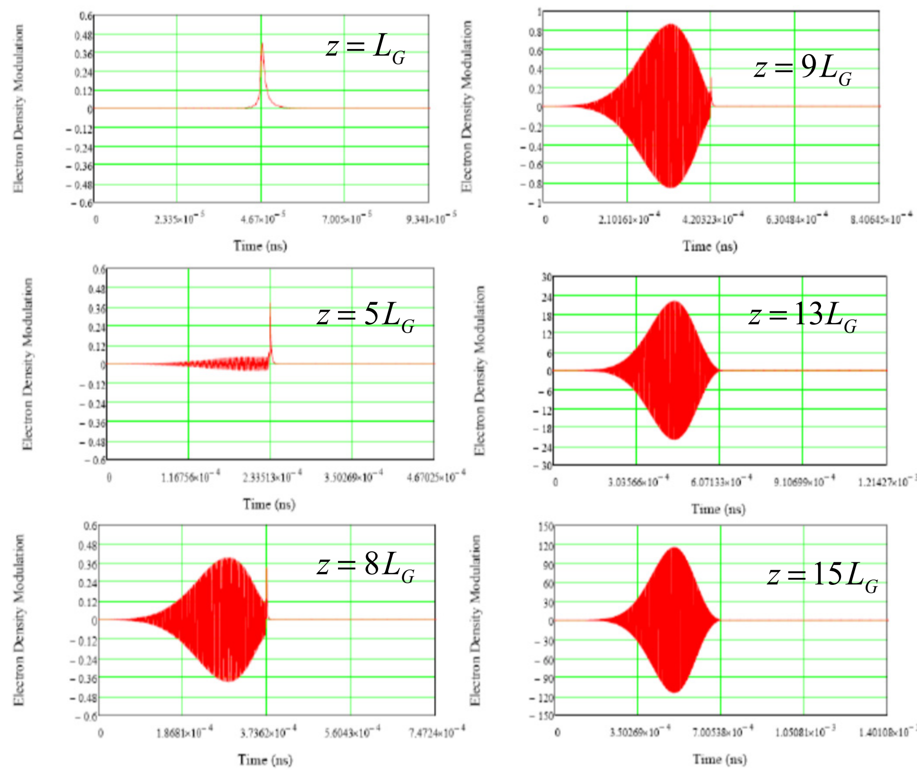
- In high gain regime, the eigenvalues can be expanded into quadratic order in the detuning parameter $\hat{C}$, i.e.

$$\lambda\left(\hat{C}\right) = c_0 + c_1\left(\hat{C} - \hat{C}_0\right) + c_2\left(\hat{C} - \hat{C}_0\right)^2$$

which make it possible to obtain analytical form of the wave-packet

$$j_1(z,t) = \frac{Z_i eck_0}{S\sqrt{\pi}} B_1\left(\hat{C}_0\right)\lambda_0 \frac{e^{2\lambda_0 \rho k_w z}}{\sqrt{-\lambda_2}}\sqrt{\frac{\rho}{2k_w z}}\, e^{ik_w z} e^{ik_0(z-ct)} e^{-\frac{\left(t - t_p(z)\right)^2}{2\sigma_t^2}}$$

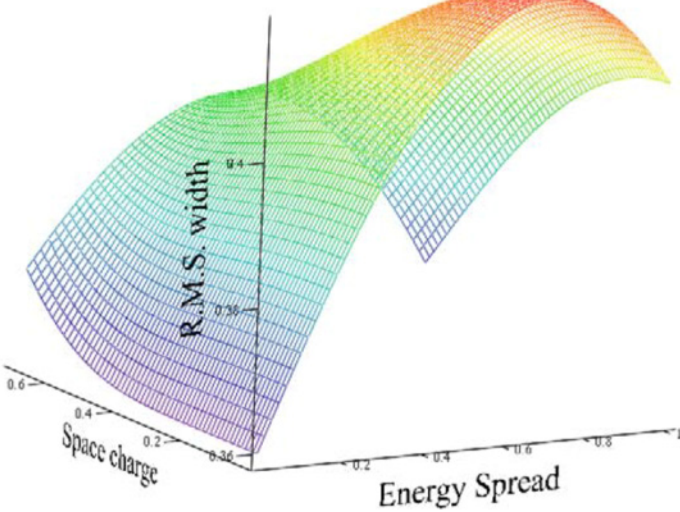
Evolution of wave-packet along undulator as calculated from 1-D theory, showing that it take a few gain length for the wave-packet to overtake the initial modulation.

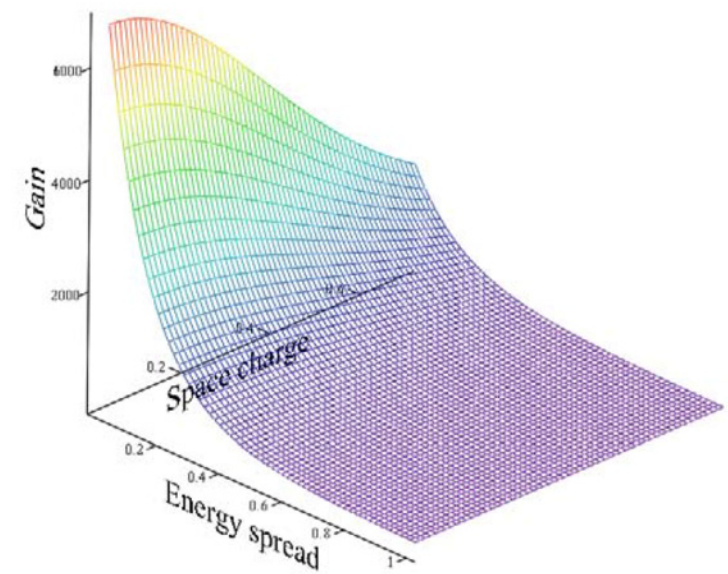


Analytical tools for FEL amplifier II

- The 1-D FEL model with uniform beam provides us with some insights as well as scaling laws of how the wave-packets depends on various beam parameters

Units: $\lambda_{fel}\sqrt{k_w z}\Big/\left(2\pi c\sqrt{\rho}\right)$



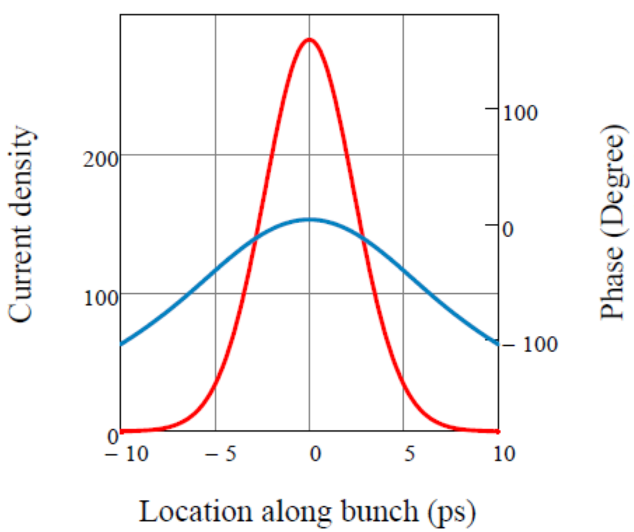


- We also used the 1-D FEL model to study beam conditioning for CeC.

