A METHOD TO OPTIMIZE THE AMPLITUDES OF HARMONICS IN BEAM BUNCHERS

P. R. Sarma, V. S. Pandit and R. K. Bhandari

Variable Energy Cyclotron Centre
Bhabha Atomic Research Centre
1/AF Bidhan Nagar, Calcutta - 700 064, INDIA

ABSTRACT

Bunchers used for pulsing the dc beams are fed with a combination of the fundamental rf wave with its various harmonics. In order to maximize the ion current in a bunch the amplitudes of the harmonics should be optimized. A method based on the linear programming technique has been developed for this optimization. Calculations have been presented for bunchers using upto four harmonics. Optimization has been done for a double drift bunching system also.

1. INTRODUCTION

In an rf particle accelerator using external injection system the beam current can be increased by bunching the injected dc beam suitably with the help of a beam buncher. The rf voltage wave form applied to the buncher should ideally be a sawtooth, which is, however, very difficult to generate for this purpose. Therefore bunchers are fed with a sinusoidal waveform or a nearly sawtooth like waveform obtained by combining fundamental wave with its various higher harmonics. In cases where a sawtooth waveform is approximated with two or more harmonics, the general effort is aimed at making the waveform more and more linear over the rf cycle. However, the bunching efficiency is not necessarily maximum when this criterion is applied. The bunching efficiency can be maximized by optimizing the amplitudes of the constituent harmonics. Such an optimization can be done by analytical methods for upto second harmonic[1]. If more harmonics are present the analytical approach becomes complicated and computational methods become necessary. The present work describes a method of optimizing the amplitudes of harmonics with the help of linear programming technique.

We have presented here the calculations for the bunchers containing upto four harmonics. Computations for a double drift buncher have also been carried out. In our treatment we have omitted the effect of space charge and, hence, the results are valid for low beam currents.

2. METHOD

Basically a buncher modulates the dc beam velocity in such a way that portions of the beam initialy separated in space and time are made to arrive at the time focus nearly at the same time. If $V = \sum V_n \sin(n\phi)$ be the voltage applied on the buncher to produce bunching at a distance L from the gap, then the phase of an arbitrary ion with respect to unmodulated ion at the time focus is given by[1],

$$\theta = \phi - \sum \mu_n \sin(n\phi) \qquad (1)$$

where,

$$\mu_n = 2\pi f L (2qV_o/m)^{-1/2} [\, T(d) F(a) V_n / V_o] \qquad (2)$$

Here, qV_o and m are the energy and mass of the particle respectively; ϕ is the phase of the rf voltage of frequency f when the particle crosses the gap and V_n is the amplitude of the n-th harmonic. $T(d)$ is the well known term due to transit time effect for a gap of width d and $F(a) = J_o(2\pi f a/c)/I_o(2\pi f a/c\gamma)$ is the correction factor needed for the finite dimension of the buncher electrode of radius a[2]. An accelerator has a fixed phase acceptance $2\theta_m$ within which particles are captured by the rf wave. This phase acceptance largely depends on the geometry of the accelerating structures. The amplitude and shape of the voltage waveform applied on the buncher are adjusted in such a way that the largest possible number of particles arrive in the specified phase acceptance of the accelerator. Thus the problem reduces to finding out the maximum value of ϕ ($=\phi_m$) for a given value of θ_m in equation (1) and which satisfies the condition that $|\theta| \leq \theta_m$ for all $|\phi| \leq \phi_m$. This problem can be easily solved analytically for upto two harmonics. For higher harmonics the easier way is to take help of the computational methods. Since equation (1) is linear in terms of μ_n, linear programming technique can be used to minimize θ for a given maximum value of ϕ_m. This is the dual problem to that of maximizing ϕ_m for a given value of θ_m and both are equivalent. The linear constraints are that for each value of ϕ, upto ϕ_m, the value of $|\theta|$ should be less than $|\theta_m|$. Thus,

$$(\phi-\phi_m)-\Sigma\mu_n[\sin(n\phi)-\sin(n\phi_m)] \leq 0 \qquad (3)$$

for $-\phi_m \leq \phi \leq \phi_m$

In the computations it suffices to use the constraint (3) only at some values of ϕ so that the number of constraints is finite and is workable. The complete table of constraints is then given by:

$$-2\phi_m - \Sigma\mu_n[-2\sin(n\phi_m)] \leq 0$$

$$(\phi_1-\phi_m)-\Sigma\mu_n[\sin(n\phi_1)-\sin(n\phi_m)] \leq 0$$

....

....

$$(\phi_i-\phi_m)-\Sigma\mu_n[\sin(n\phi_i)-\sin(n\phi_m)] \leq 0$$

..............etc...............

Our computations show that taking ϕ values at 2 degree interval yield satisfactory results. Now we can apply the standard simplex method to minimize θ for a given number of constraints. While using the simplex algorithm care should be taken to see that correct signs of various harmonics are used, otherwise the minimum one gets is not the true minimum. It has been found that all the odd harmonics are positive while the even ones are negative in sign. This is expected as a sawtooth wave also shows similar pattern.