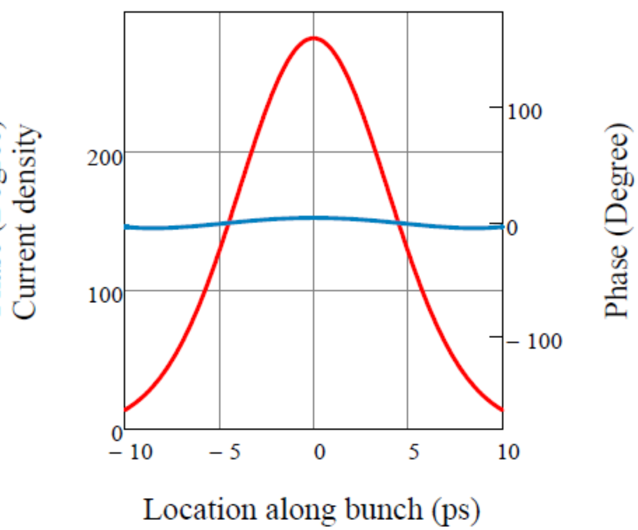
Analytical tools for FEL amplifier III
“Beam conditioning”

- As the amplitude and phase of the wave-packets depends both on the local beam current and the local beam energy, it is possible to optimize beam parameters such that the cooling efficiency over the whole ion beam can be improved.

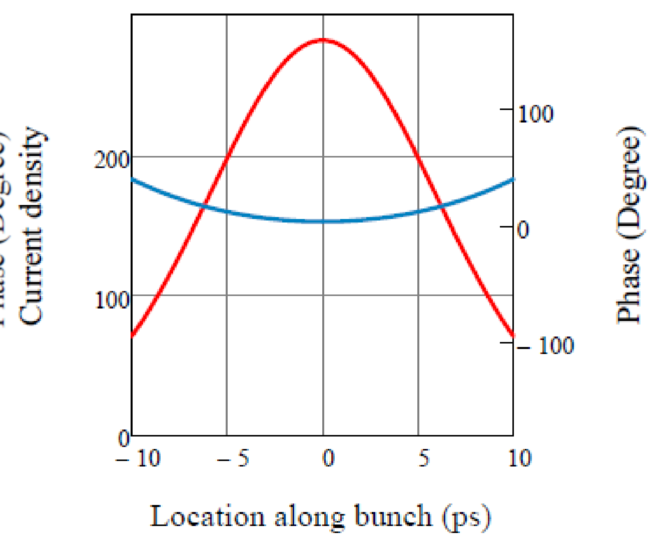
5 ps rms bunch length



8 ps rms bunch length



12 ps rms bunch length



- With the same peak current density, there is an optimum bunch length to minimize phase variation of the wave packet with respect to ions located at different portion of the electron bunch.

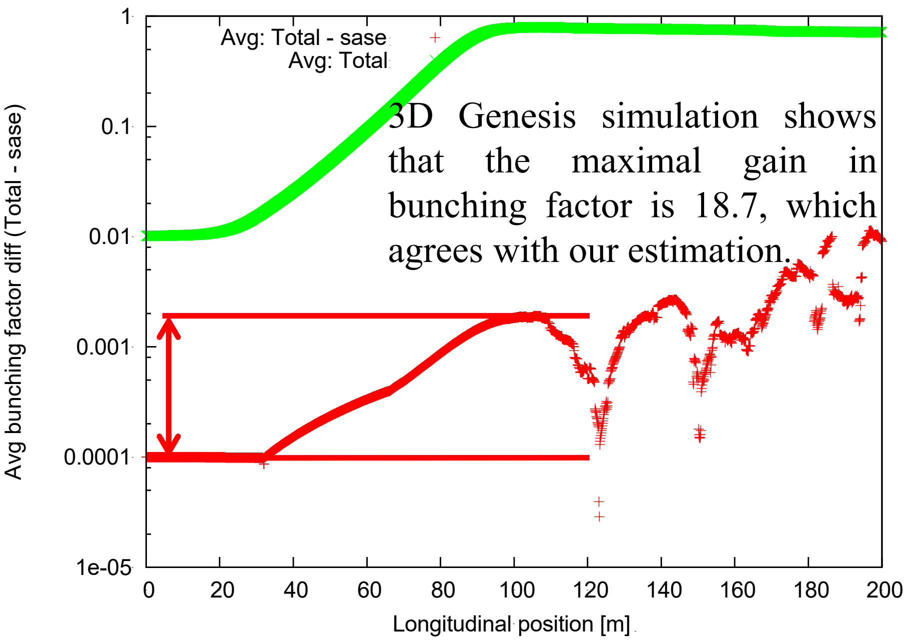
Analytical tools for FEL amplifier IV

- By requiring the relative density variation is smaller than one, we derived the upper limit of the FEL gain for the amplifier to work in the linear regime

$$\left|\delta\hat{n}/n_0\right|_{\max} < 1 \Rightarrow \quad |g|_{\max} < \frac{\lambda_o}{2}\sqrt{\frac{I_e}{ecL_c}} \Rightarrow \quad g_{\max} \sim 72\cdot\sqrt{\frac{I_e[A]\cdot\lambda_o[\mu m]}{M_c}} = 14.1$$

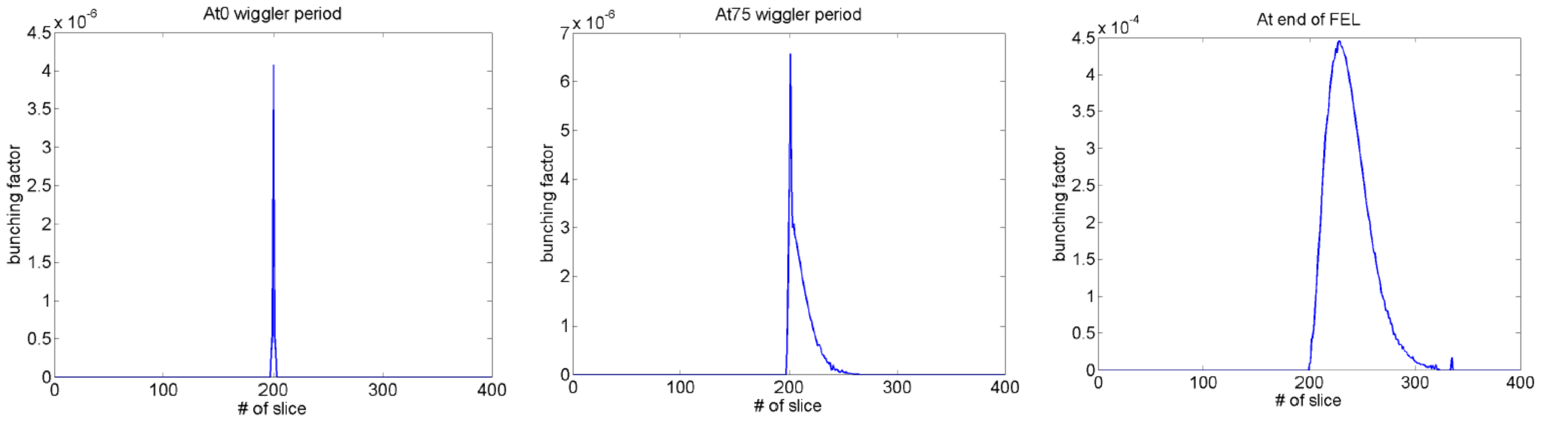
(© Y. Jing, with code GENESIS)

- γ=7460.52
- Peak current: 30 A
- Norm emittance 1 mm mrad
- RMS energy spread 2.5e-5
- λw=10 cm
- $a_w = 10$
- λo=90.73 nm
- Mc = 70.6



Simulation tools for FEL amplifier

- We use GENESIS 1.3 to simulate the amplification process in the FEL amplifier.
- Following the approach of perturbative trajectories, we run two sets of FEL simulation: one with shot noise plus modulation induced by the ion and the other one with shot noise only. The wave-packet due to the ion is extracted from the difference of the two sets of simulation.



Analytical tools for kicker I

- Dynamic equation in Kicker is very similar to that in the modulator except the initial modulation in 6D phase space dominates the process. For κ-2 velocity distribution, the electron density perturbation is determined by:

$$\frac{d^2}{dt^2}\tilde{R}_1\left(\vec{k},t\right)+\omega_p^2\tilde{R}_1\left(\vec{k},t\right)=Z_i\omega_p^2 e^{-\lambda\left(\vec{k},\vec{v}_0\right)\cdot t}-\omega_p^2\int_{-\infty}^{\infty}\tilde{f}_1\left(\vec{k},\vec{v},0\right)e^{-\lambda\left(\vec{k},\vec{v}\right)t}d^3v$$

with $\tilde{R}_1\left(\vec{k},t\right)\equiv\tilde{n}_1\left(\vec{k},t\right)e^{-\lambda\left(\vec{k}\right)t}-\int_{-\infty}^{\infty}\tilde{f}_1\left(\vec{k},\vec{v},0\right)e^{-\lambda\left(\vec{k},\vec{v}\right)t}d^3v$ and

$$\lambda\left(\vec{k},\vec{v}\right)\equiv i\vec{k}\cdot\vec{v}-\sqrt{\left(k_x\sigma_x\right)^2+\left(k_y\sigma_y\right)^2+\left(k_z\sigma_z\right)^2}$$

The solution of this inhomogeneous 2nd order differential equation reads

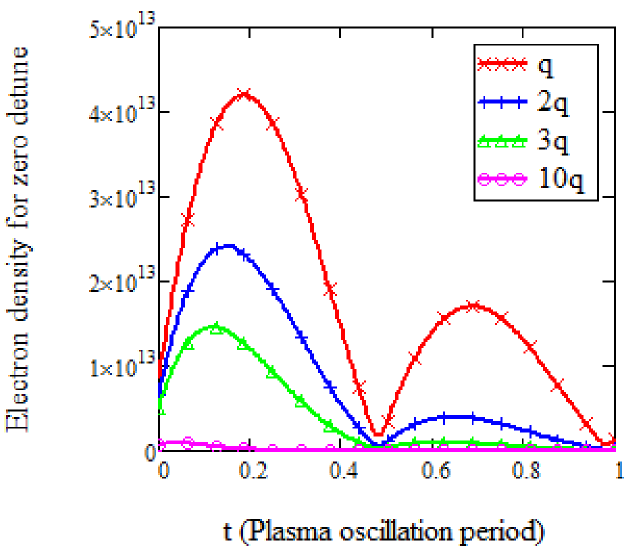
$$\tilde{R}_1\left(\vec{k},t\right)=c_1\cos\left(\omega_p t\right)+c_2\sin\left(\omega_p t\right)$$
$$+\frac{1}{\omega_p}\int_{-\infty}^{\infty}\frac{\omega_p e^{-\lambda t}+\lambda\sin\left(\omega_p t\right)-\omega_p\cos\left(\omega_p t\right)}{\lambda^2+\omega_p^2}\tilde{f}_1\left(\vec{k},\vec{v},0\right)d^3v$$