2.1 Double Drift Bunching System

Instead of applying all the harmonics on a single buncher one can use a double drift bunching system consisting of two bunchers that are separated in space and are phase locked together. The second buncher is driven at twice the frequency to that of the first. Such a bunching system was discussed by Goldstein and Laisne[3] and detailed numerical calculations were performed by Milner[4]. This system gives bunching efficiency better than what is obtainable with three harmonics on a single buncher. Here the final phase of a particle can be written as

$$\theta = \phi-\mu_1\sin\phi-\mu_2(1-b)\sin2(\phi-b\mu_1\sin\phi) \qquad (4)$$

where, b is the ratio of the distance between the two bunchers to the distance of the time focus from the first buncher. Here the two bunchers should be phase locked either by using a phase shifter or by keeping the proper distance between the two. Equation (4) is not linear in μ_1 and μ_2 and thus linear programming can not be directly used. We have used the simplex algorithm in this problem iteratively by assuming an initial value of μ_1. The convergence is fast and three or four iterations give quite accurate results.

2.2 Bunching Efficiency and Shape of the Pulse at the Time Focus

The bunching efficiency ε defined as the ratio of the number of particles

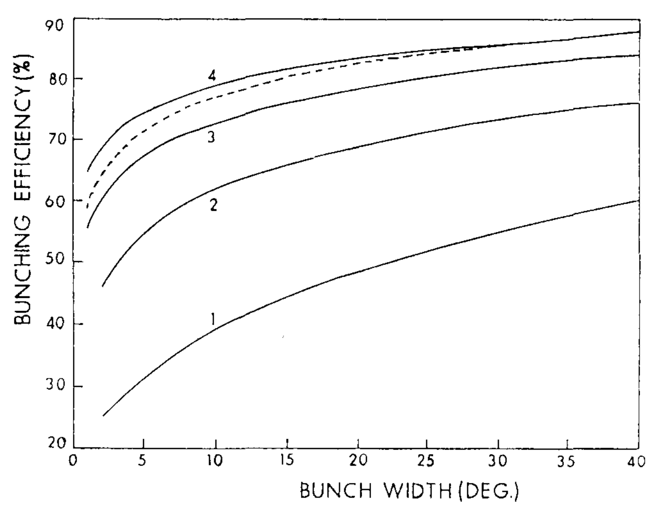

Fig. 1. Bunching efficiency as a function of bunch width for bunchers excited with 1. Sinusoidal wave (Fundamental) 2. Fundamental and second harmonic 3. Fundamental, second and third harmonics 4. Fundamental and three higher harmonics. The dotted curve shows the bunching efficiency for a double drift buncher.

within the acceptable bunch width to the total number of particles in an rf cycle is given by

$$\epsilon = \phi_m/\pi$$

Figure 1 shows the bunching efficiency as a function of bunch width for various cases. It can be seen that the bunching efficiency for a double drift buncher is better than that for a buncher with three harmonics and is comparable to that for a buncher with four harmonics.

The intensity distribution of particles $I(\theta)$ at the time focus for harmonic bunchers can be written as

$$I(\theta) = \Sigma\left|\frac{d\phi}{d\theta}\right| = \Sigma\left|\frac{K}{1 - \Sigma n\mu_n \cos(n\phi)}\right| \qquad (5)$$

where the first summation extends over all the values of ϕ which give same value of θ. Here $2\pi K$ is the intensity of the incoming beam in one rf cycle. For the double drift buncher the shape can be found out similarly by differentiating equation (4). Figure 2 shows the shape of pulse for the bunchers with two and three harmonics and for the double drift buncher.

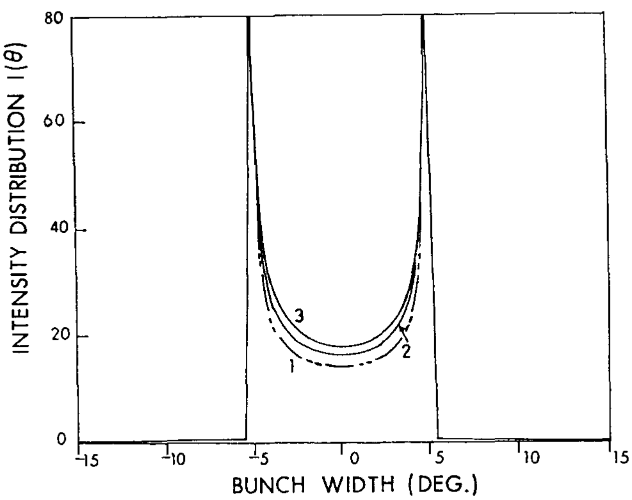

Fig. 2. Comparison of the shapes of the bunched beam optimized for a bunch width Of ±5 deg. in the cases 1. buncher with two harmonics 2. buncher with three harmonics and 3. double drift buncher.