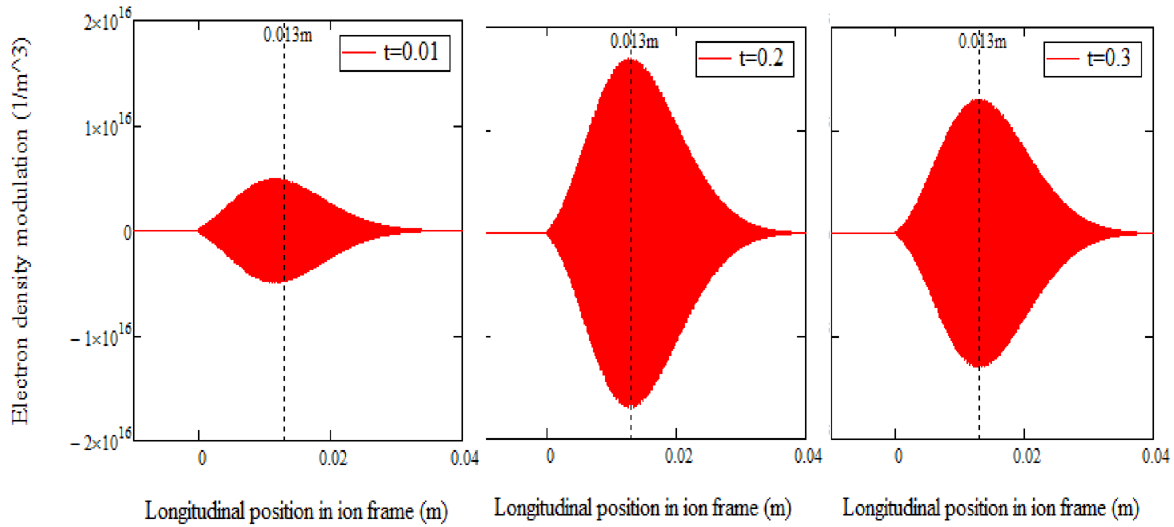
Analytical tools for kicker II

- For 1D FEL output with the following initial transverse perturbation,

$$\tilde{f}_1(x,y,z,v_x,v_y,v_z,0)=\frac{1}{\pi\beta_x\beta_y}\tilde{f}_1(z,v_z,0)\left(1+\frac{v_x^2}{\beta_x^2}+\frac{v_y^2}{\beta_y^2}\right)^{-\frac{3}{2}}$$

$$\boxed{\tilde{n}_1(k_z,t)=-Z_i\tilde{\Lambda}_{drive}(k_z)e^{ik_zv_{0z}t}e^{-|k_z|\sigma_{vz}t}\left[\cos(\omega_p t)+\frac{(\lambda_1+i\hat{C})\beta_z c\gamma_z\Gamma-ik_zv_{0z}+|k_z|\sigma_{vz}}{\omega_p}\sin(\omega_p t)\right]}$$





- For an initial perturbation with finite transverse size, the formalism should also apply. The transverse profile of the modulation can affect the longitudinal field of the wave-packet (in progresses).

Field Reduction due to Finite Transverse Modulation Size

$$\rho(\vec{r}) = \rho_o(r)\cdot\cos(kz);$$

$$\Delta\varphi = 4\pi\rho \Rightarrow \varphi(\vec{r}) = \varphi_o(r)\cdot\cos(kz);$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi_o}{dr}\right) - k^2\varphi_o = 4\pi\rho_o(r)$$

$$\varphi(\vec{r}) = -4\pi\cos(kz)\left\{I_0(kr)\int_r^\infty \xi K_0(k\xi)\cdot\rho_o(\xi)\,d\xi + K_0(kr)\int_0^r \xi I_0(k\xi)\cdot\rho_o(\xi)\,d\xi\right\}$$

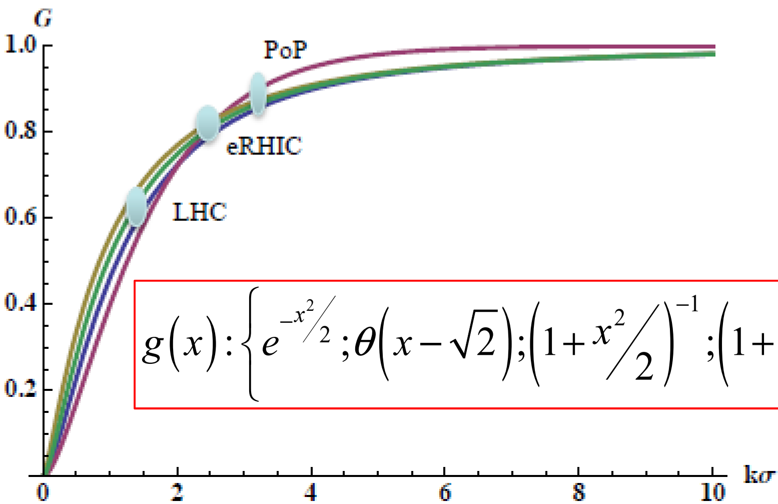
$$E_z = -\frac{\partial\varphi}{\partial z} = -4\pi k\sin(kz)\left\{I_0(kr)\int_r^\infty \xi K_0(k\xi)\cdot\rho_o(\xi)\,d\xi + K_0(kr)\int_0^r \xi I_0(k\xi)\cdot\rho_o(\xi)\,d\xi\right\}$$

$$E_r = -\frac{\partial\varphi}{\partial r} = 4\pi k\cos(kz)\left\{I_1(kr)\int_r^\infty \xi K_0(k\xi)\cdot\rho_o(\xi)\,d\xi - K_1(kr)\int_0^r \xi I_0(k\xi)\cdot\rho_o(\xi)\,d\xi\right\}$$

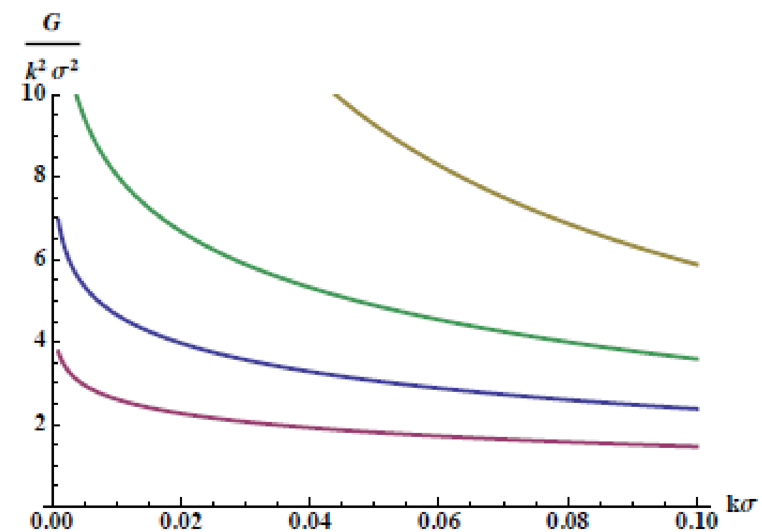
$$\rho(r) = \rho(0)\cdot g(r/\sigma)$$

$$E_{zo}(r=0) \propto -\frac{4\pi\tilde{q}}{\sigma^2}G(k_{cm}\sigma)$$

$$\boxed{k_{cm}\sigma_\perp = \frac{k_o}{\gamma_o}\sqrt{\frac{\beta_\perp\varepsilon_{n\perp}}{\gamma_o}} = \sqrt{\gamma_o}\sqrt{\beta_\perp\varepsilon_{n\perp}}\,\frac{k_w}{2(1+a_w^2)}}$$



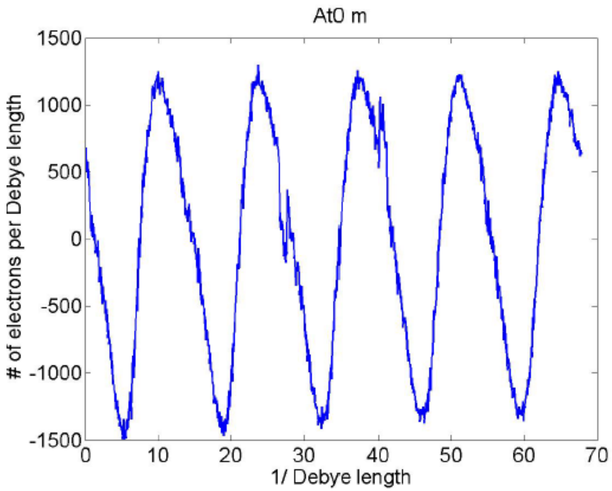
$$\boxed{g(x):\left\{e^{-x^2/2};\theta\left(x-\sqrt{2}\right);\left(1+x^2/2\right)^{-1};\left(1+x^2/4\right)^{-2}\right\}}$$

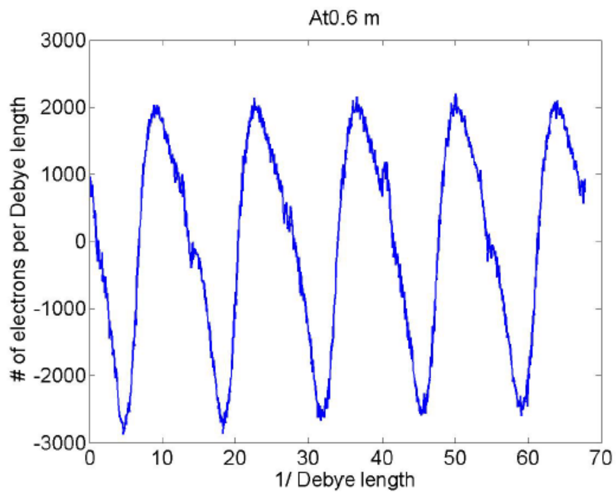


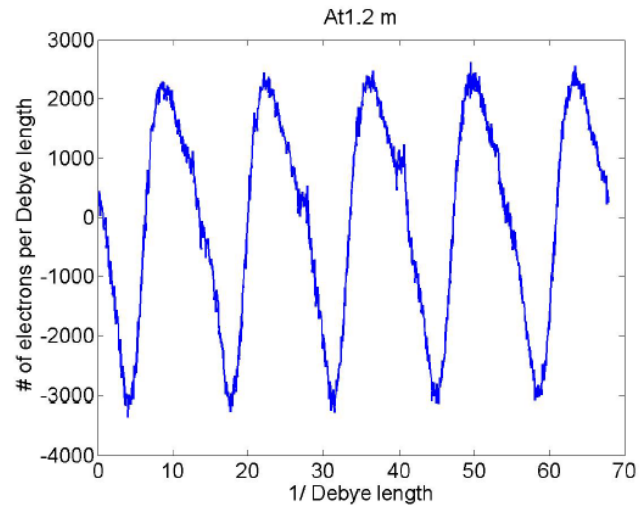
Reduction coefficient for four distribution functions: Gaussian, bear-can, κ-1, κ-2

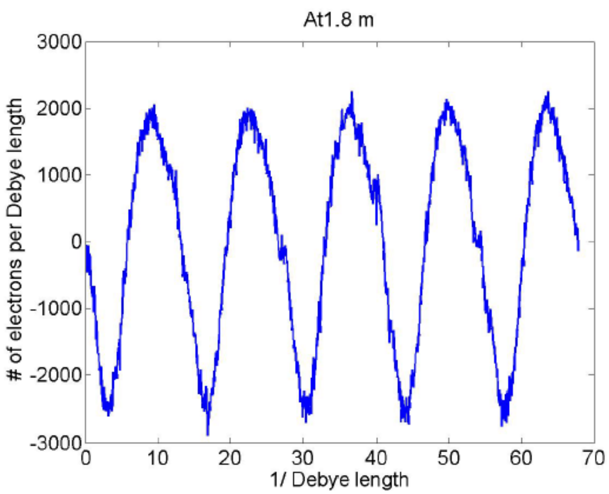
Simulation tools for kicker

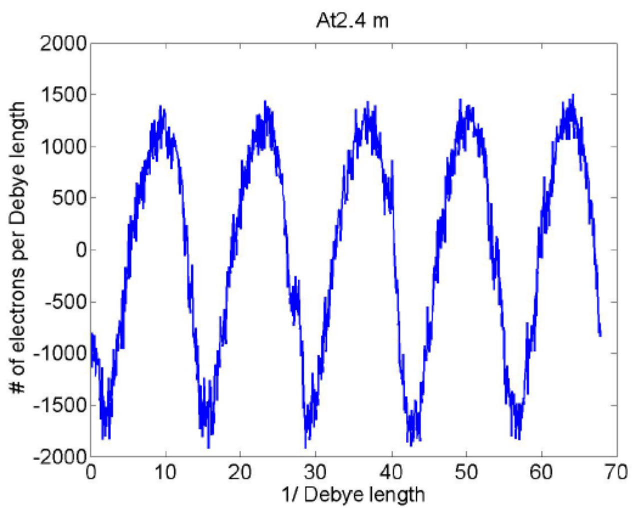
- The macro-particles from GENESIS simulation are imported into SPACE for the kicker simulation.(© J. Ma, with code SPACE)











The evolution of the wave-packet shows similar behavior as that obtained from analytical model, i.e. the wave-packet amplitude increases initially and then starts to decrease.