3. RESULTS AND DISCUSSION

For use with an accelerator the geometrical parameters of a buncher and the accelerator operational parameters are fixed a priori, and the parameters which can be adjusted are the amplitudes and phases (as defined in sec 2.) of the harmonics in the voltage waveforms. These can be determined from the μ value. The μ's defined in equation (2) contain two factors $T(d)$ and $F(a)$ which are somewhat approximate, but the ratios of amplitudes of harmonics to that of the fundamental contain no such factor and so these ratios remain fixed while the fundamental amplitude needs to be adjusted a little bit while tuning a buncher for optimum performance. Table 1 gives the values of μ_1 and the ratios μ_n/μ_1 (= V_n/V_1) for three and four harmonic bunchers as a function of the bunch width. Figure 3 shows the

Table 1
Parameters for various bunch widths

Bunch width $2\theta_m$	Buncher with 3 harmonics μ_1	μ_2/μ_1	μ_3/μ_1	Buncher with 4 harmonics μ_1	μ_2/μ_1	μ_3/μ_1	μ_4/μ_1
1	1.69	.290	.065	1.80	.360	.131	.036
2	1.72	.309	.080	1.83	.376	.150	.049
4	1.76	.333	.103	1.85	.395	.171	.068
6	1.79	.349	.122	1.87	.406	.186	.085
8	1.80	.361	.139	1.88	.414	.197	.100
10	1.82	.370	.154	1.89	.421	.206	.113
12	1.83	.379	.169	1.90	.426	.214	.127
14	1.84	.385	.182	1.91	.431	.221	.139
16	1.85	.392	.196	1.91	.435	.227	.152
18	1.86	.398	.208	1.92	.439	.233	.164
20	1.87	.403	.221	1.92	.442	.238	.175

variation of the buncher parameters for a sinusoidal and a two harmonic bunchers.

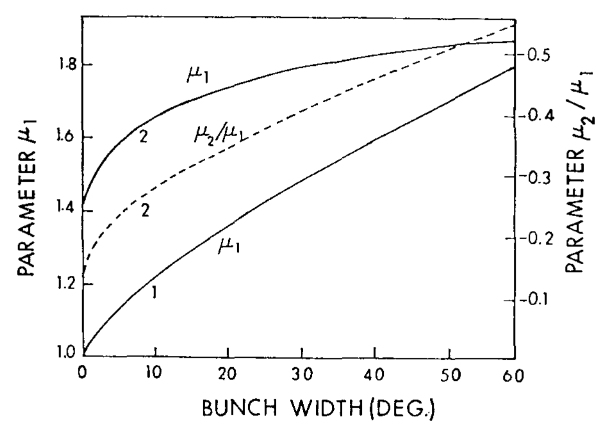

Fig. 3. Variation of the buncher parameters as a function of bunch width for 1. a sinusoidal buncher and 2. a buncher with two harmonics.

As pointed out by Milner[4)], the bunching efficiency for a double drift buncher depends not only on the μ values but also on the separation between the two bunchers compared to the distance between the first buncher to the time focus. So the parameter b should also be kept to the proper value as far as possible. Figure 4 shows the variation of the μ and b values as a function of the bunch width.

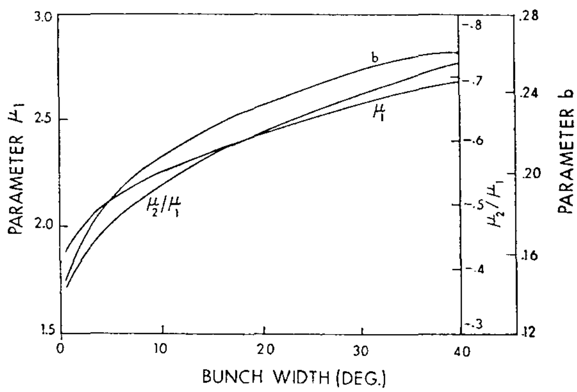

Fig. 4. Variation of the buncher parameters as a function of bunch width for a double drift buncher.

We point out here that when the parameters are optimized the peak shape curve for all the bunchers show two peaks as shown in Figure 2. If the harmonic amplitudes are not properly optimized microstructures appear[1,5].

For certain types of experiments a single peak in the pulse is more useful. To ensure this the set of constraints which need to be satisfied are given by

$$\frac{d\theta}{d\phi} \geq 0 \quad \text{for } -\phi_m \leq \phi \leq \phi_m$$

In this case the earlier set of constraints become redundant.

REFERENCES

1) Pandit, V.S.; Sarma, P.R. and Bhandari, R.K.; Optimization of the Parameters of an Ion Beam Buncher, Nucl. Instr. and Meth. in Phys. Res. Vol.276A, 21 (1989).

2) Dome, G., Theory of RF Acceleration, Proc. CERN Accelerator School, CERN, Geneva, 1983, p.110.

3) Goldstein, C. and Laisne, A., Third Harmonic Simulated Buncher, Nucl. Instr. and Meth. Vol.61, 221 (1968).

4) Milner, W.T., Double Drift Bunching Systems, IEEE Trans. Nucl. Sci. Vol.NS-26, 1445 (1979).

5) Lynch, F.J.; Lewis, R.N.; Bollinger, L.M.; Henning, W. and Despe, O.D., Beam Buncher for Heavy Ions, Nucl. Instr. and Meth. Vol.159, 245 (1979).