Start-to-end simulation for the single pass I

Steps for single pass start-to-end simulation:

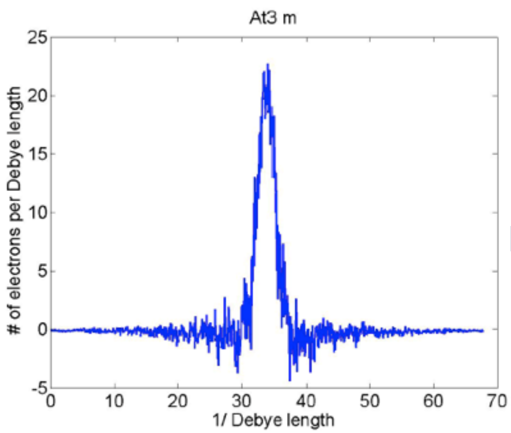
1. At the entrance of the FEL, create macro-particles for the whole electron beam with proper shot noise. The 6-D distribution of the particles is determined by the beam dynamic simulation.
2. At the entrance of modulator, create one slice of macro-particles (with duration of one optical wavelength) with proper shot noise and 6-D distribution.
3. Run modulator simulation with the slice created in step 2. Due to periodic condition, the shot noise of the slice will stay correct.
4. Replace the corresponding slice created from step 1 with that output from step 3.
5. Run Genesis simulation. (Need to add macro-particles with negative energy to tail and head slices to make it work as Genesis require each slice has the same number of macro-particles).
6. Take a proper portion of macro-particles output from GENESIS and import them into SPACE for kicker simulation.
7. Repeat step 1-6 but without the ion. The difference of step 6 and step 7 provides the single-pass coherent kick solely due to the ion.

Start-to-end simulation for the single pass II

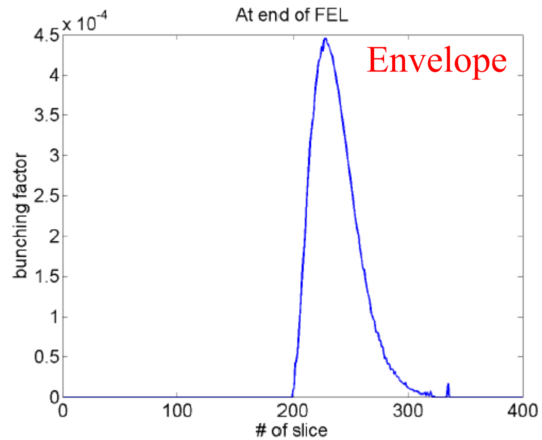
(© J. Ma, with code SPACE and GENESIS)

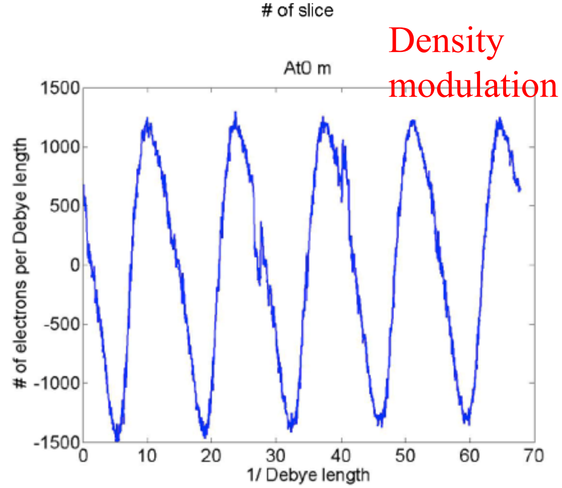
- One example of start-to-end simulation

Modulator

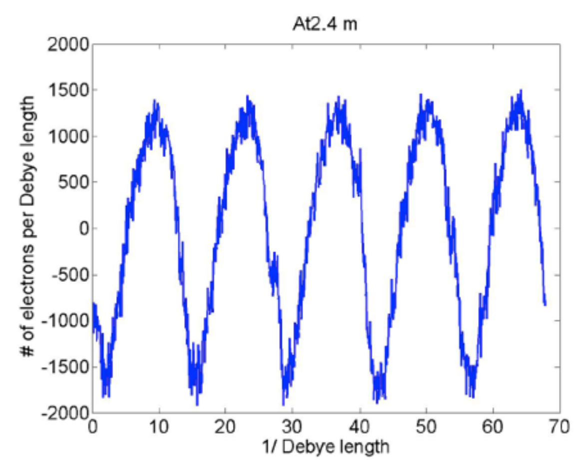


Amplifier





Kicker



Analytical tools for predicting the influences of CeC on a circulating ion beam I

- Evolution of the longitudinal phase space density of the ion bunch, after averaging over the synchrotron oscillation phase, follows the 1-D Fokker-Planck equation

$$\frac{\partial}{\partial t}F(I,t)-\frac{\partial}{\partial I}\left(\zeta(I)\cdot I\cdot F(I,t)\right)-\frac{\partial}{\partial I}\left(I\cdot D(I)\cdot\frac{\partial F(I,t)}{\partial I}\right)=0$$

- In the limit of $D(I)=0$, an analytical solution can be derived for the following form of cooling profile and initial condition,

$$\zeta(I)=\zeta_0\frac{I_e}{I+I_e} \qquad F_0(I)=\exp\left(-\frac{I}{I_{ion}}\right)$$

$P_{\log}(x)$ is called product logarithm function and can be directly evaluated in Mathematica.

as

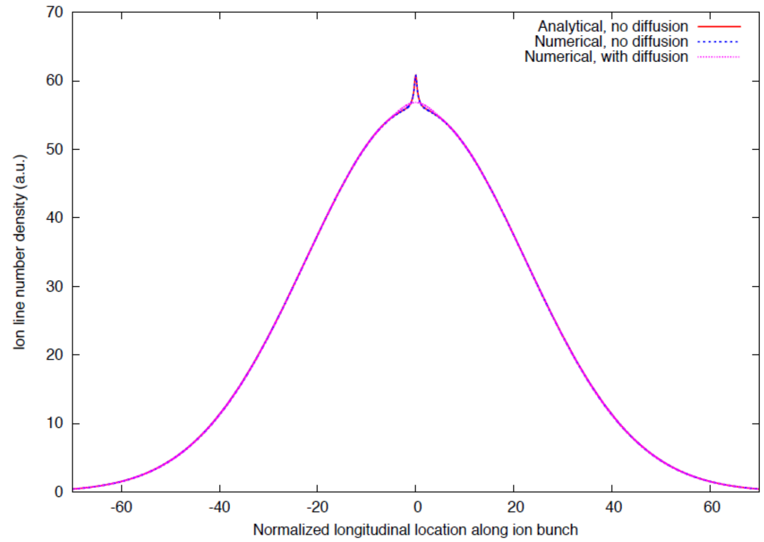
$$F(I,t)=\left(1+\frac{I_e}{I}\right)\frac{P_{\log}\left(\frac{I}{I_e}\exp\left(\zeta_0 t+\frac{I}{I_e}\right)\right)\exp\left(\frac{-I_e}{I_{ion}}P_{\log}\left(\frac{I}{I_e}\exp\left(\zeta_0 t+\frac{I}{I_e}\right)\right)\right)}{1+P_{\log}\left(\frac{I}{I_e}\exp\left(\zeta_0 t+\frac{I}{I_e}\right)\right)}$$

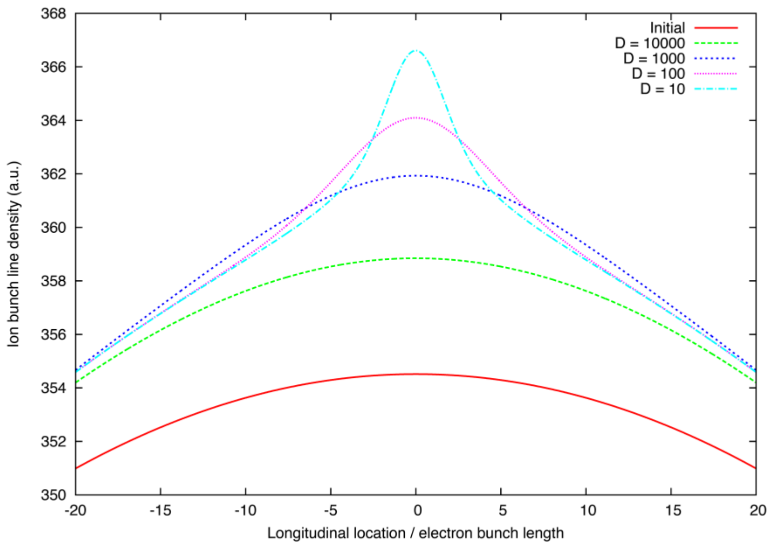
Analytical tools for predicting the influences of CeC to a circulating ion beam II

- The longitudinal line density of ion bunch is given by

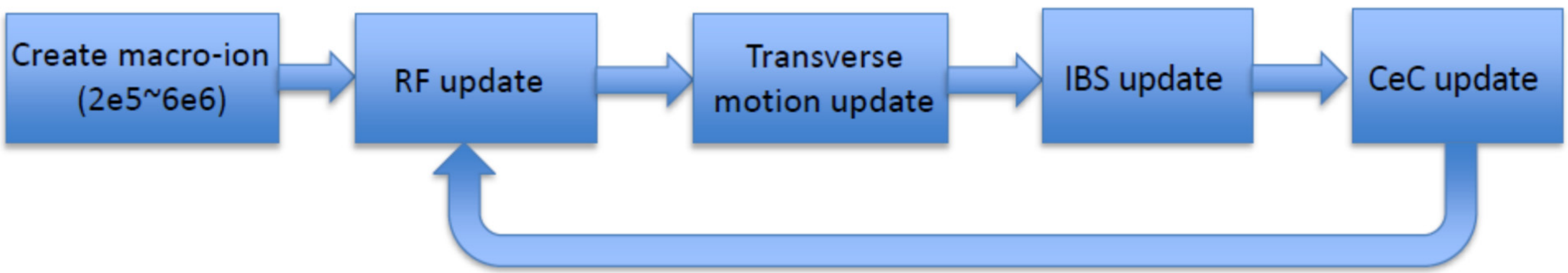
$$\rho_{ion}(t,z) = \int_{-\infty}^{\infty} F\left(z^2+\delta^2,t\right) d\delta$$

- For $D(I) \neq 0$, the 1-D Fokker-Planck equation can be solved numerically with arbitrary form of cooling rate and initial ion distribution.
- The analytical studies reveals the fact that the central blips due to local cooling tends to be smeared out by diffusive kicks from IBS and more significantly, from incoherent kicks induced by neighbor ions.





Simulation tools for predicting the influences of CeC to a circulating ion beam I



Energy kicks from CeC is $\Delta E_j = \Delta E_{coh,j} + \Delta E_{inc,j}$

Coherent kick induced by the ion itself $\Delta E_{coh,j} \equiv -Z_i e E_p l \sin\left(k_0 D \cdot \delta_j\right)$

Incoherent kick induced by the neighbor ions (using the Gaussian profile as obtained by quadratic expansion of FEL eigenvalues)

$$\Delta E_{inc,j} \equiv -Z_i e E_p l \sum_{i \neq j} e^{-\frac{\left(\varsigma_j - \varsigma_i\right)^2}{2\sigma_{z,rms}^2}} \sin\left(k_0\left(D\delta_j + \varsigma_j - \varsigma_i\right) - k_2^2\left(\varsigma_j - \varsigma_i\right)^2\right)$$

Since there is no correlation between any successive incoherent kicks, one can use a random kick to represent the incoherent kicks

For a random number uniformly distributed between -1 and 1 $\left\langle X^2 \right\rangle = \frac{1}{2}\int_{-1}^{1} X^2 dX = \frac{1}{3}$

$$\Delta E_{j,N} \approx -Z_i e E_p l_1 \sin\left(k_0 D \cdot \delta_j\right) + \sqrt{\frac{\left\langle \Delta E_{inc,j}^2 \right\rangle}{\left\langle X^2 \right\rangle}} \cdot X_{j,N}$$

Simulation tools for predicting the influences of CeC to a circulating ion beam II

- Assuming the ion density does not vary significantly over the width of the wave-packet

$$\left\langle \Delta E_{inc,j}^{2} \right\rangle = \frac{\left(Z_i e E_p l_1 \right)^2}{2} \int_{-\infty}^{\infty} \rho_{ion}\left(\varsigma_i \right) e^{-\frac{\left(\varsigma_i - \varsigma_j \right)^2}{\sigma_{z,rms}^2}} d\varsigma_i \approx \frac{\left(Z_i e E_p l_1 \right)^2}{2} \sqrt{\pi} \rho_{ion}\left(\varsigma_j \right) \sigma_{z,rms}$$

- The one-turn energy kick due to CeC is

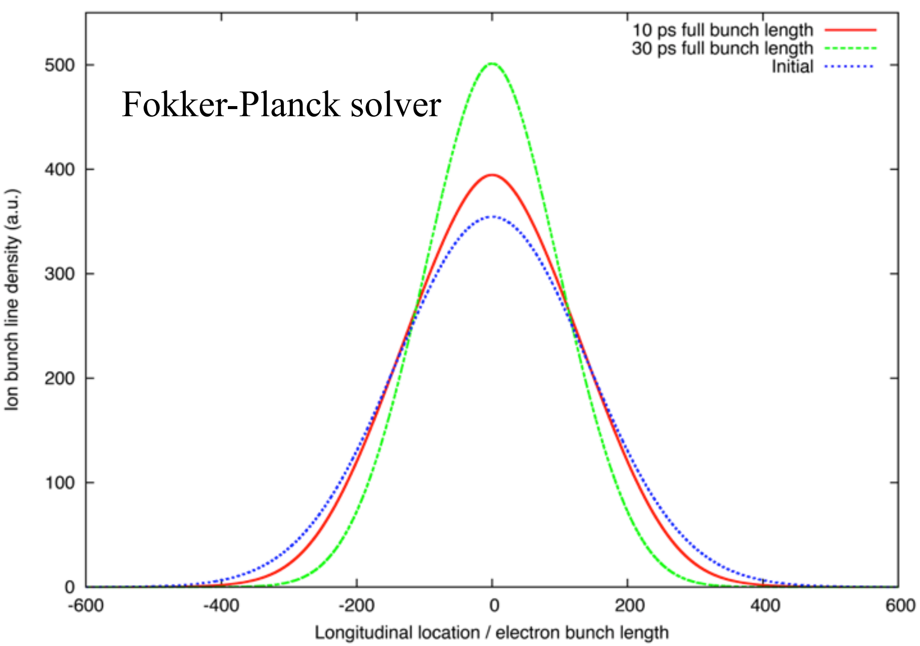
$$\Delta E_{j,N} \approx -Z_i e E_p l \sin\left(k_0 D \cdot \delta_j \right) + Z_i e E_p l \sqrt{\frac{3}{2} \sqrt{\pi} \rho_{ion}\left(\varsigma_j \right) \sigma_{z,rms}} \cdot X_{j,N} + \Delta E^{e}{}_{j,N}$$

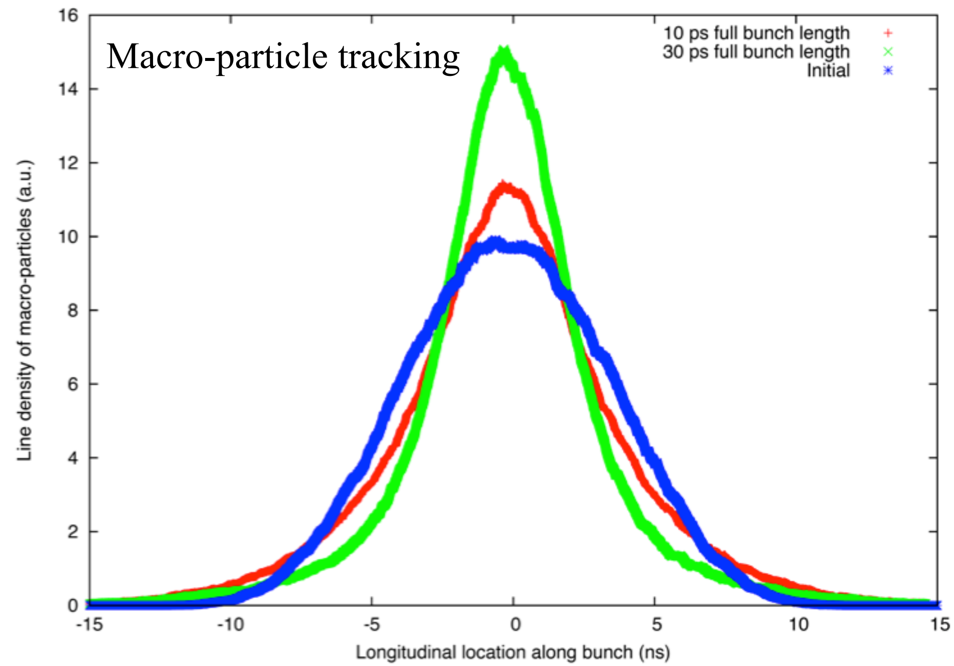
Diffusive kick induced by neighbor electrons, i.e. electrons' shot noise

$$\Delta E_{j,N}^{e} \approx e E_p l \sqrt{\frac{3}{2} \sqrt{\pi} \rho_e\left(\varsigma_j \right) \sigma_{z,rms}} \cdot Y_{j,N}$$

Simulation tools for predicting the influences of CeC to a circulating 40 GeV/u Au ion beam III

The plots show how the longitudinal profile of ion bunch evolves after 40 minutes of cooling with 10 ps (red) and 30 ps (green) electron bunch. The left plot is generated by solving Fokker-Planck equation as described in the previous slides and the right plot is created by the macro-particle tracking code. The parameters applied are for cooling 40 GeV RHIC gold ion beam.





Status of simulation for the proof of CeC principle experiment

- The start-to-end single pass simulation have been mostly developed and benchmarked with theory.
- With inputs of initial distribution of the electron beam, the single pass simulation can provide the density variation induced by an ion at any section of the CeC system.
- It should be straightforward to obtain the momentum kick received by the ion while it traveling through the kicker section and we are currently working on it.
- Simulation tools for the circulating ion beam has been developed. Once we obtain the momentum kick received by an ion at any location of the 6-D space, we will be able to predict how ion beam evolves under the influence of cooling.
- We are also working on refining the simulation such as implementing a more realistic model for the three undulator sections (We currently treating it as a single long section.).

Conclusions

- We had develop a tool-box of analytical and numerical tools either for estimating or predicting performance of Coherent electron Cooling
  - with the final goal to compare our perditions with the proof-of-principle CeC experiment
- We did not discover any show stoppers in this process, while we included
  - space charge, energy spread, realistic e-beam parameters, finite transverse size of the beam, resistive wall wake-fields ....
- Next step – post-CeC-demonstration - will be in-depth studies of CeC with micro-bunching amplifier

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